

# The Dynamics of Firm-Level Adjustment to Trade Liberalization\*

James A. Costantini  
INSEAD

Marc J. Melitz  
Princeton University, CEPR and NBER

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## Abstract

We build a dynamic model of firm-level adjustment to trade liberalization that jointly incorporates the main salient features highlighted by recent empirical micro-level studies of firms and trade. Our model captures the joint entry, exit, export, and innovation decisions (subject to sunk costs) of heterogeneous firms as they adjust to trade liberalization. We characterize this industrial evolution over its entire transition path to a new steady state with lower trade costs - starting from the time that trade liberalization is first announced (but not necessarily yet implemented). We rely on numerical methods to solve for these equilibrium paths. In order to more accurately capture the dynamics of firm adjustments to trade, we model the sunk nature of market entry costs for both the domestic and export market - as well as the per-unit and additional fixed costs of exporting incurred in every period. Firm-level productivity evolves stochastically, and innovation involves a trade-off between its cost and a return in terms of a “better” distribution of future productivity draws.

Although the empirical micro-level studies of firms and export status initially emphasized the selection effects of more productive firms into export markets, several recent studies have highlighted a separate channel for the effects of trade on productivity operating through firm-level improvements in productivity. Our model captures both of these channels for the productivity enhancing effects of trade - and analyzes their interactions over the adjustment path to lower trade costs. In particular, we highlight how the relative timing and magnitude of firm-level productivity improvements and export market entry decisions are also determined by non-technological factors such as the timing of trade liberalization announcements and the speed of liberalization. Under all these different trade liberalization scenarios (anticipated versus surprise, gradual versus sudden), we characterize both the distributional effects across firms as well as their aggregate effects for industrial performance. We find that the anticipation of upcoming liberalization, and a more gradual path of liberalization (once implemented) induces firms to innovate ahead of export market entry.

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## 1 Introduction

Recent evidence on firm-level adjustments to trade liberalization have documented that firms jointly make innovation and export market participation decisions (see Aw, Roberts, and Winston (2007), Bustos (2006), Trefler (2004), Verhoogen (2006)). Another now large and established research agenda using micro-level production data has confirmed time and again the strong self-selection of more productive firms into export markets. More recently, another branch of this literature has found some evidence for a “learning-by-exporting” phenomenon, whereby firms improve their productivity subsequent to export market participation (see for instance, Delgado, Farinas, and Ruano (2002), De Loecker (2006), Girma, Greenaway, and Kneller (2004), Topalova (2004), and the survey in Lopez(2005)).

In this paper, we build a dynamic model of firm-level adjustment to trade liberalization that jointly addresses all of these features. Our model captures the self-selection of more productive firms into export markets, the joint export market participation and innovation decisions, and the continuing innovation of other firms following their entry into export markets.<sup>1</sup> We thus explain how some potentially conflicting results concerning the direction of causation between export participation and productivity (based on whether productivity improvements are observed prior to or subsequent to export market participation) can be reconciled within a single model that recognizes that these decisions are jointly considered by firms, and jointly affected by trade liberalization.

Our model shows how some important non-technological factors, such as the pace and anticipation of trade liberalization, can fundamentally affect this perceived causation link between export status and productivity. For example, we show that the anticipation of trade liberalization tends to bring forward the decision to innovate relative to the export market participation, and how a more abrupt pace of liberalization amplifies these effects. Our model also highlights the potential empirical pitfalls of analyzing current period industrial performance as a response to the concurrent trade costs during periods of trade liberalization. Our model shows how the current industrial response not only depends on those concurrent trade costs, but also, inextricably, on the firms’ prior

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<sup>1</sup> Alvarez and Lopez (2005) find empirical support for all three types of firm behavior. Yeaple (2005) and Ederington and McCalman (2006) have theoretically analyzed the joint technology adoption and export decisions of firms, when there are no ex-ante differences between them. Chaney (2005) investigates the dynamics of firm entry and productivity with heterogeneous firms, immediately following an unanticipated opening to trade (from autarky). He shows how this leads to an overshooting of productivity due to the sluggish response of exit. Our model confirms these findings following certain types of trade liberalization, starting from an open economy environment. Our paper is most closely related to Atkeson and Burstein (2006), who also analyze the joint innovation and export decisions of firms in a dynamic model of trade with heterogeneous firms. Atkeson and Burstein (2006) consider a continuous type of innovation activity (performed by all firms at varying intensities) whereas we consider a one-off innovation opportunity, such as the adoption of a new technology or a major product quality upgrade/redesign.

expectations about those current trade costs, and their expectations for their changes in the future. Although we emphasize that “true” learning-by-exporting (in the literal sense that firms learn from their foreign market experience) can not be inferred based on the timing of export market entry, we do not mean to dismiss the potential relevance of such a learning channel.<sup>2</sup>

In order to capture these complex dynamic effects involving changes in expectations about future outcomes, we build a model with heterogeneous firms that incorporates both idiosyncratic firm uncertainty (future productivity is stochastic) and forward looking decisions subject to sunk costs. All of the forward looking firm decisions concerning entry, exit, export, and innovation incorporate a sunk cost component. We focus on the interaction between the firm decisions to undertake a large, one-off innovation and the decision to enter the export market. The benefit of this innovation is a one-time stochastic jump in productivity (relative to the default productivity transition). Given firm heterogeneity, this leads to a sorting of firms into innovators and non-innovators in a very similar way to the sorting that occurs with export market participation.<sup>3</sup> Naturally, this induces a large amount of overlap between exporting and innovation across the productivity distribution (high productivity firms undertake both). We will therefore focus on the transition paths into these activities, and the relative timing of the export and innovation decisions.

We characterize both the stationary equilibrium given an invariant level of trade costs, as well as the equilibrium transition path given any arbitrary path of trade liberalization from a high trade cost stationary state to a new one with lower cost. The initial and final stationary states do not necessarily have the same sorting of firms into innovation and export. For example, some firms with intermediate productivity levels innovate ahead of the export decision when facing sufficiently high trade costs but reverse this ordering in an environment with low enough trade costs.

We analyze the transition dynamics between two such stationary states from high to low trade costs. The transition dynamics are largely shaped by the way the firms’ value of undertaking innovation and/or exports is affected by the future evolution of trade costs, and the associated anticipated industry response. As these firm decisions are subject to sunk costs (irreversibility) along with idiosyncratic firm uncertainty, they will critically depend on the firms’ anticipation concerning future liberalization (when it is announced, and the pace of liberalization once undertaken). We

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<sup>2</sup>We intentionally constructed our model without any such learning channel to highlight that this is not needed in order to explain the micro-level evidence on the relative timing of export market entry and productivity gains. The existence of such a channel remains an open empirical question. To our knowledge, only Crespi, Criscuolo, and Haskel (2006) find some evidence for learning-by-exporting that is not based on the timing of export market entry.

<sup>3</sup>Bustos (2006) and Verhoogen (2004) both report how measures of firm innovation, for example the reported adoption of new technologies or ISO certification, are strongly correlated with firm productivity and export status.

will contrast trade liberalization scenarios that differ along these timing dimensions.

We rely on numerical methods to solve for these equilibria. Since we do not know of any previously developed computational methods to solve this type of dynamic problem with a large number of firms, we develop a general computational algorithm that can be used to solve a wide set of related dynamic industry evolution models.<sup>4</sup> We describe this algorithm in detail in the appendix.

## 2 Model Setup

As highlighted above, we develop our model to analyze the evolution of an industry comprised of heterogeneous firms in response to trade liberalization. The firms in the industry are distinguished by their productivity. We focus on two interdependent firm policies: innovation involving a stochastic jump in firm productivity, and export market participation. Clearly, these policies are both affected by trade liberalization, which increases the returns to both activities. We investigate how these policy choices respond to the timing of trade liberalization: whether the liberalization is anticipated by the firms, and whether the liberalization, once started, occurs abruptly or gradually. We analyze this model in a partial equilibrium setting with respect to the industry: we assume a demand system for the industry as a whole, and a perfectly elastic labor supply to the industry at the economy wide wage.

The core elements of the model are based on Melitz (2003), re-introducing the stochastic evolution of firm productivity from Hopenhayn (1992). In addition, we add the innovation option subject to sunk costs. We then computationally solve this extended model for its stationary equilibrium along with any transition paths between two stationary states. We next describe each part of the model, the equilibrium, and how we calibrate the model based on the empirical literature.

### Demand

Consumer preferences for the differentiated varieties in the industry are C.E.S. with elasticity  $\sigma > 1$ . There is a continuum of varieties  $\omega \in \Omega$ . Let  $P_t = [\int_{\omega \in \Omega} p_t(\omega)^{1-\sigma}]^{1/(1-\sigma)}$  be the C.E.S. price index

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<sup>4</sup>A seminal contribution to the computation of such equilibria with a small number of firms under oligopoly is Pakes and McGuire (1994), following the development of the theoretical version of the model in Erikson and Pakes (1995). Both papers study an industry closed to foreign trade. Erdem and Tybout (2003) extend the work of Pakes et al (1994) to the case of an import competing industry. The computational methods we develop in the current paper are radically different as they apply to a monopolistically competitive sector with a large number of competing firms (where the mass of firms evolves endogenously). These methods have also been concurrently used in Costantini (2006) to study the effects of credit constraints on industrial evolution. Similar methods applied to a continuous innovation decision in a general equilibrium setting have also recently been developed by Atkeson and Burstein (2006).

for the aggregated differentiated good  $Q_t \equiv [\int_{\omega \in \Omega} q_t(\omega)^{(\sigma-1)/\sigma}]^{\sigma/(\sigma-1)}$  at time  $t$ , where  $p_t(\omega)$  and  $q_t(\omega)$  are the price and quantity consumed of the individual varieties  $\omega$ . We further assume that overall demand for the differentiated good  $Q_t$  is generated by  $Q_t = AP_t^{1-\eta}$ , where  $A$  is an exogenous demand parameter for the industry (constant over time), and  $\eta < \sigma$  is the industry price elasticity of demand. Demand for an individual variety  $\omega$  is then  $q_t(\omega) = Q_t P_t^\sigma p_t(\omega)^{-\sigma} = AP_t^{\sigma-\eta} p_t(\omega)^{-\sigma}$ .

## Production

Each variety is produced by a firm with productivity  $v \in \Upsilon$  (which we now also use as the index for varieties). Labor is the only factor of production, and we normalize the wage level to unity. Firms produce with a technology exhibiting constant marginal cost  $1/v$ , along with an overhead per-period fixed cost  $F$  (measured in labor units). Given the demand system and a continuum of competing firms, all firms set a constant markup  $\sigma/(\sigma-1)$  over marginal cost. The per-period profit for production sold in the domestic market is:

$$\pi_t^D(v) = \frac{(\sigma-1)^{\sigma-1}}{\sigma^\sigma} AP_t^{\sigma-\eta} v^{\sigma-1} - F. \quad (1)$$

## Productivity Evolution and Innovation

The firms' productivity evolves stochastically in each time period with a known martingale process. In addition, firms have a one-time opportunity to innovate. We denote the firms that have not innovated as  $A$  firms, and firms that have exercised their innovation option as  $B$  firms.<sup>5</sup> The benefit of innovation is an one-time evolution of productivity based on a more favorable (and known) probability distribution than the alternative of no innovation. Note that a firm only benefits from this better productivity draw once. However, given the subsequent martingale evolution process, this productivity gain is long-lasting. We choose the same martingale process for both  $A$  and  $B$  firms. New innovators expect a subsequent productivity draw at a given percentage above their current productivity. The sunk innovation cost is stochastic, but i.i.d. in every period (with different realizations across firms). The realization of this sunk cost is either  $S^B$  with probability  $\gamma^B$  or infinite: in the latter case the firm can not innovate. Hence, only a proportion  $\gamma^B$  of firms can consider innovating in any given period.<sup>6</sup>

<sup>5</sup>For simplicity, we allow  $A$  firms only a one time choice to become a  $B$  firm. A more general set-up could allow repeated innovation by firms, or possibly allow  $B$  firms to invest to scrap their innovation and transition back to an  $A$  firm.

<sup>6</sup>We introduce  $\gamma^B$  only to generate a smoother firm distribution across productivity.

## Trade

Firms have the opportunity to export. We consider a symmetric two-country world where exporters incur three additional costs: a per-unit (iceberg) trade costs,  $\tau_t$ , a per period fixed cost to export,  $F^X$ , and a sunk cost to enter the export market,  $S^X$ . There are no sunk costs to exit the export market (though subsequent re-entry into the export market would require, again, payment of the sunk cost to enter the export market). In this paper, we only consider trade liberalization involving the per-unit trade cost; hence, this is the only trade cost indexed by time. Based on the symmetric foreign demand set-up discussed above, in each time period that a firm chooses to export, the firm generates profits from export  $\pi_t^X(v)$  given by

$$\pi_t^X(v) = \frac{(\sigma - 1)^{\sigma-1}}{\sigma^\sigma} AP_t^{\sigma-\eta} \left( \frac{v}{\tau_t} \right)^{\sigma-1} - F^X. \quad (2)$$

## Value functions and Firm Policy Decisions

We now discuss the firm policy decisions. Within each time period, the timing of events is as follows, and illustrated in Figure 1. First, firms decide whether to continue in the industry or exit. This is based on the maximization of firm value  $V_t(v, z)$ , with state  $z \in \{AD, BD, AX, BX\}$  referring to firm technology and export status. The firm compares the value of continuing,  $V_t^C(v, z)$ , to the value of exit, which we set to zero for simplicity:

$$V_t(v, z) = \max [0, V_t^C(v, z)]. \quad (3)$$

Continuing firms maximize their value by optimally choosing innovation and export policies. Firms operating with technology  $A$  have the option of innovating and switching to technology  $B$ . Firms also choose whether to start exporting, continue to export, or to stop exporting. Lastly, firms discount next period profits at the exogenous rate  $\beta$ , and internalize the exogenous probability  $\delta$  of a death-inducing shock (which is independent of productivity  $v$  and state  $z$ ).<sup>7</sup> The firm policy choices must satisfy the Bellman equation:

$$V_t^C(v, z) = \max_{z'} \left\{ \pi_t(v, z') - S^B I^B(z, z') - S^X I^X(z, z') + \beta (1 - \delta) \int_{v \in \Upsilon} V_{t+1}(v', z') dG[v' | v, I^B(z, z')] \right\}, \quad (4)$$

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<sup>7</sup>Thus, there is both endogenous exit (due to a bad productivity shock) and exogenous exit due to the death shock.

where

$$\pi_t(v, z') = \begin{cases} \pi_t^D(v) & \text{if } z' \in \{AD, BD\} \\ \pi_t^D(v) + \pi_t^X(v) & \text{if } z' \in \{AX, BX\} \end{cases}$$

represents total firm per-period profit, and

$$I^B(z, z') = \begin{cases} 1 & \text{if } z \in \{AD, AX\}, z' \in \{BD, BX\} \\ 0 & \text{otherwise} \end{cases} \quad \text{and} \quad I^X(z, z') = \begin{cases} 1 & \text{if } z \in \{AD, BD\}, z' \in \{AX, BX\} \\ 0 & \text{otherwise} \end{cases}$$

are indicators for new innovators and exporters.  $G[v' | v, I^B(z, z')]$  is the firm productivity stochastic evolution process, where  $E[v' | v, 0] = v$ .

The optimal firm policies from (4) can be summarized by a set of transition productivity cutoffs from state  $z$  to  $z'$  or exit. For a given  $z$ , the range of possible productivity levels  $v$  will be partitioned into segments representing next period choice of  $z'$  or exit. Clearly, the exit segment will always encompass the lowest productivity levels up to a cutoff. Some other rankings are clear: the choice to transition to, respectively,  $AX$  or  $BX$  (whenever feasible) will always occur at higher  $v$  than the transition to, respectively,  $AD$  or  $BD$  (again, whenever feasible). In other words, among both innovators and non-innovators, relatively more productive firms choose to export. Absent an upper-bound on productivity  $v$ , the same logic applies to the  $B$  states relative to their respective  $A$  states (relatively more productive firms choose to innovate). However, since we use a finite productivity grid in our numerical methods (with an upper-bound on productivity), the dominance of the  $B$  state is reversed for productivity levels right below the upper bound: there is no incentive to innovate for a firm whose productivity can not increase any further. In all our numerical simulations, we choose a high enough upper-bound on the productivity grid such that there are essentially no firms in this situation. The dominance of the  $B$  states over their respective  $A$  states is thus quantitatively verified.

Yet, there are also some rankings of transition states that will critically vary with the level of trade costs. Most prominently, the transitions from the  $AD$  state to either the  $BD$  or  $AX$  states. In an environment with high enough trade costs, firms who are just productive enough to transition out of their current  $AD$  state will choose to innovate ( $ADBD$  transition). As trade costs fall, the export option becomes relatively more attractive, and those firms who are just productive enough to transition out of  $AD$  may choose to transition to  $AX$  instead (export, but not innovate).<sup>8</sup> Our

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<sup>8</sup>Clearly, those  $AD$  firms receiving high enough productivity transition draws will choose to both innovate and export, the  $ADBX$  transition.

numerical calibration will feature this reversal in the transition rankings from  $AD$ . We describe and motivate these transitions in greater detail later, along with the other simulation results.

### Entrants

At the start of each period, new entrants can potentially enter the industry. An entrant pays a sunk cost of entry,  $S$ , and then realizes its initial productivity draw from a known invariant distribution  $G_E(v)$ . Entry is not otherwise restricted. Entrants arrive into the industry in the  $AD$  state. Thereon, entrants are indistinguishable from incumbent firms with the same productivity and policy state. A prospective entrant therefore faces a net value of entry

$$V_t^E = \int_{v \in \Upsilon} V_t(v, AD) dG_E(v) - S.$$

When this value is negative, entry is unprofitable.

## 3 Equilibrium

Let  $\mu_{z,t}$  represent the measure function for producing firms with state  $z$  in period  $t$ , over sets of productivity levels  $v$ . This function summarizes all information on the distribution of producing firms across productivity levels, as well as the total mass of producing firms in state  $z$ ,  $M_{z,t} = \mu_{z,t}(\Upsilon)$ . A dynamic equilibrium is characterized by a time path for the price index  $\{P_t\}$ , the measure of firms in each state,  $\{\mu_{z,t}\}$ , and the mass of entrants  $\{M_{E,t}\}$ . Note that a choice of  $\{P_t\}$  uniquely determines the time path for  $\{V_t^C(v, z)\}$  and thus determines all the optimal choices for any firm, given its productivity  $v$  and state  $z$ . An equilibrium  $\{P_t\}$ ,  $\{\mu_{z,t}\}$ , and  $\{M_{E,t}\}$  must then satisfy the following three conditions:

**Firm Value Maximization** All firms' choices for exit/continuation, and given the latter, for  $z'$  (export/innovation) conditional on  $v$  and  $z$  must satisfy (3) and (4). In the aggregate, this means that  $\mu_{z,t}$  is entirely determined by  $\mu_{z,t-1}$  and the choices for  $\{P_t\}$  and  $\{M_{E,t}\}$ : Starting with a mass and distribution of firms at time  $t-1$ , a share  $\delta$  of firms receive the exogenous death shock, while the remaining  $(1-\delta)$  share of firms receive their new productivity draw. To these firms are added the  $M_{E,t}$  new entrants to the  $AD$  state, with a distribution determined by  $G_E(v)$ . All firms make their endogenous exit decisions and the continuing firms make their innovation/export decision (the optimal choice for  $z'$  given  $v$  and  $z$ ). This entails a



distribution and mass of firms for every state. In equilibrium this must match the chosen  $\mu_{z,t}$ .

**Free Entry** In equilibrium, the net value of entry  $V_t^E$  must be non-positive, since there is an unbounded pool of prospective entrants and entry is not restricted beyond the sunk entry cost. Furthermore, entry must be zero whenever  $V_t^E$  is negative.

**Aggregate Industry Accounting** The mass and distribution of firms over productivity levels (aggregating over states) implies a mass and distribution of prices (applying the profit maximizing markup rule to firm marginal cost  $1/v$ ). Aggregating these prices into the C.E.S. price index must yield the chosen  $P_t$  in every period.

### Stationary Equilibrium

A time invariant level of trade costs  $\tau$  leads to a stationary equilibrium with a time invariant price index  $P$ , measure of firms  $\mu_z$ , and mass of entrants  $M_E$ . In such a stationary equilibrium, entry must be positive since there is always an exogenous component to exit. Thus  $V_t^E$  must be zero in this equilibrium. Although an equal mass of firms enter and exit, their distributions over productivity and states will not generally match. This is due to the productivity transition dynamics among incumbent firms (which also induce transitions across states). Jointly, these productivity and state transitions, along with the distribution of entrants and exiting firms, lead to a stationary distribution of firms for every state.

### Equilibrium Along Trade Liberalization Transition

We consider the equilibrium along a transition driven by a liberalization of trade that reduces the trade cost  $\tau_t$ . Firms are initially in a stationary equilibrium, as described above. At the end of one of these stationary time periods, trade liberalization is announced. We analyze the anticipation effects of such an announcement by varying the time between it and the time the first drop in trade cost occurs. The trade cost can then drop abruptly or gradually. Thereafter, the trade cost remains constant at its new lower level. The equilibrium then converges to this new stationary equilibrium with lower  $\tau$ .

We compare three trade liberalization scenarios that vary along these timing dimensions, while keeping the same pre-liberalization and post-liberalization stationary states (thus, the same cumulative drop in  $\tau$ ). 1) an unanticipated, abrupt drop in trade cost ( $UA$ ): there is no anticipation

period, and the full difference in  $\tau$  between the two stationary states occurs from one period to the next. 2) An anticipated, abrupt drop in trade costs (*AA*): very similar to the unanticipated case, but with a lag period between the time the liberalization is announced, and the time it takes effect. 3) An anticipated gradual drop in trade costs (*AG*): the same anticipation period as in the previous *AA* scenario, but then followed by a gradual drop in the trade costs until the same final post-liberalization  $\tau$  is reached. We consider how firm policies and hence the industry equilibrium evolve over a long enough set of time periods such that, by the final period, the industry is arbitrarily close to its stationary equilibrium consistent with the new lower trade costs. Thus, a summary description of the total long-run change in the industry is provided by a comparison of the stationary states generated by the initial and final set of parameters. The equilibrium path for the price index  $\{P_t\}$ , measure of firms  $\{\mu_{z,t}\}$ , and entrants  $\{M_{E,t}\}$  will thus begin at their old stationary levels, remain constant until trade liberalization is announced, then follow a transition path until they reach the new stationary state levels, and remain constant thereafter. During the transition, as opposed to the stationary states, the net value of entry may be negative resulting in periods of zero entry. Since we will consider scenarios with fewer firms (across states) in the post-liberalization stationary state – relative to the pre-liberalization stationary state – such periods of zero entry will occur whenever the change in trade cost is large and abrupt.

## Calibration

We search for the equilibrium paths of  $\{P_t\}$ ,  $\{\mu_{z,t}\}$ , and  $\{M_{E,t}\}$  using numerical methods. The appendix provides a description of the algorithm used. In essence: we first compute the values of  $P$ ,  $\mu_z$ , and  $M_E$  in the old and new stationary equilibria. The algorithm then iterates over candidate equilibrium paths for  $\{P_t\}$  and  $\{M_{E,t}\}$ . The choice for  $\{P_t\}$  determines all of the policy choices for any incumbent firm (this is the crucial benefit of abstracting from strategic interactions in our monopolistic competition equilibrium). Since  $\mu_z$  in the old stationary state is known, we can thus compute  $\{\mu_{z,t}\}$  based on those policy choices, and the choice for the number of entrants. In turn, we can then compute a new price index  $\{P_t\}$  based on the distribution and mass of firms (which implies a distribution of prices). We iterate until this new price path  $\{P_t\}$  matches the prior choice of the candidate  $\{P_t\}$ .

We next describe how we set the parameters of the model to run the model simulations. The model is calibrated to reflect the findings from the recent empirical literature on the effect of trade on firms and industries, and typical patterns of firm dynamics within industries, in particular:

Bartelsman et. al. (2000); Bernard and Jensen (1999); Bernard and Jensen (2006); Bustos (2006); de Loecker (2006); and Trefler (2004).

We first describe the grid over time periods and productivity levels on which to run the model (Table 1). We set each time period to correspond to one month. This is relatively short, as compared to the typical time unit used empirically of one year, thus smoothening out the dynamic processes. We set the total number of time periods to 200 (i.e., around 17 years) as this is long enough to ensure that by the final period the industry has converged close to the stationary equilibrium corresponding to the final set of parameters. Note that we do not impose this final stationary state as the end point: rather, we allow the industry to evolve towards it.

We consider two aspects of the timing of the change in trade policy: the extent of pre-announcement of the policy, and the time required to implement the change in policy. Reflecting the timing of policy changes discussed in the empirical literature, we compare the three scenarios previously described (UA, AA, AG). In all three scenarios, the trade costs initially change after a period of three years. For the UA scenario, this change is unanticipated. For the AA and AG scenarios, this change is anticipated at the beginning of year one (with year zero the old stationary state). Once the trade costs change, in the UA and AA cases the drop in  $\tau$  occurs immediately, over a period of one month. For the AG case, the gradual change in  $\tau$  is linear over a period of three years, and then  $\tau$  is constant at the new stationary state thereafter.

We set the productivity range to  $v = [0.7, 3]$ . This grid size is exogenous to any firm decisions. Hence, we set a sufficiently wide range such that this encompasses the range of productivity levels relevant to firms in the endogenous equilibrium size distribution. In particular, this grid is set such that the exit cutoffs are sufficiently above the lower bound, and that virtually no firms have productivity levels close to the upper bound. The resulting size distribution of firms (for the range of trade costs considered) exhibits a 75th to 25th percentile size ratio around 2, a 90th to 10th percentile size ratio around 4, and standard deviation of log size around 0.6. This represents a smaller amount of size heterogeneity than found in most empirical studies of firm size across industries.<sup>9</sup> We stick with this lower level of size heterogeneity for two reasons: first, our model is meant to capture the equilibrium dynamics for a narrowly defined industry, probably narrower than the classifications used in economy-wide studies of firm size distributions. Second, we acknowledge that there are many other sources of heterogeneity across firms that induce differences in firm size

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<sup>9</sup>For instance, the standard deviation of log size ranges from 0.79 to 1.27 for selected 5-digit Portuguese manufacturing industries (Cabral and Mata (2003)), and varies between 0.9 to 0.95 for Dutch manufacturers over the period 1978 to 1998 (Marsili (2006)).

not directly related to the productivity/product quality channel from our model. Our model is therefore intended to capture a subset of firm-level differences that induce the empirical heterogeneity in firm size. We set the number of productivity grid points to 600; high enough that there are sufficient grid points to minimize any effects from the discreteness of the grid. For instance, a finer grid allows for the productivity cutoffs to more smoothly adjust over time.

We next we discuss the demand and production parameters (see Table 2 for details). The main demand parameters are the price elasticity of demand for the aggregate industry output, which we set to  $\eta = 1.5$ , and elasticity of substitution between varieties, which we set to  $\sigma = 4$ . The overhead cost  $F = 9$  is set such that, on average, firms devote 20% of their labor cost to overhead.

We set the range of variable trade costs to  $\tau = 1.35$  and  $\tau = 1.05$  in, the initial and final stationary states. This reduction of 30% corresponds to the typical reduction in trade costs for industries most affected by trade reforms, though we intentionally choose to model a substantial liberalization reform. In addition, we set the relative levels of variable, fixed and sunk costs of trade to lead to a range of 15% to 45% of exporting firms, across the two stationary states. These percentages are within empirical ranges for the proportion of exporters across sectors. Specifically, we set the fixed costs of trade to  $F^X = 10$  and the sunk costs of trade to  $S^X = 2.4$ . Hence, the fixed cost of trade are comparable in magnitude to the per-period fixed costs. The relatively low sunk costs of trade ease firms' shift in and out of export status, as there is empirical evidence substantial transitions by firms in and out of the export market.

Next we discuss our choices for the productivity transitions. First the death shock: we set this at 15% per year, which is higher than the firm level exit rates observed empirically (of around 3-7% per year). However, as mentioned above, the model could be interpreted as focused on product lines, and we would expect higher exit rates for these more disaggregated units of analysis. For firms that do not innovate, we set the stochastic productivity transition based on a lognormal distribution. For each firm, the draw is from a distribution with mean corresponding to the current firm's productivity. The standard deviation is the same across all firms (with truncation of extreme changes in productivity, in part to avoid accumulation of firms at the edge of the productivity grid). Thus each firm has the same probability of experiencing a similar percent increase or decline in productivity<sup>10</sup>. For firms that do innovate, we set the average increase in productivity to 10% above

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<sup>10</sup>Note that, although the productivity transition has no effect on the firm's expected productivity, this is not true for the effect on firm's expected profit. The profit function rises steeply with productivity - hence in expectation, profits rise with a productivity shock. The death shock in part compensates for this effect (otherwise no firm would ever want to exit). Another modeling alternative would be to specify a mean reverting productivity transition.

their current productivity. The new productivity is again drawn from a lognormal distribution (with truncation of extreme outcomes). The expected increase in productivity is comparable to the empirical evidence on the increase in productivity for firms entering the export market. We set the sunk costs of innovation to  $S^B = 300$  whenever innovation is feasible, which occurs with a  $\gamma_B = .5$  probability (i.i.d. over time for every potential innovator). This sunk cost is equivalent to a per-period interest charge of around 14% of the level of fixed costs (at our 5% annual discount rate  $\beta$ ). If innovation is not feasible, then firms choose their next best alternative. Note that this effect of  $\gamma_B$ , which is modeled to induce some smoothing in the distribution of firms close to the innovation cutoff, will be short lived: over the course of one year, virtually all firms ( $1 - .5^{12} > .999$ ) who wish to innovate (in each month of the year) will be able to do so.

Finally, we specify the distribution of potential entrants over productivity levels as lognormal with  $\log(.8)$  mean and .2 standard deviation, restricted to the  $[\cdot 7, 3]$  productivity grid: see Figure 2. The endogenous exit productivity cutoffs will always be above the .7 productivity lower bound so that some entrants with low productivity draws around this cutoff choose to immediately exit and not produce. Overall, in our simulations, entrants enter with an average productivity lower than that of incumbent firms (whose average productivity is always above  $v = 1.15$ ). Thus, our simulations replicate the robust empirical findings that recent entrants are on average smaller, and exhibit higher exit rates than incumbent firms. The entry sunk cost is set to  $S = 60$ , which is equivalent to a per-period interest charge of 3% of fixed costs (at our 5% annual discount rate  $\beta$ ).

## 4 Simulated Results

We now describe the numerical properties of the two stationary states with the pre-liberalization high trade cost  $\tau = 1.35$  and the post-liberalization low trade cost  $\tau = 1.05$ . One defining characteristic of these equilibria concern the cutoff productivity levels that determine the key firm policy decisions for exit, export, and innovation. These cutoffs are shown in Table 4. The first two columns of each panel show the productivity range along with the transition decision (the range for which no transition occurs is indicated by a dash, -). The third column indicates the yearly transitions flows to each of the other possible states. These flows are expressed as a percentage of the number of firms in the originating state.<sup>11</sup> As we previously discussed, the rankings of the firm decision rules across productivity are such that exit (the endogenously driven component) is always at the

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<sup>11</sup>Note that these flows represent the accumulation of transitions over twelve monthly simulation periods, so these flows can be greater than 100% for originating states that are transitory for most firms. The exogenous exit flows due to the death shock  $\delta$  (independent across productivity) are not represented.

bottom, and the  $BX$  option is always at the top.<sup>12</sup> However, the level of trade cost significantly affects the relative benefits of innovation and export market participation. This is most clearly seen in the optimal decision of  $AD$  firms who have not yet undertaken either activity and jointly consider both of these options following favorable productivity shocks (top part of Table 4). The productivity cutoffs in the left panel show that the pre-liberalization trade cost  $\tau = 1.35$  is high enough that exporting is relatively unattractive prior to innovation:  $AD$  firms receiving positive productivity shocks will predominantly choose to innovate, but not export. A tiny fraction of firms receiving large positive shocks choose to undertake both. Still, no firms choose to export but not innovate – and thus all exporters in the pre-liberalization stationary state have innovated.<sup>13</sup> With lower trade costs, exporting becomes more attractive to mid-level productivity firms. Given our calibration for the post-liberalization stationary state, there are then some  $AD$  firms who choose to export but not innovate, following a productivity increase. Given large enough productivity increases, these  $AD$  firms then choose to jointly innovate and export. There are thus no longer any transitions from  $AD$  to  $BD$ : a decision to innovate but not export by some  $AD$  firms.<sup>14</sup> The transitions from  $AX$  in Table 4 show that this state is mostly transitional, with firms either moving on to  $BX$  (if things go well) or back down to  $AD$  (if they do not): the annual flows from  $AX$  to either of these states are large (recall that the annual flows are cumulations of twelve monthly simulation periods) as the productivity band to remain in  $AX$  is relatively narrow.

Since firms do not have an option to ‘un-innovate’ (transition from a  $B$  state back to  $A$ ), the rankings of the transition decisions for innovators in the bottom half of Table 4 are straightforward: the lowest productivity firms exit, the highest productivity firms export, and the remaining firms choose to only serve their domestic market (including the decision to exit the export market and forego the sunk export cost for  $BX$  firms receiving bad productivity shocks). The cutoff ranges for the transitions between  $BD$  and  $BX$  also highlight the important effects arising from the

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<sup>12</sup>As we previously noted, the use of a productivity upper bound implies that the benefits of innovation are worthless to a firm with productivity at the upper bound threshold. This implies that  $BX$  is not the best option for non-innovators with productivity levels right below this threshold. Our calibration is such that the productivity upper bound  $v \leq 3$  is high enough that there are virtually no firms close to this productivity range (see distribution plots, discussed later on in this section). Thus, the effect of the upper bound on the incentives to innovate is immaterial in our calibrations – and is consequently ignored in the transition decisions in Table 4.

<sup>13</sup>To be precise, the number of  $AX$  firms is not exactly zero: Due to the i.i.d. probability of infeasible innovation ( $\gamma_B$  draw), a tiny fraction of  $AD$  firms, who wish to both export and innovate, are constrained to only export until their  $\gamma_B$  draw is reversed. However, the fraction of firms concerned is so minute, and the transition via  $AX$  so transitory, that the number of  $AX$  firms is essentially zero up to a rounding error.

<sup>14</sup>The bottom half of Table 4 shows that there are still some  $BD$  firms in the post-liberalization stationary state. These firms all transition to  $BD$  from the  $BX$  state: some  $BX$  firms subsequently receive bad productivity shocks and exit the export market, transitioning to  $BD$ .

combination of sunk export market costs and firm-level uncertainty concerning future productivity: these jointly generate an export market hysteresis band. Both the pre- and post-liberalization stationary states feature such a band, within which non-exporters ( $BD$ ) and exporters ( $BX$ ) both choose to maintain their current state. There is also a very similar export market hysteresis band between  $AD$  and  $AX$  firms in the post-liberalization stationary state.  $AD$  firms also face option values to waiting when considering the decision to innovate in the pre-liberalization stationary state. Absent any uncertainty concerning future productivity, some  $AD$  firms would choose to innovate: innovation generates a positive net present value activity for those firms (but one that is less than the option value of waiting).<sup>15</sup>

Figures 3 and 4 show the distribution of firms over productivity for each state in the pre- and post-liberalization stationary states. Figure 5 combines the firm productivity distributions across states. The top panel of Table 5 quantitatively summarizes how the firms are distributed across the two stationary states, and also shows that there are fewer total firms (across states) in the post-liberalization stationary state (although there is a larger number of competing imported varieties in the latter). The overall striking difference between the two firm distributions across states is the substantial shift from  $AD$  firms to  $BX$  firms (in terms of both the range and mass of the distributions), when moving from the pre- to post-liberalization stationary state. When we analyze the dynamics along the trade liberalization equilibrium between these two stationary states, we will pay particular attention to the transition paths for this substantial group of firms moving between the two states. The right hand side of Table 5 highlights the subset of transitions most relevant for the trade liberalization equilibrium: transitions out of  $AD$  and into  $BX$ . In order to make these firm transition flows comparable, they are expressed as percentages of the total number of firms across states (and not relative to the number of firms in the originating state, as was the case for Table 4). As was the case for Table 4, the flows are annual, accumulating twelve monthly simulation periods.

## Trade Liberalization

The contrast between the two stationary states already highlights how trade costs affect the relative benefits of innovation and export market participation (and hence also highlights how the decision to innovate is inextricably linked to the export decision). We now analyze the transition dynamics between these two stationary states, paying particular attention to how these dynamics are affected

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<sup>15</sup>There is no hysteresis band associated with this option value as  $BD$  firms can not un-innovate.

by the anticipation and pace of liberalization. We investigate the effects of the three different trade liberalization scenarios that involve the same long run transition between the two stationary states, as previously described. The first scenario is an unanticipated abrupt lowering of the trade costs,  $UA$ ; the second scenario involves the same abrupt change in trade costs, but anticipated three years ahead,  $AA$ ; and the third scenario is an anticipated but gradual change in trade costs,  $AG$ . In all cases, variable trade cost  $\tau$  first decreases at the beginning of the fourth year. In the case of the anticipated liberalizations, this is announced at the beginning of the first year. The gradual liberalization occurs over a period of three years, whereas the abrupt liberalization occurs from one month to the next.

As described above, our model numerically solves for the full range of firm responses to the liberalization at monthly frequencies. Figure 6 shows the response of the price index  $P_t$  at this monthly frequency. Table 5 reports annual averages for the evolution in the number of firms and their distributions across each state ( $AD, BD, AX, BX$ ), as well as the key transition flows out of  $AD$  and into  $BX$  at annual frequencies for all three scenarios. In all cases, year zero represents the old stationary state. Although we simulate the response over a period of seventeen years (to make sure that the end of the simulation corresponds nearly exactly to the new stationary state), we only report the first eight years as, in all three scenarios, that is enough to get back to an industry equilibrium that is very close to the new stationary state (with very little year to year variations in later years).

In the case of the unanticipated liberalization ( $UA$ ), all years preceding the drop in trade costs follow the old stationary state equilibrium. Entry drops to zero for ten months following the drop in trade costs to accommodate the lower number of producers in the new long run stationary state. Immediately following the drop in trade costs, over a third of the  $AD$  firms immediately enter the export market. Of these new exporters, a third innovate jointly with the export decisions ( $ADBX$  transition) while two thirds do not innovate immediately upon entering the export market ( $ADAX$  transition). The transitions into  $BX$  show that a large portion of  $BD$  firms also immediately begin exporting (this flow represents almost all of the existing  $BD$  firms). Lastly, a large portion of the new  $AX$  exporters choose to innovate shortly after their export market entry decision: the large  $ADAX$  transitions are accompanied by large  $AXBX$  transitions, with very few firms remaining as  $AX$  for very long. Thus, we can summarize the transition paths of the new exporters by noting that most of them innovate either concurrent with, or subsequent to, their export market entry decision.



We now describe the transition path for a similar case of abrupt liberalization, but where this liberalization is now anticipated three years ahead of time. It is clear from the response of the price index that this anticipation not only affects the firms' expectations for the future, but also their current behavior. This is most drastically exhibited by the behavior of the low productivity  $AD$  firms, that start exiting as soon as the news of trade liberalization is announced. Figure 7 shows the response of the exit productivity cutoff for  $AD$  firms. The main force behind the exit of these low productivity firms is not the endogenous response of the price, but the direct effect of the change in those firms' option value to remain active in the industry – given future liberalization. In fact, even if the price index did not change at all during the first three years prior to liberalization (stayed fixed at the old stationary equilibrium), the response of the exit cutoff would be essentially as it is depicted. The news of future liberalization has a dramatic effect on the low productivity firms' option value to remain active. The probability that these firms survive past the first three years is now drastically reduced, and hence the option value of waiting for better productivity draws is driven to zero (the closer to the expected liberalization, the closer the option value is driven to zero).

For similar reasons, the option values to invest for  $AD$  firms and to export for  $BD$  firms are substantially affected by the prospect of future liberalization.<sup>16</sup> The high productivity  $AD$  firms know that they will almost surely be innovating post liberalization (and exporting), and the high productivity  $BD$  firms know that they will almost surely be exporting post liberalization. Thus, their option value of waiting to do so also goes to zero as the liberalization date approaches inducing those  $AD$  firms to innovate (transition to  $BD$ ) and those  $BD$  firms to export (transition to  $BX$ ). This is clearly reflected in the transitions in Table 5, although our parametrization suggests that this effect is only quantitatively important for the year preceding liberalization (the percentage of firms that transition from  $AD$  to  $BD$  increases from 4% to 9%, and that of firms that transition from  $BD$  to  $BX$  increases from 5% to 8%). After liberalization in year four, almost all the  $BD$  firms enter the export market, and the transitions flows and distribution of firms across states very quickly approximates that of the unanticipated liberalization scenario. The crucial difference is that the anticipation of future liberalization induces many new exporters to innovate ahead of liberalization, and thus also ahead of their anticipated, but yet unrealized, entry into the export market.

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<sup>16</sup>There are virtually no  $AX$  firms, so no purpose in considering the innovation option for those firms. Also, the  $AX$  state is not attractive to  $AD$  firms, who would rather innovate first at the higher trade costs – hence no reason to consider the option value of exporting for  $AD$  firms.

In this case of anticipated, abrupt trade liberalization, entry drops to zero for several periods: for the six months preceding the drop in  $\tau$ , and for just over a year following the drop. Preceding the drop in  $\tau$ , the exit threshold rises. Just after the drop in  $\tau$ , the exit threshold continues to rise and overshoots the long run level, only returning down to the long-run level nine months after the drop in  $\tau$ . As entrants are typically smaller than incumbents (and initially start out as *AD* firms), the value of entry is sensitive to this pattern in the exit cutoff. The lack of entry contributes to the reduction in the number of firms in the industry (and, in particular, at lower productivity levels): hence the steep rise in the price index prior to liberalization. Post liberalization the price drops below the long-run level. One reason is that there is an immediate large switch of *BD* firms into the export market, which drives down industry price. In effect, “too many” incumbent firms remain in the industry, in the hope that their productivity evolves favorably: for instance, there is no sudden shift of *AD* firms into exit. With the prospect of the shakeout of *AD* firms, the value of entry is negative. Over time, this firm “overhang” is whittled away and entry eventually resumes.

Lastly, we turn to the scenario of the anticipated gradual liberalization. If this liberalization is gradual enough, as is the case with our current calibration, then the anticipation of liberalization does not have any noticeable effects on the firms’ transitions between states prior to the start of liberalization – although it is still important enough to be significantly reflected in the endogenous price index response and in the exit behavior of the least productive firms. Also, the transition process is sufficiently gradual that entry remains positive in all periods, unlike the other two scenarios with abrupt changes in trade costs.

However, the anticipated future course of liberalization does significantly affect the firm’s transitions between states once liberalization begins. A series of unanticipated drops in trade costs would induce firms to start exporting whenever profitable to do so (and innovate concurrently or later as that becomes profitable too), but would not induce firms to innovate ahead of their expected entry into export markets. Yet, this is exactly what is reflected in the transitions reported in Table 5 for this scenario. As liberalization begins, we witness the same substantial response in the transitions from *AD* to *BD*. These firms then subsequently enter the export market and transition from *BD* to *BX*. As in the case of the anticipated abrupt liberalization, some *BD* firms are induced to enter the export market as their option value of waiting to export falls, though this effect is harder to separate from the direct effect of the drop in trade costs in the current scenario.

## 5 Conclusion

In this paper, we build a dynamic model of firm-level adjustment to trade liberalization that jointly addresses firm's decisions to innovate and/or to enter the export market. We analyze the equilibrium transition from an initial stationary state with high trade costs, comparing scenarios that differ in the extent to which the trade liberalization is anticipated by firms and how fast the trade costs drop once the liberalization starts. The comparison across the trade liberalization scenarios shows how some important non-technological factors, such as the pace and anticipation of trade liberalization, can fundamentally affect the perceived causation link between export status and productivity. For example, we show that the anticipation of trade liberalization tends to bring forward the decision to innovate relative to the export market participation, and how a more abrupt pace of liberalization amplifies these effects. Thus, our model shows how the current industrial response not only depends on concurrent trade costs, but also, inextricably, on the firms' prior expectations about those current trade costs, and their expectations for future trade costs. More generally, our model highlights the potential empirical pitfalls of analyzing current period industrial performance as a response to the concurrent trade costs during periods of trade liberalization.

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Table 1: Calibration: Model Timing and Productivity Grid

<b>Variable</b>	<b>Empirical evidence*</b>	<b>Explanation of model calibration</b>
<b>Time path of trade policy</b>		
Duration of time periods $t$	Most empirical data is annual data	Set each time period to correspond to one month, so time period short relative to typical empirical unit of analysis. Set total number of time periods such that the by the final period the industry has converged close to the final stationary state
Timing of policy change	Typical timeframes are: Zero to three years anticipation of policy change [T]; and around two to four years to implement	Set Announced policy change to start after 36 periods (i.e., three years) after announcement. Set Abrupt change as one period (i.e., one month) transition, and Gradual as 36 period transition (i.e., three years)
Discount rate $\beta$		5% per year, thus 0.4% per month (i.e., per time period)
<b>Productivity grid</b>		
Productivity $v$	Relative size of largest to smallest firms often over 100x [dL], [B et al]	Set $v$ range to [0.7,3] to allow a sufficiently broad range of firm sizes, as relative firm size determined by productivity $v$ to the power $\sigma-1$
<b>Normalization</b>		
Wage per period $w$		Normalize monthly wage to one

\* The reference to the empirical literature is as follows: [B et al] Bartelsman (2003); [B,J 99] for Bernard and Jensen (1999); [B,J 06] for Bernard and Jensen (2006); [B] for Bustos (2006); [dL] for de Loecker (2006); and [T] for Trefler (2004)

Table 2: Calibration: Demand, Production and Trade Costs

Variable	Empirical evidence*	Explanation of model calibration
<b>Demand</b>		
Elasticity of demand	$\eta$	Set to 1.5
Elasticity of substitution	$\sigma$	Set to 4
<b>Production</b>		
Fixed costs	F	Skilled workers ~16% of workforce, with an additional 6% in continuing exporters [B]
<b>Trade costs</b>		
Variable, fixed and sunk costs of trade	$\tau$ , FX and SX	Range of evidence on extent of drop in trade costs: maximum drop in duty is from over 10% to 1% over 8 years [T]; maximum 30% to 20% drop in freight and duty costs over 10 years, with average 10% to 8% drop [B,J 06]; and drop from 12% to 11% average external tariff over 3 years, but within Mercosur drop is from range of 0-22% down to around 0% [B].
Proportion of exporters		Set $\tau$ to drop from 1.35 to 1.05. Set FX to 10, and SX to 2.4.
		Exporters rise for [dL] from 33% to 48% of firms, and for [B] from 38% to 54%. Proportion of firms per year becoming exporters: 11% for [dL], 10% for [B,J 06] and 4% for [B] (in sample of mostly large firms). Proportion of exporters per year that stop exporting: 15% for [B,J 06] and 0.5% for [B] (in sample of mostly large firms)
		Set trade cost to result in rise in proportion of exporters from around 15% to 45% of firms

\* The reference to the empirical literature is as follows: [B et al] Bartelsman (2003); [B,J 99] for Bernard and Jensen (1999); [B,J 06] for Bernard and Jensen (2006); [B] for Bustos (2006); [dL] for de Loecker (2006); and [T] for Treffer (2004)

Table 3: Calibration: Evolution of Productivity and Entry

<b>Variable</b>	<b>Empirical evidence*</b>	<b>Explanation of model calibration</b>
<b>Productivity transitions</b>		
Death shock $\delta$	Exit rate ~3% per year [dL], and ~3-7% per year [B et al]	Set to 15% per year (with additional exit from firm productivity dropping below exit productivity cutoff), thus 1.4% per month (i.e., per time period)
Transition for firms (except for A firms investing to become B)		Productivity evolves according to truncated lognormal evolution with mean $\log(v)$ and 0.02 standard deviation (hence, mean zero change in productivity). Also, truncate increase/decrease to future productivity to within +/- 20% of current $v$
<b>Investment</b>		
Transition for A firms that invest to become B	Trade opening impact on plants of ~14% improvement in productivity [T]. Also, ~9% immediate impact on productivity [dL]. Note these include the impact of any change in technology that occurred so as to export. Indeed, many new exporters seem to increase technology spending [B].	Productivity evolves according to truncated lognormal evolution with mean $\log(1.1v)$ and 0.02 standard deviation (hence, mean +10% change in productivity). Also, truncate increase/decrease to future productivity to within +/- 50% of current $v$
Sunk cost of investment	SB and $\gamma^B$	Set SB=300 with probability $\gamma^B=0.5$ , which corresponds to a monthly interest charge of 1.3 (i.e., around 14% of per period fixed costs); otherwise SB infinite
<b>Entrants</b>		
Entrant size	Entrants smaller than incumbents on average. Also, around 50% of entrants survive to 7 years, with 20% hazard in year 1 and around 10% hazard thereafter [B et al]	Set entrants as distributed lognormal, with mean $\log(0.8)$ and std dev=0.2, over the productivity grid (hence generating truncated distribution). This results in entrants with, relative to incumbents, lower average productivity and higher exit rates.
Entry sunk cost	S	Set S=60, which corresponds to a monthly interest charge of 0.2 (i.e., around 3% of per period fixed costs).

\* The reference to the empirical literature is as follows: [B et al] Bartelsman (2003); [B,J 99] for Bernard and Jensen (1999); [B,J 06] for Bernard and Jensen (2006); [B] for Bustos (2006); [dL] for de Loecker (2006); and [T] for Trefler (2004)



Table 4: Firm Cutoff Rules and Transition Flows in the Stationary States

Transitions from AD					
Pre			Post		
v range	To	Flow	v range	To	Flow
0.70–0.88	Exit	14.3%	0.70–0.91	Exit	22.2%
0.88–1.30	--	--	0.91–1.17	--	--
1.30–1.40	BD	4.6%	1.17–1.19	AX	16.8%
1.40+	BX	0.2%	1.19+	BX	4.1%

Transitions from AX				
Pre		Post		
		v range	To	Flow
(No AX firms)		0.70–0.91	Exit	0.0%
		0.91–1.13	AD	80.8%
		1.13–1.19	--	--
		1.19+	BX	201.7%

Transitions from BD					
Pre			Post		
v range	To	Flow	v range	To	Flow
0.70–0.89	Exit	0.7%	0.70–0.92	Exit	6.9%
0.89–1.42	--	--	0.92–1.17	--	--
1.42+	BX	65.7%	1.17+	BX	35.3%

Transitions from BX					
Pre			Post		
v range	To	Flow	v range	To	Flow
0.70–0.89	Exit	0.0%	0.70–0.92	Exit	0.0%
0.89–1.37	BD	20.0%	0.92–1.13	BD	6.1%
1.37+	--	--	1.13+	--	--

Table 5: Distribution of Firms and Transitions Across States

Stationary States	# firms	Firm Distribution				Transitions				
		AD	BD	AX	BX	ADB	ADAX	ADB	BDB	AXB
Pre	570	77%	8%	0%	15%	4%	0%	0%	5%	0%
Post	446	51%	4%	3%	42%	0%	9%	2%	2%	6%

UA	year	# firms	Firm Distribution				Transitions				
			AD	BD	AX	BX	ADB	ADAX	ADB	BDB	AXB
0	570	77%	8%	0%	15%	4%	0%	0%	5%	0%	
1	570	77%	8%	0%	15%	4%	0%	0%	5%	0%	
2	570	77%	8%	0%	15%	4%	0%	0%	5%	0%	
3	570	77%	8%	0%	15%	4%	0%	0%	5%	0%	
4	473	59%	2%	4%	35%	0%	16%	6%	8%	10%	
5	441	55%	2%	3%	40%	0%	10%	2%	1%	7%	
6	438	53%	2%	3%	42%	0%	9%	2%	1%	6%	
7	439	52%	3%	3%	42%	0%	9%	2%	1%	6%	
8	440	52%	3%	3%	42%	0%	9%	2%	1%	6%	

AA	year	# firms	Firm Distribution				Transitions				
			AD	BD	AX	BX	ADB	ADAX	ADB	BDB	AXB
0	570	77%	8%	0%	15%	4%	0%	0%	5%	0%	
1	619	79%	8%	0%	13%	3%	0%	0%	4%	0%	
2	628	78%	9%	0%	13%	4%	0%	0%	5%	0%	
3	597	75%	10%	0%	15%	9%	0%	0%	8%	0%	
4	461	57%	2%	3%	39%	0%	11%	4%	14%	6%	
5	412	50%	2%	3%	45%	0%	10%	2%	1%	6%	
6	421	51%	2%	3%	44%	0%	9%	2%	1%	6%	
7	428	51%	3%	3%	43%	0%	9%	2%	1%	6%	
8	433	51%	3%	3%	43%	0%	9%	2%	1%	6%	

AG	year	# firms	Firm Distribution				Transitions				
			AD	BD	AX	BX	ADB	ADAX	ADB	BDB	AXB
0	570	77%	8%	0%	15%	4%	0%	0%	5%	0%	
1	599	78%	8%	0%	14%	3%	0%	0%	5%	0%	
2	605	78%	8%	0%	14%	3%	0%	0%	5%	0%	
3	615	78%	8%	0%	14%	4%	0%	0%	5%	0%	
4	592	76%	7%	0%	17%	6%	0%	0%	9%	0%	
5	529	69%	5%	0%	26%	8%	1%	1%	10%	1%	
6	452	58%	3%	1%	38%	2%	6%	5%	4%	3%	
7	417	51%	2%	3%	44%	0%	10%	2%	1%	6%	
8	424	51%	2%	3%	44%	0%	9%	2%	1%	6%	

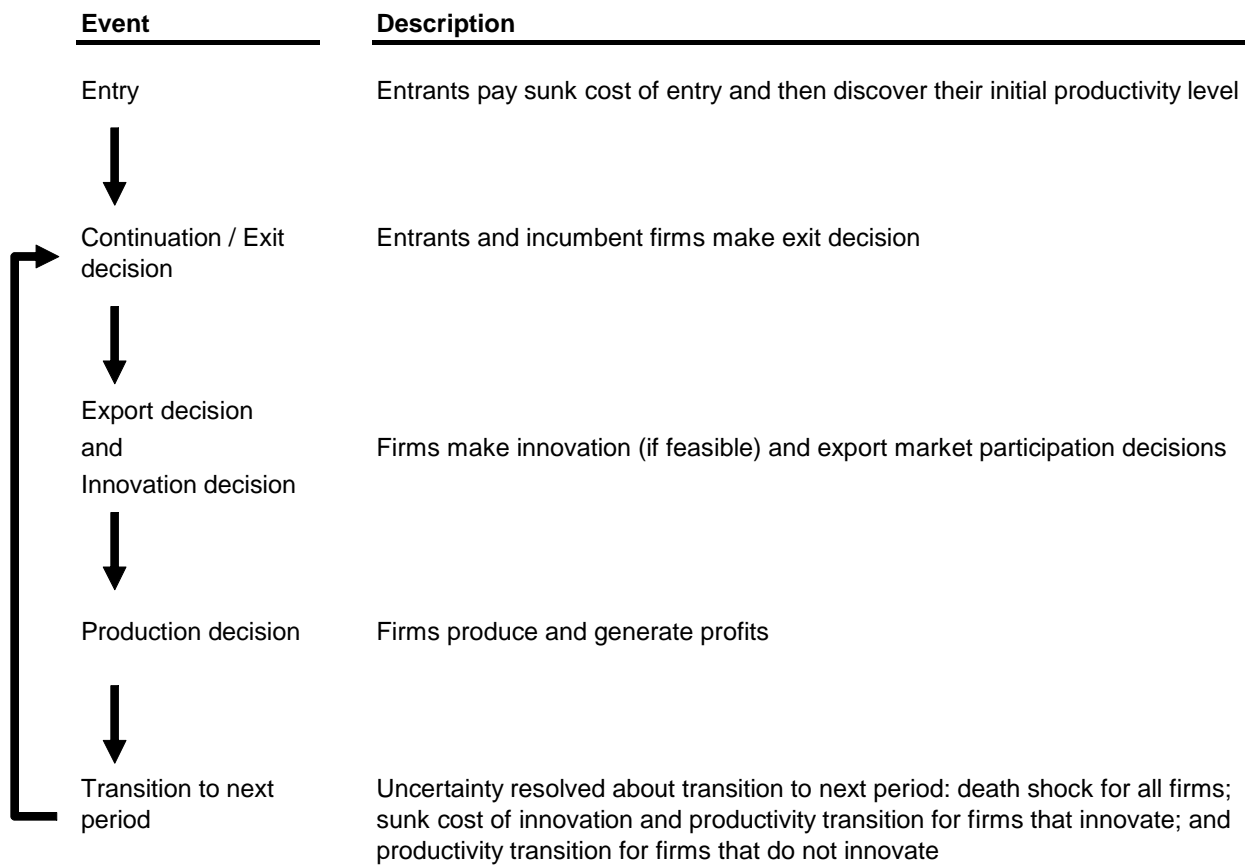


Figure 1: Timing and Description of Events Within Time Periods

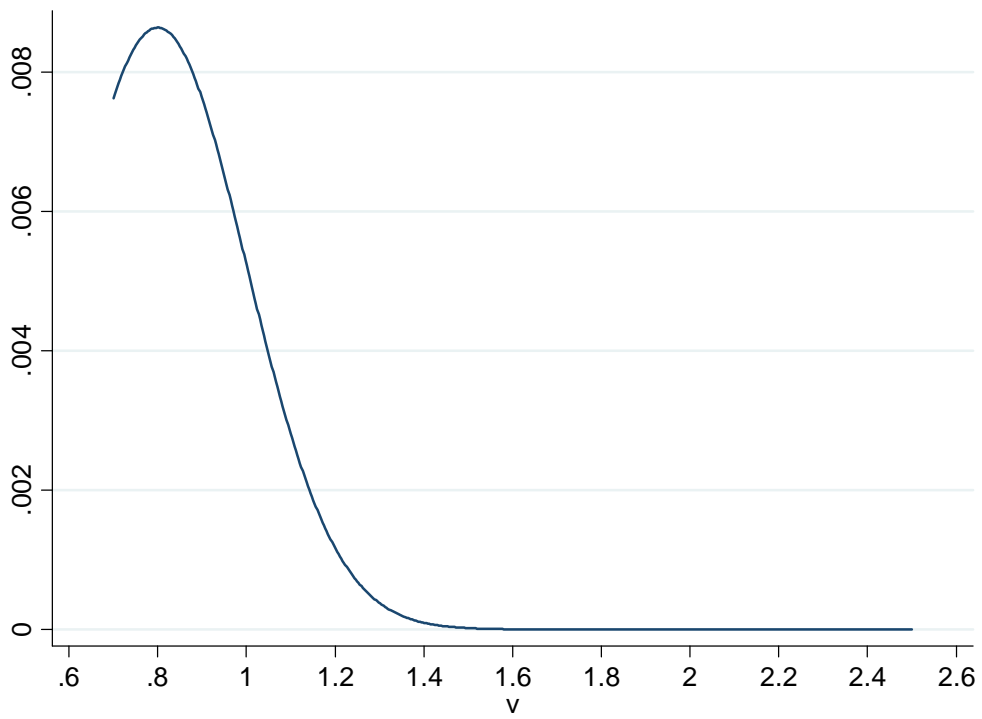


Figure 2: Productivity Distribution of Entrants

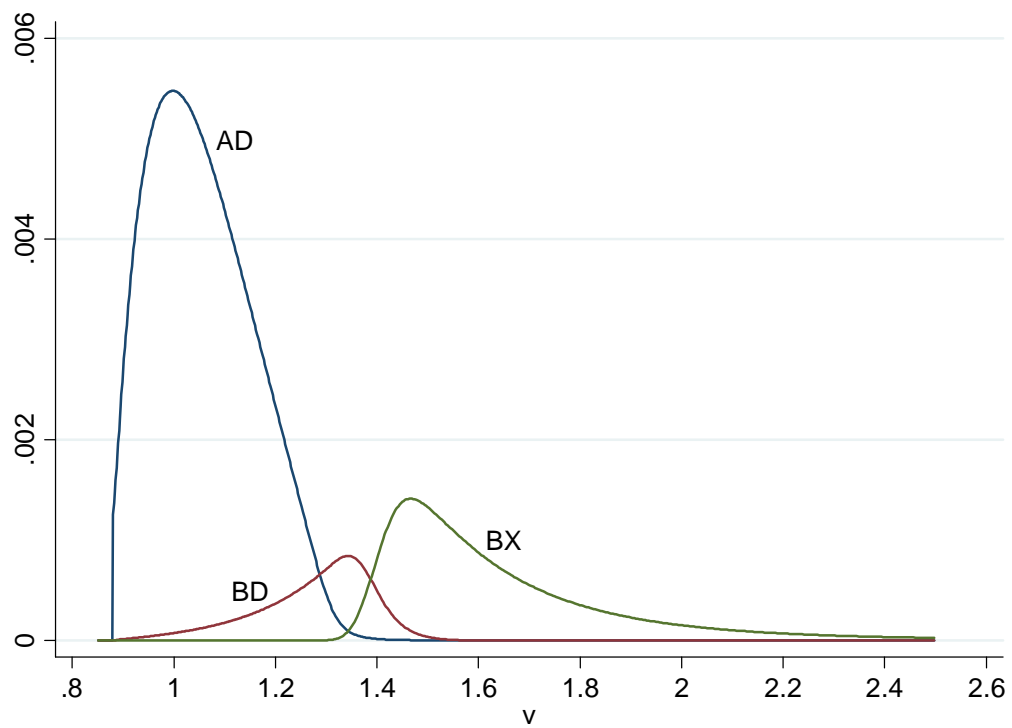


Figure 3: Pre-Liberalization Steady State Distribution of Firm Productivity

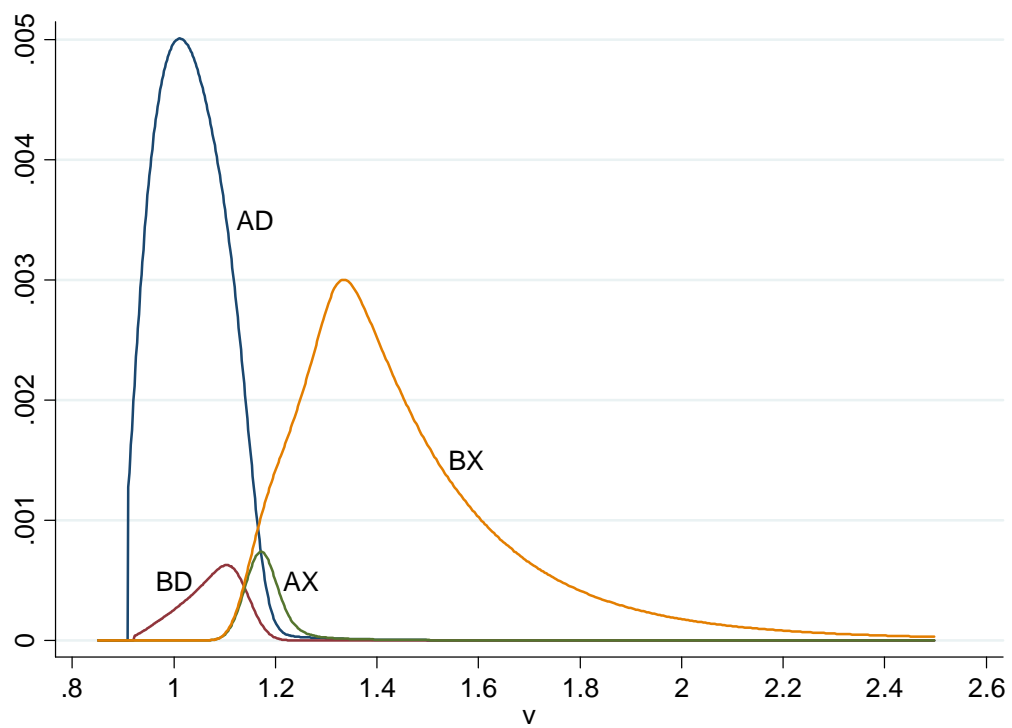


Figure 4: Post-Liberalization Distribution of Firm Productivity

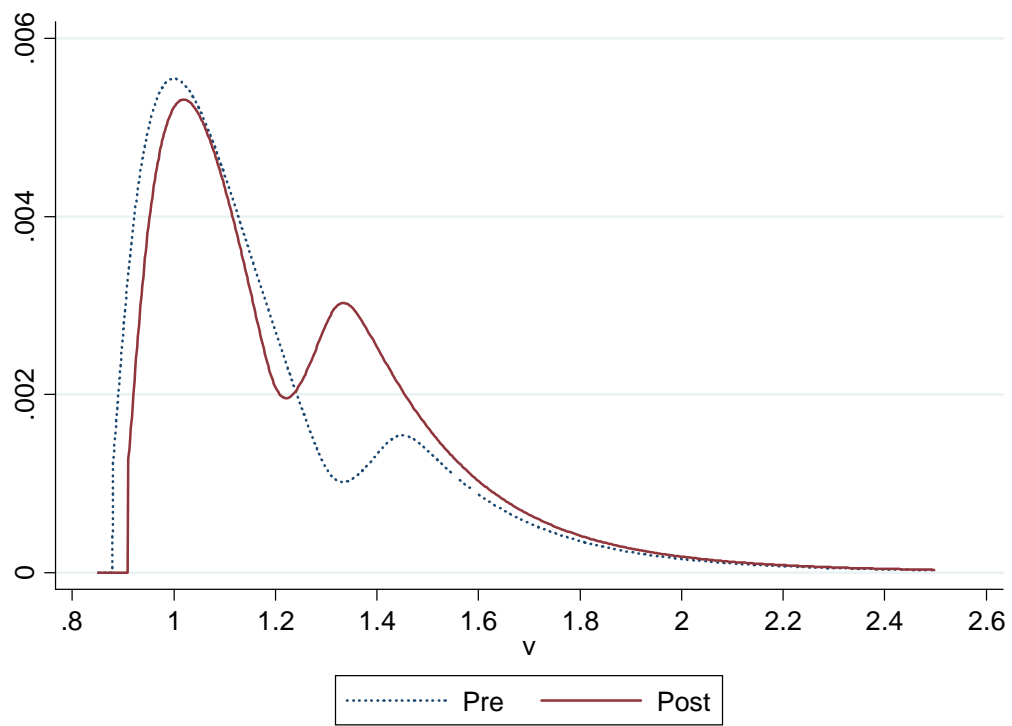


Figure 5: Combined Distribution of Firm Productivity Across Steady States

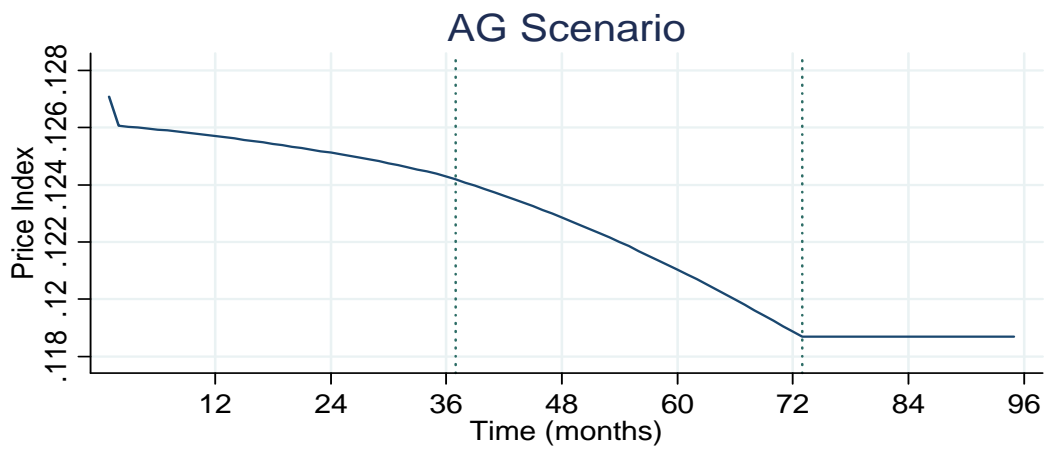
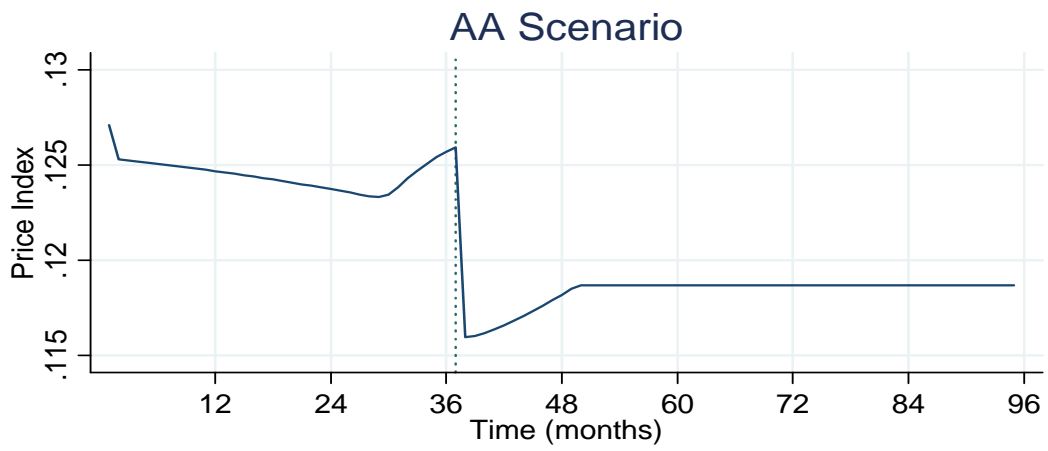
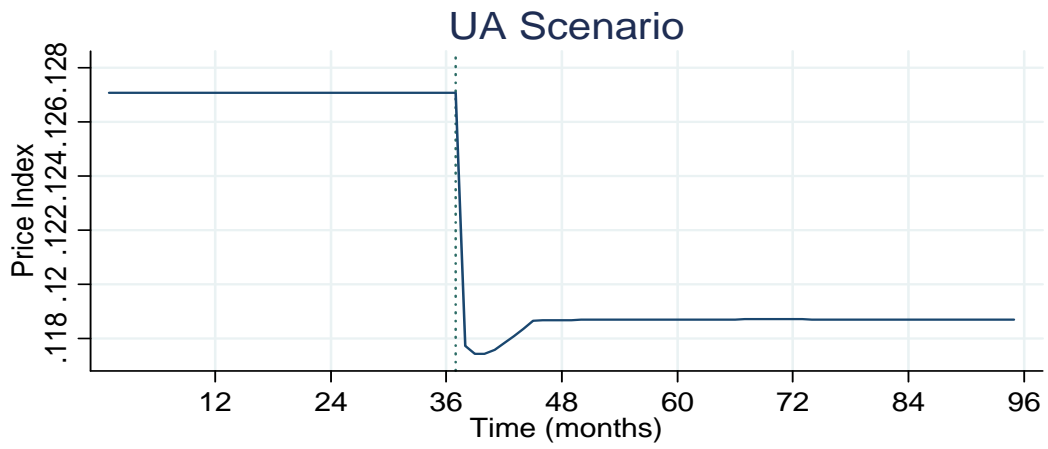


Figure 6: Equilibrium Price Index Over Time



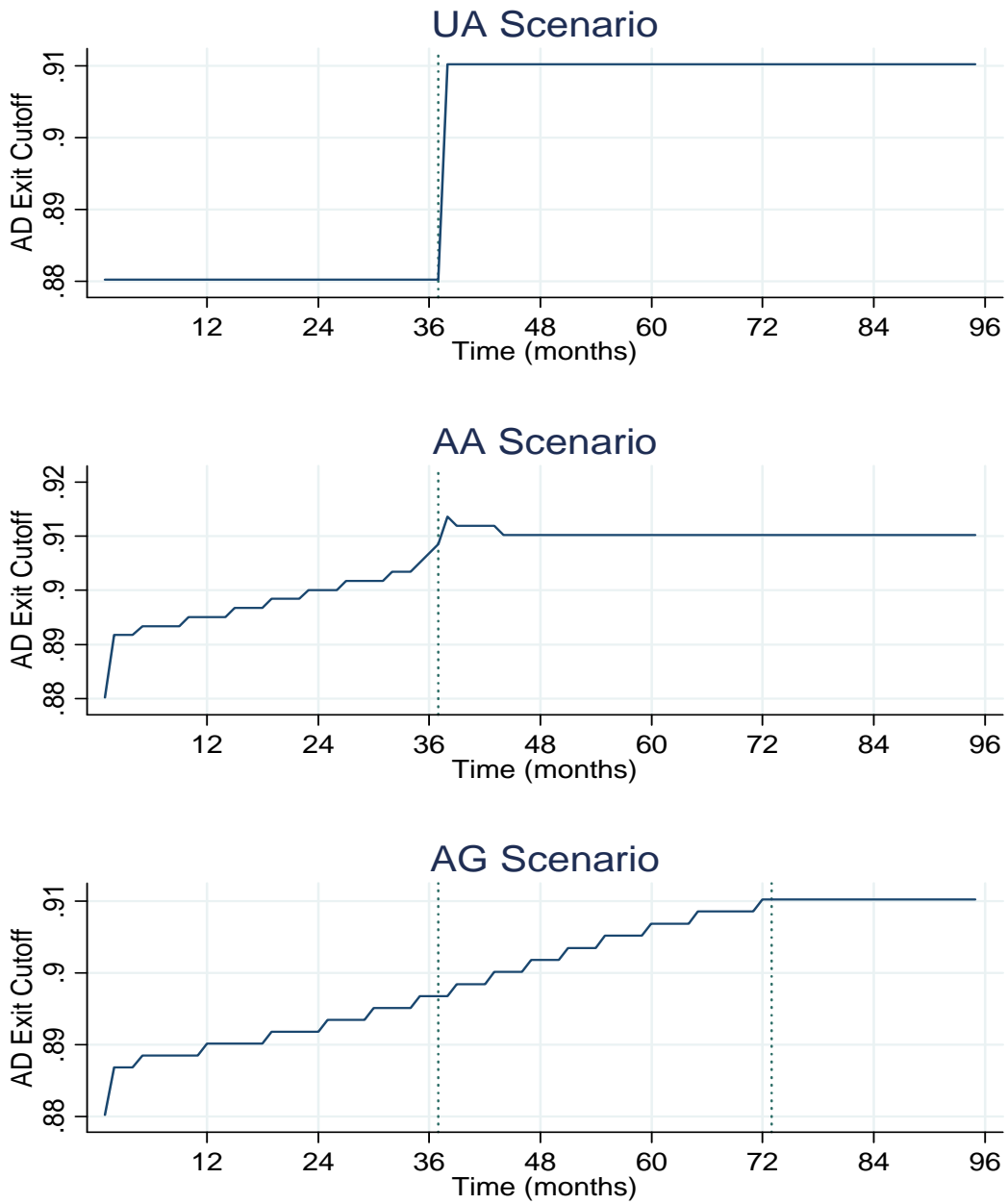


Figure 7: *AD* Firm Exit Cutoff Over Time

## Appendix

### A Model Algorithm

Following we describe the algorithm for numerically solving the model, focusing on the equilibrium conditions required and the sequence of calculations performed. The demand structure leads to monopolistic competition. In particular, this means that each firm in each time period  $t$  need only know industry aggregate outcomes for industry price  $P$  from time  $t$  onwards,  $\{P_t, \dots, P_T\}$ , to determine its specific policies conditional on its current productivity  $v$  and policy state  $z = \{AD, BD, AX, BX\}$ , where  $A$  and  $B$  refer to firm choice of technology, and  $D$  and  $X$  to firm choice of whether to export or not. Firm policy choices are whether to  $\{Continue, Exit\}$ , and, if continuing, whether to switch policy state.

The algorithm comprise three steps. Step 1 is to set parameters. Step 2 is to compute the firm policies and firm-size distribution  $\mu_{z,1}$  corresponding to the initial parameter values, the initial stationary state equilibrium at  $t = 1$ . Within Step 2, there is an iteration over the aggregate price for the stationary state  $P_1$ . Step 3 computes the firm policies and firm-size distribution for the evolution from the initial stationary state through to period  $T$ . Within the Step 3, there is an iteration over the price path  $\{P_2, \dots, P_T\}$ .

- 1) Set initial parameters, including for industry characteristics and grid structure.
- 2)  $P_1$  iteration:

- Choose candidate value for  $P_1$ .
- Firm Value and Policy Iteration:
  - Compute profit  $\pi(v)$  at each productivity  $v$ , based on the specific demand system and production function chosen.
  - Pick a candidate value function  $V_1(v, z)$ .
  - Determine  $\{Continuation/Exit\}$  and choice of policy state  $z$  at each  $\{v, z\}$ .
  - The set of firm policies over continuation and choice of policy state imply a next iteration value for the value function,  $V_1'(v, z)$ , based on computing the value of continuing and comparing to the value of exit.
  - Check whether new  $V_1'(v, z)$  is sufficiently close to  $V_1(v, z)$ .
    - \* If not, continue iteration with  $V_1'(v, z)$ .
    - \* If close enough, return to  $P_1$  iteration.
- Check the value of entry. As seek equilibria with positive entry the condition should be close to zero. Compute firm-size distribution  $\mu_{z,1}$ .
  - If close enough to zero,  $P_1$  iteration is complete.
  - If not, then adjust candidate  $P_1$  accordingly: if condition is positive lower  $P_1$ , if negative raise  $P_1$ .

- 3)  $\{P_2, \dots, P_T\}$  iteration:

- Choose candidate value for  $\{P_2, \dots, P_T\}$ .

- Compute price corresponding to stationary state at final parameter values.
- Set initial guess for  $\{P_2, \dots, P_T\}$  based on prices corresponding to initial and final parameter values.
- Firm value and policy iteration
  - Firm Value and Policy Iteration for  $t = T$ :
    - \* Compute profit  $\pi_T(v)$  at each productivity  $v$ , based on the specific demand system and production function chosen.
    - \* Pick a candidate value function  $V_T(v, z)$ .
    - \* Determine  $\{Continuation/Exit\}$  and choice of policy state  $z$  at each  $\{v, z\}$ .
    - \* The set of firm policies over continuation and choice of policy state imply a next iteration value for the value function,  $V_T'(v, z)$ , based on computing the value of continuing and comparing to the value of exit.
    - \* Check whether new  $V_T'(v, z)$  is sufficiently close to  $V_T(v, z)$ .
      - If close enough, return to  $\{P_2, \dots, P_T\}$  iteration.
      - If not, continue iteration with  $V_T'(v, z)$ .
  - Firm Value and Policy Iteration for  $t = \{2, \dots, T - 1\}$ :
    - \* Compute profit  $\pi_t(v)$  at each productivity  $v$ , based on the specific demand system and production function chosen.
    - \* Iterate back to compute  $V_{T-1}(v, z)$  based on  $\pi_t(v)$  and  $V_T'(v, z)$ , and period T policies, based on computing the value of continuing and comparing to the value of exit. Hence, determine period  $T - 1$  policies  $\{Continuation/Exit\}$  and choice of policy state  $z$ .
    - \* Iterate back to period  $t = 2$ .
- Compute value of entry.
- Compute the size-distribution of firms  $\mu_z = \{\mu_{z,2}, \dots, \mu_{z,T}\}$  consistent with the computed firm policies.
  - Compute  $\mu_{z,2}$  based on  $\mu_{z,1}$  and firm policies computed for  $t = 2$ .
  - Determine number of entrants:
    - \* If value of entry negative for  $t = 2$ , set entry to zero.
    - \* If value of entry is non-negative, set entry such that:
      - Case 1: If the distribution of incumbents implies a price below  $P_2$  then entry is zero, as adding entrants would further distance the firm distribution from the current value of price path
      - Case 2: If the distribution of incumbents implies a price above  $P_2$ , then add entrants until the firm distribution (including entrants) implies a price equal to  $P_2$
  - Iterate forward to compute  $\mu_z = \{\mu_{z,3}, \dots, \mu_{z,T}\}$ .
- Check whether price path  $\{P_2, \dots, P_T\}$  is close enough to an equilibrium:

- Objective function to assess equilibrium comprised of two parts:
  - \* The first part measures the distance between the price path and firm distribution:  $(P^{max} - P)$
  - \* The second part measures an equivalent gap based on the value of entry:  $(P^{fe} - P)$ .
    - This is zero if value of entry is negative (to capture instances when this value is close to zero but negative, we consider this to be zero if value of entry/sunk cost of entry is larger than  $-10^{-4}$ ).
    - This is negative if the value of entry is positive. We calculate  $P^{fe}$  as what the price in the time period in question would need to change to in order to close part of the gap in value of free entry. Hence, if value of entry is positive the price change is negative so as to lower profitability and thus lower the value of entry. The adjustment is moderated by the extent to which price adjustments for future periods (which have been determined as the algorithm work backs through time periods) are for increases or decreases in prices.
  - \* The objective function is then the Euclidian distance of these two measures:  $((P^{max} - P)^2 + (P^{fe} - P)^2)^{\frac{1}{2}}$
- If objective function not sufficiently small, construct new candidate price path.
  - \* The suggested price adjustment is the average of  $(P^{max} - P)$  and  $(P^{fe} - P)$ .
  - \* The actual price adjustment is only part of the suggested price adjustment, to reduce the risk of cycling over successive iterations of the price path.