PEDRO GETE

MICHAEL REHER

Two Extensive Margins of Credit and Loan-to-Value Policies

We analyze a model of mortgage markets, housing tenure choice, heterogeneous agents, and default with closed form solutions. We uncover new insights which may inspire empirical work, and we ground already established insights in a series of tractable expressions. Then we study optimal loan-to-value (LTV) regulation and show that the choice of an LTV cap should balance the opposing forces of access to homeownership and the negative externalities associated with default. Homeownership affordability concerns induce procyclical elements into optimal regulation which attenuate the countercyclical regulation justified by the negative default externalities.

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IN THIS ARTICLE, WE IDENTIFY and characterize the key economic mechanisms driving mortgage markets and housing tenure choice, many of which are closely linked and thus difficult to disentangle in the data or in quantitative exercises. We take an analytic approach, developing a tractable model of mortgage markets with heterogeneous agents. The model admits a series of closed-form expressions that describe the relevant economic mechanisms, as well as their interplay. We uncover several new insights that may inspire empirical work, and we ground already

established insights in a series of tractable expressions. An additional advantage of our approach is that it enables us to study loan-to-value (LTV) regulation incorporating housing tenure choice. This is important because it allows us to show that optimal LTV regulation should have procyclical elements because of concerns related to home affordability. Such elements counterbalance the countercyclical regulation required by default externalities, fire sales, and government guarantees.

We propose a model of collateralized lending with households, who are heterogeneous in their expected income growth, and perfectly competitive lenders, who can observe household heterogeneity. In the first period, households decide whether to rent or purchase a home, which requires mortgage credit. House prices and future income depend on an aggregate business cycle shock, and following adverse shocks, some households must default on their mortgages. Default probability is a key driver of the model, and, as we shall show, it operates somewhat differently for recourse versus nonrecourse loans. The mechanisms at play become evident in a series of closed-form expressions for the following endogenous variables: mortgage rates, credit ceilings, households' housing tenure choice, LTV, loan-to-income (LTI) ratios, and the intensive and extensive margins of credit.

In particular, we characterize two extensive margins of credit. The first extensive margin is supply driven. For risky borrowers, default is so likely that, at a given LTV and LTI, there is no feasible interest rate at which the lender would extend credit to the borrower. Thus, lenders ration credit. The second extensive margin is demand driven. It emerges because of a household's preference for renting versus homeownership. That is, households may choose not to participate in mortgage markets because renting is relatively more attractive, not because they cannot obtain a loan. The two extensive margins are not disjoint and are affected by many of the same factors.

Our model generates several predictions, some of which are new, and others of which are already-established, but grounded in a series of tractable expressions. These are our three most novel predictions:

First, with respect to credit supply, we characterize and analyze the properties of what Geanakoplos (2014) calls "the credit surface," that is, the interest rate offered by lenders as a function of the LTV, the LTI, and the income growth of the borrower. Low risk-free rates (which translate into a lower cost of funds for the lenders) lead to lower mortgage rates, especially for high-LTI households. Moreover, high-LTI households suffer the greatest credit tightening in response to reduced expectations of house price growth. Ehrmann and Ziegelmeyer (2014) and Gerlach-Kristen and Lyons (2014) provide recent cross-country evidence suggesting that the impact of house prices, monetary policy, and income shocks on credit depends on LTV and LTI ratios. Lax monetary policy leads to higher leverage ratios, lower mortgage spreads, and higher default risk in the pool of financed borrowers. This result suggests that the period of low interest rates in the run-up to the recent financial crisis may have been related to the high LTV ratios that lenders accepted over that period, an argument supported by evidence from Jimenez et al. (2014).

Finally, increases in the cost of default for lenders, such as the large fines several U.S. banks have paid for delinquent loans they issued during the housing boom,

translate into tighter lending conditions. The pass-through is stronger when monetary policy is lax. This may explain why lending conditions in mortgage markets have been tight during the current recovery.

Second, with respect to credit demand, higher expectations for house price growth encourage more households to apply for a mortgage provided the rent-to-price ratio does not react too steeply to these expectations. Otherwise, it is possible that the home unaffordability resulting from this increase in expectations will discourage mortgage applications. Moreover, households demand a higher LTV when lenders' loan recovery rate is high, as this recovery rate is passed onto borrowers in the form of lower mortgage rates.

Third, with respect to the structure of the loan contract, nonrecourse mortgages lead to a demand for larger LTVs than recourse mortgages, because they allow a convex gamble on house prices where the maximum loss is the down payment. Relatedly, nonrecourse mortgages should correlate with higher default rates, a prediction documented by Ghent and Kudlyak (2011). Furthermore, nonrecourse mortgages should also correlate with reduced ex-post heterogeneity in down payments and in mortgage costs, because lenders place smaller weight on measures of borrower quality (such as FICO scores) than with recourse mortgages. However, in nonrecourse jurisdictions, a tighter credit ceiling and higher mortgage rates lead to greater ex-ante heterogeneity in the demand for credit. In this situation, households with lower present income but higher income growth suffer the most and instead decide to rent.

Besides making predictions about mortgage markets in the positive sense, our model also allows us to study optimal LTV regulation from a normative perspective. Consider a regulator who chooses an LTV cap to maximize households' utility, taking into account that some households will be renters and that lenders need to break even. Moreover, the regulator incorporates negative externalities from larger default rates associated with high LTV loans. For example, these externalities are pecuniary externalities or lost government revenue from loan guarantees or bank bailouts. Neither households nor lenders internalize these default externalities, and thus there is a role for government regulation. We show that optimal LTV policy should balance the opposing forces of access to homeownership and the costs of default. The macroprudential literature has shown that the negative externalities justify countercyclical LTV regulation. Our value added is to demonstrate why optimal LTV regulation should have some procyclical elements, even taking into account the costs of default. Specifically, during an expansion, home affordability concerns (which can be measured from price-to-rent ratios) rise. Moreover, the optimal LTV depends directly on lenders' cost of funds, which suggests a close link between monetary policy and financial regulation, and on the costs of default, which suggests that optimal LTV policies should condition on factors like lenders' loan recovery rate.

In terms of model, the article most closely related to ours is Eaton and Gersovitz (1981). They show the existence of credit ceilings in a model without the asymmetric information that Stiglitz and Weiss (1981) use in their work on credit rationing.

We differ from Eaton and Gersovitz (1981) because they focus on strategic default (lenders do not have the ability to take possession of a borrower's assets in case she defaults on payments), and on noncollateralized debt. We focus on nonstrategic default (default is negative net worth) and on collateralized debt with recourse. Thus, in our model, the value of the collateral works along with borrower quality to determine which borrowers have access to credit.

In terms of content, we connect with a variety of literatures. First, our model complements the growing literature analyzing optimal macroprudential regulation. This literature often models housing markets using a DSGE approach based on Iacoviello (2005), where house prices are endogenous, there are pecuniary externalities, and there is only one borrower; that is, the models focus only on the intensive margin of credit. Korinek and Mendoza (2014) is a recent survey of the literature on pecuniary externalities, and Lambertini, Mendicino, and Punzi (2013) or Rubio and Carrasco-Gallego (2014) are recent studies of optimal LTV regulation.[1] We highlight a channel absent in those papers: the extensive margin, and how the welfare benefits associated with homeownership push optimal regulation to consider affordability problems, thereby relaxing LTV constraints in periods of high price-to-rent ratios to allow more households to obtain a mortgage.

Second, we complement quantitative models of mortgage markets. For example, Campbell and Cocco (2015) solve a model of exogenous house prices with a household who borrows at endogenously determined mortgage rates and can default. Corbae and Quintin (2015) study a model of heterogenous agents with exogenous house prices. They show that exogenous changes in approval standards increased the number of high-leverage loans prior to the crisis, and this can explain over 60% of the rise in foreclosure rates. Chatterjee and Eyigungor (2015) endogenize house prices in a model of heterogenous agents and long-term debt, studying three shocks that can account for the dynamics of house prices and foreclosures. Gete and Zecchetto (2015) study quantitatively how loan guarantees affect credit supply and demand in a model with endogenous house prices and rents. Our theoretical analysis provides new insights that complement this quantitative literature.

Finally, we also relate to models of household debt and rationing. Zinman (2015) provides an excellent survey of this ample literature. Most of this literature is based on models with asymmetric information, like Harrison, Noordewier, and Yavas (2004), who propose a signaling model of LTV ratios and default risk.

The rest of the article is organized as follows. Section 1 presents the model. Sections 2, 3, and 4 characterize, respectively, credit supply, credit demand, equilibrium in mortgage markets and housing tenure choice. Section 5 studies optimal LTV regulation. Section 6 analyzes nonrecourse mortgages. Section 7 concludes. Appendix A contains the parameterization of the model. Appendix B has the proofs.

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1. Lin (2014) is a recent study of optimal LTV in a two-period model of corporate debt with only one bank.

1. THE SETUP

It is a two-period model with lenders and heterogeneous households. House prices and rents are exogenous. Real house price growth across periods is governed by an aggregate business cycle shock ϵ which follows a Pareto distribution on $[\underline{\epsilon}, \infty)$ with cumulative density function $F(\epsilon) = 1 - (\frac{\underline{\epsilon}}{\epsilon})^2$. Second-period house prices are

$$p'(\epsilon) = pB\epsilon, \tag{1}$$

where B is a parameter. We denote period-two variables with a prime, and the expected value of the aggregate shock as

$$\epsilon_0 \equiv \mathbb{E}[\epsilon] = 2\underline{\epsilon}.$$

1.1 Households

There is a continuum of households. They differ in income growth across periods (A), which is distributed according to the cumulative density function $G(A)$ with lower bound $\underline{A}$ and probability density function $g(A)$. For example, A is human capital. Households know their type. In period 1, all households have the same income (y). Second-period income for a household of type A is also subject to the business cycle shock:

$$y'(A, \epsilon) = yA\epsilon. \tag{2}$$

Households have quasilinear utility. In period 1, they choose to rent ($I = 0$) or buy ($I = 1$) housing (h if owned, h_r if rented). Homeowners have access to mortgage credit. There is a homeownership premium (k) to capture factors such as intrinsic preferences for homeownership, the favorable tax treatment of owning a house, or transaction costs in the rental market (Henderson and Ioannides 1983). In period 2, households enjoy nonhousing consumption (c' or c'_r), which serves as a numeraire. Household preferences are

$$u(c, I, h) = I[k \log(h) + \beta\mathbb{E}[c']] + (1 - I)[\log(h_r) + \beta\mathbb{E}[c'_r]], \tag{3}$$

where β is the discount factor.

When considering the decision whether to rent or own, households compare the utility from the two decisions. We define the value function of a renter of type A as $W(A)$,

$$W(A) = \max_{\{h_r, d_r, c'_r\} \geq 0} \log(h_r) + \beta\mathbb{E}[c'_r] \tag{4}$$

$$s.t.$$

$$rh_r + d_r = y, \tag{5}$$

$$c'_r = y' + R^D d_r, \tag{6}$$

where d_r are deposits which return gross interest R^D, and r is the rental price of housing. Renters do not have access to mortgage credit.

As a benchmark we analyze recourse mortgages. That is, in case of a borrower's default, the lender can go after the borrower's house and other assets, or she can sue to have borrower's wages garnished. In Section 6 we analyze nonrecourse mortgages. With a nonrecourse mortgage, the lender can only seize the house in the event of default.

We denote by m the mortgage borrowings, and by $R(A, m)$ the mortgage rate, which, as we will discuss below, it is a function of the borrower's type and the loan size. There is an LTV limit imposed by the government, Θ. There is also an endogenous credit limit imposed by lenders which we will discuss in Section 2. Our approach to the household's problem follows Eaton and Gersovitz (1981) in that households internalize the credit surface $R(\cdot)$ and the regulatory cap Θ but do not internalize the lender's credit ceiling, $\bar{\theta}(A)$. Otherwise no loan application would be rejected because, by construction, households would not apply for LTVs larger than what lenders would grant.

The value function of an owner with income growth A is

$$U(A) = \max_{\{h,m,d,c'\} \geq 0} k \log (h) + \beta \mathbb{E}[c'] \tag{7}$$

$$s.t.$$

$$ph + d = y + m, \tag{8}$$

$$\frac{m}{ph} \leq \Theta, \tag{9}$$

$$c' = \max\{0, y' + R^D d + p'h - mR(A, m)\}. \tag{10}$$

The max operator in (10) captures how a household who cannot cover her mortgage debt will default and consume nothing.[2,3]

Whether households default depends on the business cycle. For a household of type A, borrowing m, at rate R, and with collateral h in a recourse mortgage, we denote by $\epsilon^*(A, R, m, h)$ the threshold for the aggregate shock such that the household defaults

2. Alternatively, we could assume that households who default consume some minimum consumption level provided by a government's transfer. The results would be very similar.

3. Gerardi et al. (2013) provide evidence that strategic default during the 2007–9 recession was relatively rare.

for any business cycle below the threshold. The threshold $\epsilon^*(A, R, m, h)$ is implicitly defined as

$$y'\left(A, \epsilon^*\right) + p'\left(\epsilon^*\right)h = mR\left(A, m\right). \tag{11}$$

Or, using (1) and (2),

$$\epsilon^*\left(A, R, m, h\right) = \frac{mR\left(A, m\right)}{phB + yA}, \tag{12}$$

where we are already using the result that households with recourse mortgage debt do not hold deposits if they face mortgage rates higher than deposit rates ($R(A, m) > R^D$). This will be the case as long as the household has some positive probability of default ($\epsilon^*(.) > \underline{\epsilon}$). We show both results in Lemma 1.[4]

We can define the difference between the owner and renter's utility for a household of type A as

$$F(A) = U(A) - W(A). \tag{13}$$

The household will own if $F(A) > 0$, provided that the household can obtain credit from the lender, as we discuss below.

1.2 Lenders

There is a continuum of identical, risk-neutral lenders that compete loan by loan and can see the borrower's type. Lenders only originate mortgages that in expectation allow them to cover their cost of funds R^D. When making their origination decisions, lenders understand that households may default. The foreclosure technology is inefficient so that the lender only recovers a fraction $\gamma < 1$ of the house value. The recovery rate γ on the defaulter's assets captures how the foreclosed property may require significant repairs, or how the lender might suffer lawsuits or future regulatory burdens because of default.[5]

When the deadweight loss of default is high ($\gamma \leq 0.5$), the revenue of the lender is strictly decreasing in the loan interest rate as long as there is positive probability of the borrower's default. In that case, decreasing default probability by reducing the mortgage rate increases the revenue to the lender. Thus, for $\gamma \leq 0.5$, lenders will lower the mortgage interest rate until the borrower's default probability is zero, and they will only extend mortgages with no default.[6] Because we want to analyze equilibria with default, we focus on $\gamma > 0.5$.

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4. In our framework, because the support of the shock ϵ is bounded below, it is possible for a household to have zero default probability, and thus mortgages would not command a spread over lender's cost of funds. Bounded support is a feature that helps us solve the entire model in closed form.

5. Examples of the latter are the fines on Bank of America and Citibank in the summer of 2014.

6. The proof of Proposition 1 discusses this result.

Lenders' expected profits from lending m to a borrower of type A with collateral h are

$$\mathbb{E}[\Pi(h, m, A)] = \int_{\underline{\epsilon}}^{\max\{\underline{\epsilon}, \epsilon^*(.)\}} \gamma[hp'(\epsilon) + y'(A, \epsilon)]dF(\epsilon)$$

$$+ \int_{\max\{\underline{\epsilon}, \epsilon^*(.)\}}^{\infty} mR(A, m)\, dF(\epsilon) - R^D m. \tag{14}$$

The first term of the right-hand side is the expected recovered value of a defaulter's assets, because the household defaults for business cycles worse than $\epsilon^*(A, R, m, h)$. The second term is the loan's expected payout if the borrower repays. The last term is the cost of funds for the lender. Perfect competition among lenders implies that lenders' expected profits on borrower A should be zero:

$$\mathbb{E}[\Pi(h, m, A)] = 0. \tag{15}$$

2. CREDIT SUPPLY

Mortgage supply reacts via changes in mortgage rates and credit ceilings. Proposition 1 characterizes the key elements in credit supply. There is a maximum credit limit $\bar{m}(h, A)$ that lenders would provide to a borrower of type A posting collateral h. Above the credit limit there is no mortgage rate at which lenders would extend credit as default is too likely. Below the credit limit, lenders propose an interest rate function $R(A, \theta, \lambda)$ that is a function of the borrower's expected future income (A), of the LTV (θ), and of the loan-to-current-income ratio (LTI), which we denote $\lambda = \frac{m}{y}$. The credit limit can also be expressed as the maximum LTV at which the lender is willing to extend credit. We denote this endogenous LTV ceiling as $\bar{\theta}(A, \lambda)$.

PROPOSITION 1. *There exists a maximum mortgage size that a borrower of type A with collateral h can receive:*

$$\bar{m}(h, A) = \frac{\epsilon_0 \gamma (phB + yA)}{R^D}. \tag{16}$$

Likewise, for a borrower of type A with LTI ratio λ, there is a maximum LTV ratio

$$\bar{\theta}(A, \lambda) = \frac{\epsilon_0 \gamma \lambda B}{\lambda R^D - \epsilon_0 \gamma A} \tag{17}$$

above which the lender would not lend.

For all $m \in (0, \bar{m}(h, A))$ there is a mortgage rate satisfying the lender's zero profit condition (15). In particular,

$$R(A, \theta, \lambda) = \begin{cases} \dfrac{(2\gamma - 1)(\lambda B + \theta A)^2 \underline{\epsilon}^2}{\lambda \theta \left(\epsilon_0 \gamma (\lambda B + \theta A) - R^D \lambda \theta\right)} & \text{if } \max\left\{\underline{\epsilon}, \epsilon^*(.)\right\} = \epsilon^*(.) \\ \\ R^D & \text{if } \max\left\{\underline{\epsilon}, \epsilon^*(.)\right\} = \underline{\epsilon} \end{cases}. \quad (18)$$

Moreover, the lender's expected revenue

$$\Omega(h, A, m, R) = \int_{\underline{\epsilon}}^{\max\{\underline{\epsilon}, \epsilon^*(.)\}} \gamma [hp'(\epsilon) + y'(A, \epsilon)] dF(\epsilon)$$

$$+ \int_{\max\{\underline{\epsilon}, \epsilon^*(.)\}}^{\infty} Rm \, dF(\epsilon)$$

is increasing, concave, and bounded above in mortgage rates R.

2.1 The Credit Ceiling

The credit ceiling comes from the fact that lender's expected revenue $\Omega(.)$ is bounded above in mortgage rates $R(.)$. The credit ceiling $\bar{m}(h, A)$ is defined by

$$\Omega(h, A, \bar{m}(h, A), R) = R^D \bar{m}. \quad (19)$$

Lending above $\bar{m}(h, A)$ makes default too likely, and lenders cannot balance cost with expected revenue from the borrower. In that case, lenders ration credit. Proposition 2 summarizes the main properties of the credit ceiling and Figure 1 displays them. Figure 1 is based on the parameter values in Table 1, and the details of the calibration are presented in Appendix A.

PROPOSITION 2. *The LTV ceiling, $\bar{\theta}(A, \lambda)$, satisfies the following properties:*

$$\frac{\partial \bar{\theta}}{\partial A} > 0, \ \frac{\partial \bar{\theta}}{\partial B} > 0, \ \frac{\partial \bar{\theta}}{\partial R^D} < 0, \ \frac{\partial \bar{\theta}}{\partial \gamma} > 0, \ \frac{\partial \bar{\theta}}{\partial \lambda} < 0,$$

with the following interactions:

$$\frac{\partial^2 \bar{\theta}}{\partial \gamma \partial R^D} < 0, \ \frac{\partial^2 \bar{\theta}}{\partial B \partial A} > 0.$$

Lenders accept higher leverage ratios when borrowers have better income growth (higher A), or higher expected house price growth (higher B). Borrowers' life-time income depends on y and A; both raise the credit ceiling in (16).

A lower cost of funds for lenders, which is directly related to monetary policy, allows lenders to extend credit to riskier borrowers (higher LTV). Thus, there is a direct link between monetary policy and leverage ratios in mortgage markets. This
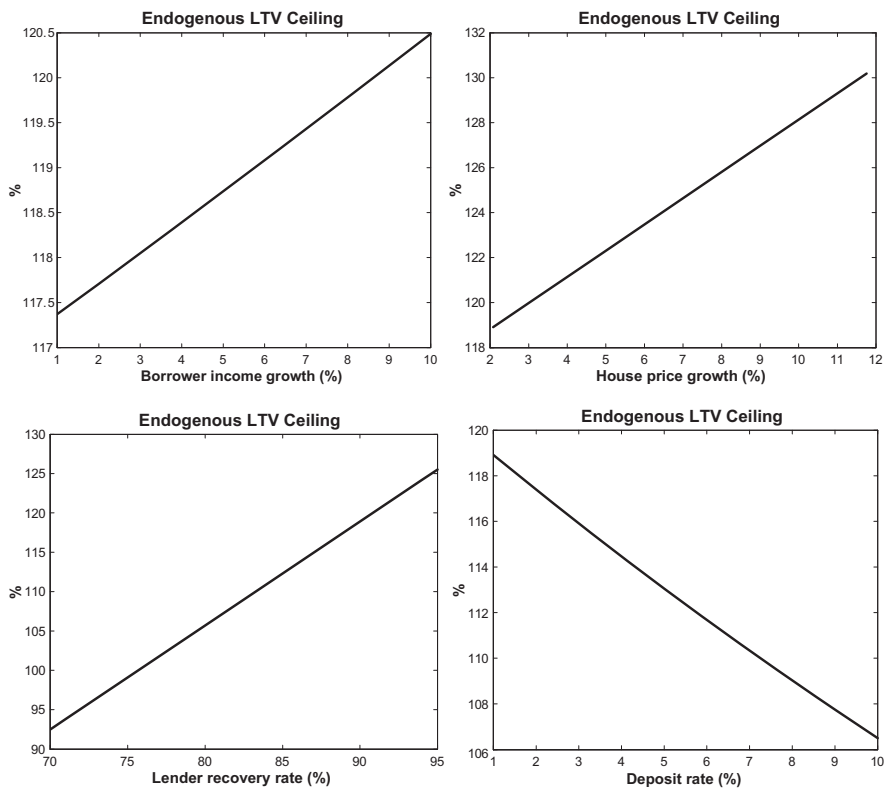
Fig. 1. Comparative Statics on the Loan-to-Value Ceiling.

Note: This figure plots the loan-to-value ceiling $\bar{\theta}(A, \lambda)$ characterized in Proposition 1 as a function of borrower's income growth, house price growth, lender's cost of funds, and recovery rates.

TABLE 1

PARAMETERS

$R^D = 1.01$	$B = 1.16$	$\beta = 0.99$
$k = 1.04$	$\gamma = 0.9$	$\theta = 0.8$
$p = 1.46$	$y = 0.91$	$r = \frac{p}{10.5}$
$\underline{\epsilon} = 0.44$	$\alpha = 1.1$	$\underline{A} = 0.49$

result suggests that the period of low interest rates in the run-up to the recent financial crisis may have been related to the high LTV ratios that lenders accepted over that period.

Reductions in the recovery rate (γ) can be interpreted as increases in the penalties the lenders face in case of loan default. For example, during the last several years, lenders have had to pay fines for nonperforming loans that they granted. As $\frac{\partial \bar{\theta}}{\partial \gamma} > 0$

shows, lenders translate those fines into tighter lending conditions. Interestingly, the pass-through from recovery rates to credit conditions is stronger when monetary policy is relatively lax, $\frac{\partial^2 \bar{\theta}}{\partial \gamma \partial R^D} < 0$. Intuitively, loose monetary policy makes it easier for lenders to break even on average, so that a reduction in default costs makes them even more willing to extend credit.

The result $\frac{\partial \bar{\theta}}{\partial \lambda} < 0$ says that households with higher LTI ratios have more difficulty accessing credit. The property $\frac{\partial^2 \bar{\theta}}{\partial B \partial A} > 0$ says that positive house price growth benefits the leverage capacity of high-income-growth households more than of low-growth households. Gerlach-Kristen and Lyons (2014) provide empirical confirmation that the impact of house prices and income shocks on credit depends on LTV and LTI ratios.

2.2 The Credit Supply Surface and Mortgage Spreads

Equation (18) characterizes the credit surface, that is, the interest rate offered by lenders as a function of the LTV, LTI, and the income growth of the borrower. Geanakoplos (2014) provides evidence that the credit surface in mortgage markets is a very important object in the economy as a whole. The next proposition discusses the most important and interesting properties of the credit supply surface.

PROPOSITION 3. *As long as there is some positive default probability* (max $\left\{ \underline{\epsilon}, \epsilon^*(.) \right\} = \epsilon^*(.))$, *then* $\frac{\partial R}{\partial A} < 0$, $\frac{\partial R}{\partial \lambda} > 0$ *and* $\frac{\partial R}{\partial \theta} > 0$. *Moreover,*

$$\frac{\partial R}{\partial \lambda} = \left(\frac{\theta}{\lambda} \right)^2 \frac{A}{B} \frac{\partial R}{\partial \theta}, \tag{20}$$

$$\frac{\partial R}{\partial \lambda} = -\frac{A}{\lambda} \frac{\partial R}{\partial A}. \tag{21}$$

$$\frac{\partial^2 R}{\partial A \partial R^D} < 0, \quad \frac{\partial^2 R}{\partial \lambda \partial R^D} > 0, \quad \frac{\partial^2 R}{\partial \theta \partial R^D} > 0. \tag{22}$$

$$\frac{\partial^2 R}{\partial A \partial B} > 0, \quad \frac{\partial^2 R}{\partial \lambda \partial B} < 0, \quad \frac{\partial^2 R}{\partial \theta \partial B} < 0. \tag{23}$$

$$\frac{\partial^2 R}{\partial A \partial \gamma} > 0, \frac{\partial^2 R}{\partial \lambda \partial \gamma} < 0, \frac{\partial^2 R}{\partial \theta \partial \gamma} < 0 \quad \text{if } \gamma > \frac{3 R^D \lambda \theta \epsilon_0 (\lambda B + \theta A) - 4 \left(R^D \lambda \theta \right)^2}{\epsilon_0^2 (\lambda B + \theta A)^2}. \tag{24}$$

Lenders price the higher default probability induced by a higher LTV, higher LTI, or lower income growth. Properties (20) and (21) describe what the credit surface looks like. The ratio $(\frac{\theta}{\lambda})^2 \frac{A}{B}$ controls whether mortgage rates react more to changes in LTI or LTV. The ratio $\frac{A}{\lambda}$ controls whether rates react more to income growth or to LTI.

Property (22) shows how monetary policy affects the credit supply surface. It lowers lenders' cost of funds, thus increasing their ability to break-even with lower mortgage rates while supporting higher LTV, higher LTI, or lower income growth. In that regard, a loose monetary policy cushions decreases in income and house prices. Ehrmann and Ziegelmeyer (2014) provide supporting evidence that the transmission of monetary policy to mortgage rates is especially beneficial for high LTI households $(\frac{\partial^2 R}{\partial \lambda \partial R^D} > 0)$.

Property (23) shows a similar result for high expected house price growth (B). The property $\frac{\partial^2 R}{\partial B \partial A} > 0$ means high-income-growth households will enjoy larger interest rate reductions when house prices are expected to grow. Intuitively, lenders are less willing to lower rates for low-income-growth households even during expected house price booms, because these households are inherently riskier. Also, $\frac{\partial^2 R}{\partial B \partial \lambda} < 0$ has an interesting empirical implication, as it shows how, keeping the LTV constant, households with high LTI ratios will benefit more from increases in house prices. This is because those households have larger positions in real estate.

Property (24) shows that if the recovery rate is sufficiently high, then a reduction in default costs cushions low-income-growth borrowers. That is, for low default costs (high γ), the slope of R with respect to A is less steep. Likewise, the slope of R with respect to λ or θ becomes less steep with low default costs.

The next proposition characterizes the mortgage spread, defined as $\Delta^R(A, \theta, \lambda) = R(A, \theta, \lambda) - R^D$.

PROPOSITION 4. *For a borrower of type A with LTV θ and LTI λ, the mortgage spread, $\Delta^R(A, \theta, \lambda) = R(A, \theta, \lambda) - R^D$ is increasing in lenders' cost of funds. That is,*

$$\frac{\partial \Delta^R}{\partial R^D} > 0.$$

The reduction in default probability associated with a lower cost of funds (R^D) is more than one-to-one, and, as a result, loose monetary policy (R^D falls) lowers mortgage spreads; that is, Δ^R falls.

2.3 Supply-Driven Extensive Margin of Credit

The credit ceiling allows us to define a *lender-driven extensive margin of credit*. That is, we can define the minimum income growth that a borrower must have for lenders to issue her a mortgage at a given LTI and LTV. We denote this borrower type

as $A_L(\lambda, \theta)$. Any borrower with income growth below this threshold cannot secure a mortgage with that LTI and LTV. Using $A_L(\lambda, \theta)$ to integrate over households' types gives the share of households who would be rejected mortgage credit, which we denote as $\Psi^L(\lambda, \theta)$.

DEFINITION 1. *The borrower with the minimum income growth such that lenders would finance her mortgage with a given LTI and LTV is*

$$A_L(\lambda, \theta) = \lambda \left[\frac{R^D}{\epsilon_0 \gamma} - \frac{1}{\theta} B \right]. \tag{25}$$

If $A > A_L(\lambda, \theta)$, the household can obtain a mortgage from the lender, and otherwise she cannot. Thus, for given LTV and LTI ratios, we can define the fraction of lender-rationed households as

$$\Psi^L(\lambda, \theta) = \int_{\underline{A}}^{\max\{A_L(\lambda,\theta),\underline{A}\}} g(A), \tag{26}$$

*with $\frac{\partial \Psi^L(\lambda,\theta)}{\partial B} < 0$, $\frac{\partial \Psi^L(\lambda,\theta)}{\partial \epsilon_0} < 0$, $\frac{\partial \Psi^L(\lambda,\theta)}{\partial \theta} > 0$, $\frac{\partial \Psi^L(\lambda,\theta)}{\partial R^D} > 0$, and $\frac{\partial \Psi^L(\lambda,\theta)}{\partial \lambda} > 0$ if $[\frac{R^D}{\epsilon_0 \gamma} - \frac{1}{\theta} B] > 0$. Moreover, $\frac{\partial \Psi^L(\lambda,\theta)}{\partial \lambda \partial \theta} > 0$. The fraction $[1 - \Psi^L(\lambda, \theta)]$ is the **lender-driven extensive margin of credit**: that is, how many borrowers qualify for a mortgage with an LTV equal to θ and an LTI of λ.*

The condition $[\frac{R^D}{\epsilon_0 \gamma} - \frac{1}{\theta} B]$ in (25) relates the difference between financing costs ($R^D \theta$) to expected home price growth multiplied by the recovery rate ($B\epsilon_0 \gamma$). If this difference is high, then conditions are not good for lenders, and so rationing happens at a higher rate. The opposite occurs if it is low.

The following effects lower $A_L(\lambda, \theta)$, and thus induces lenders to extend credit to riskier households: a reduction in R^D, higher expected house price growth (B), or a better expected business cycle (ϵ_0). This result is supported by evidence from Jimenez et al. (2014) that expansive monetary policy alters the risk composition of the supply of credit and, in particular, banks' risk taking. The lender's cutoff A_L does not depend on the current price p, but rather on the expected price growth B. The idea is that lenders care about future prices of collateral, which are better captured by B than p.

Changes in LTV have larger effects the larger the LTI ($\frac{\partial \Psi^L(\lambda,\theta)}{\partial \lambda \partial \theta} > 0$). Regulations imposing lower LTV ratios constrict credit per capita, but they also expand the range of individuals who qualify for credit ($\frac{\partial \Psi^L(\lambda,\theta)}{\partial \theta} > 0$). That is, they constrict the intensive margin of credit, but expand the extensive margin of credit.

3. CREDIT DEMAND

Households of type A compare the utility from being a renter with the utility of homeownership. If the utility of homeownership is larger, the household applies for a mortgage. The next lemma characterizes the solution to the household's problem.[7]

LEMMA 1. *Renter's choices ($I = 0$) are*

$$h_r = \frac{y}{r}, \tag{27}$$

$$W(A) = \log\left(\frac{y}{r}\right) + \beta y \epsilon_0 A. \tag{28}$$

Homeowner's choices ($I = 1$) are

$$h(\theta^*(A)) = \frac{y}{(1 - \theta^*(A))p}, \tag{29}$$

$$m(\theta^*(A)) = \frac{\theta^*(A)y}{1 - \theta^*(A)}, \tag{30}$$

$$U(A, \theta^*(A)) = k \log\left(\frac{y}{p(1 - \theta^*(A))}\right)$$
$$+ y\beta\underline{\epsilon}^2 \left[\frac{(B + [1 - \theta^*(A)]A)^2}{\theta^*(A)[1 - \theta^*(A)]R(A, \theta^*(A)))}\right], \tag{31}$$

$$\theta^*(A) = \min\left\{\hat{\theta}(A), \Theta\right\}, \tag{32}$$

where $\hat{\theta}(A)$ solves

$$\frac{k}{1 - \hat{\theta}(A)} = \frac{y\beta\underline{\epsilon}^2(B + A[1 - \hat{\theta}(A)])}{\hat{\theta}(A)[1 - \hat{\theta}(A)]R(A, \hat{\theta}(A))}$$
$$\times \left[2A + \left(\frac{\partial R}{\partial \theta} - [2\hat{\theta}(A) - 1]\right)\frac{(B + A[1 - \hat{\theta}(A)])}{\hat{\theta}(A)(1 - \hat{\theta}(A))}\right]. \tag{33}$$

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7. Without loss of generality we assume that first-period income is sufficiently low, and, in particular, $y \leq \frac{1}{\beta R^D}$. This condition rules out the case of renters saving in deposits, as we prove in the Appendix.

The difference between an owner's (U) and renter's utility (W) for a household of type A is

$$F(A, \theta^*) = U(A, \theta^*) - W(A) = \log\left(\frac{r y^{k-1}}{p^k (1-\theta^*)^k}\right)$$

$$+ y\beta\epsilon_0 \left[\frac{\epsilon(B + A[1-\theta^*])^2}{2\theta^*[1-\theta^*]R} - A\right], \tag{34}$$

where θ^* denotes $\theta^*(A)$, and R denotes $R(A, \theta^*)$. The household would prefer to own if $F(A, \theta^*) > 0$.

Expression (33), which is the first-order condition of $U(A, \theta)$ with respect to θ, characterizes the household's target LTV, $\hat{\theta}(A)$. Households demand a higher LTV when house prices are expected to grow (high B), as this raises the upside of a real estate gamble. A lower homeownership premium (low k) lowers the LTV. If the slope of lenders' mortgage rate curve, as a function of loan size, gets steeper, then households' target LTV is smaller. Proposition 5 will derive these properties using the closed-form, equilibrium expression for $\hat{\theta}(A)$.

Our approach in (32) to the borrower's LTV choice is similar to Eaton and Gersovitz (1981). Households internalize the credit surface $R(\cdot)$ and the regulatory cap Θ, but they do not internalize the lender's credit ceiling. If households internalized this ceiling, then (32) becomes $\min\left\{\hat{\theta}(A), \Theta, \bar{\theta}(A, \lambda)\right\}$. Thus, no mortgage application would be rejected because, by construction, households would not apply for an LTV larger than what lenders would grant. Our expression (32) allows for the existence of mortgage rejections and a lender-driven extensive margin of credit.

Interestingly, reducing a binding down-payment constraint has the largest effects on homeownership demand from households with less current income. That is, $\frac{\partial F(A, \Theta)}{\partial y \partial \Theta} \leq 0$.[8] This is an empirical fact documented by Landvoigt, Piazzesi, and Schneider (2015), and by Fuster and Zafar (2014).

3.1 The Borrower-Driven Extensive Margin of Credit

A credit demand, or borrower-driven, extensive margin of credit is defined because some household prefer to rent than to apply for a mortgage. Formally, we define this extensive margin in the following proposition.[9]

DEFINITION 2. *For a given $\theta^* = \min\left\{\hat{\theta}, \Theta\right\}$, where $\hat{\theta}$ is implicit in (33) and Θ is the regulatory LTV cap, define $A_B(\theta^*)$ such that*

$$F(A_B(\theta^*), \theta^*) = 0. \tag{35}$$

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8. The proof is with the proof of Lemma 1 and requires $\hat{\theta}(A) \leq 1$, which is a condition guaranteed if expected house price growth is below a certain threshold, $B < \frac{R^D}{\epsilon_0 \gamma}$.

9. We characterize this definition in the Appendix.

Then, if $A > A_B(\theta^*)$, the household would like to own, and otherwise she rents. Therefore, we can consider

$$\Psi^B\left(\theta^*\right) = \int_{\underline{A}}^{\max\left\{A_B(\theta^*),\underline{A}\right\}} g\left(A\right) dA, \tag{36}$$

*and define the **borrower-driven extensive margin of credit** as the share of households who would choose to seek out a mortgage with an LTV of θ^*, that is, $[1 - \Psi^B(\theta^*)]$.*

If the LTV is too low from the household's perspective, such as when LTV regulation is tight, then households would prefer to rent than to pay a large down payment or seek out a smaller, lower-quality house. In that case, raising the LTV increases the number of households willing to be homeowners; that is, it increases the borrower-driven extensive margin of credit. In addition, higher mortgage rates discourage households from homeownership, holding the LTV constant.

4. EQUILIBRIUM IN MORTGAGE MARKETS AND HOUSING TENURE CHOICE

4.1 Mortgage Markets

For exogenous rental rates (r), house prices (p) and lenders' cost of funds (R^D), we define an equilibrium in mortgage markets as a set of functions $\{R^*(A), m^*(A), \theta^*(A)\}$ characterized in the next proposition.

PROPOSITION 5. *Each household solves her problem (3)–(10) such that she applies for a mortgage if $A > A_B(\theta^*)$. The lender's zero-profit condition (15) holds for each mortgage applicant. The share of mortgage applications rejected is*

$$\Gamma = \frac{\int_{A_B(\theta^*)}^{A_L(\theta^*)} g\left(A\right) dA}{\left[1 - \Psi^B\left(\theta^*\right)\right]}, \tag{37}$$

with

$$\theta^* = \min\{\hat{\theta}, \Theta\}, \tag{38}$$

$$\hat{\theta} = 1 - \frac{y\beta\left(R^D - \epsilon_0\gamma B\right)}{k(2\gamma - 1)}, \tag{39}$$

$$m^* = \frac{\theta^* y}{1 - \theta^*}, \tag{40}$$

$$\lambda^* = \frac{m^*}{y}. \tag{41}$$

The mortgage rate for financed households of type A is:

$$R^*(A) = \begin{cases} \dfrac{(2\gamma - 1)(B + A(1 - \theta^*))^2 \underline{\epsilon}^2}{\theta^*(\epsilon_0 \gamma(B + A(1 - \theta^*)) - R^D \theta^*)} & \text{if } \max\{\underline{\epsilon}, \epsilon^*(.)\} = \epsilon^*(.) \\[4mm] R^D & \text{if } \max\{\underline{\epsilon}, \epsilon^*(.)\} = \underline{\epsilon} \end{cases}. \quad (42)$$

The target LTV $\hat{\theta}$ satisfies $\frac{\partial \hat{\theta}}{\partial \epsilon_0} > 0$, $\frac{\partial \hat{\theta}}{\partial \gamma} > 0$, $\frac{\partial \hat{\theta}}{\partial R^D} < 0$, $\frac{\partial \hat{\theta}}{\partial y} < 0$, $\frac{\partial \hat{\theta}}{\partial k} > 0$ and $\frac{\partial^2 \hat{\theta}}{\partial \epsilon_0 \partial \gamma} < 0$.

Lastly, when $\hat{\theta} > \Theta$, the rejection rate Γ satisfies $\frac{\partial \Gamma}{\partial \Theta} > 0$. It satisfies $\frac{\partial \Gamma}{\partial R^D} > 0$ if and only if $\frac{g(A_L)}{g(A_B)} > (1 - \Gamma)\frac{\gamma}{1 - \gamma}$.

For households at their desired LTV ($\theta^* = \hat{\theta}$), this LTV is procyclical ($\frac{\partial \hat{\theta}}{\partial \epsilon_0} > 0$) because households can sustain higher leverage when house prices and income are expected to grow. Similarly, households with higher initial incomes are better-off with a lower LTV ($\frac{\partial \hat{\theta}}{\partial y} < 0$). Higher default costs correspond to a lower LTV, because the costs of default are directly transferred to the household through the equilibrium interest rate ($\frac{\partial \hat{\theta}}{\partial \gamma} > 0$). Thus, reforms which improve foreclosure processes should allow mortgage markets to sustain higher LTV ratios. The procyclicality of the optimal LTV depends on the level of default costs ($\frac{\partial^2 \hat{\theta}}{\partial \epsilon_0 \partial \gamma} < 0$). That is, if foreclosures impose significant costs on lenders, then, during a downward business cycle, the LTV will fall more quickly than when the costs of default are lower.

On the other hand, if households are bound by the regulatory LTV cap ($\theta^* = \Theta$), then raising the LTV cap will increase the number of rejected mortgage applications ($\frac{\partial \Gamma}{\partial \Theta} > 0$). This happens because reducing a binding down payment threshold brings lower-quality borrowers into the application pool (A_B falls).

When a lender's borrowing cost R^D increases, it becomes more stringent and rejects $g(A_L)$ more applicants; at the same time, households understand that this increase in R^D will be passed on to them in the form of a higher mortgage rate, which prompts $g(A_B)$ fewer households to apply for a mortgage. Thus, if $\frac{g(A_L)}{g(A_B)}$ exceeds a threshold $(1 - \Gamma)\frac{\gamma}{1 - \gamma}$, then the mortgage rejection rate increases ($\frac{\partial \Gamma}{\partial \Theta} > 0$), and otherwise it will fall ($\frac{\partial \Gamma}{\partial \Theta} \leq 0$).

The next proposition pins down the equilibrium thresholds for the lender and borrower-driven extensive margins of credit as a function of the equilibrium LTV θ^* characterized above.

PROPOSITION 6. *The thresholds for the lender and borrower-driven extensive margins of credit are*

$$A_B(\theta^*) = \frac{(1 - \theta^*)(2\gamma - 1)\log\left[\frac{p^k(1-\theta^*)^k}{ry^{k-1}}\right] - y\beta\left[\epsilon_0 \gamma B - R^D \theta^*\right]}{(1 - \theta^*)(1 - \gamma)\epsilon_0 y\beta}, \quad (43)$$

$$A_L(\theta^*) = \frac{\theta^*}{(1 - \theta^*)} \left[\frac{R^D}{\epsilon_0 \gamma} - \frac{B}{\theta^*} \right]. \tag{44}$$

The extensive margin of credit is lender-driven $(A_B \leq A_L)$ if and only if

$$\frac{p}{r} \leq \left[\frac{\mu^{1-k}}{(1-\theta^*)^k} \right] \exp \left\{ \frac{y\beta \left[\epsilon_0 \gamma B - R^D \theta^* \right]}{\gamma(1-\theta^*)} \right\}, \tag{45}$$

where $\mu = \frac{p}{y}$ represents the price-to-current-income ratio.

The inequality (45) characterizes when there is credit rationing, that is, when households willing to buy are rejected credit. First, the price-to-rent ratio cannot be sufficiently high or potential homebuyers would not apply for credit and rent instead. Thus, for high price-to-rent ratios the extensive margin is driven by borrowers, not by lenders. Second, lenders ration credit more when house prices are expected to grow less, in bad business cycles, with lower recovery rates, and with a higher cost of funds. However, the two extensive margins are strongly connected. Households care about the same variables as lenders, because those variables alter their mortgage rates. Lower B, ϵ_0, and γ, and higher R^D have a negative impact on borrowers, and in such a way that makes the borrower-driven margin more likely to dominate. To further explore this issue, using (43) and (44), we can write one threshold as a function of the other

$$A_B = \frac{2\gamma - 1}{1 - \gamma} \frac{1}{y\beta\epsilon_0} \log \left[\frac{p}{r} \mu^{k-1} (1 - \theta^*)^k \right] + \frac{\gamma}{1 - \gamma} A_L. \tag{46}$$

Then we can see that, as long as $\gamma > 0.5$, the derivative of A_B with respect to B is always greater, in absolute terms, than the derivative of A_L with respect to B. This is because, $\frac{\partial A_B}{\partial B} = \frac{\gamma}{(1-\gamma)} \frac{\partial A_L}{\partial B} < \frac{\partial A_L}{\partial B} < 0$. One implication of this relationship is that an increase in home price growth can switch the dominant margin from borrower-driven to lender driven, but the converse is not possible. So, in periods of home price acceleration, it is increasingly likely that credit supply is the driver of the rent-own decision.

4.2 Housing Tenure Choice

Now we introduce a housing tenure choice rule, which will be important for our analysis of optimal LTV policy in Section 5. Suppose that a household decides to apply for a mortgage $(A > A_B(\theta^*))$. There are two cases: either (i) the household is granted a mortgage with LTV θ^* and becomes a homeowner $(A > A_L(\theta^*))$, or (ii) she is rejected by the lender (when $A \leq A_L(\theta^*)$), learns her credit ceiling $\bar{\theta}(A)$, and re-solves her problem (3)–(10), replacing (9) with her LTV ceiling $\bar{\theta}(A)$.[10] In

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10. We can express the lender's LTV ceiling as a function of only the borrower's type because, as shown in (40), the household's LTI is a one-to-one mapping with the LTV.

the second case, the household will accept mortgage credit if and only if borrowing at her credit ceiling is preferred to renting, that is, if $A > A_B(\bar{\theta}(A))$. Formally, the tenure choice rule can be defined as follows:

$$I = \begin{cases} 1 & if & A > A_R(\Theta) \\ 0 & if & A \leq A_R(\Theta) \end{cases}, \tag{47}$$

where $A_R(\Theta)$ is the rent threshold, defined by

$$A_R(\Theta) = A_B \left(\min \left\{ \hat{\theta}, \Theta, \bar{\theta}(A_R(\Theta)) \right\} \right). \tag{48}$$

Thus, the household's tenure choice features a cutoff rule, where households with productivity above the rent threshold $A_R(\Theta)$ defined in (48) own, and the remaining households rent.

Price-to-rent and tenure choice. Now we explore the role of the price-to-rent ratio in the choice of housing tenure. First, we can rearrange (43) to derive a user-cost formula akin to Poterba (1984):

$$\log \left(\frac{r}{p} \right) = \log(1 - \theta) - \beta \left[\epsilon_0 y \frac{\epsilon(B + (1-\theta)A_B)^2}{2R(\theta^*, A_B)\theta^*(1-\theta^*)} - \epsilon_0 y A_B \right]. \tag{49}$$

We have focused on the case where the homeownership premium k is unity, and, for tractability, shall do so for the rest of this section.[11] Expression (49) says that the marginal homeowner, A_B, must be indifferent between buying a house and renting, which is the essence of the user cost approach. In particular, note that, holding θ^* and $R(\cdot)$ fixed in (49), the rent-price ratio is decreasing in B. That is, the expected capital gain from house price growth reduces the user cost of housing, and thus pushes down the rental rate.

Following the previous result, let us suppose that the rent-price ratio is a decreasing function of expected house price growth. That is, $\frac{r}{p} = \frac{r}{p}(B)$, with $\frac{\partial (\frac{r}{p})}{\partial B} < 0$. This leads to the following proposition.

PROPOSITION 7. *The mortgage application threshold $A_B(\theta^*)$ satisfies $\frac{\partial A_B}{\partial B} < 0$ if and only if*

$$\left| \frac{\partial \left(\frac{r}{p} \right)}{\partial B} \right| < \frac{\gamma}{1 - \theta^*} \frac{y\beta\epsilon_0}{2\gamma - 1} \frac{r}{p}. \tag{50}$$

In words, the condition (50) says that an increase in expected house price growth will lead to an increase in mortgage applications only when the rent-price ratio does not react too steeply to these expectations. Otherwise, the cost of homeownership

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11. Our results are qualitatively unchanged if $k \neq 1$, but the exposition is clearer when $k = 1$. Also, for ease of notation, we have simply written $A_B(\theta^*)$ as A_B.

today ($\frac{p}{r}$) outweighs both the gains from more favorable credit conditions (lower R) and capital appreciation (higher B).

5. OPTIMAL LTV POLICY

LTV caps are among the most popular policies to regulate mortgage markets.[12] A major motivation for these caps are the negative externalities from larger default rates associated with high LTV loans. For example, these externalities are pecuniary externalities or lost government revenue from loan guarantees or bank bailouts. Neither households nor lenders internalize these default externalities, and thus there is a role for government regulation.

In the model analyzed so far, the market allocation is socially optimal. In this section, we introduce default externalities to analyze optimal LTV policy. We denote by Λ the reduced-form social cost per default. We will show that the optimal LTV policy reflects a trade-off between the benefits from reducing default and the gains from expanded access to homeownership.

First, we define $\rho(A, \theta)$ as the probability that homeowner A receiving an LTV of θ will default. We can express this probability as

$$\rho(A, \theta) = \int_{\underline{\epsilon}}^{\max\{\underline{\epsilon}, \epsilon^*(A)\}} dF(\epsilon) = 1 - \frac{1}{(2\gamma - 1)^2}\left[2\gamma - \frac{R^D\theta}{\underline{\epsilon}(B + A(1 - \theta))}\right]^2. \quad (51)$$

The default probability is increasing in θ, as a higher LTV makes default more likely, because borrowers are more exposed to harmful aggregate shocks. That is,

$$\frac{\partial\rho}{\partial\theta} = \frac{2}{(2\gamma - 1)^2}\left[2\gamma - \frac{R^D\theta}{\underline{\epsilon}(B + A(1 - \theta))}\right]\left[\frac{R^D(B + A)}{\underline{\epsilon}(B + A(1 - \theta))^2}\right] > 0, \quad (52)$$

provided $\theta < \bar{\theta}(A)$.[13]

The policymaker internalizes default externalities and can choose to impose an LTV cap, Θ. The policymaker follows a utilitarian criteria and maximizes the sum of household utilities, $\bar{u}(\Theta)$, net of default cost and subject to the lenders' zero profit condition:

$$V(\Theta) = \bar{u}(\Theta) - \Lambda \int_{\max\{\underline{A}, A_R(\Theta)\}}^{\infty} \rho(A, \min\{\hat{\theta}, \Theta, \bar{\theta}(A)\})g(A), \quad (53)$$

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12. According to Claessens, Ghosh, and Mihet (2014) at least 24 countries have LTV regulations. For example, Hong Kong has a maximum LTV ratio of 70% or 60%, depending on the value of the property, but mortgage loans with an LTV of up to 90% are available for homebuyers who meet certain eligibility criteria. Malaysia and Korea have a 70% maximum. In the U.S., GSE conforming mortgages and Qualified Mortgage loans are allowed LTV ratios of 80%, and FHA loans can go up to 96.5%.

13. If instead $\theta = \bar{\theta}(A)$, then, from Section 2, $2\gamma\underline{\epsilon}(B + A(1 - \theta)) = R^D\theta$, and so $\frac{\partial\rho}{\partial\theta} = 0$. Even so, $\frac{\partial\rho}{\partial\theta}$ is weakly increasing in θ.

with

$$\bar{u}(\Theta) = \int_{\underline{A}}^{\max\{\underline{A}, A_R(\Theta)\}} W(A)g(A)$$

$$+ \int_{\max\{\underline{A}, A_R(\Theta)\}}^{\infty} U(A, \min\{\hat{\theta}, \Theta, \bar{\theta}(A)\})g(A).$$

The policymaker takes into account that some households will be renters and others will be owners according to the housing tenure rule (47). The lenders' expected profits equation drops from (53) because it is zero. However, it will affect the optimal LTV through determining mortgage rates for homebuyers and through mortgage rejections, both of which are reflected in $U(\cdot)$. The following proposition characterizes the optimal cap.[14]

PROPOSITION 8. *The optimal LTV cap Θ^* maximizes V and satisfies*

$$0 = \int_{A_R}^{\infty} \frac{\partial U}{\partial \Theta} g(A) - \Lambda \left(-\rho(A_R, \min\{\hat{\theta}, \Theta^*, \bar{\theta}(A_R)\})) \frac{\partial A_R}{\partial \Theta} g(A_R) \right.$$

$$\left. + \int_{A_R}^{\infty} \frac{\partial \rho}{\partial \Theta} g(A) \right). \tag{54}$$

In particular, the LTV cap satisfies $\frac{\partial \Theta^*}{\partial \Lambda} < 0$. Additionally, $\frac{\partial \Theta^*}{\partial k} > 0$ when some households are credit constrained ($\Theta^* > \bar{\theta}(A_R)$).

The optimal LTV, as described by (54), reflects a trade-off between default externalities and allowing households access to homeownership. On the one side, when the costs of default are large, the LTV cap should be smaller to reduce default rates. That is, $\frac{\partial \Theta^*}{\partial \Lambda} < 0$. However, lowering the LTV cap reduces household utility by making it difficult for households to access homeownership, which delivers more utility per unit of housing services than renting, as captured by k. That is, $\frac{\partial U}{\partial \Theta} \geq 0$ and $\frac{\partial A_R}{\partial \Theta} \leq 0$, which reflects how households are more likely to prefer owning to renting when they have access to high LTV mortgages. When the benefits from being a homeowner increase (larger k), optimal policy recommends higher LTV caps to facilitate households' access to homeownership, so that $\frac{\partial \Theta^*}{\partial k} > 0$.[15]

Figure 2 displays graphically the trade-off between credit risk and home affordability. It plots the homeownership rate and the default probability (for a household

14. We focus on the more interesting case $\underline{A} \leq A_R(\Theta)$ where not all households are homeowners.

15. When $\Theta^* \leq \bar{\theta}(A_R)$, it is possible that with a higher k the optimal LTV cap should be lower. This is because higher k induces lower-quality borrowers to become homeowners and thus raises default. It may then be needed to lower the optimal LTV cap to discourage these lower-quality households from taking out a mortgage. Thus, when $\Theta^* \leq \bar{\theta}(A_R)$, the sign of $\frac{\partial^2 V}{\partial \Theta \partial k}$ would, in general, depend on parameter values. However, when some households are credit constrained ($\Theta^* > \bar{\theta}(A_R)$) then we can be sure the LTV cap is increasing in k as some of the riskier households would not qualify even if the cap increases.
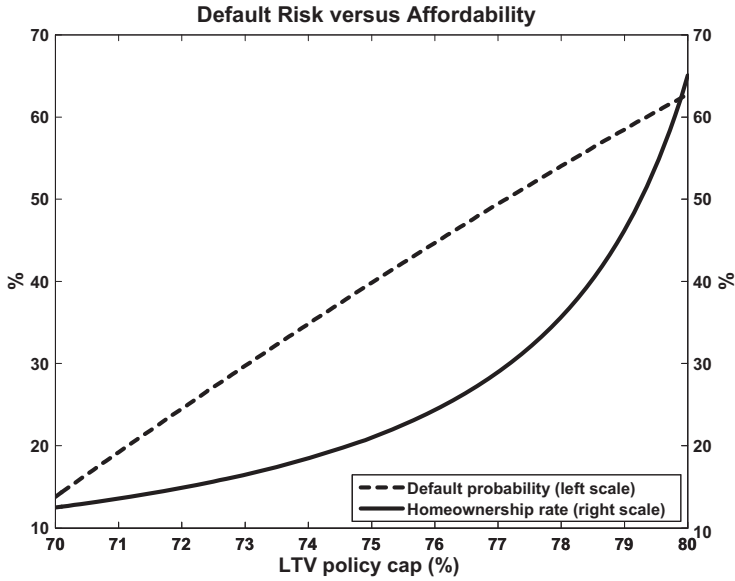
FIG. 2. Default Risk versus Home Affordability.

NOTE: This figure plots borrower's default risk (for a household with 5% income growth) and the homeownership ratio for different values of the binding LTV cap.

with 5% income growth) as a function of the LTV cap. A higher LTV cap increases default probability, but also allows households access to homeownership.

These remarks suggest that the optimal LTV cap has procyclical elements in the following sense. During expansions, price-to-rent ratios typically grow, and so households can purchase less housing per unit of leverage. Yet the existence of a homeownership premium means that it is socially optimal to accommodate the rising affordability barrier with higher leverage ratios. Of course, with greater leverage, the expected external cost of default also grows, and so the choice of a cap reflects a balance between the cost of default and the cost of unaffordability, as captured by the first-order condition in (54). Moreover, we can also infer from (54) that the optimal LTV cap will depend on lenders' cost of funds (R^D) and loan recovery rate (γ) through the rent-own cutoff (A_R) and through household decision making ($\frac{\partial U}{\partial \Theta}$), which highlights a link between monetary policy and financial regulation.

Before concluding, it is worth discussing that LTV regulation impact the intensive and extensive margins of credit in different fashions. For example, if a binding LTV cap is increased, households will take advantage of the additional leverage to either purchase larger, higher-quality houses (the intensive margin), or to switch their housing tenure and become homeowners (the extensive margin). However, the extensive margin effect will be muted when the lenders' extensive margin is dominant ($A_R = A_B(\bar{\theta}(A_R))$), because the regulatory LTV cap does not enter into the lender's

expected profits. In other words, the credit supply surface does not change with the regulatory cap Θ.

6. NONRECOURSE MORTGAGES

To this point, we have focused on recourse mortgages. Now we analyze nonrecourse mortgages; that is, in the case of default, the lender only has recourse to the borrower's house, hp'. We summarize the main new results in the following proposition.

PROPOSITION 9. *A household with a nonrecourse mortgage will default whenever the value of her house is lower than the mortgage balance to repay, that is, $hp' \leq Rm$. Lenders will reject applications with an LTV greater than*

$$\bar{\theta} = \frac{\epsilon_0 \gamma B}{R^D}, \tag{55}$$

and charge the same mortgage rate to all financed households with an LTV of θ,

$$R^* = \begin{cases} \dfrac{(2\gamma - 1)B^2 \underline{\epsilon}^2}{\theta(\epsilon_0 \gamma B - R^D \theta)} & \text{if } \max\left\{\underline{\epsilon}, \epsilon^*(.)\right\} = \epsilon^*(.) \\ R^D & \text{if } \max\left\{\underline{\epsilon}, \epsilon^*(.)\right\} = \underline{\epsilon} \end{cases}. \tag{56}$$

At a given LTV, θ, the nonrecourse mortgage rate is higher than the recourse mortgage rate for every A. Additionally, at a given θ, the default probability of any household A is greater under a nonrecourse mortgage than under a recourse mortgage.

Lastly, if $k \geq 1$, so that households place a utility premium on owner-occupied shelter, then there exists an initial income threshold y_R such that households will own if $y > y_R$, and otherwise they will rent. The threshold y_R solves $F^*(y_R) = 0$, where F^* is the difference between equilibrium utility of owners and renters.

Proposition 9 shows that lenders and borrowers behave differently with nonrecourse mortgages than with recourse mortgages. Because a borrower's type A neither reduces default probability nor affects the value of assets recovered in the event of default, this variable plays no role from a lender's perspective. Thus, neither mortgage rates nor credit ceilings depend on a household's income.[16] In nonrecourse jurisdictions, variables such as the FICO score should not have as significant an impact on access to credit or mortgage costs. In this light, nonrecourse mortgages dampen credit market heterogeneity among financed households; it is as if, from the lenders' perspective, all households were of type $A = 0$.

With nonrecourse mortgages, borrowers act differently in several dimensions. First, because lenders have no recourse to a household's deposits, borrowers with sufficient

16. This result would change with reputational costs from default (e.g., exclusion from all credit markets) because the value of future access to credit, and hence the default decision, depends on a household's income.

first-period resources will find it optimal to invest in deposits as a way to hedge house price risk.[17] In particular, a homeowner's second-period consumption now becomes

$$c' = \max \left\{ 0, hp' - mR\left(m\right) \right\} + y' + R^D d. \tag{57}$$

Second, because lenders ignore expected income growth (A) when lending, non-recourse mortgages favor households with a high initial income (y) but low expected income growth (A) over those with low initial income but high expected income growth (probably young households). This last group suffers the most in terms of down payments, credit limits, and mortgage rates. Many of these high expected income households may decide to rent. Thus, nonrecourse mortgages may generate larger inequality in access to credit.

Third, with nonrecourse mortgages, a household will default whenever the value of her house is lower than the mortgage balance to repay. *Ceteris paribus*, this leads households to default more often with nonrecourse mortgages. Ghent and Kudlyak (2011) provide empirical evidence of this result.

Lastly, absent some form of punishment for default, nonrecourse loans are, from the household's perspective, basically a convex gamble on house prices where the maximum loss is the down payment. If prices increase quickly enough ($p' > \frac{Rm}{h}$) households repay and enjoy the capital gains. If prices fall too much, households default, and the lender absorbs the loss except for the down payment. Thus, nonrecourse mortgages encourage demand for higher LTV ratios.

7. CONCLUSIONS

In this article, we analyzed a model with heterogeneous agents, default and closed form solutions. We studied the main drivers of credit demand and supply, highlighting new interesting empirical predictions. Then, we used the model to analyze optimal LTV policy. The macroprudential literature has focused on the importance of regulation as a means to internalize the costs from higher default associated with high LTV. Our results show that it is important not to neglect the beneficial role of mortgage debt as a gateway to homeownership. Optimal policy is therefore a trade-off between the two objectives. Exploring these insights in general equilibrium is a topic we leave for future research.

APPENDIX A: PARAMETERIZATION

We divide the parameters into two groups. The first group of parameters are decided exogenously directly from the data. The second group of parameters are chosen for the model to match some empirical targets. Income growth follows a Pareto distribution

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17. To show this result we depart from Section 3 and relax the assumption that $y \leq \frac{1}{\beta R^D}$.

with cumulative distribution $G(A) = 1 - (\frac{\underline{A}}{A})^{\alpha}$. Table 1 summarizes the parameters used in the figures.

A. *Exogenously selected parameters.* Lenders' cost of funds is assumed 1% ($R^D = 1.01$); the LTV in steady state is $\theta = 0.8$; the curvature in the Pareto distribution for income growth is set to the value estimated by Clementi and Gallegati (2005) for the U.S. between 1980 and 2001 ($\alpha = 1.1$).

B. *Endogenously selected parameters:* The remaining parameters are selected for the model to match the following targets that are consistent with U.S. averages from 1991 to 2013 obtained from FRED.

1) Homeownership ratio, defined as $1 - G(A^*)$, equal to 65%.
2) Loan default rate of 4%. The default rate in the model is

$$\frac{\int_{\max\{A^*, \underline{A}\}}^{\max\{A^*, \underline{A}, \widetilde{A}\}} dG(A)}{\int_{\max\{A^*, \underline{A}\}}^{\infty} dG(A)},$$

with $\widetilde{A}$ being the income growth threshold for default when aggregate shocks are at their mean, that is, $\epsilon^*(\widetilde{A}) = \epsilon_0$.

3) An average mortgage rate of 6%. The average mortgage rate is

$$\int_{\max\{A^*, \underline{A}\}}^{\infty} R^*(A) dG(A|A > \max\{A^*, \underline{A}\}),$$

where $G(A|A > \max\{A^*, \underline{A}\})$ is the conditional density of a borrower receiving credit.

4) An average charge-off rate of 0.5%. Charge-offs are the value of loans, net of recoveries, considered as a loss for the bank. The model equivalent is

$$\int_{\max\{A^*, \underline{A}\}}^{\max\{\max\{A^*, \underline{A}, \widetilde{A}\}\}} \frac{R^*(A)m - \gamma[hp'(\epsilon_0) + y'(A, \epsilon_0)]}{R^*(A)m} dG(A|A > \max\{A^*, \underline{A}\}).$$

5) House price growth is on average 2% ($\frac{\mathbb{E}[p'(\epsilon)]}{p} = 1.02$).

APPENDIX B: PROOFS

B.1 Proof of Proposition 1

We use the result (which we prove with Lemma 1) that as long as there is a positive probability of a borrower's default (i.e., $\epsilon^*(A, R, m, h) > \underline{\epsilon}$), mortgage rates are higher than deposit rates (i.e., $R(A, \theta, \lambda) > R^D$ for any A, θ and λ), and in that case households who borrow to buy a house do not save in deposits (i.e., $m > 0 \Rightarrow d = 0$).

For the case $\epsilon^*(A, R, m, h) > \underline{\epsilon}$, because the aggregate shock follows a Pareto-2 distribution, we can write the lender's expected profits as

$$\mathbb{E}[\Pi(m, R)] = 2\underline{\epsilon}^2 \gamma \int_{\underline{\epsilon}}^{\epsilon^*} \frac{[phB + yA]}{\epsilon^2} d\epsilon + 2\underline{\epsilon}^2 Rm \int_{\epsilon^*}^{\infty} \frac{d\epsilon}{\epsilon^3} - R^D m$$

$$= 2\gamma(phB + yA)\underline{\epsilon} - \frac{\underline{\epsilon}^2(phB + yA)^2}{Rm}(2\gamma - 1) - R^D m.$$

Imposing the zero-expected-profit condition, $\epsilon_0 \equiv 2\underline{\epsilon}$ and rearranging terms, we obtain

$$R = \frac{(2\gamma - 1)(phB + yA)^2 \underline{\epsilon}^2}{m(\epsilon_0 \gamma(phB + yA) - R^D m)}, \tag{B1}$$

or, using the definition of LTV ($\theta = \frac{m}{ph}$), and LTI ($\lambda = \frac{m}{y}$).

$$R(A, \theta, \lambda) = \frac{(2\gamma - 1)(\lambda B + \theta A)^2 \underline{\epsilon}^2}{\lambda \theta \left(\epsilon_0 \gamma(\lambda B + \theta A) - R^D \lambda \theta\right)} \quad \text{if } \max\{\underline{\epsilon}, \epsilon^*(.)\} = \epsilon^*(.).$$

If $\epsilon^*(A, R, m, h) \leq \underline{\epsilon}$ there is no risk of borrower's default and $R = R^D$.

Using (B1), $\gamma > \frac{1}{2}$, and because mortgage rates cannot be negative, then $m \in (0, \frac{\epsilon_0 \gamma(phB + yA)}{R^D})$. Therefore, we define the credit ceiling for borrower A as

$$\bar{m}(h, A) = \frac{\epsilon_0 \gamma(phB + yA)}{R^D}. \tag{B2}$$

The result for $\bar{\theta}(A, \lambda)$ follows from rearranging terms in the expression for the credit ceiling (B2), and using the definitions of LTV and LTI. That is,

$$\frac{\bar{m}}{ph} = \frac{\epsilon_0 \gamma \left(\lambda B + \frac{\bar{m}}{ph} A\right)}{\lambda R^D} \iff \bar{\theta}(A, \lambda) = \frac{\epsilon_0 \gamma \lambda B}{\lambda R^D - \epsilon_0 \gamma A}.$$

We can write expected revenue as

$$\Omega(h, m, R, A) = \epsilon_0 \gamma(phB + yA) - \frac{\underline{\epsilon}^2(phB + yA)^2}{Rm}(2\gamma - 1),$$

and given $\gamma > \frac{1}{2}$, we obtain

$$\frac{\partial \Omega}{\partial R} = \frac{\underline{\epsilon}^2(phB + yA)^2}{mR^2}(2\gamma - 1) > 0,$$

$$\frac{\partial^2 \Omega}{\partial R^2} = (-2)\frac{\underline{\epsilon}^2(phB + yA)^2}{mR^3}(2\gamma - 1) < 0,$$

and, moreover,

$$\lim_{R \to \infty} \Omega = \epsilon_0 \gamma (phB + yA) > 0.$$

Hence, $\Omega(.)$ is increasing and concave in R on $\mathbb{R}^+$. Moreover, because Ω is increasing and has a finite limit as $R \to \infty$, then it is bounded above in R.

Lastly, as to why $\gamma \leq \frac{1}{2}$ relates to a situation of no-default, we shall see in Lemma (1) that the default threshold for a mortgage applicant of type A requesting an LTV of θ can be written as

$$\epsilon^*(A) = \frac{R\theta}{B + A(1-\theta)} = (2\gamma - 1) \frac{(B + A(1-\theta)) \underline{\epsilon}^2}{\epsilon_0 \gamma (B + A(1-\theta)) - R^D \theta},$$

which is nonpositive when $\gamma \leq \frac{1}{2}$ and $m < \bar{m}(A)$. Thus, $\max\{\epsilon^*(\cdot), \underline{\epsilon}\} = \underline{\epsilon}$ and default probability is zero. Intuitively, the costs of default are so great that, if the lender is to offer a loan, it will be at such a low interest rate as to eliminate the possibility of default.

B.2 Proof of Proposition 2

We obtain $\frac{\partial \bar{\theta}}{\partial A} = \frac{(\epsilon_0 \gamma)^2 \lambda B}{(\lambda R^D - \epsilon_0 \gamma A)^2} > 0$, $\frac{\partial \bar{\theta}}{\partial B} = \frac{\epsilon_0 \gamma \lambda}{\lambda R^D - \epsilon_0 \gamma A} > 0$, $\frac{\partial \bar{\theta}}{\partial R^D} = -\frac{\lambda^2 \epsilon_0 \gamma B}{(\lambda R^D - \epsilon_0 \gamma A)^2} < 0$, $\frac{\partial \bar{\theta}}{\partial \lambda} = \frac{-(\epsilon_0 \gamma)^2 AB}{(\lambda R^D - \epsilon_0 \gamma A)^2} < 0$, and $\frac{\partial \bar{\theta}}{\partial \gamma} = \frac{\epsilon_0 \lambda^2 B R^D}{(\lambda R^D - \epsilon_0 \gamma A)^2} > 0$. Moreover, the interaction of the properties are $\frac{\partial^2 \bar{\theta}}{\partial B \partial A} = \frac{(\epsilon_0 \gamma)^2 \lambda}{(\lambda R^D - \epsilon_0 \gamma A)^2} > 0$, and $\frac{\partial^2 \bar{\theta}}{\partial \gamma \partial R^D} = \frac{-\epsilon_0^2 \lambda^2 \gamma AB - \epsilon_0 \lambda^3 B R^D}{(\lambda R^D - \epsilon_0 \gamma A)^3} < 0$.

B.3 Proof of Proposition 3

First, the derivative of R over A is:

$$\frac{\partial R}{\partial A} = \frac{1}{\lambda} \frac{(2\gamma - 1) \underline{\epsilon}^2 (\lambda B + \theta A) \left[\epsilon_0 \gamma (\lambda B + \theta A) - 2R^D \lambda \theta \right]}{\left[\epsilon_0 \gamma (\lambda B + \theta A) - R^D \lambda \theta \right]^2}.$$

Given $\gamma > \frac{1}{2}$ then $\frac{\partial R}{\partial A} < 0 \iff \epsilon_0 \gamma (\lambda B + \theta A) - 2R^D \lambda \theta < 0 \iff m > \frac{\bar{m}(A)}{2}$, where $\bar{m}(A)$ is defined in (B2). However, $\frac{\bar{m}(A)}{2} = \gamma \tilde{m}$, where

$$\tilde{m} = \frac{\epsilon_0 (phB + yA)}{2R} \tag{B3}$$

is the mortgage size at which there is no risk of borrower's default, that is $\epsilon^*(A, R, \tilde{m}, h) = \underline{\epsilon}$. Because $\epsilon^*(A, R, m, h)$ is increasing in m, then $\epsilon^*(A, R, m, h) > \underline{\epsilon} \iff m > \tilde{m}$. Thus, $\gamma < 1$ guarantees that $\tilde{m} > \gamma \tilde{m}$, and we can conclude that $\frac{\partial R}{\partial A} < 0$ as long as $m > \tilde{m}$, that is, as long as there is some probability of borrower's default.

The derivative of R over λ is $\frac{\partial R}{\partial \lambda} = -\frac{A}{\lambda^2}\frac{(2\gamma-1)\underline{\epsilon}^2(\lambda B+\theta A)[\epsilon_0\gamma(\lambda B+\theta A)-2R^D\lambda\theta]}{[\epsilon_0\gamma(\lambda B+\theta A)-R^D\lambda\theta]^2}$. Its sign depends on $[\epsilon_0\gamma(\lambda B + \theta A) - 2R^D\lambda\theta]$. Thus,

$$\epsilon_0\gamma(\lambda B + \theta A) - 2R^D\lambda\theta < 0 \iff m > \tilde{m} \iff \frac{\partial R}{\partial \lambda} > 0.$$

Similarly, we can show that $\frac{\partial R}{\partial \theta} = -\frac{B}{\theta^2}\frac{(2\gamma-1)\underline{\epsilon}^2(\lambda B+\theta A)[\epsilon_0\gamma(\lambda B+\theta A)-2R^D\lambda\theta]}{[\epsilon_0\gamma(\lambda B+\theta A)-R^D\lambda\theta]^2}$, and therefore $\frac{\partial R}{\partial \theta} > 0$ if $m > \tilde{m}$.

$\frac{\partial R}{\partial \lambda}$ and $\frac{\partial R}{\partial \theta}$ are linked to $\frac{\partial R}{\partial A}$ as :

$$\frac{\partial R}{\partial \lambda} = -\frac{A}{\lambda}\frac{\partial R}{\partial A}, \tag{B4}$$

$$\frac{\partial R}{\partial \theta} = -\frac{\lambda B}{\theta^2}\frac{\partial R}{\partial A}. \tag{B5}$$

Using (B4) and (B5), we can compute the cross-derivatives with respect to R^D. That is, $\frac{\partial^2 R}{\partial A\partial R^D} = -\frac{2R^D\theta^2\lambda\underline{\epsilon}^2(2\gamma-1)(\lambda B+\theta A)}{[\epsilon_0\gamma(\lambda B+\theta A)-R^D\theta\lambda]^3} < 0$, $\frac{\partial^2 R}{\partial \lambda\partial R^D} = -\frac{A}{\lambda}\frac{\partial^2 R}{\partial A\partial R^D} > 0$, and $\frac{\partial^2 R}{\partial \theta\partial R^D} = -\frac{\lambda B}{\theta^2}\frac{\partial^2 R}{\partial A\partial R^D} > 0$. Then we can show the cross-derivatives with respect to B. That is, $\frac{\partial^2 R}{\partial A\partial B} = \underline{\epsilon}^2(2\gamma-1)\frac{2(R^D\lambda\theta)^2}{[\epsilon_0\gamma(\lambda B+\theta A)-R^D\lambda\theta]^3} > 0$, $\frac{\partial^2 R}{\partial \lambda\partial B} = -\frac{A}{\lambda}\frac{\partial^2 R}{\partial A\partial B} < 0$, and $\frac{\partial^2 R}{\partial \theta\partial B} = -\frac{\lambda}{\theta^2}\frac{\partial R}{\partial A} - \frac{\lambda B}{\theta^2}\frac{\partial^2 R}{\partial A\partial B} < 0$.

Finally, we can compute the cross-derivatives with respect to γ. That is, $\frac{\partial^2 R}{\partial A\partial \gamma} = \frac{\underline{\epsilon}^2(\lambda B+\theta A)}{\lambda}\{\frac{[\epsilon_0^2\gamma(\lambda B+\theta A)^2-3R^D\lambda\theta\epsilon_0(\lambda B+\theta A)+4(R^D\lambda\theta)^2]}{[\epsilon_0\gamma(\lambda B+\theta A)-R^D\lambda\theta]^3}\}$, $\frac{\partial^2 R}{\partial \lambda\partial \gamma} = -\frac{A}{\lambda}\frac{\partial^2 R}{\partial A\partial \gamma}$, and $\frac{\partial^2 R}{\partial \theta\partial \gamma} = -\frac{\lambda B}{\theta^2}\frac{\partial^2 R}{\partial A\partial \gamma}$. The sign depends on the value of γ, with $\gamma > \frac{3R^D\lambda\theta\epsilon_0(\lambda B+\theta A)-4(R^D\lambda\theta)^2}{\epsilon_0^2(\lambda B+\theta A)^2} \iff \frac{\partial^2 R}{\partial A\partial \gamma} > 0, \frac{\partial^2 R}{\partial \lambda\partial \gamma} < 0, \frac{\partial^2 R}{\partial \theta\partial \gamma} < 0$.

B.4 Proof of Proposition 4

The mortgage spread is $\Delta^R(A, \theta, \lambda) = R(A, \theta, \lambda) - R^D = \frac{(2\gamma-1)(\lambda B+\theta A)^2\underline{\epsilon}^2}{\lambda\theta(\epsilon_0\gamma(\lambda B+\theta A)-R^D\lambda\theta)} - R^D$. We can show that $m > \tilde{m}$, with $\tilde{m}$ defined in (B3), implies that Δ^R is increasing in R^D. To show this, first consider the percentage mark-up of the mortgage rate over the risk free rate: $\frac{\Delta^R}{R^D}$. On the one hand,

$$\frac{\partial\left[\frac{\Delta^R}{R^D}\right]}{\partial R^D} = \frac{\partial}{\partial R^D}\left[\frac{(2\gamma-1)(\lambda B+\theta A)^2\underline{\epsilon}^2}{\lambda\theta\left(\epsilon_0\gamma(\lambda B+\theta A)R^D - (R^D)^2\lambda\theta\right)} - 1\right]$$

$$= -\frac{(2\gamma-1)(\lambda B+\theta A)^2\underline{\epsilon}^2}{\lambda\theta\left(\epsilon_0\gamma(\lambda B+\theta A)R^D - (R^D)^2\lambda\theta\right)^2}[\epsilon_0\gamma(\lambda B+\theta A)-2(R^D)\lambda\theta] > 0,$$

because $\epsilon_0 \gamma (\lambda B + \theta A) - 2(R^D)\lambda\theta < 0 \iff m > \tilde{m}$. However, on the other hand, we can equivalently write

$$\frac{\partial\left[\frac{\Delta^R}{R^D}\right]}{\partial R^D} = \frac{\partial\Delta^R/\partial R^D}{R^D} - \frac{\Delta^R}{(R^D)^2},$$

which, from above, must be positive, and thus we infer $\frac{\partial\Delta^R}{\partial R^D} > 0$. So the credit spread Δ^R is increasing in R^D if $m > \tilde{m}$.

B.5 Characterizing Definition 1

Fixing a mortgage size m in (16) we obtain $\bar{m}(A_L(\lambda, \theta)) = m$, and from there it follows $A_L(\lambda, \theta) = \lambda[\frac{R^D}{\epsilon_0\gamma} - \frac{1}{\theta}B]$. Because the credit ceiling $\bar{m}$ is strictly increasing in A, if $A < A_L$, then $\bar{m}(A) < \bar{m}(A_L) = m$. This implies that for $A \leq A_L$, the borrower's application for mortgage m is rejected. Taking derivatives we obtain $\frac{\partial A_L(\lambda,\theta)}{\partial B} = -\frac{\lambda}{\theta} < 0$, $\frac{\partial A_L(\lambda,\theta)}{\partial\epsilon_0} = -\frac{\lambda R^D}{(\epsilon_0)^2\gamma} < 0$, $\frac{\partial A_L(\lambda,\theta)}{\partial\theta} = \frac{\lambda B}{\theta^2} > 0$, $\frac{\partial A_L(\lambda,\theta)}{\partial R^D} = \frac{\lambda}{\epsilon_0\gamma} > 0$, and $\frac{\partial A_L(\lambda,\theta)}{\partial\lambda} = \frac{R^D}{\epsilon_0\gamma} - \frac{1}{\theta}B > 0$ if $(\frac{R^D}{\epsilon_0\gamma} - \frac{1}{\theta}B) > 0$.

Using Leibniz rule, we can show $\frac{\partial\Psi^L(\lambda,\theta)}{\partial B} = -g(A_L(\lambda, \theta))\frac{\lambda}{\theta} < 0$, $\frac{\partial\Psi^L(\lambda,\theta)}{\partial\epsilon_0} = -g(A_L(\lambda, \theta))\frac{\lambda R^D}{(\epsilon_0)^2\gamma} < 0$, $\frac{\partial\Psi^L(\lambda,\theta)}{\partial\theta} = g(A_L(\lambda, \theta))\frac{\lambda B}{\theta^2} > 0$, $\frac{\partial\Psi^L(\lambda,\theta)}{\partial R^D} = g(A_L(\lambda, \theta))\frac{\lambda}{\epsilon_0\gamma} > 0$, $\frac{\partial\Psi^L(\lambda,\theta)}{\partial\lambda\partial\theta} = \frac{g(A_L(\lambda,\theta))B}{\theta^2} > 0$ and $\frac{\partial^2\Psi^L(\lambda,\theta)}{\partial\lambda} = g(A_L(\lambda, \theta))(\frac{R^D}{\epsilon_0\gamma} - \frac{1}{\theta}B) > 0 \iff (\frac{R^D}{\epsilon_0\gamma} - \frac{1}{\theta}B) > 0$.

B.6 Proof of Lemma 1

Derivation of Renter's Solution. The problem's setup is detailed in (4)–(6). First, we substitute (6) into (4) so the renter's problem is a two-variable optimization in h_r and d_r. The Karush-Kuhn-Tucker conditions for these two variables are, respectively

$$\lambda r = \frac{1}{h_r},$$

$$\lambda = \beta R^D + \mu_d.$$

The multiplier λ corresponds to the constraint (5), and μ_d corresponds to $d_r \geq 0$. Because of log utility in housing services we know $h_r > 0$ and we can omit the associated multiplier for h_r. If $d_r > 0$, we obtain $h_r = \frac{1}{r\beta R^D}$, $d_r = y - rh_r$. However, this requires $y > \frac{1}{\beta R^D}$, for otherwise we have $d_r \leq 0$. If y is less than this threshold, the solution is $h_r = \frac{y}{r}$. In this case, substituting h_r into (4) and using the fact that $\mathbb{E}[y'] = yA\epsilon_0$, we obtain the value function $W(A) = \log(\frac{y}{r}) + \beta yA\epsilon_0$.

Derivation of Owner's Solution. The problem's setup is detailed in (7)–(10). Using the Principle of Optimality, we begin by optimizing over all decisions for a given θ, and then optimize over θ. Also, until the last step when we optimize over θ, we

shall simply write $R(A, m)$ as R. The household internalizes the credit supply surface $R(A, m)$ in its choice of LTV.

First, we show that if there is a positive probability of borrower's default, that is, if $\epsilon^*(A) > \underline{\epsilon}$, then $R > R^D$. This result gives that households with mortgage debt do not hold deposits. We prove the result by contradiction. We write lender's expected profits as

$$\mathbb{E}[\Pi(A)] = \int_{\underline{\epsilon}}^{\epsilon^*(A)} \gamma X(\epsilon) dF(\epsilon) + \int_{\epsilon^*(A)}^{\infty} Rm dF(\epsilon) - R^D m,$$

where $X(\epsilon)$ represents household's period-two resources: $X(\epsilon) = hp'(\epsilon) + y'(A, \epsilon)$. The default-inducing shock threshold for borrower A, denoted $\epsilon^*(A)$, is defined as

$$Rm = X(\epsilon^*(A)). \tag{B6}$$

If there is a positive probability of default, that is, if $\epsilon^*(A) > \underline{\epsilon}$, and if $R \leq R^D$, then

$$\mathbb{E}[\Pi(A)] = \int_{\underline{\epsilon}}^{\epsilon^*(A)} \gamma X(\epsilon) dF(\epsilon) + \int_{\epsilon^*(A)}^{\infty} Rm dF(\epsilon) - R^D m \tag{B7}$$

$$\leq \gamma \int_{\underline{\epsilon}}^{\epsilon^*(A)} Rm dF(\epsilon) + \int_{\epsilon^*(A)}^{\infty} Rm dF(\epsilon) - R^D m \tag{B8}$$

$$< Rm - R^D m \tag{B9}$$

$$\leq 0,$$

where (B8) uses the fact that p', y' are increasing in ϵ, and ϵ^* satisfies (B6). And $\gamma < 1$ guarantees (B9). Hence the zero-profit condition cannot hold if $R \leq R^D$, and thus we conclude $R > R^D$ if $\epsilon^*(A) > \underline{\epsilon}$.

To show that if $m > 0$, then $d = 0$, suppose there exists an allocation with $m > 0, d > 0$, for a given h. Then, using the budget constraint $d + ph = m + y$, utility is given by

$$U_1 = k \log \left(\frac{m + y - d}{p} \right)$$

$$+ \beta \int_{\epsilon^*}^{\infty} [R^D d + hp'(\epsilon) + y'(\epsilon) - Rm] dF(\epsilon), \tag{B10}$$

$$\epsilon^* = \frac{Rm - R^D d}{phB + yA}, \tag{B11}$$

where the expression for the default threshold ϵ^* used the fact that $p'(\epsilon) = pB\epsilon$, $y'(\epsilon) = yA\epsilon$. We have also used the fact that expected consumption can be written as

$$\mathbb{E}[\max\{0, c'\}] = \int_{\max\{\underline{\epsilon}, \epsilon^*(.)\}}^{\infty} \left[R^D d + h p'(\epsilon) + y'(\epsilon) - Rm \right] dF(\epsilon), \quad \text{(B12)}$$

where $\underline{\epsilon}$ is the worst possible case business cycle. Now, let us construct a new allocation with $\hat{d} = 0$, $\hat{m} = m - \frac{R^D}{R}d$. We know from the lender's problem that $R^D < R$. We also know that if $y < ph$, that is, if the household cannot purchase the house without a mortgage, then $m - d > 0$. These two facts establish that $\hat{m}$ is positive. By construction, $\epsilon_m^* = \epsilon_{\hat{m}}^*$. That is, the new allocation does not change the default threshold. Moreover, also by construction, $c_m' = c_{\hat{m}}'$ for any p', y', h, so that the new allocation does not change second-period consumption when the household does not default. Therefore, writing utility under the new allocation as

$$U_2 = k \log \left(\frac{\hat{m} + y}{p} \right) + \beta \int_{\epsilon^*}^{\infty} [h p'(\epsilon) + y'(\epsilon) - Rm] dF(\epsilon),$$

and taking the difference with utility under the original allocation, we obtain

$$U_2 - U_1 = k \log \left(\frac{\hat{m} + y}{m - y - d} \right) = k \log \left(\frac{m + y - \frac{R^D}{R}d}{m - y - d} \right) > 0$$

because $R^D < R$. Thus, utility was improved under the allocation with $\hat{d} = 0$ and $\hat{m} > 0$. Therefore, we conclude that, if the optimal plan for the household is to have $m > 0$, then $d = 0$. That is, borrowers do not also save in deposits.

Because $R > R^D$ implies $d = 0$, then from the first-period budget constraint

$$h = \frac{y}{p(1 - \theta)}. \quad \text{(B13)}$$

Substituting this h into to (12), and using $m = \theta ph$, we obtain the default threshold

$$\epsilon^*(A) = \frac{Rm}{ph B + yA} = \frac{R\theta}{B + A(1 - \theta)}.$$

We can obtain the value function for homeowners:

$$U(A) = k \log(h) + \beta \int_{\epsilon^*}^{\infty} [h p'(\epsilon) + y'(\epsilon) - Rm] dF(\epsilon) = k \log \left(\frac{y}{p(1 - \theta)} \right)$$

$$+ y\beta \underline{\epsilon}^2 \left[\frac{(B + (1 - \theta)A)^2}{R\theta(1 - \theta)} \right].$$

In the general case, showing $\frac{\partial^2 F}{\partial y \partial \theta} < 0$ is quite difficult, although we can show this holds in an equilibrium with $\hat{\theta} < 1$. Specifically, differentiating (B17) :

$$\frac{\partial F}{\partial y} = \frac{k-1}{y} + \frac{\beta}{2\gamma - 1}\left[-\frac{R^D \theta - \epsilon_0 \gamma B}{1 - \theta} + A\epsilon_0(1-\gamma)\right],$$

$$\frac{\partial^2 F}{\partial y \partial \theta} = -\frac{\beta}{2\gamma - 1}\left[\frac{R^D - \epsilon_0 \gamma B}{(1-\theta)^2}\right] < 0,$$

because $\hat{\theta} < 1$ implies $R^D > \epsilon_0 \gamma B$.

Finally, we optimize over θ. In particular, as a function of θ, the household's utility can be expressed

$$U(A, \theta) = k \log\left(\frac{y}{p(1-\theta)}\right) + y\beta\underline{\epsilon}^2\left[\frac{(B + (1-\theta)A)^2}{R(A, \theta)\theta(1-\theta)}\right],$$

where we are writing R as $R(A, \theta)$ to acknowledge that the household internalizes the credit supply surface in its choice of LTV. We assume that $U(A, \theta)$ is single-peaked in θ, which we shall then verify in the proof of Proposition (5). It then follows that the unconstrained maximum of U with respect to θ can be uncovered via the first-order condition of U with respect to θ,

$$0 = \frac{k}{1-\theta} - \frac{y\beta\underline{\epsilon}^2(B + A(1-\theta))}{\theta(1-\theta)R}\left[2A + \left(\frac{\partial R}{\partial \theta} - [2\theta - 1]\right)\right.$$
$$\left. \times \frac{(B + A(1-\theta))}{\theta(1-\theta)}\right].$$

Let $\hat{\theta}$ denote the unique solution to this expression. Strictly speaking, there is nothing that prevents $\hat{\theta}$ from being negative, in which case the household would like to short-sell. However, we do not wish to consider such corner solutions here. Therefore, the only situation in which the household's LTV constraint binds is when $\hat{\theta} > \Theta$. In such a situation, the household will choose Θ. Thus, the household's solution features a cutoff rule: $\theta^*(A) = \min\{\hat{\theta}, \Theta\}$.

B.7 Characterizing Definition 2

Note first that from (B17) we know that in equilibrium F is strictly increasing in A. Because $A_B(R, \theta)$ is defined such that $F(A_B(R, \theta)) = 0$, we infer that for $A \leq A_B(R, \theta)$, $F(A) \leq 0$, and so the household rents. Moreover, we can express the fraction of borrowers who would choose to rent rather than own as $\Psi^B = \int_{\underline{A}}^{A_B} g(A)dA$.

To show $\frac{\partial A_B(R,\theta)}{\partial R} > 0$, we use the Implicit Function Theorem:

$$\frac{\partial A_B(R,\theta)}{\partial R} = -\frac{\frac{\partial F(A_B(R,\theta))}{\partial R}}{\frac{\partial F(A_B(R,\theta))}{\partial A_B(R,\theta)}} = \frac{\left[y\beta\epsilon_0 \frac{\underline{\epsilon}(B+A_B(1-\theta))^2}{2R^2\theta(1-\theta)}\right]}{y\beta\epsilon_0\left[\frac{\underline{\epsilon}(B+A_B(1-\theta))}{R\theta} - 1\right]} > 0,$$

where we can show that the denominator is positive (weakly) by substituting R by (42). Namely, the bracketed term in the denominator becomes,

$$\frac{\underline{\epsilon}(B+A(1-\theta))}{R\theta} - 1 = \underline{\epsilon}(B+A(1-\theta))\frac{\epsilon_0\gamma(B+(1-\theta)A) - R^D\theta}{\underline{\epsilon}^2(2\gamma-1)(B+(1-\theta)A)^2} - 1,$$

$$= \frac{\underline{\epsilon}(B+(1-\theta A)) - R^D\theta}{\underline{\epsilon}(2\gamma-1)(A+(1-\theta)B)},$$

which is non-negative if and only if

$$\underline{\epsilon} \geq \frac{R^D\theta}{B+(1-\theta)A},$$

where the right-hand side corresponds to the value of ϵ^* when the lender charges $R = R^D$. But the lender only charges such a rate if there is no risk of default, that is, if and only if

$$\frac{R^D\theta}{B+(1-\theta)A} \leq \underline{\epsilon}.$$

Continuing, for $\frac{\partial A_B(R,\theta)}{\partial\theta}$ we have

$$\frac{\partial A_B(R,\theta)}{\partial\theta} = -\frac{\frac{\partial F(A_B(R,\theta))}{\partial\theta}}{\frac{\partial F(A_B(R,\theta))}{\partial A_B}} = -\frac{\left[\frac{k}{1-\theta} + \frac{y\beta\epsilon_0\underline{\epsilon}}{2R}\left[\frac{(B+A_B(1-\theta))[(2\theta-1)B+(\theta-1)A_B]}{[\theta(1-\theta)]^2}\right]\right]}{y\beta\epsilon_0\left[\frac{\underline{\epsilon}(B+A_B(1-\theta))}{R\theta} - 1\right]}.$$

Given a positive (weakly) denominator as before, $\frac{k}{1-\theta} > 0$, $\frac{y\beta\epsilon_0\underline{\epsilon}}{2R} > 0$, and $\frac{(B+A_B(1-\theta))}{[\theta(1-\theta)]^2} > 0$, the sign of $\frac{\partial A_B(R,\theta)}{\partial\theta}$ depends on the level of LTV; for example, if $[(2\theta-1)B+(\theta-1)A_B] > 0$ then $\frac{\partial A_B(R,\theta)}{\partial\theta} < 0$.

B.8 Proof of Proposition 5

Let us assume that $A > A_B(\theta)$. Then we know that such a household will apply for a mortgage. Specifically, by substituting (B13) into the relationship $m = \theta ph$ we obtain borrower's requested loan amount, m, and then by substituting m into the

bank's zero-profit interest rate curve, (B1). Specifically,

$$R(m, A) = \frac{(2\gamma - 1)(phB + yA)^2 \underline{\epsilon}^2}{m(\epsilon_0 \gamma (phB + yA) - R^D m)} \tag{B14}$$

$$= \frac{(2\gamma - 1)(B + A(1 - \theta))^2 \underline{\epsilon}^2}{\theta(\epsilon_0 \gamma (B + A(1 - \theta)) - R^D \theta)}. \tag{B15}$$

If, on the other hand, $A \leq A_B(\theta)$, we have that the household does not apply for a loan. As a result, the household is not a participant in the mortgage market and instead rents a house.

As to the determination of θ^*, we can use the expression for $R(\cdot)$ in (B15) to write U in terms of θ as

$$U(A) = k \log\left(\frac{y}{p(1 - \theta)}\right) + y\beta \left[\frac{\epsilon_0 \gamma (B + (1 - \theta)A) - R^D \theta}{(2\gamma - 1)(1 - \theta)}\right], \tag{B16}$$

and obtain a first-order condition with respect to θ,

$$0 = k(1 - \theta) + \frac{y\beta}{2\gamma - 1}\left[\epsilon_0 \gamma B - R^D\right] \iff \theta = 1 - y\beta \frac{R^D - \epsilon_0 \gamma B}{k(2\gamma - 1)} = \hat{\theta}.$$

In particular, we have verified the hypothesis of Lemma (1) that U is single-peaked in θ. Therefore, using the result of Lemma (1), we conclude $\theta^* = \min\{\hat{\theta}, \Theta\}$.

As to characterizing $\hat{\theta}$, the derivatives are $\frac{\partial \hat{\theta}}{\partial \epsilon_0} = \frac{y\beta \gamma B}{k(2\gamma - 1)} > 0$, $\frac{\partial \hat{\theta}}{\partial R^D} = -\frac{y\beta}{k(2\gamma - 1)} < 0$, $\frac{\partial \hat{\theta}}{\partial y} = -\beta \frac{R^D - \epsilon_0 \gamma B}{k(2\gamma - 1)} < 0$, $\frac{\partial \hat{\theta}}{\partial k} = y\beta \frac{R^D - \epsilon_0 \gamma B}{k^2(2\gamma - 1)} > 0$, $\frac{\partial^2 \hat{\theta}}{\partial \epsilon_0 \partial \gamma} = \frac{-y\beta B}{k(2\gamma - 1)^2} < 0$ and $\frac{\partial \hat{\theta}}{\partial \gamma} = \frac{2y\beta}{k}\left(\frac{R^D - \frac{1}{2}\epsilon_0 B}{(2\gamma - 1)^2}\right) > \frac{2y\beta}{k}\left(\frac{R^D - \gamma \epsilon_0 B}{(2\gamma - 1)^2}\right) > 0$ as $\gamma > \frac{1}{2}$. We have also assumed $B < \frac{R^D}{\epsilon_0 \gamma}$, which means that the household's target LTV $\hat{\theta}$ is less than 1.

To characterize Γ when $\theta^* = \Theta$, we can compute

$$\frac{\partial \Gamma}{\partial \Theta} = \frac{1}{1 - \Psi^B(\Theta)}\left(\frac{\partial A_L(\cdot, \Theta)}{\partial \Theta} g(A_L) - [1 - \Gamma]\frac{\partial A_B(\Theta)}{\partial \Theta} g(A_B)\right),$$

where $\frac{\partial A_L(\cdot, \Theta)}{\partial \Theta} > 0$ from Section 2, and $\theta^* = \Theta < \hat{\theta}$ implies $\frac{\partial A_B(\Theta)}{\partial \Theta} < 0$. We can also compute

$$\frac{\partial \Gamma}{\partial R^D} = \frac{\Theta}{\epsilon_0 \gamma (1 - \Theta)[1 - \Psi^B(\Theta)]}\left(g(A_L) - [1 - \Gamma]\frac{\gamma}{1 - \gamma} g(A_B)\right),$$

which is positive provided

$$\Gamma > 1 - \frac{1 - \gamma}{\gamma}\frac{g(A_L)}{g(A_B)}.$$

Otherwise, it is negative. Note that $\gamma > \frac{1}{2}$ and $g(A_L) < g(A_B)$ (which is reasonable if G is Pareto and $A_L > A_B$) imply that the right-hand side of the above inequality is between 0 and 1.

B.9 Proof of Proposition 6

Substituting (42) into (34) we obtain

$$F(A) = \log\left(\frac{ry^{k-1}}{p^k(1-\theta^*)^k}\right)$$
$$+ y\beta\left[\frac{\epsilon_0\gamma B - R^D\theta^* + A\epsilon_0(1-\gamma)(1-\theta^*)}{(1-\theta^*)(2\gamma-1)}\right]. \tag{B17}$$

That is, F is a linear function of A. We find $A_B(R,\theta)$ by setting $F(A) = 0$ and solving for A. That is,

$$A_B\left(\theta^*\right) = \frac{(1-\theta^*)(2\gamma-1)\log\left[\frac{p^k(1-\theta^*)^k}{ry^{k-1}}\right] - y\beta\left[\epsilon_0\gamma B - R^D\theta^*\right]}{(1-\theta^*)(1-\gamma)\epsilon_0 y\beta}. \tag{B18}$$

Moreover, substituting (40) into (25) we can write the threshold for lender-driven credit rationing as

$$A_L\left(\theta\right) = \frac{\theta^*}{(1-\theta^*)}\left[\frac{R^D}{\epsilon_0\gamma} - \frac{1}{\theta^*}B\right]. \tag{B19}$$

The condition $A_B \leq A_L$ is obtained from comparing (B18) and (B19)

$$\frac{\theta R^D - \epsilon_0\gamma B}{\epsilon_0\gamma(1-\theta^*)} \geq \frac{(1-\theta^*)(2\gamma-1)\log\left[\frac{p^k(1-\theta^*)^k}{ry^{k-1}}\right] - y\beta\left[\epsilon_0\gamma B - R^D\theta^*\right]}{(1-\theta^*)(1-\gamma)\epsilon_0 y\beta}$$
$$\Longleftrightarrow$$
$$\frac{p}{r} \leq \left[\frac{y^{k-1}}{p^{k-1}(1-\theta^*)^k}\right]\exp\left\{y\beta\frac{\epsilon_0\gamma B - \theta^* R^D}{\gamma(1-\theta^*)}\right\}.$$

B.10 Proof of Proposition 7

Consider the equilibrium mortgage application threshold A_B as written in (46),[18]

$$A_B = \frac{2\gamma-1}{1-\gamma}\frac{1}{y\beta\epsilon_0}\log\left[\frac{p}{r}(1-\theta^*)\right] + \frac{\gamma}{1-\gamma}A_L. \tag{B20}$$

---

18. We continue to focus on the case where $k = 1$ because it leads to a cleaner exposition. It is straightforward to show that the results are qualitatively the same when $k \neq 1$ and r is a decreasing function of B.

Then, differentiating (B20) with respect to expected house price growth B and using (44), we obtain

$$
\begin{aligned}
\frac{\partial A_B(\theta^*)}{\partial B} &= -\frac{2\gamma - 1}{1 - \gamma} \frac{1}{y\beta\epsilon_0} \frac{p}{r} \frac{\partial (r/p)}{\partial B} + \frac{\gamma}{1 - \gamma} \frac{\partial A_L(\theta^*)}{\partial B} \\
&= -\frac{2\gamma - 1}{1 - \gamma} \frac{1}{y\beta\epsilon_0} \frac{p}{r} \frac{\partial (r/p)}{\partial B} - \frac{\gamma}{1 - \gamma} \frac{1}{1 - \theta^*},
\end{aligned}
$$

which is negative if and only if

$$
\left| \frac{\partial \left(\frac{r}{p} \right)}{\partial B} \right| < \frac{\gamma}{1 - \theta^*} \frac{y\beta\epsilon_0}{2\gamma - 1} \frac{r}{p}.
$$

B.11 Proof of Proposition 8

First, note that the planner's function $V(\Theta)$ in (53) is concave in Θ, because aggregate household utility $\bar{u}(\Theta)$ is concave in Θ, and default probability ρ is increasing in Θ when Θ binds, using (52). Therefore, a solution to the first-order condition of V with respect to Θ corresponds to a local maximum. Differentiating V, we obtain

$$
\begin{aligned}
\frac{\partial V}{\partial \Theta} &= \left[W(A_R) - U \left(A_R, \min\{\hat{\theta}, \Theta, \bar{\theta}(A_R)\} \right) \right] \frac{\partial A_R}{\partial \Theta} g(A_R) + \int_{A_R}^{\infty} \frac{\partial U}{\partial \Theta} g(A) \, dA \\
&\quad + \Lambda \left(\rho \left(A_R, \min\{\hat{\theta}, \Theta, \bar{\theta}(A_R)\} \right) \frac{\partial A_R}{\partial \Theta} g(A_R) - \int_{A_R}^{\infty} \frac{\partial \rho}{\partial \Theta} g(A) \, dA \right), \quad \text{(B21)}
\end{aligned}
$$

where $A_R = A_R(\Theta)$ is the marginal homeowner indifferent between owning and renting. Therefore, $W(A_R) = U(A_R, \min\{\hat{\theta}, \Theta, \bar{\theta}(A_R)\})$ as in definition (48). The expression (B21) can then be simplified to

$$
\begin{aligned}
\frac{\partial V}{\partial \Theta} &= \int_{A_R}^{\infty} \frac{\partial U}{\partial \Theta} g(A) \, dA - \Lambda \left(-\rho \left(A_R, \min\{\hat{\theta}, \Theta, \bar{\theta}(A_R)\} \right) \right) \frac{\partial A_R}{\partial \Theta} g(A_R) \\
&\quad + \int_{A_R}^{\infty} \frac{\partial \rho}{\partial \Theta} g(A) \, dA \right). \quad \text{(B22)}
\end{aligned}
$$

As to characterizing Θ^*, the Implicit Function Theorem implies that for any x such that $\frac{\partial^2 V}{\partial \Theta \partial x} > 0$, we have $\frac{\partial \Theta^*}{\partial x} > 0$. Continuing,

$$
\frac{\partial^2 V}{\partial \Theta \partial \Lambda} = \rho \left(A_R, \min\{\hat{\theta}, \Theta, \bar{\theta}(A_R)\} \right) \frac{\partial A_R}{\partial \Theta} g(A_R) - \int_{A_R}^{\infty} \frac{\partial \rho}{\partial \Theta} g(A) \, dA < 0,
$$

using the fact that $\frac{\partial A_R}{\partial \Theta} \leq 0$, which means that households cannot be forced to borrow more than they would like, and $\frac{\partial \rho}{\partial \Theta} > 0$, which uses (52) and assumes that there exists a household receiving an LTV of Θ^*.[19]

Lastly, suppose $\bar{\theta}(A_R) = \min\{\hat{\theta}, \Theta^*, \bar{\theta}(A_R)\}$, which means that some households are credit constrained. Economically, this is not an unreasonable case to consider. It then follows that A_R is the solution to $A_R = A_B(\bar{\theta}(A_R))$ which, notably, is independent of Θ^*, so that $\frac{\partial A_R}{\partial \Theta} = 0$ and thus

$$
\begin{aligned}
\frac{\partial^2 V}{\partial \Theta \partial k} &= \int_{A_R}^{\infty} \frac{\partial^2 U}{\partial \Theta \partial k} g(A)\, dA \\
&\quad + \left(\frac{\partial U(A_R, \min\{\hat{\theta}, \Theta, \bar{\theta}(A_R)\})}{\partial \Theta} \right. \\
&\quad \left. - \Lambda \frac{\partial \rho(A_R, \min\{\hat{\theta}, \Theta, \bar{\theta}(A_R)\})}{\partial \Theta} \right) \frac{\partial A_R}{\partial k} g(A_R) \\
&= \int_{A_R}^{\infty} \frac{\partial^2 U}{\partial \Theta \partial k} g(A)\, dA \\
&= \frac{1}{1 - \Theta^*} \int_{A_R}^{\infty} \mathbf{1}\left\{ \Theta^* = \min \hat{\theta}, \Theta^*, \bar{\theta}(A_R) \right\} g(A)\, dA > 0.
\end{aligned}
$$

In the second line, we used the fact that, because A_R is independent of Θ^*, the marginal household is not bound by the LTV cap, and so $\frac{\partial U(\cdot)}{\partial \Theta}$ and $\frac{\partial \rho(\cdot)}{\partial \Theta}$ equal zero when evaluated at $A = A_R$. The indicator function $\mathbf{1}\left\{ \Theta^* = \min\{\hat{\theta}, \Theta^*, \bar{\theta}(A_R)\} \right\}$ means that the LTV cap is binding for household A, and we assume that this holds for a nonzero measure of households.

We conclude by remarking that, when we do not restrict ourselves to the case $\bar{\theta}(A_R) = \min\{\hat{\theta}, \Theta^*, \bar{\theta}(A_R)\}$, it is possible that with a higher k, lower-quality borrowers are induced to become homeowners. This puts downward pressure on the optimal LTV cap, to discourage these lower-quality households from taking out a mortgage, as they have a greater default probability. This is because k and θ are complements for the household. Thus, the sign of $\frac{\partial^2 V}{\partial \Theta \partial k}$ would, in general, depend on parameter values.

B.12 *Proof of Proposition 9*

First, consider the lender's decision. The lender's zero-profit condition for a non-recourse loan is

$$
\mathbb{E}[\Pi(h, m)] = \int_{\underline{\epsilon}}^{\max\{\underline{\epsilon}, \epsilon^*(.)\}} \gamma h p'(\epsilon) dF(\epsilon) + \int_{\max\{\underline{\epsilon}, \epsilon^*(.)\}}^{\infty} Rm\, dF(\epsilon) - R^D m. \quad \text{(B23)}
$$

---

19. Otherwise, the cap is not binding, and because no household receives an LTV of Θ^*, $\frac{\partial \rho}{\partial \Theta} = 0$ for every A.

Note that (B23) implies that the nonrecourse zero-profit condition corresponds to the recourse zero-profit condition for the case of a borrower with $A = 0$. Thus, the lender's credit supply surface is described by

$$
R^* = \begin{cases} \dfrac{(2\gamma - 1)B^2 \underline{\epsilon}^2}{\theta^*(\epsilon_0 \gamma B - R^D \theta^*)} & \text{if } \max\{\underline{\epsilon}, \epsilon^*(.)\} = \epsilon^*(.) \\[2ex] R^D & \text{if } \max\{\underline{\epsilon}, \epsilon^*(.)\} = \underline{\epsilon} \end{cases}.
\tag{B24}
$$

The denominator of (B24) implies the following credit ceiling,

$$
\bar{\theta} = \frac{\epsilon_0 \gamma B}{R^D}.
\tag{B25}
$$

To show that default probability is greater under a nonrecourse loan, note that the productivity threshold for default for a given A and θ under a recourse loan is

$$
\epsilon^*(A, \theta^*) = \frac{R(A, \theta)\theta}{B + A(1 - \theta)}.
$$

Because we saw in Proposition 2 that $\frac{\partial R}{\partial A} < 0$, it is clear from above that $\frac{\partial \epsilon^*}{\partial A} < 0$, holding θ fixed. With a nonrecourse loan, all households behave as type $A = 0$ in their propensity to default. Thus, ϵ^* will be strictly greater for a nonrecourse loan than for a recourse loan for any household A at a given θ.[20] Consequently, default probability, expressed as $\rho(A, \theta) = \int_{\max\{\epsilon^*, \underline{\epsilon}\}}^{\epsilon^*} dF(\epsilon)$, will be strictly higher for every household A in the nonrecourse case. Likewise, $\frac{\partial R}{\partial A} < 0$ implies that the mortgage rate for a nonrecourse mortgage will be higher for every borrower A.[21]

As for households, because we have relaxed the assumption that $y \le \frac{1}{\beta R^D}$, it is now possible that renters will choose to hold deposits. Using (28), a renter's utility can be written

$$
W(A) = \log\left(\frac{y - d}{r}\right) + \beta R^D d + \beta \epsilon_0 y A,
$$

so that solving the first-order condition with respect to d yields the renter's choice of deposits,

$$
d_r = \max\left\{0, y - \frac{1}{\beta R^D}\right\}.
$$

20. We are assuming $\underline{A} > 0$, so that future labor income is always positive; a household does not enter the second period with nonmortgage debt to pay off.

21. We have assumed that there is positive probability of default, so that $\epsilon^* > \underline{\epsilon}$ for both mortgage structures; otherwise the probability of default equals zero in both cases.

For homeowners, we showed in Lemma 1 that it is not optimal for households to choose deposits $d > 0$ when mortgages are recourse. Now, though, we can write the homeowner's utility, as a function of A and θ^*, as

$$U(A, \theta^*) = k \log \left(\frac{y - d}{p(1 - \theta^*)} \right) + \beta R^D d + \beta \epsilon_0 y A$$

$$+ \beta \int_{\epsilon^*}^{\infty} [ph B\epsilon - R\theta^* ph] dF(\epsilon)$$

$$= k \log \left(\frac{y - d}{p(1 - \theta^*)} \right) + \beta R^D d + \beta \epsilon_0 y A$$

$$+ \beta \underline{\epsilon}^2 (y - d) \left[\frac{B^2}{R\theta^*(1 - \theta^*)} \right]$$

$$= k \log \left(\frac{y - d}{p(1 - \theta^*)} \right) + \beta R^D d + \beta \epsilon_0 y A$$

$$+ \beta(y - d) \left[\frac{\epsilon_0 \gamma B - R^D \theta^*}{(2\gamma - 1)(1 - \theta^*)} \right] \tag{B26}$$

Differentiating (B26) with respect to d and solving the first-order condition, we obtain

$$d^* = \max \left\{ 0, y - k \left[\beta R^D - \beta \left(\frac{\epsilon_0 \gamma B - R^D \theta^*}{(2\gamma - 1)(1 - \theta^*)} \right) \right]^{-1} \right\}.$$

Using d^*, it is straightforward to solve the first-order condition of (B26) with respect to θ to show that the target LTV is

$$\hat{\theta} = 1 - \frac{\beta(y - d^*) \left(R^D - \epsilon_0 \gamma B \right)}{k(2\gamma - 1)},$$

and thus the optimal LTV is

$$\theta^* = \min \left\{ \hat{\theta}, \Theta \right\}.$$

Lastly, to show that households will own if $y > y_R$ and otherwise they will rent, it suffices to show that $\frac{\partial F^*}{\partial y} \geq 0$, where $F^* = U^* - W^*$ is the difference between owners' and renters' utility in an equilibrium of mortgage markets and housing tenure choice. Conditional on showing $\frac{\partial F^*}{\partial y} \geq 0$, we can define y_R such that $F^*(y_R) = 0$.[22] If $F^*(y) < 0$ for every $y \in \mathbb{R}^+$ then we shall set $y_R = \infty$, and likewise if $F^*(y) > 0$ for every $y \in \mathbb{R}^+$, then we shall set $y_R = 0$.

---

22. If there exists a nondegenerate interval $Y \subseteq \mathbb{R}^+$ such that $\frac{\partial F^*}{\partial y} = 0$ for all $y \in Y$ and $F^*(y) = 0$ for all $y \in Y$, then y_R is not unique, and it suffices to set $y_R = \sup Y$.

Continuing from (B26), (28), and the expression for d_r, we can express F^* as a function of first-period income y,

$$F^*(y) = \log\left(\frac{r}{y - d_r}\left[\frac{y - d^*}{p(1 - \theta_R)}\right]^k\right) + \beta R^D\left(d^* - d_r\right)$$

$$+ \beta(y - d^*)\left[\frac{\epsilon_0\gamma B - R^D\theta_R}{(2\gamma - 1)(1 - \theta_R)}\right], \tag{B27}$$

where $\theta_R = \min\left\{\bar{\theta}, \Theta, \hat{\theta}\right\}$, representing the LTV a homeowner would receive in an equilibrium of housing tenure choice. Note that, using an envelope theorem, $\frac{\partial F^*}{\partial d^*} = \frac{\partial F^*}{\partial d_r} = 0$ and likewise $\frac{\partial F^*}{\partial \theta_R} = 0$ when $\theta_R = \hat{\theta}$. If $\theta_R \neq \hat{\theta}$, then θ_R is independent of y, as it either equals the lender's ceiling in (B25) or Θ. Therefore, differentiating (B27) with respect to y gives

$$\frac{\partial F^*}{\partial y} = \frac{k}{y - d^*} - \frac{1}{y - d_r} + \beta\frac{\epsilon_0\gamma B - R^D\theta_R}{(2\gamma - 1)(1 - \theta_R)}. \tag{B28}$$

Note that, from the derivation of (B26), the third term in (B28) equals

$$\beta\frac{\mathbb{E}\left[\max\left\{0, hp' - mR(m)\right\}\right]}{y - d^*} > 0.$$

Proceeding, and denoting proportionality as $\propto$, we can rewrite (B28) as

$$\frac{\partial F^*}{\partial y} \propto k - \frac{y - d^*}{y - d_r} + \left(y - d^*\right)\beta\frac{\epsilon_0\gamma B - R^D\theta_R}{(2\gamma - 1)(1 - \theta_R)},$$

$$= k - \frac{y - d^*}{y - d_r} + \left(y - d^*\right)\beta\Phi, \tag{B29}$$

where

$$\Phi = \frac{\epsilon_0\gamma B - R^D\theta_R}{(2\gamma - 1)(1 - \theta_R)}.$$

Using the expressions for d_r and d^* above, we can write

$$y - d_r = \min\left\{y, \frac{1}{\beta R^D}\right\},$$

$$y - d^* = \min\left\{y, k\left[\beta R^D - \beta\Phi\right]^{-1}\right\}.$$

Because $k \geq 1$ and $\Phi > 0$, there are three cases. In Case 1, $y - d_r = y = y - d^*$. In this case, (B29) implies

$$\frac{\partial F^*}{\partial y} \propto k - 1 + y\beta\Phi > 0.$$

In Case 2, $y - d_r = \frac{1}{\beta R^D}$ and $y - d^* = k[\beta R^D - \beta\Phi]^{-1}$. In this case, (B29) becomes

$$\frac{\partial F^*}{\partial y} \propto k - k\frac{R^D}{R^D - \Phi} + k\frac{\Phi}{R^D - \Phi} = 0.$$

In Case 3, $y - d_r = \frac{1}{\beta R^D}$ and $y - d^* = y$. In this case, (B29) becomes

$$\frac{\partial F^*}{\partial y} \propto k - y\left(\beta R^D + \beta\Phi\right),$$

$$\geq k - k = 0.$$

The last line used the fact that $y - d^* = y$ implies $y \leq k[\beta R^D - \beta\Phi]^{-1}$. This establishes that $\frac{\partial F^*}{\partial y} \geq 0$, which is what needed to be shown.

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