Seeing Through Colorblindness: Social Networks as a Mechanism for Discrimination[*]

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Abstract

I study labor markets in which firms both (1) hire via referrals and (2) are race-blind or "colorblind." I develop an employment model showing that—despite initial equality in ability, employment, wages, and network structure—minorities receive disproportionately fewer jobs through referral and lower expected wages, simply because their social group is smaller. This discriminatory outcome, which I term "social network discrimination," arises from homophily and falls outside the dominant economics discrimination models—taste-based and statistical. I calibrate the model using a nationally-representative U.S. sample and estimate the lower-bound welfare gap caused by social network discrimination is up to 3.2 percent, disadvantaging black workers. This paper isolates a potential underlying mechanism for inequality, adding to the understanding of labor market disparities that have been widely studied across the social sciences. In doing so, the paper disproves the proposition that "colorblind" policies are inherently merit-promoting, thereby introducing a new rationale for race-conscious policy.

Keywords: discrimination, diversity, employment, homophily, inequality, labor market, social networks, racial justice, affirmative action

1   Introduction

In the U.S. context, the vestiges of slavery and state-enforced racial segregation directly and indirectly have contributed to racial disparities, confounding efforts to foster equality. Suppose, however, that the circumstances of racial groups were equalized—that the slate was wiped clean. In a world without prejudice—and one beginning in a state of equality—would racial disparities still arise between majority and minority workers?

In this paper, I develop an employment model and assume equal ability, employment, and network structure between majority and minority workers in the initial time period. Workers are observationally equivalent, so employers cannot determine the race of job candidates (among other things). Despite these equalizing assumptions, I find that if majority and minority workers have (1) an equal chance of having a social tie (i.e., an equivalent network density) and (2) an equal bias in favor of forming same-group social ties (i.e., an equivalent type in-group bias), then the probability of a firm offering a job through referral to minority workers is *lower* than their share of the labor force. For minority workers to have a proportional chance of receiving job offers through referral, they must compensate with a stronger network density and/or type in-group bias. The estimated welfare gap increases in a convex way with the majority group share of the labor force, suggesting that the disadvantage of minority workers magnifies with the degree of their minority status in the labor market. Finally, this paper calibrates the model, estimating the lower bound welfare gap between black and white workers caused by social network discrimination.

While the model may be generalizable for arbitrary assignments of majority and minority groups, social science research suggests that race and ethnicity create the greatest divide socially (McPherson, Smith-Lovin, and Cook 2001).[1] Hence, this paper's findings impact debates surrounding the merits of race-conscious policies. Many who are critical of such policies share a common assumption: that in a world without historical discrimination—

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1. According to this research, after race and ethnicity, the greatest social divides are created by age, religion, education, occupation, and gender, in approximately that order.

without Jim Crow or implicit biases—"no policy" would be the best policy. No policy would yield the most meritocratic outcome, with opportunities distributed according to corresponding ability or "merit."[2] Yet the findings in this paper suggest otherwise. Achieving equality of opportunity may remain elusive even in the absence of psychological prejudice, historical wrongs, and differences in ability or education among racial groups.[3] Advantages from being within a larger or more strongly-connected social network may persist, despite one's talent. All else equal, outcomes may remain unequal.

This paper makes four main contributions. First, the paper presents a novel theoretical approach for uncovering discriminatory outcomes independent from discriminatory motives (i.e., independent from both prevailing models of discrimination in economics: taste-based and statistical)—these discriminatory outcomes arise even under an initial state of equality between majority and minority groups, unlike in previous work. In other words, not only does this paper develop a standard labor-market model that reveals the limitations of past economic models of discrimination, but also this work offers a direct rigorous account of what I term "social network discrimination" using mainstream economic theory.[4] Second, the model in the paper predicts that in the presence of race-blind hiring policies, minorities will be inherently disadvantaged, all else equal. This prediction applies to cases in which social structure and human capital are initially equal between majority and minority workers. Hence, the prediction may not apply to cases in which minority groups begin

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2. Various members of the Supreme Court have voiced this sentiment, along with some legal scholars, particularly in the context of "colorblind" policies. For economic analysis of the implications of various color-blind policies, see, e.g., Chan and Eyster (2003) and Ray and Sethi (2010).

3. Notwithstanding little change in recent support for redistribution despite rises in inequality (Ashok, Kuziemko, and Washington 2015), greater understanding of the fairness (or lack thereof) of the economic system might influence some people's preferences for redistribution (Alesina, Stantcheva, and Teso 2018).

4. "Social network discrimination" is a new term this paper introduces to the economics literature that captures the phenomenon in which racial minorities are treated differently in the distribution of opportunity due to distortions from social network dynamics. In this paper, one way social network discrimination manifests is as minorities receiving fewer job referrals, all else equal, simply because their social group is smaller. Though capturing a different concept, social network discrimination may relate to "institutional discrimination" in certain contexts, which Small & Pager Small and Pager 2020 defines as differential treatment by race that is either perpetrated by organizations or codified into law. Yet the social network discrimination introduced in this paper does not rely on organizations beginning as racially homophilous, unlike the example of institutional discrimination their paper presents. Their paper also provide a lengthy discussion on the divergence between economic and sociological approaches to discrimination.

with—for example—*more* education, employment, or social ties. Third, this paper performs equilibrium analysis of employment and wage differences caused by homophily along majority/minority status, which is a contribution to both the economic and sociological literatures on labor markets. In doing so, the paper isolates a potential underlying mechanism for inequality, adding to our understanding of racial disparities that have been widely studied across the social sciences. Fourth, the paper calibrates the model using a nationally-representative sample from the National Longitudinal Study of Adolescent to Adult Health. In this calibration exercise, white respondents represent the majority group and black respondents represent the minority group. Through calibration, I estimate that the lower bound welfare gap—i.e., the difference in expected wages—caused by social network discrimination is up to 3.2 percent, with black respondents disadvantaged compared to white ones. To the best knowledge of the author, there has not been any prior quantification of the magnitude of impact social network discrimination has in isolation on racial disparities.

Contributions to Literature. The model in this paper extends the one from Montgomery (1991) to incorporate two-dimensional heterogeneity: while the original model only groups workers by ability, this paper also groups them by majority/minority status. Doing so yields findings that go far beyond a mere application of the base model. The original model does not focus on outcomes when homophily (the well-documented tendency for people to associate more with others similar to themselves) exists along characteristics *uncorrelated* with ability—namely, on what effects emerge when social ties are formed along dimensions orthogonal to productivity. The original model does not incorporate demographic considerations.

Filling this gap is important. Within sociology, research has explored homophily in various contexts, including its causes (Wimmer and Lewis 2010; Leszczensky and Pink 2019) as well as how it influences friendships (Blau 1977; Syed and Juan 2012), interethnic marriages (Skvoretz 2013), and social inequality (DiMaggio and Garip 2012), among other areas. Rubineau and Fernandez (2013) shows how referral behavior can segregate jobs beyond the

influence of homophily. Within economics, there is an extant literature on homophily (see, e.g., Golub and Jackson 2012), and on the impact of referrals on inequality,[5] yet many inequality findings have relied on the existence of some degree of prior period inequality beyond majority/minority group size—for example, that if a demographic group has higher past employment, then that would yield an advantage in securing future jobs. This paper adds to insights from both fields, demonstrating that referral advantages may still unequally accrue over time even under initial equality between majority/minority groups, due to homophily. In particular, this paper adds a theoretical foundation for why homophily may contribute to inequality in referral markets,[6] as well as predicts under what conditions such disparities will *not* surface.

There has been increasing focus on uncovering mechanisms behind racial disparities in labor market outcomes (Bayer and Charles 2018; Chetty et al. 2020) and wage inequalities (Card and Lemieux 1994; Lemieux 2006). Evidence for racial differences in networking outcomes exists (Korenman and Turner 1996; Lalanne and Seabright 2011; Mengel 2015; Lindenlaub and Prummer 2016). Jackson (2009) discusses how homophily leads to segregation of groups, which leads to different equilibrium investment decisions in areas like education. Yet this paper introduces a more direct mechanism for inequality from homophily: that even given *equal* investments in human capital, homophily may still directly foster disparities through hiring dynamics.[7] Bolte, Immorlica, and Jackson (2020) finds that inequality due to homophily arises as a function of historical employment, whereas this paper does not rely on historical differences in labor market conditions. Similarly, unlike in Calvo-Armengol and Jackson (2004), which provides a network-based mechanism for inequality deriving from

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5. For some of the evidence, see Arrow and Borzekowski (2004), Ioannides and Datcher Loury (2004), Bayer, Ross, and Topa (2008), Hellerstein, McInerney, and Neumark (2011), Renneboog and Zhao (2011), Burks et al. (2015), Pallais and Sands (2016), Chandrasekhar, Morten, and Peter (2020), and Miller and Schmutte (2021). In a different setting than referrals, Dasaratha (2020) finds disproportionate benefits from peer effects due to homophily.

6. In contrast, Zeltzer (2020) presents empirical evidence to suggest that gender homophily is a significant factor in explaining the gender-wage gap among medical professionals.

7. Early versions of these findings can be found in the author's undergraduate honors thesis (see Okafor (2007)). Buhai and Leij (2020) explores how inequality can result from different choices in skill specialization, i.e., different investments in human capital.

differential drop-out rates, this paper uncovers how homophily can lead to inequality through the channel of employment opportunities themselves. Thus, this paper introduces a *direct* mechanism through which homophily can generate inequality that does not rely on historical differences between social groups, a contribution not previously found in the literature to the best knowledge of the author. In addition, this paper's findings add to ones from sociology that link homophily to social inequality (see, e.g., DiMaggio and Garip 2012); in this paper, in contrast, homophily's impact on inequality does not operate through a mechanism that exacerbates individual level differences (recall our model assumes groups have *equal* ability and initial employment). Notably, this paper's findings are also distinct from past economics and sociology research on the influence of more traditional discrimination in hiring (see, e.g., Bertrand and Mullainathan 2004; Pager, Bonikowski, and Western 2009). Unlike those articles, this one uncovers disparities even in contexts in which discriminatory motives and implicit biases are not only absent but impossible, as the model in this paper does not allow firms to distinguish who is a majority and who is a minority worker. Significantly, if the biases suggested by such prior research are incorporated—biases which tend to favor the majority group—the size of the disparities predicted in this paper's model would be even larger.

This paper proceeds as follows: Section 2 introduces a formal setup of the model. Section 3 presents the model's key findings for majority and minority workers. Section 4 provides discussion, including a description of (1) why the particular parameterization of homophily is used and is consistent with the contact hypothesis from psychology; (2) why other parameterizations are not used; and (3) model considerations in light of historical context. Section 5 performs a calibration of the model under simplifying assumptions to estimate the lower bound magnitude of social network discrimination in real-world settings. Section 6 concludes, introducing some of the key theoretical, practical, and legal implications of the findings.

2 Model

Here, I extend the Montgomery ([1991](#)) two-period model to incorporate two-dimensional heterogeneity: while the original model only groups workers by ability, this one also groups them by majority/minority status.[8]

Workers: I consider a labor market with two time periods ($t = 1$ and $t = 2$) and many workers, with an equal measure in each period.[9] Each worker works one period, and is one of two types: majority or minority. Each worker's type is predetermined and assigned before the period in which he or she enters the market. The fraction of majority workers is $\delta > \frac{1}{2}$, while $1 - \delta < \frac{1}{2}$ are minority. Similar to Montgomery ([1991](#)), I assume that $\frac{1}{2}$ of the workers within each type are high-ability, while $\frac{1}{2}$ are low-ability. High-ability workers produce one unit of output, while low-ability workers produce zero units. Workers are observationally equivalent: firms neither know what ability workers possess (before production), nor whether workers are of the majority or minority type (at any time).[10]

Firms: Firms are free to enter the market in either period. At most, each firm may employ one worker. A firm's profit in each period is equal to the productivity of its worker minus the wage paid.[11] Each firm must set wages before it learns the productivity of its worker. There are no output-contingent contracts.[12]

Structure of Social Network: As the focus of the model is referrals, now I describe how the social network through which referrals occur is drawn. As described, there are four categories of workers: high-ability majority, high-ability minority, low-ability majority, and low-ability minority.

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8. Most of the model's assumptions are standard in labor-market models of adverse selection, especially that of Greenwald ([1986](#)) (Montgomery [1991](#)).

9. Similar to Montgomery ([1991](#)), I simplify the analysis by examining the model as the number of workers approaches infinity.

10. If the majority/minority type is based on race, then this environment would correspond with a "race-blind" setting. Later we see that period-1 workers' actions are nonstrategic; hence, no assumption needs to be made on their knowledge of their own or of period-2 workers' types.

11. Product price is exogenously determined and normalized to unity.

12. This assumption captures a significant rationale for screening of job applicants and the use of referrals: the inability to fully tie compensation to productivity. See Montgomery ([1991](#)) and Greenwald ([1986](#)) for further discussion of this assumption.

Now let us represent each period-2 worker as an urn, and each social tie that a period-1 worker possesses as a ball.[13] The assignment of social ties is equivalent to a scenario where the balls are randomly dropped into the urns.[14] A period-1 worker possesses a social tie ("ball") with a probability equal to its majority/minority type's network density (denoted by τ_{maj} or τ_{min}).[15] A period-1 worker's sole social tie, if they have one, is dropped into an "urn" (period-2 worker) which is: (1) of the same ability with probability $\alpha \in (\frac{1}{2}, 1)$[16]; and (2) of the same majority/minority type with a probability determined by the period-1 worker's in-group bias.[17] The network structure is thus characterized by three parameters: network density (τ_{maj} and τ_{min}), majority/minority type in-group bias (denoted by ψ_{maj} and ψ_{min}, respectively), and ability in-group bias (α).

Timing: Each firm hires a period-1 worker through the market and learns his or her ability. As period-1 workers are observationally equivalent (and cannot be referred for jobs since there is no previous time period), each firm hiring through the market receives a high-ability worker with probability $\frac{1}{2}$.

After learning the ability of its current worker, each firm may set a referral offer to be relayed to the worker's social tie. Whether the referral offer is relayed is conditional on the firm's worker holding a social tie. If he or she does hold one, then the firm will only attract the acquaintance if the referral offer exceeds both the period-2 market wage and all other referral offers received by the acquaintance. A firm not wishing to hire through referral will set no referral offer (or might just set a referral offer below the period-2 market wage, which has no probability of acceptance). Period-2 workers then compare all offers received, accepting the highest.

All period-2 workers who receive no referral offers must find employment through the

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13. The "urn-ball" model is standard in probability theory and has been used in various economics models. For more background on the "urn-ball" model, see Shimer (2007).

14. Hence, period-2 workers can have zero, one or more than one social tie across period-1 workers.

15. If the period-1 worker is a majority worker, he or she possesses a social tie with probability $\tau_{maj} \in (0, 1)$; a minority worker possesses a social tie with probability $\tau_{min} \in (0, 1)$.

16. The matched period-2 worker is hence of a different ability with probability $1 - \alpha \in (0, \frac{1}{2})$.

17. The following subsection explains both the conceptualization and the parameterization of "in-group bias."

general market.

In summary, the timing of the game is as follows.

1. Each firm hires period-1 workers through the market at a wage of w_{M1}.

2. Period-1 production occurs, after which each firm learns the productivity of its worker.

3. Social ties are determined.

4. If a firm wishes to hire through referral, it sets a referral offer. I denote firm i's referral offer by w_{Ri}. (Conditional on having a social tie, each period-1 worker then relays his or her firm's wage offer (w_{Ri}) to their period-2 acquaintance.)

5. Each period-2 worker compares all wage offers received. They either accept one or wait to find employment through the general market.

6. Any period-2 worker with no offers (or who refuses all offers) goes on the market. Wages in this market are denoted w_{M2}.

7. Period 2 production occurs.

2.1 Implied Model of Referrals

A simplified implied model of referrals from the formal exposition of the social network structure above is as follows:

1. If asked to provide a referral by the employer, the employed worker in period 1 provides the referral offer to their social tie, if they have a tie.

2. The period-1 worker has a social tie with probability τ_{maj} (if period-1 worker is majority) or τ_{min} (if period-1 worker is minority).

3. The worker's social tie is determined from their direct contacts.

4. The pool of direct contacts for a period-1 high-ability *majority* worker is determined as follows:

    a. First, sample potential contacts at random from the population.

    b. For each potential contact, if they are also from the majority, keep them with probability ψ_{maj}. If they are in the minority, keep them with probability $(1 - \psi_{maj})$.

    c. The expected share of majority contacts is therefore:

$$\phi_{maj} = \frac{\delta \cdot \psi_{maj}}{[\delta \cdot \psi_{maj}] + [(1 - \delta)(1 - \psi_{maj})]}$$

    d. Finally, for each contact, if they are of high ability, keep them with probability α. If not, keep them with probability $1 - \alpha$.[18]

    e. The social tie is determined at random from their pool of contacts.

5. The pool of direct contacts for a period-1 high-ability *minority* worker follows the same steps (a)-(e), except for (b) and (c), which are modified accordingly:

    b. For each potential contact, if they are also from the minority, keep them with probability ψ_{min}. If they are in the majority, keep them with probability $(1 - \psi_{min})$.

    c. The expected share of minority contacts is therefore:

$$\phi_{min} = \frac{(1 - \delta) \cdot \psi_{min}}{[(1 - \delta) \cdot \psi_{min}] + [\delta \cdot (1 - \psi_{min})]}$$

6. Therefore, the probability that a high-ability majority worker has a social tie with another high-ability majority worker is $\tau_{maj}\phi_{maj}\alpha$. Similarly, the probability that a

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18. Since high-ability and low-ability workers each occupy $\frac{1}{2}$ of the population, the expected share of same-ability contacts simply equals α.

high-ability minority worker has a social tie with another high-ability minority worker is $\tau_{min}\phi_{min}\alpha$.

2.2   Note on Type In-Group Bias

Type "in-group bias" (ψ_{maj} and ψ_{min}) can be conceptualized as capturing the fact that there are shared attributes that simply make it easier for some workers to form social ties with each other than with others. For example, for a given chance encounter, one is more likely to form a social tie with another worker who shares a more similar background, because there are simply more elements in common to establish the foundation of a relationship—for example, consider the natural bonding that occurs when two people meet who grew up in the same neighborhood or who experienced the same cultural traditions. Hence, "in-group bias" does not represent favoritism toward a demographic group: in the model, any given worker views members of one's own majority/minority group and members of the other group equivalently, conditional on having a social tie. Similarly, conditional on *not* having a social tie, any given worker views members of the same majority/minority group and members of the other group with the same level of (dis)interest. Hence, although "in-group bias" deeply impacts network formation, it is fully distinct from (racial or group) animus, in-group favoritism, or traditional conceptions of taste-based preferences in economics.[19]

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19. Proposition 1 and the Discussion Section describe why the following specification for "in-group bias" is used: $\Pr\{\text{period-1 worker knows own majority/minority type}\} = \frac{w\cdot\psi}{[w\cdot\psi]+[(1-w)(1-\psi)]}$, where w is the share of the labor force for the worker type (either δ or $1-\delta$) and ψ is the type in-group bias of the worker type (either ψ_{maj} or ψ_{min}).

Figure I: Model Parameters

Parameter	Name	Description	Range
δ	Majority share	Share of the total labor force comprised of majority workers	$\delta \in (\frac{1}{2}, 1)$
$1 - \delta$	Minority share	Share of the total labor force comprised of minority workers	$1 - \delta \in (0, \frac{1}{2})$
α	Ability in-group bias	Probability a worker's social tie is with another worker of equal ability	$\alpha \in (\frac{1}{2}, 1)$
τ_{maj}	Network density (majority group)	Probability a majority worker has a social tie	$\tau_{maj} \in (0, 1)$
τ_{min}	Network density (minority group)	Probability a minority worker has a social tie	$\tau_{min} \in (0, 1)$
ψ_{maj}	Type in-group bias (majority group)	Bias a majority worker exhibits toward workers of the same group in favor of forming social ties. Value of 1/2 means no bias (probability of social tie with another majority worker = δ). Value of 1 means full bias (probability of social tie with another majority worker = 1).* I assume neither full nor no bias.	$\psi_{maj} \in (\frac{1}{2}, 1)$
ψ_{min}	Type in-group bias (minority group)	Bias a minority worker exhibits toward workers of the same group in favor of forming social ties. Value of 1/2 means no bias (probability of social tie with another minority worker = $1 - \delta$). Value of 1 means full bias (probability of social tie with another minority worker = 1).* I assume neither full nor no bias.	$\psi_{min} \in (\frac{1}{2}, 1)$

*$\psi \in [0, \frac{1}{2})$ represents heterophily, a rare social network phenomenon outside the scope of this paper.

3   Equilibrium

I examine a competitive equilibrium of the economy in which firms seek to maximize profits.

The first subsection below presents basic equilibrium properties shared with Montgomery (1991). The second subsection presents new propositions that I have added to the model by incorporating two-dimensional heterogeneiy (i.e., categorization of workers by majority

and minority type in addition to (instead of solely by) ability level). These new propositions relate to discrimination and labor market disparities.

3.1 Basic Equilibrium Properties

Basic equilibrium properties can be expressed via the following three lemmas, which establish that (1) referral wage offers are dispersed within an interval between the period-2 market wage and a maximum referral wage offer; (2) a firm will only hire through referral if it employs a high-ability worker in period 1; and (3) the period-1 market wage is greater than the expected period-1 productivity. More discussion on each of these points is included below. Omitted proofs are found in the Appendix.

Lemma 1 *Referral wage offers lie within the interval between w_{M2} and a maximum referral wage offer $\overline{w}_R$; hence, $w_R \in [w_{M2}, \overline{w}_R]$. The density of the referral wage offer distribution is positive across this entire range.*

Proof. Claim 4 in Burdett and Judd (1983) establishes the existence and uniqueness of an equilibrium, while Theorem 4 proves wage dispersion exists in the equilibrium (since the probability that a period-2 worker receives exactly one referral offer is strictly between 0 and 1). Given this wage dispersion, the market wage (w_{M2}) must coincide with the bottom of the referral wage distribution, as any referral offer below the market wage will necessarily be rejected by workers in favor of going to the general market. At the maximum referral wage offer (denoted $\overline{w}_R$ and derived in Appendix Equation A.7), the probability a worker accepts the referral offer is 1. Hence, firms will not offer a referral wage above this amount as it will necessarily reduce profits. Proposition 2.2 in Butters (1977) proves that there are no gaps in the wage distribution. If there were a gap between some w_1 and w_2, then a firm offering the higher wage could reduce its offer by ϵ without reducing the probability its offer is accepted, thereby increasing its profits. ∎

Lemma 2 *A firm will attempt to hire through referral if and only if it employs a high-ability worker in period 1.*

This result follows from the ability in-group bias (α). Hiring through the market yields zero expected profit (due to the free entry of firms and the symmetric lack of information on the ability of workers). For firms employing high-ability workers in period 1, an accepted referral offer yields constant positive profit over the range of the referral offer distribution $[w_{M2}, \overline{w}_R]$. Higher wage offers yield a higher probability of attracting a period-2 worker. Firms employing low-ability workers in period 1 will not hire through the referral market, since the ability in-group bias means the referred worker will more likely also be low-ability.

As a result of this lemma, a disproportionately high number of low-ability workers find employment through the general market. This drives the market wage below the average productivity of the entire population. However, adverse selection does not eliminate the market. Since some high-ability workers are not "well-connected," they fail to receive referral wage offers, which leads them to find employment in the general market. Thus, the market wage remains above zero.

Lemma 3 *The period-1 market wage is greater than the expected period-1 productivity.*

If a firm obtains a high-ability worker in period 1, they expect positive period-2 profits. This fact drives the period-1 market wage higher than the productivity of the population. This wage can be viewed as comprising the average productivity of the worker plus an "option value" of a period-2 referral. This option will be exercised if the period-1 worker reveals themselves to be high-ability (which occurs after period-1 production concludes, if the worker is in fact high-ability).

3.2 Propositions on Social Network Discrimination

New propositions reflecting social network discrimination and inequality are detailed below.[20] The propositions establish that minority workers receive a disproportionately low fraction of job offers through referral and a lower expected wage, all else equal. Recall that the market wage lies below the referral offer distribution. Hence, all these effects taken together yield a welfare gap between minority and majority workers in period 2 that did not exist in period 1. Omitted proofs are in the Appendix.

Proposition 1A *In an environment with equal magnitude of majority/minority network parameters ($\tau_{maj} = \tau_{min}$ and $\psi_{maj} = \psi_{min}$), the probability that, among referral offers, a minority worker is referred is lower than their share of the labor force. Conversely, the probability that, among referral offers, a majority worker is referred is larger than their share of the labor force.*

Proposition 1B *The inequality in the distribution of referral job offers can be eliminated by minority workers having a sufficiently higher type in-group bias (ψ_{min}).*

Simplified Numerical Example: First I explore the intuition of Proposition 1A by using a simple numerical example that relaxes some of the assumptions of the model to illustrate how type in-group bias operates.[21] Suppose there are two majority workers in both periods, and one minority worker in both periods—i.e., $\delta = 2/3$. Also suppose there is type in-group bias: each worker has a bias in favor of forming social ties with workers of the same majority/minority type. To illustrate this bias, let us say that for encounters between majority period-1 and majority period-2 workers, there is a 2/3 chance of forming a tie, whereas encounters between a majority period-1 and minority period-2 worker has a 1/3 chance of

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20. An interactive plot that shows how social network parameters influence the distribution of referral offers received between majority and minority groups can be found at https://justiceinsights.shinyapps.io/social_network_discrimination/

21. This example is illustrative and does not incorporate ability heterogeneity (i.e., it assumes all workers are high-ability), so is simplified relative to the main model. Furthermore, the example includes a finite number of workers whereas the model includes a continuum.

forming a tie—i.e., $\psi = 2/3$. Suppose all three period-1 workers encounter all three period-2 workers. The expected number of ties period-1 majority workers form with their own type is thus 4/3 (while the expected number of ties with minority workers is 1/3); this means the fraction of same-type social ties for majority workers is 4/5. Let $\phi_{maj} = 0.8$ denote this fraction of same-type social ties (note that ϕ is not a parameter of the model and can be calculated directly from ψ via Equation 1 below). Similarly, it is straightforward to calculate that the fraction of same-type social ties for the period-1 minority worker is $\phi_{min} = 0.5$.

Now let us apply these bias dynamics to a case that more closely conforms with the assumptions of the model—in which period-1 workers each have exactly one social tie with probability 1 (i.e., $\tau_{maj} = \tau_{min} = 1$). The fraction of referral job offers going to majority workers is simply a weighted sum:

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\begin{aligned}
&\Pr\{\text{referral job offer goes to majority per-2 worker}\} \\
&= \Pr\{\text{per-1 worker is majority}\} \cdot \Pr\{\text{per-1 majority knows per-2 majority}\} \\
&\quad + \Pr\{\text{per-1 worker is minority}\} \cdot \Pr\{\text{per-1 minority knows per-2 majority}\} \\
&= \delta \cdot \tau_{maj} \cdot \phi_{maj} + (1 - \delta) \cdot \tau_{min} \cdot (1 - \phi_{min}) \\
&= 2/3 \cdot 1 \cdot 0.8 + 1/3 \cdot 1 \cdot 0.5 \\
&= 0.7
\end{aligned}
$$

Hence, only 0.3 of referral job offers go to minority workers, even though they occupy 1/3 of the labor force. This simple example illustrates the distorting influence of having the same magnitude of bias operating on groups of different sizes. The bias in favor of a same-group social tie for a majority worker extends toward a greater fraction of the population than does the same magnitude of bias for a minority worker. Hence, equal magnitudes of bias *unequally* impact the respective chances of knowing workers of the same type. [**End of Example**]

I can generalize both the reasoning of the simplified numerical example above and the

type-in group bias of the model as follows:

(1) $\Pr\{\text{period-1 worker knows own maj/min type}\} = \phi = \dfrac{w \cdot \psi}{[w \cdot \psi] + [(1-w)(1-\psi)]}$

where w is the share of the labor force for the worker type (either δ or $1 - \delta$) and ψ is the in-group bias of the worker type (either ψ_{maj} or ψ_{min}). $\psi = 0.5$ reflects no bias—i.e., a proportional chance of a social tie being with another of the same type—and $\psi = 1$ reflects when all ties are with members of the same type. ϕ represents either ϕ_{maj} or ϕ_{min}, depending on whether the period-1 worker belongs to the majority or minority group, respectively. The Discussion section further explores the relationship between ψ and ϕ, and its implications on the results.

In the Appendix, I prove that parity in the distribution of job offers between high-ability majority and minority workers is accomplished only when:

(2) $(1-\delta)\left[\delta\tau_{maj}\phi_{maj} + (1-\delta)\tau_{min}(1-\phi_{min})\right] = \delta\left[\delta\tau_{maj}(1-\phi_{maj}) + (1-\delta)\tau_{min}(\phi_{min})\right]$

where ϕ_{maj} and ϕ_{min} are calculated from Equation 1 above. From Equation 2, one can calculate what magnitude other parameters must be for parity. I denote such parameters for minority workers as $\tau_{min}^=$ (compensating network density) and $\psi_{min}^=$ (compensating in-group bias). Figure II illustrates that a sufficiently high network density or type in-group bias can mitigate the disproportionality in the distribution of job offers through referral. All else equal, minority workers can either have more social ties ($\tau_{min}^= > \tau_{maj}$) or a "stronger-knit" social network ($\psi_{min}^= > \psi_{maj}$).

The network density required to eliminate the inequality ($\tau_{min}^=$) increases in τ_{maj}, ψ_{maj}, and δ. It decreases in ψ_{min}, which can readily be understood intuitively. The greater the probability of majority workers having social ties (and/or the greater the degree of their homophily), the greater minority workers' compensating parameters ($\tau_{min}^=$ or $\psi_{min}^=$) must be to achieve a proportional amount of all job offers through referrals. In Figure II, the plot

Figure II: Magnitude of Minority Group Network Parameters
Required for Parity in Referral Job Offers[*]
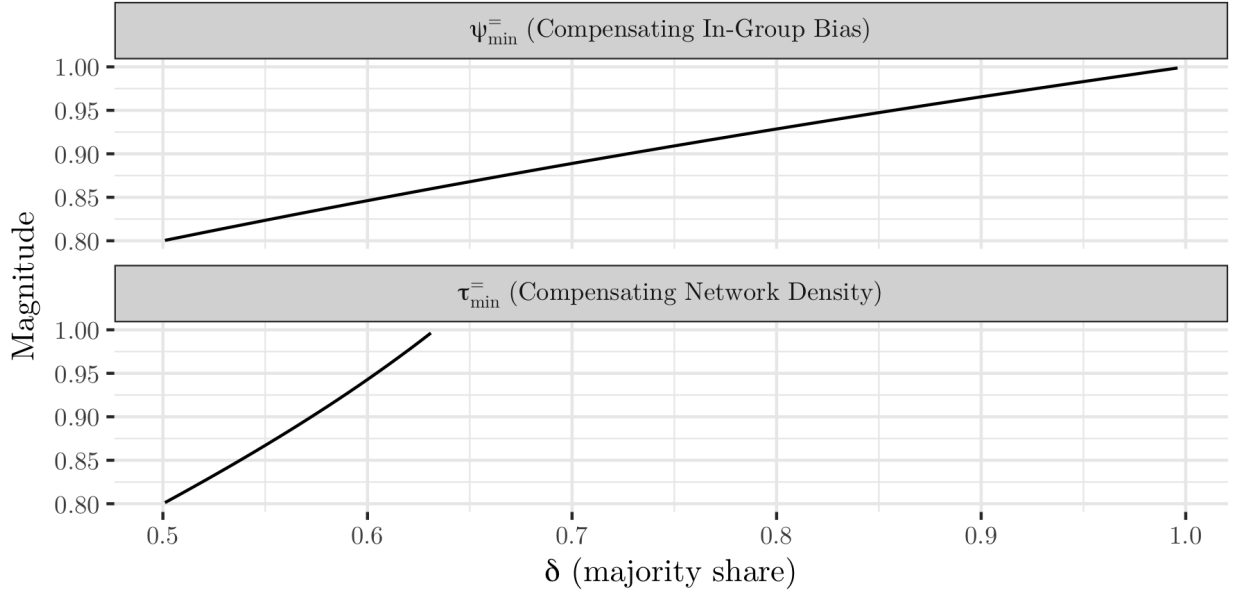
[*]In each chart, all other relevant network parameters $= 0.8$.

of $\tau_{min}^{=}$ has no values when δ is greater than approximately 0.63. This is because there is no attainable magnitude of network density that will yield parity in the distribution of job offers when δ surpasses this threshold.

Proposition 2 *In an environment with equal magnitude of majority/minority network parameters ($\tau_{maj} = \tau_{min}$ and $\psi_{maj} = \psi_{min}$), the period-2 market wage (w_{M2}) decreases as majority workers occupy a greater fraction of the labor force. w_{M2} also decreases in the ability in-group bias α.*

Recall that workers who do not receive jobs through referral must find employment through the market. Proposition 1 shows that minority workers, all else equal, receive a disproportionately low fraction of job offers through referral, and thus disproportionately find employment through the market. Decreases in the market wage (w_{M2}) thereby hurt the average welfare of minority workers, relative to that of majority workers.

The Appendix includes the expression for w_{M2}. Given $\alpha > \frac{1}{2}$, w_{M2} is always less than $\frac{1}{2}$, the average productivity of the population. Analysis shows that w_{M2} is decreasing in α. For

all $\psi_{maj} = \psi_{min}$ and $\tau_{maj} = \tau_{min}$, w_{M2} also decreases in δ.

Proposition 3 *In an environment with equal magnitude of majority/minority network parameters ($\tau_{maj} = \tau_{min}$ and $\psi_{maj} = \psi_{min}$), the referral wage and welfare (i.e., average expected wage) for minority workers is lower than for majority workers.*

Much of the intuition behind this finding follows from Proposition 1 (that majority workers receive a disproportionately high number of job offers through referral). There are two margins to consider. First, the extensive margin: majority workers disproportionately get hired through the referral market (which provides higher wages than the general market), driving up expected welfare for the majority group. Second, the intensive margin: recall that workers accept the maximum referral wage offer received. Hence, by majority workers receiving a higher number of referral offers, their expected maximum offer increases, thereby also driving up their relative welfare.

Let $E \prod_H (w_R)$ denote the expected period-2 profit earned by a firm employing a high-ability worker and setting a referral wage. To maintain equilibrium wage dispersion, firms must earn the same expected profit on each referral wage offered:

$$E \prod_H (w_R) = c \qquad \forall w_R \in [w_{M2}, \overline{w}_R]$$

Given the expression for c derived in the Appendix, firms with high-ability workers who have social ties earn positive expected profits as long as $\alpha > \frac{1}{2}$. Analysis shows that c is increasing in τ_{maj} and τ_{min}. Furthermore, the equilibrium referral-offer distribution $F(\bullet)$ may be determined by setting $E \prod_H (w_R)$ equal to c for all potential wage offers w_R.

Unfortunately, doing so does not yield a closed-form solution for $F(w_R)$. Given a continuum of firms, the equilibrium referral-offer distribution $F(\bullet)$ can be interpreted as either: 1) each firm randomizes over the entire distribution; or 2) a fraction $f(w_R)$ of firms offers each wage for sure. From the second interpretation, I denote these referral wages with w_{Rk}. One can then derive an expression for $w_{Rk}(\alpha, \delta, \tau_{maj}, \tau_{min}, \psi_{maj}, \psi_{min}, F(w_{Rk}))$ and calculate

an average referral wage received by a majority worker (denoted $E(w_{R_{Hmaj}})$) vs. a minority worker (denoted $E(w_{R_{Hmin}})$), for any given δ, α, τ_{maj}, τ_{min}, ψ_{maj}, and ψ_{min}.

Analysis shows that when majority/minority network parameters are the same magnitude, if $\alpha > \frac{1}{2}$ and $\delta > \frac{1}{2}$, $E(w_{R_{Hmaj}}) > E(w_{R_{Hmin}})$. In other words, the expected referral wage for high-ability majority workers is greater than for high-ability minority workers.

Figure III: Estimated Welfare Gap of Minority Workers[*]
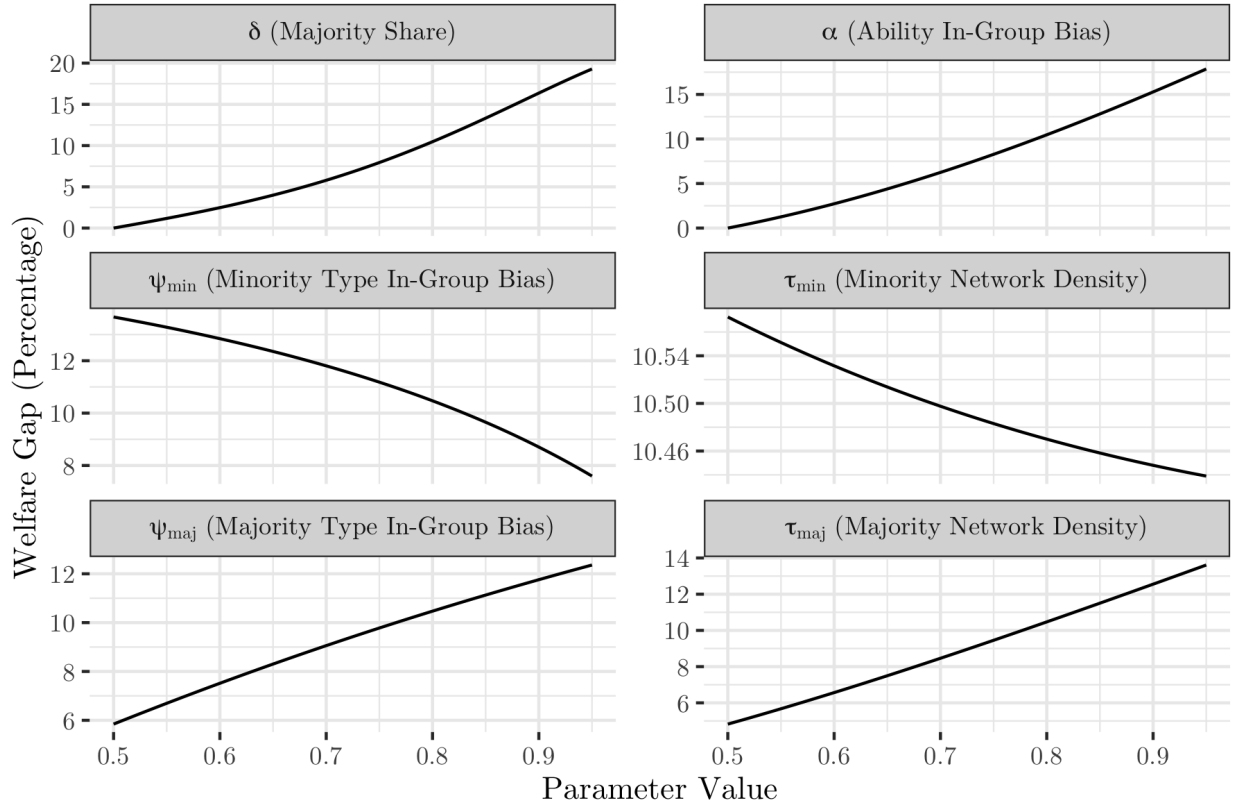
Welfare (i.e., expected wage) is calculated by summing the market wage (w_{M2}) and expected referral wage $(E(w_{R_{Hmaj}})$ or $E(w_{R_{Hmin}}))$, weighted by the likelihood of the worker's type gaining employment through the market or through referrals, respectively. Figure III plots the estimated welfare gap between majority and minority workers as a function of various network parameters.[22] The welfare gap increases in δ, α, ψ_{maj}, and τ_{maj}. It decreases

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22. Estimation of wage gap normalizes to 1 the number of offers that referred high-ability minority workers receive and assumes a uniform distribution across the referral wage distribution ($w_R \sim U(w_{M2}, \overline{w}_R)$).

in ψ_{min} and τ_{min}. Of note, the welfare gap increases convexly as the majority group occupies a greater share of the labor force.

4 Discussion

If the majority/minority type in the model is based on race, then the setting would correspond with a "race-blind" or "colorblind" setting. This is because workers are observationally equivalent, and so firms cannot incorporate race into hiring decisions. Due to the historical context of race in the United States, past research has found bias *in favor of* the majority group (see, e.g., Bertrand and Mullainathan (2004)). The presence of such bias would exacerbate the disparities already predicted by the model in this paper. The rest of this discussion section will first consider the choice behind the parameterization of homophily (i.e., in-group bias). Next, this section will consider why potential alternative parameterizations were not used. Lastly, this section will consider the model's implications given the historical context of inequality.

4.1 Why this homophily parameterization is used

A key assumption of this paper's parameterization of homophily is that the probability of connection with a particular social group is increasing in that group's population share. This assumption is reasonable due to at least two points: (1) the intuition introduced by the simplified numerical example following Proposition 1 in Section 3.2; and (2) empirical research on the contact hypothesis from psychology. Regarding the first point: as the numerical example illustrates, since social connections form from interpersonal interactions, the more interpersonal interactions an individual has with members of a particular social group, the more social connections the individual is expected to form with that group. Hence, the key assumption holds: the probability of connection with a particular social group would be increasing in that group's population share. Regarding the second point: the key parame-

terization assumption is consistent with empirical research on the classic contact hypothesis from psychology. The contact hypothesis suggests that prejudice and segregation is decreased through increased contact between in-group members and out-group members (Allport 1954). Empirical research has found that as the quantity of contact with other social groups increases, cross-group relationships are fostered (see, e.g., Hewstone et al. 2018; Laurence et al. 2019). Hence, the parameterization assumption that the probability of a connection with a particular social group is increasing in that group's population share holds.

Given the key assumption, this paper employs a parameterization of homophily that implies the relationship between Pr{Period-1 worker knows own majority/minority type} and in-group bias is not linear. Instead, the relationship depends on population share. To understand why, consider an individual worker from each social group, as was done in the Simplified Numerical Example in Section 3.2. For a majority worker, any given magnitude of bias in favor of same-group social ties should operate on a greater share of the population than it does for a minority worker, since by construction a greater proportion of the population consists of majority workers. Hence, one would expect the parameterization of homophily to have some multiplicative relationship with the share of the labor force, which a simple linear parameterization would not have.[23] Furthermore, one would expect any given magnitude of homophily to have an amplified effect on the likelihood of having a same-group social tie for a given majority worker compared to a given minority worker.

The relationship between this paper's parameterization of in-group bias and the incidence of social ties can be seen in Figure IV. Specifically, Figure IV illustrates the sensitivity of same-group social ties (ϕ) to the in-group bias parameter (ψ). This relationship is based on the expression linking share of the labor force (w) to in-group bias described in Equation 1

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23. See the Conclusion for several examples of common real-world settings in which one would expect these social network dynamics to take place.

Figure IV: Social Tie Sensitivity to In-Group Bias[*]
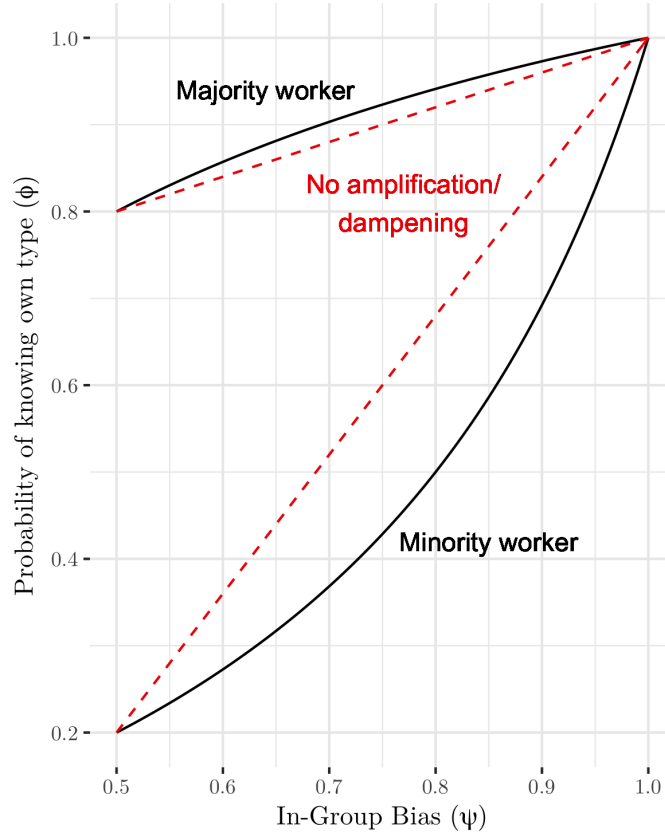
[*]where majority share of the labor force is 80%

of Proposition 1:

$$\Pr\{\text{period-1 worker knows own maj/min type}\} = \phi = \frac{w \cdot \psi}{[w \cdot \psi] + [(1 - w)(1 - \psi)]}$$

(The Simplified Numerical Example immediately following Proposition 1 in Section 3.2 illustrates the logic of this expression.) When $\psi = 0.5$, there is no bias (i.e., probability of social ties with the same type $= w$); when $\psi = 1$, there is full bias (i.e., probability of social ties with the same type $= 1$). The dashed line represents a linear scaling, in which there is no amplification/dampening effect for majority and minority workers. Though not included in the graph below, the specification used in this article yields a linear relationship if both workers' groups occupy 50% of the labor force (i.e., when social groups are the same size there is no amplification/dampening effect).

The findings in this paper related to the unequal distribution of referral offers are robust to specifications for bias—e.g., Equation 1 from Proposition 1—where the relationship between the probability of knowing one's own type (ϕ) and the bias (ψ) is more concave for majority workers than for minority workers. In other words, it is robust to specifications where there is a comparative amplification effect on the incidence of same-group social ties for majority workers (as seen in Figure IV). This relationship captures the fact that an equal magnitude of bias has a disproportionately larger impact on majority workers than on minority workers.

4.2   Why another homophily parameterization is not used

One might propose that instead of the parameterization I employ, one could simply define the homophily measures ψ_{maj} and ψ_{min} as the probability of referring another worker of the same type, conditional on having a social tie.

Under this alternative parameterization, for a given period-2 majority worker of high ability, the probability of receiving a referral from a specific period-1 majority worker would be $\frac{\tau_{maj}\alpha\psi_{maj}}{\delta N}$. The numerator measures the probability that the referral is made to a period-2 majority high-ability worker, and the denominator is the probability that the offer is made to the specific period-2 worker from the pool of all available period-2 high-ability majority workers, of which there are δN.

There are δN period-1 high-ability majority workers, so the expected number of referrals to a given period-2 worker of the same type is $\tau_{maj}\alpha\psi_{maj}$. This is the expected value of δN Bernoulli trials where each trial succeeds with probability $\frac{\tau_{maj}\alpha\psi_{maj}}{\delta N}$.

By similar logic, the expected number of referrals from period-1 high-ability minority workers to period-2 high-ability majority workers is:

$$\frac{(1-\delta)\tau_{min}\alpha(1-\psi_{min})}{\delta}$$

The expected total number of referrals made to any period-2 high-ability majority worker is the sum:

$$(1) \qquad \tau_{maj}\alpha\psi_{maj} + \frac{(1-\delta)\tau_{min}\alpha(1-\psi_{min})}{\delta}$$

By similar logic, the expected number of offers made to any given period-2 high-ability minority worker is:

$$(2) \qquad \tau_{min}\alpha\psi_{min} + \frac{(\delta)\tau_{maj}\alpha(1-\psi_{maj})}{1-\delta}$$

If we set $\tau_{maj} = \tau_{min}$ and $\psi_{maj} = \psi_{min}$, it is easy to see that the expression in (2) is greater than (1) when $\delta > \frac{1}{2}$. However, this alternative parameterization of homophily suffers serious shortcomings. To see why, consider the following two scenarios: (A) the majority group occupies 99% of the population and $\psi_{maj} = \psi_{min} = 0.99$, which would mean under this alternative parameterization that workers refer the same type 99% of the time; and (B) the majority group only occupies 51% of the population and $\psi_{maj} = \psi_{min} = 0.99$.

Despite capturing the likelihood of same-group referrals in both scenario (A) and (B), ψ *under this alternative parameterization does not meaningfully reflect the level of bias exhibited by workers in forming same-group social ties.* For example, a 99% likelihood of referring the same group when one is the overwhelming majority has a significantly different interpretation than a 99% likelihood of referring the same group when one is barely the majority, let alone when one is the minority. As such, $\psi_{maj} = \psi_{min}$ does not reflect equal homophily between groups under this alternative model. Thus, a homophily parameterization that does not explicitly incorporate the relative sizes of the in-group and out-group in determining the likelihood of same-group social ties suffers severe shortcomings. The parameterization I use in this paper avoids these shortcomings through the relationship of ψ with ϕ, and represents a far superior measure.

4.3 Model implications in historical context

Recall that the predicted inequality between majority and minority workers is based on several assumptions, which include: (1) majority and minority workers have no labor market disparities in the initial time period; (2) the only distinguishable difference between groups is relative size (i.e., ability, network density, and in-group biases are all equivalent); (3) workers are more likely to know others with similar characteristics; and (4) there is no psychological prejudice. These assumptions are critical when considering historical examples where minority workers enjoy *greater* welfare than majority workers (e.g., white South Africans), or when particular demographic groups who comprise a majority of a local labor market face worse outcomes (e.g., black workers in a variety of U.S. metropolitan areas). These cases do not undermine the accuracy of the model, not only because their circumstances clearly violate the model's assumptions (e.g., that there is full equality between groups in the initial time period), but also because these cases intimately involve the distorting influence or legacy of psychological prejudice, which the employment model explicitly and intentionally does not incorporate.

5 Calibration

This section calibrates the model to assess, in one application, the magnitudes of model parameters—such as majority and minority type in-group bias and network density—and to estimate a lower bound for the magnitude of inequality arising from social network discrimination. The setting for this calibration exercise is the Public Use data from the National Longitudinal Study of Adolescent to Adult Health, 1994–2008 (Harris and Udry 2022). This data consists of a nationally representative sample of U.S. adolescents in grades 7 through 12 during the 1994–1995 school year. In the friendship section of the in-school questionnaire, which was administered to over 90,000 students attending 145 schools in 80 communities, the respondents were asked to nominate up to five male and five female friends from the roster

of all students enrolled. This information was used to construct social network parameters, which are adapted below to estimate the parameters in this paper's model. Further details about the overall sample and design of the study are provided in Resnick et al. (1997). I make some simplifying assumptions for the purpose of this calibration, such as assuming that the social network parameters of the respondents do not change from childhood through their eventual entry into the labor market.

5.1   Calibrated Social Network Parameters

For the purpose of this calibration exercise, white respondents represent the majority and black respondents represent the minority. Here I define some of the terms used below in the calibration data: *ego* means respondent; *alter* means the student in the same school as ego who is eligible to be nominated as a friend; *node* means a unique member of a network; and *ego's send-network* means the ego and the set of alters nominated by the ego as friends. Social network parameters are calculated as follows:

Majority share (δ)—This parameter is calculated by taking the total count of white respondents in the calibration data and dividing it by the total count of white respondents plus black respondents. Based on this definition, $\delta = 0.70$.

Ability in-group bias (α)—I vary this parameter between 0 and 1.[24]

Network density (τ_{maj} and τ_{min})—Marsden (1987) estimates the network density for whites at 0.61 and blacks at 0.63.[25]

Type in-group bias (ψ_{maj} and ψ_{min})—These parameters are calculated based on the calibration data parameter "ego-network heterogeneity measure for race." The ego-network

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24. As illustrated in the Proof for Proposition 1B in the Appendix, the ability in-group bias does not impact the magnitude of social network parameters that is needed to achieve parity between majority and minority workers in the distribution of referral offers.

25. This may represent a conservative difference between τ_{maj} and τ_{min}. Marsden (1987) also estimates the white population has significantly larger network sizes (mean size 3.1) than the black population (mean size 2.25). This means that the average white worker has a greater probability of having a social tie (and thus receiving a referral). Since our model sets a maximum of one social tie per period-1 worker, to account for this real-life difference in network sizes between races one might widen the gap in the network density parameters τ_{maj} and τ_{min} (which has not been done for the purposes of this calibration).

heterogeneity assesses the heterogeneity of the respondent's network with respect to race. The formula used to calculate the ego-network heterogeneity for respondent i is:

$$HETEROGENEITY_{iR} = 1 - \left[\sum_{k=1}^{n} \left(\frac{R_k}{d} \right)^2 \right] ,$$

where:

R = the race attribute

R_k = the number of nodes with race k in the ego network

d = the number of nodes in the ego network with valid data on R

n = the total number of races of R represented in the ego network

If all members of the respondent's network who have valid data on race share the same race, $HETEROGENEITY_{iR} = 0$.[26] I perform a simple transformation of $HETEROGENEITY_{iR}$ and take the mean to estimate the parameters ϕ_{maj} and ϕ_{min} (which represents the probability that a period-1 worker knows own maj/min type):

$$\phi_{maj} = mean \left(1 - HETEROGENEITY_{iR \text{ for } i \in \text{white respondents}} \right) , \text{ and}$$

$$\phi_{min} = mean \left(1 - HETEROGENEITY_{iR \text{ for } i \in \text{black respondents}} \right)$$

This gives $\phi_{maj} = 0.74$ and $\phi_{min} = 0.66$. Then, I solve for ψ_{maj} and ψ_{min} by adapting Equation 1 from Proposition 1:

(1) Pr{white respondent refers white person} $= \phi_{maj} = \dfrac{\delta \cdot \psi_{maj}}{[\delta \cdot \psi_{maj}] + [(1 - \delta)(1 - \psi_{maj})]}$

(2) Pr{black respondent refers black person} $= \phi_{min} = \dfrac{(1 - \delta) \cdot \psi_{min}}{[(1 - \delta) \cdot \psi_{min}] + [\delta \cdot (1 - \psi_{min})]}$

Substituting parameter values and solving gives $\psi_{maj} = 0.55$ and $\psi_{min} = 0.82$.

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26. $HETEROGENEITY_{iR}$ is missing if respondent is the only member of their underlying network, or if all members of the respondent's network (including themselves) have missing data on race.

5.2   Inequality Arising From Social Network Discrimination

Let us now take the calibrated model parameters, calculated above from a nationally-representative sample of social networks among white and black respondents, to estimate the degree of economic inequality deriving purely from social network discrimination. It is worth noting that the inequality predicted in this subsection is based on the counterfactual world introduced by the model: namely, that there is equal average ability, education, and initial employment between white and black workers. I assume that half of white respondents and half of black respondents are high-ability, and half of both racial groups are low-ability. Hence, the assumption moving forward is that the only difference between racial groups is in their social network parameters, not in the distribution of any attribute correlated with productivity or ability.

I have already calculated all relevant unknown parameters of our model in the previous subsection: $\delta = 0.70$, $\tau_{maj} = 0.61$, $\tau_{min} = 0.63$, $\psi_{maj} = 0.55$, and $\psi_{min} = 0.82$.

Next, I seek to determine the welfare gap—the difference in expected wage—between high-ability white and black respondents. Let H_{maj} denote high-ability majority worker, H_{min} denote high-ability minority worker, L_{maj} denote low-ability majority worker, and L_{min} denote low-ability minority worker. The steps I take to calculate the expected wage gap between high-ability black and white workers is as follows:

1. Estimate the likelihood that a high-ability white worker accepts a referral wage, relative to the likelihood a high-ability black worker accepts a referral wage;

2. Estimate the market wage and the expected referral wage for both racial groups;

3. Estimate the expected welfare gap between black and white workers by summing the market wage w_{M2} and expected referral wage $E(w_{R_k})$, weighted by the likelihood of the worker's racial group gaining employment through the general market or through the referral market, respectively.

Computational details on these steps can be found in Appendix 6.

After completing Steps 1 through 3, substituting calibration parameter values gives a welfare gap—which is the difference in expected wages in this model. As I allow ability in-group bias to vary, the welfare gap varies in Figure V from 0 to 3.2 percent between racial groups, with black workers being disadvantaged compared to white workers. This welfare gap is driven by two factors: (1) the disproportionately high likelihood that black workers will be hired through the (non-referral) general market, which pays the lowest wage on the equilibrium wage distribution; and (2) the disproportionately high volume of referrals white workers receive, which increases the maximum referral wage white workers are able to eventually accept compared to black workers.

Figure V: Lower-Bound Welfare Gap from Social Network Discriminatin (Black vs. White Workers)[*]
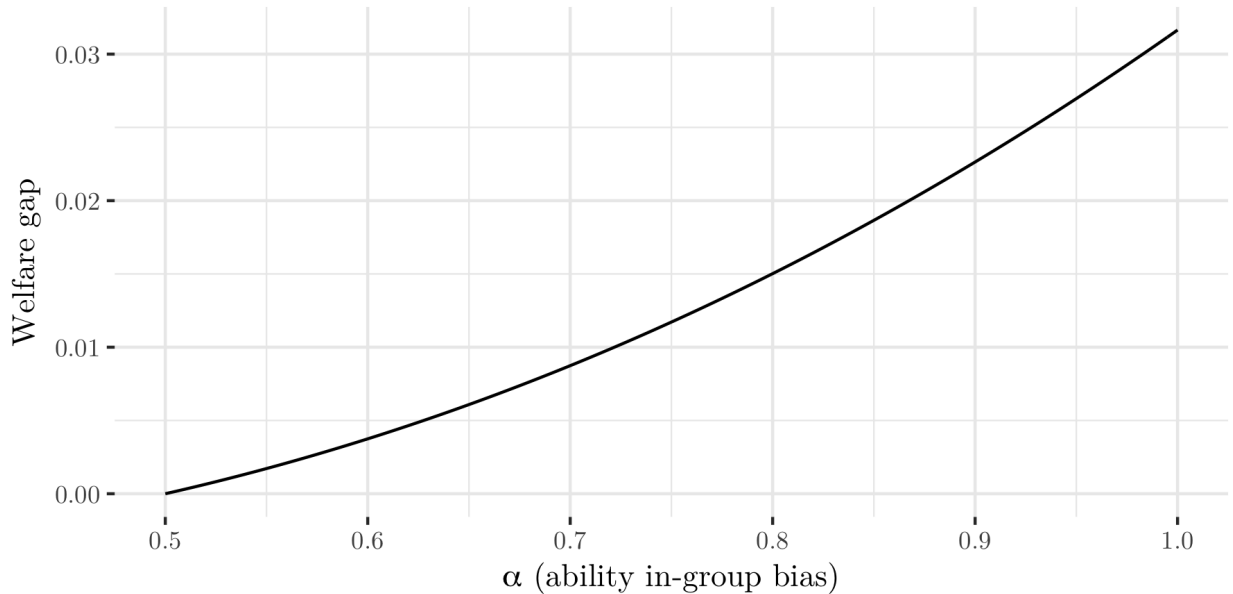
It is important to remember that the estimated welfare gap between black and white workers is based on a key simplifying assumption of the model: namely, that both the majority and minority groups begin the first period in a state of equality. In other words, the maximum welfare gap in Figure V of 3.2 percent is what the model estimates over time given an initial state of equality between racial groups. Yet in the U.S. context, initial

equality between racial groups did not exist; black workers not only may be negatively impacted today by persisting social network discrimination (as suggested above), but also they have been harmed historically by the remnants of past discrimination (which is outside the scope of this paper's analysis). Historical discrimination would plausibly exacerbate the inequalities in referral opportunities between black and white workers—even conditional on the same magnitude of the underlying social network parameters. As such, the estimated welfare gap from the calibration exercise may represent a lower bound of the true extent to which social network discrimination contributes to racial inequality in the U.S. context.

6 Conclusion

The findings in this paper have significant theoretical, practical, and legal implications.

Theoretical. This paper names the phenomenon called social network discrimination and reveals the shortcomings of referral-based systems and related hiring practices from a racial justice perspective. Despite initial equality in ability, employment, wages, and network structure—minorities receive fewer jobs through referral and lower expected wages, simply because their social group is smaller. This discriminatory outcome falls outside the dominant economics discrimination models—taste-based and statistical. I calibrate the model and estimate the lower-bound welfare gap caused by social network discrimination is up to 3.2 percent, disadvantaging black workers. This paper isolates a potential underlying mechanism for inequality, adding to the understanding of labor market disparities that have been widely studied across the social sciences.

Practical. By identifying and terming "social network discrimination," this paper introduces further important practical implications. Research has shown that theories of discrimination, and the language used in communicating them, can rationalize or challenge stereotypes, as well as shape the attitudes and behaviors of influential members of society (Tilcsik 2021). Much popular U.S. debate already derides race-conscious policies as a

deviation from merit or fairness; a wider awareness of social network discrimination could challenge these perceptions, highlighting that the root of persistent racial disparities may be more complex than previously understood.

Social network discrimination is applicable to a wide variety of real-world contexts beyond simply referrals for job opportunities. Opportunities within a school can be unfairly distributed due to social network effects; the findings from this paper can be reasonably generalized to other settings in which opportunities are distributed based on informal information channels. For example, the basic dissemination of personal, educational, and career guidance and wisdom that is a hallmark of the college experience can similarly be unfairly distributed due to social network discrimination. Some peers may have greater access to information simply because they belong to a larger social group.

Important social ties are formed in postsecondary schooling (e.g., colleges and universities), in professional schools (e.g., business schools, law schools, and medical schools), and in workplaces. This paper suggests there are inherent social network advantages associated with belonging to a larger demographic group, all else equal, since the formation of such ties are influenced by social phenomena like homophily. In short, social network discrimination may distort the allocation of personal, educational, and professional opportunities in significant ways that foster racial disparities—an important insight that not only may help explain substantial inequalities that persist between demographic groups today, but also may help inform efforts to promote diversity and equitable access to opportunity in the future.

Legal. This paper disproves a persisting argument that race-blind or "colorblind" policies are inherently merit-enhancing, introducing a new rationale for race-conscious policy. In this paper's model, employers cannot identify which workers are majority and which ones are minority—i.e., employers operate fully under color-blind hiring policies—yet still disparities arise between demographic groups. In enforcing Title VII of the Civil Rights Act, the EEOC prohibits employers from using neutral employment policies and practices that have a disproportionately negative impact on applicants or employees of a particular color, race,

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age, sex, national origin, or disability status. Referral-based hiring may constitute in some contexts such a policy with disproportionately negative impact. Furthermore, social network discrimination is relevant for ongoing legal (and popular) debates surrounding affirmative action—both in the workplace and perhaps also in higher education. To date, the Court has not meaningfully considered remedying social network discrimination as a rationale for race-conscious policy. The specific contours of this argument I leave for future research; yet at bare minimum, simply recognizing the distorting influences of social network discrimination can better inform how we approach matters of race and justice.

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Appendix A: Model Proofs

Lemma 2 *A firm will attempt to hire through referral if and only if it employs a high-ability worker in period 1.*

Proof. Firms employing high-ability workers in period 1 will make referral offers for two reasons. First, hiring through the market yields zero expected profit (given the assumption of free entry of firms). Second, an accepted referral offer yields constant positive profit over the range of the referral offer wage distribution $[w_{M2}, \overline{w}_R]$. (If it did not yield constant positive profits, it would be impossible to maintain equilibrium wage dispersion; the profit-maximizing firms would offer only a subset of the distribution—i.e., those wages that maximized profits.) An offer below w_{M2} will never be accepted, while an offer above $\overline{w}_R$ increases the wage without increasing the probability of attracting a worker. To complete the proof of this lemma, I show that firms employing low-ability workers in period 1 will hire through the market (i.e., not rely on referrals).

If a firm employing a low-ability worker did deviate from this Lemma and made a referral offer w_R, its expected profit (denoted $E\prod_L(w_R)$) would be represented by a slight modification to the expression for the expected profit for a firm employing a high-ability worker (denoted $E\prod_H(w_R)$ and derived in Equation A.6 of Proposition 3). The expression for $E\prod_L(w_R)$ differs from that of $E\prod_H(w_R)$ in that the incidences of α in the first p^{HMAJ}, p^{HMIN}, p^{LMAJ}, and p^{LMIN} terms are replaced with $(1-\alpha)$, and vice versa.

In this scenario, as long as $\alpha > \frac{1}{2}$, $E\prod_L(w_R) < 0$. Hence, expected profits are less than those from hiring in the general market, which equals zero due to free entry of firms. The lemma is thus proved: a firm employing a low-ability worker in period-1 prefers to hire through the market, maximizing expected profit. ∎

Lemma 3 *The period-1 market wage is greater than the expected period-1 productivity.*

Proof. Firms hiring in the period-1 market earn an expected period-2 profit equal to the probability of obtaining a high-ability period-1 worker times the expected profit (denoted c) from a referral. Free entry thus drives the wage above expected period-1 productivity:

$$w_{M1}(\alpha, \delta, \tau_{maj}, \tau_{min}, \psi_{maj}, \psi_{min})$$
$$= \frac{1}{2} + \frac{1}{2}c(\alpha, \delta, \tau_{maj}, \tau_{min}, \psi_{maj}, \psi_{min})$$
$$= \frac{1}{2}[1 + c(\alpha, \delta, \tau_{maj}, \tau_{min}, \psi_{maj}, \psi_{min})]$$

The expression for c is derived in Proposition 3. Given comparative-statics results on c, w_{M1} is increasing in τ_{maj} and τ_{min}. When both $\tau_{maj} = \tau_{min}$ and $\psi_{maj} = \psi_{min}$, w_{M1} is decreasing in δ. ∎

Proposition 1A *In an environment with equal magnitude of majority/minority network parameters ($\tau_{maj} = \tau_{min}$ and $\psi_{maj} = \psi_{min}$), the probability that, among referral offers, a minority worker is referred is lower than their share of the labor force. Conversely, the probability that, among referral offers, a majority worker is referred is larger than their share of the labor force.*

Proposition 1B *The inequality in the distribution of referral job offers can be eliminated by minority workers having a sufficiently higher type in-group bias (ψ_{min}).*

Proof. Let us first consider a given high-ability period-2 worker (H). Since all referral wage offers are above the period-2 market wage, the probability that H would accept a referral wage offer w_{Ri} from firm i can be expressed:

$$\Pr\{H \text{ accepts } w_{Ri}\} = \Pr\{H \text{ receives no higher offer } w_{Rj} \ \forall \text{ firm } j \neq i\}$$

Since referral offers are allocated independently,

$$\Pr\{H \text{ accepts } w_{Ri}\} = \prod_{j \neq i} \Pr\{H \text{ receives no higher offer } w_{Rj}\}$$
$$= \prod_{j \neq i} [1 - \Pr\{H \text{ receives an offer } w_{Rj} > w_{Ri}\}]$$

The probability that firm j offers a wage $w_{Rj} > w_{Ri}$ to H is the product of two independent probabilities:

$$\Pr\{H \text{ receives an offer } w_{Rj} > w_{Ri}\} = \Pr\{\text{firm } j \text{ makes offer to } H\} \cdot \Pr\{w_{Rj} > w_{Ri}\}$$

If $2N$ workers were in period-1, free entry implies that N firms employ high-ability workers. Now I will analyze both parts of the expression from the perspective of a high-ability majority worker (H_{maj}) and high-ability minority worker (H_{min}).

The probability that firm j offers a referral to H_{maj} is a weighted average of whether firm j hired a majority or minority worker in period 1. Denote ϕ_{maj} as the probability a majority worker knows another majority worker, and ϕ_{min} as the probability a minority worker knows another minority worker, where:

$$\phi_{maj} = \frac{(\delta \cdot \psi_{maj})}{(\delta \cdot \psi_{maj}) + [(1 - \delta) \cdot (1 - \psi_{maj})]} \text{ , and}$$

$$\phi_{min} = \frac{(1 - \delta) \cdot \psi_{min}}{[(1 - \delta) \cdot \psi_{min}] + [\delta \cdot (1 - \psi_{min})]}$$

Then:

(A.1) $\Pr\{\text{firm } j \text{ makes offer to } H_{maj}\} = \delta \left(\dfrac{\alpha \tau_{maj} \phi_{maj}}{N} \right) + (1 - \delta) \left(\dfrac{\alpha \tau_{min}(1 - \phi_{min})}{N} \right)$

Likewise, for a period-2 minority high-ability worker:

$$\Pr\{\text{firm } j \text{ makes offer to } H_{min}\} = \delta \left(\frac{\alpha \tau_{maj}(1 - \phi_{maj})}{N} \right) + (1 - \delta) \left(\frac{\alpha \tau_{min} \phi_{min}}{N} \right)$$

Based on these expressions, for minority workers to have a proportional chance of receiving

job offers through referral, the following must hold:

$$\Pr\{\text{firms make offer to } H_{maj}\} \propto \Pr\{\text{firms make offer to } H_{min}\}$$

only when

$$(1-\delta) \cdot \Pr\{\text{firms make offer to } H_{maj}\} = \delta \cdot \Pr\{\text{firms make offer to } H_{min}\}$$

or

$$(1-\delta)\left[\delta\tau_{maj}\phi_{maj} + (1-\delta)\tau_{min}(1-\phi_{min})\right] = \delta\left[\delta\tau_{maj}(1-\phi_{maj}) + (1-\delta)\tau_{min}(\phi_{min})\right]$$

Given $\tau_{maj} = \tau_{min}$ and $\psi_{maj} = \psi_{min}$, analysis shows the left-hand side of the equation is always larger than the right-hand side of the equation, which means majority workers receive a disproportionately large volume of referral offers. Both the minority network density required to compensate for the inequality (denoted $\tau_{min}^=$) and the minority type in-group bias required to compensate (denoted $\psi_{min}^=$) increase in τ_{maj}, ψ_{maj}, and δ. The greater the probability of majority workers having social ties (or the greater the degree of their type in-group bias or likelihood of possessing social ties), the greater minority workers' compensating parameters ($\tau_{min}^=$ or $\psi_{min}^=$) must be to achieve a proportional amount of all job offers through referrals.

Though higher minority network density and type in-group bias can both reduce the disproportionality in the distribution of job offers through referral, analysis shows that between these two parameters only $\psi_{min}^=$ is attainable (i.e., below 1) under all possible combinations of social network parameters. ∎

Proposition 2 *In an environment with equal magnitude of majority/minority network parameters ($\tau_{maj} = \tau_{min}$ and $\psi_{maj} = \psi_{min}$), the period-2 market wage (w_{M2}) decreases as majority workers occupy a greater fraction of the labor force. w_{M2} also decreases in the ability in-group bias α.*

Proof. First I derive an expression for period-2 market wage. I build on the analysis from Proposition 1. If there were $2N$ workers in period 1, free entry implies that N firms employ high-ability workers. If firms select their referral wage offer by randomizing over the equilibrium wage distribution $F(\bullet)$ (to be derived below),

$$\Pr\{H_{maj} \text{ receives an offer } w_{Rj} > w_{Ri}\} = \Pr\{\text{firm } j \text{ makes offer to } H_{maj}\} \cdot [1 - F(w_{Ri})]$$

for all firms j who employ a high-ability worker in period 1. I have already shown that:

$$\Pr\{H \text{ accepts } w_{Ri}\} = \prod_{j \neq i} \Pr\{H \text{ receives no higher offer } w_{Rj}\}$$
$$= \prod_{j \neq i} [\, 1 - \Pr\{H \text{ receives an offer } w_{Rj} > w_{Ri}\}]$$

Substitution yields:

$$\Pr\{H_{maj} \text{ accepts } w_{Ri}\}$$
$$= \{1 - [\frac{1}{N}(\delta\alpha\tau_{maj}\phi_{maj} + (1-\delta)\alpha\tau_{min}(1-\phi_{min}))] \cdot [1 - F(w_{Ri})]\}^{N-1}$$

Since the model assumes a large number of workers, as N approaches ∞,

(A.2) $\Pr\{H_{maj} \text{ accepts } w_{Ri}\} = \exp\{-[\delta\alpha\tau_{maj}\phi_{maj} + (1-\delta)\alpha\tau_{min}(1-\phi_{min})][1 - F(w_{Ri})]\}$

Details on this step can be found in Rapoport (1963) and Montgomery (1991). One can use similar steps to obtain the probability that firm i's offer is accepted by a given high-ability majority worker (H_{maj}), low-ability majority worker(L_{maj}), and low-ability minority worker(L_{min}).

As high-ability workers tend to receive more offers, they are less likely to accept any given offer $w_{Ri} < \overline{w}_R$. Since a period-2 worker finds employment through the market only if he receives no offers (or rejects all referral offers):

$$\Pr\{\text{market} \mid H_{maj}\} = \Pr\{H_{maj} \text{ accept } w_{M2}\}$$

The market wage coincides with the bottom of the referral wage distribution, $F(\bullet)$, because any referral wage below the market wage will be rejected by period-2 workers, to gain employment through the market. Thus, given that $F(w_{M2}) = 0$:

$$\Pr\{\text{market} \mid H_{maj}\} = \exp\{-[\delta\alpha\tau_{maj}\phi_{maj} + (1-\delta)\alpha\tau_{min}(1-\phi_{min})]\}$$

I can derive similar expressions for H_{min}, L_{maj}, and L_{min}. Let:

(A.3)
$$e^{HMAJ} = \exp\{-[\delta\alpha\tau_{maj}\phi_{maj} + (1-\delta)\alpha\tau_{min}(1-\phi_{min})]\}$$
$$e^{HMIN} = \exp\{-[\delta\alpha\tau_{maj}(1-\phi_{maj}) + (1-\delta)\alpha\tau_{min}\phi_{min}]\}$$
$$e^{LMAJ} = \exp\{-[\delta(1-\alpha)\tau_{maj}\phi_{maj} + (1-\delta)(1-\alpha)\tau_{min}(1-\phi_{min})]\}$$
$$e^{LMIN} = \exp\{-[\delta(1-\alpha)\tau_{maj}(1-\phi_{maj}) + (1-\delta)(1-\alpha)\tau_{min}\phi_{min}]\}$$

I now use Bayes's rule to calculate the period-2 market wage:

$$
w_{M2}(\alpha, \delta, \tau_{maj}, \tau_{min}, \psi_{maj}, \psi_{min})
$$
$$
= E(\text{productivity} \mid \text{market})
$$

(A.4)
$$
= \frac{\Pr(\text{market} \mid H_{maj}) \cdot \Pr(H_{maj}) + \Pr(\text{market} \mid H_{min}) \cdot \Pr(H_{min})}{\Pr(\text{market} \mid H) \cdot \Pr(H) + \Pr(\text{market} \mid L) \cdot \Pr(L)}
$$
$$
= \frac{(e^{HMAJ} \cdot \delta) + (e^{HMIN} \cdot (1 - \delta))}{(e^{HMAJ} + e^{LMAJ}) \cdot \delta + (e^{HMIN} + e^{LMIN}) \cdot (1 - \delta)}
$$

Given $\alpha > \frac{1}{2}$ and both network densities (τ_{maj} and τ_{min}) greater than zero, w_{M2} is always less than $\frac{1}{2}$, the average productivity of the population. Analysis shows that w_{M2} is decreasing in α. Furthermore, for all $\psi_{maj} = \psi_{min}$ and $\tau_{maj} = \tau_{min}$, w_{M2} is also decreasing in δ. ∎

Proposition 3 *In an environment with equal magnitude of majority/minority network parameters ($\tau_{maj} = \tau_{min}$ and $\psi_{maj} = \psi_{min}$), the referral wage and the welfare (i.e., average expected wage) for minority workers is lower than for majority workers.*

Proof. Consider the expected period-2 profit earned by a firm employing a high-ability worker and setting a referral wage (recall the productivity of high-ability workers equals one, while that of low-ability workers equals zero):

$E \prod_H (w_R)$

$$
= \Pr\{\text{high-ability majority period-2 referral hired} \mid w_R\} \cdot (1 - w_R)
$$
$$
+ \Pr\{\text{high-ability minority period-2 referral hired} \mid w_R\} \cdot (1 - w_R)
$$
$$
+ \Pr\{\text{low-ability majority period-2 referral hired} \mid w_R\} \cdot (-w_R)
$$
$$
+ \Pr\{\text{low-ability minority period-2 referral hired} \mid w_R\} \cdot (-w_R)
$$

(If no referred worker is hired, perhaps because the period-1 worker possesses no social tie or because the referred acquaintance receives a better offer, the firm hires through the market and earns zero expected profit.)

The probability of hiring a high-ability majority period-2 referred worker is the product of two independent probabilities (substituting from Equations A.1 and A.2 from Propositions 1 and 2):

$$
\Pr\{\text{high-ability period-2 majority referral hired} \mid w_R\}
$$
$$
= \Pr\{\text{offer made to a high-ability majority referral}\} \cdot \Pr\{H_{maj} \text{ accepts } w_R\}
$$
$$
= \delta\alpha\tau_{maj}\phi_{maj} + (1 - \delta)\alpha\tau_{min}(1 - \phi_{min})
$$
$$
\cdot \exp\{-[\delta\alpha\tau_{maj}\phi_{maj} + (1 - \delta)\alpha\tau_{min}(1 - \phi_{min})][1 - F(w_R)]\}
$$

Similar steps can be followed to derive the respective conditional probability for high-ability minority, low-ability majority, and low-ability minority workers.

Let:

$$\begin{aligned}
p^{HMAJ} &= \delta\alpha\tau_{maj}\phi_{maj} + (1-\delta)\alpha\tau_{min}(1-\phi_{min}) \\
p^{HMIN} &= \delta\alpha\tau_{maj}(1-\phi_{maj}) + (1-\delta)\alpha\tau_{min}\phi_{min} \\
p^{LMAJ} &= \delta(1-\alpha)\tau_{maj}\phi_{maj} + (1-\delta)(1-\alpha)\tau_{min}(1-\phi_{min}) \\
p^{LMIN} &= \delta(1-\alpha)\tau_{maj}(1-\phi_{maj}) + (1-\delta)(1-\alpha)\tau_{min}\phi_{min}
\end{aligned}$$

(A.5)

So, to simplify:

$$E\prod_H(w_R)$$

(A.6)

$$\begin{aligned}
&= p^{HMAJ} \cdot \exp\{-[p^{HMAJ}][1-F(w_{Ri})]\} \cdot (1-w_R) \\
&\quad + p^{HMIN} \cdot \exp\{-[p^{HMIN}][1-F(w_{Ri})]\} \cdot (1-w_R) \\
&\quad + p^{LMAJ} \cdot \exp\{-[p^{LMAJ}][1-F(w_{Ri})]\} \cdot (-w_R) \\
&\quad + p^{LMIN} \cdot \exp\{-[p^{LMIN}][1-F(w_{Ri})]\} \cdot (-w_R)
\end{aligned}$$

To maintain equilibrium wage dispersion, firms must earn the same expected profit on each referral wage offered:

$$E\prod_H(w_R) = c \qquad \forall w_R \in [w_{M2}, \overline{w}_R]$$

To calculate this profit constant, note that the firm could deviate from the specified strategy and offer a wage of w_{M2}; in this case, the referred worker accepts the firm's offer only if they receive no other offers.

Recall that $F(w_{M2}) = 0$. The firm's expected profit is therefore given by (using terms defined in Equations A.3 and A.5):

$$\begin{aligned}
&E\prod_H(w_{M2}) \\
&= (p^{HMAJ})(e^{HMAJ})(1-w_{M2}) + (p^{HMIN})(e^{HMIN})(1-w_{M2}) \\
&\quad + (p^{LMAJ})(e^{LMAJ})(-w_{M2}) + (p^{LMIN})(e^{LMIN})(-w_{M2}) \\
&= c
\end{aligned}$$

Substituting for w_{M2} (Equation A.4), I can determine $c(\alpha, \delta, \tau_{maj}, \tau_{min}, \psi_{maj}, \psi_{min})$. Given $\alpha > \frac{1}{2}$, firms with high-ability workers who possess social ties earn positive expected profits. Analysis shows that c is increasing in α, τ_{maj}, and τ_{min}. When both $\psi_{maj} = \psi_{min}$ and $\tau_{maj} = \tau_{min}$, c is decreasing in δ.

Given the expression for $c(\alpha, \delta, \tau_{maj}, \tau_{min}, \psi_{maj}, \psi_{min})$, the equilibrium referral-offer distribution $F(\bullet)$ can be determined by setting $E\prod_H(w_R)$ (Equation A.6) equal to c for all potential wage offers w_R.

Unfortunately, doing so does not yield a closed-form solution for $F(w_R)$. Given a continuum of firms, the equilibrium referral-offer distribution $F(\bullet)$ can be interpreted as either: (1) each firm randomizes over the entire distribution; or (2) a fraction $f(w_R)$ of firms offers each wage for sure.

From the second interpretation, one can denote these referral wages with w_{Rk} and estimate the average referral wage received by a high-ability majority worker (denoted $E(w_{R_{Hmaj}})$) vs. a high-ability minority worker (denoted $E(w_{R_{Hmin}})$), for any given δ, α, τ_{maj}, τ_{min}, ψ_{maj}, and ψ_{min}. Analysis shows that in an environment with equal magnitude of majority/minority network parameters, if $\alpha > \frac{1}{2}$ and $\delta > \frac{1}{2}$, $E(w_{R_{Hmaj}}) > E(w_{R_{Hmin}})$.

Proposition 1 shows that, all else equal, minority workers receive a smaller proportion of jobs through referral than their fraction of the population. As a result, minority workers more frequently gain employment through the market, receiving the (lower) market wage. In this Proposition, I showed that even when offered a job through referral, minority workers have lower expected referral wages than majority workers. ∎

To conclude, one can derive an expression for the maximum referral wage offered $\overline{w}_R$ (where $F(\overline{w}_R) = 1$, by definition):

$$(A.7) \qquad \overline{w}_R(\alpha, \delta, \tau_{maj}, \tau_{min}, \psi_{maj}, \psi_{min}) = \frac{p^{HMAJ} + p^{HMIN} - c}{p^{HMAJ} + p^{HMIN} + p^{LMAJ} + p^{LMIN}}$$

A firm that offers a referral wage of $\overline{w}_R$ attracts a referred worker with probability 1 (conditional on its period-1 worker possessing a social tie). The firm's expected profit, c, is thus equal to $p^{HMAJ} + p^{HMIN} - \overline{w}_R(p^{HMAJ} + p^{HMIN} + p^{LMAJ} + p^{LMIN})$. $\overline{w}_R$ is increasing in α, τ_{maj}, and τ_{min}.

Appendix B: Calibration Steps

B.1 Step 1: Estimate the relative likelihood between racial groups that a high-ability worker accepts a referral wage

As Appendix 6 explains, let e^X denote the $Pr(X$ accepts the market wage), which can be calculated as follows:

$$e^{H_{maj}} = \exp\{-[\delta\alpha\tau_{maj}\phi_{maj} + (1-\delta)\alpha\tau_{min}(1-\phi_{min})]\}$$
$$e^{H_{min}} = \exp\{-[\delta\alpha\tau_{maj}(1-\phi_{maj}) + (1-\delta)\alpha\tau_{min}\phi_{min}]\}$$
$$e^{L_{maj}} = \exp\{-[\delta(1-\alpha)\tau_{maj}\phi_{maj} + (1-\delta)(1-\alpha)\tau_{min}(1-\phi_{min})]\}$$
$$e^{L_{min}} = \exp\{-[\delta(1-\alpha)\tau_{maj}(1-\phi_{maj}) + (1-\delta)(1-\alpha)\tau_{min}\phi_{min}]\}$$

Then, the likelihood that a high-ability white worker accepts a referral wage, relative to the likelihood that a high-ability black worker accepts a referral wage can be calculated as follows:

$$\Pr\{H_{maj} \text{ accepts referral relative to } \Pr\{H_{min} \text{ accepts referral}\}\} = \gamma_{maj} = \frac{1 - e^{H_{maj}}}{1 - e^{H_{min}}}$$

B.2 Step 2: Estimate the market wage and the expected referral wage for both racial groups

Market wage—As the proof for Proposition 3 in Appendix 6 illustrates, Bayes's rule allows one to calculate the period-2 market wage:

$$
\begin{aligned}
&w_{M2}(\alpha, \delta, \tau_{maj}, \tau_{min}, \psi_{maj}, \psi_{min}) \\
&= E(\text{productivity} \mid \text{market}) \\
&= \frac{\Pr(\text{market} \mid H_{maj}) \cdot \Pr(H_{maj}) + \Pr(\text{market} \mid H_{min}) \cdot \Pr(H_{min})}{\Pr(\text{market} \mid H) \cdot \Pr(H) + \Pr(\text{market} \mid L) \cdot \Pr(L)} \\
&= \frac{(e^{H_{maj}} \cdot \delta) + (e^{H_{min}} \cdot (1-\delta))}{(e^{H_{maj}} + e^{L_{maj}}) \cdot \delta + (e^{H_{min}} + e^{L_{min}}) \cdot (1-\delta)}
\end{aligned}
$$

As the exposition for Proposition 2 mentions, w_{M2} should always be below the average productivity of the workforce, which is 0.5. This is because homophily along ability (i.e., ability in-group bias) causes a disproportionately high volume of high-ability workers to be hired via referrals, which lowers the average productivity in the (non-referral) general market and drives down the equilibrium market wage.

Referral Wage—Adapting Appendix 6 Equation A.7, any given referral wage w_{R_k} can be expressed as:

(A.8)
$$
\begin{aligned}
w_{R_k} = &\frac{p^{H_{maj}} \cdot \exp\{-[p^{H_{maj}}][1 - F(w_{R_k})\} + p^{H_{min}} \cdot \exp\{-[p^{H_{min}}][1 - F(w_{R_k})]\} - c}{p^{H_{maj}} \cdot \exp\{-[p^{H_{maj}}][1 - F(w_{R_k})\} + p^{H_{min}} \cdot \exp\{-[p^{H_{min}}][1 - F(w_{R_k})]\}} \\
&\quad + p^{L_{maj}} \cdot \exp\{-[p^{L_{maj}}][1 - F(w_{R_k})]\} + p^{L_{min}} \cdot \exp\{-[p^{L_{min}}][1 - F(w_{R_k})]\}
\end{aligned}
$$

where $F(\bullet)$ represents the equilibrium wage distribution; c represents the expected firm profit, which is constant across the entire wage distribution and can thus be set equal to the expected productivity in the (non-referral) general market; and p^X is denoted as follows:

$$p^{H_{maj}} = \delta\alpha\tau_{maj}\phi_{maj} + (1-\delta)\alpha\tau_{min}(1-\phi_{min})$$
$$p^{H_{min}} = \delta\alpha\tau_{maj}(1-\phi_{maj}) + (1-\delta)\alpha\tau_{min}\phi_{min}$$
$$p^{L_{maj}} = \delta(1-\alpha)\tau_{maj}\phi_{maj} + (1-\delta)(1-\alpha)\tau_{min}(1-\phi_{min})$$
$$p^{L_{min}} = \delta(1-\alpha)\tau_{maj}(1-\phi_{maj}) + (1-\delta)(1-\alpha)\tau_{min}\phi_{min}$$

$$c = E\prod_H(w_{M2}) = (p^{HMAJ})(e^{HMAJ})(1-w_{M2}) + (p^{HMIN})(e^{HMIN})(1-w_{M2})$$
$$+ (p^{LMAJ})(e^{LMAJ})(-w_{M2}) + (p^{LMIN})(e^{LMIN})(-w_{M2})$$

For simplicity, I assume a uniform equilibrium wage distribution $F(\bullet)$ and so have normalized the value for the average referral wage for black workers—$F(w_{R_k}^{H_{min}})$—to 0.5 (since in Step 1 we normalized $\gamma_{min} = 1$). I then estimate $F(w_{R_k}^{H_{maj}})$ for the referral wage for white workers. Since workers accept the maximum referral wage offered, estimating $F(w_{R_k}^{H_{maj}})$ is akin to finding the expected value of the maximum of γ_{maj} random variables:[27]

$$F(w_{R_k}^{H_{maj}}) = \frac{\gamma_{maj}}{\gamma_{maj}+1}$$

B.3 Step 3: Estimate the expected welfare gap between racial groups

To calculate the expected welfare gap between black and white workers, I take the sum of the market wage and the expected referral wage, weighted by the likelihood of being hired via the general market or via the referral market, respectively:

$$\text{Welfare gap} = 1 - \frac{\Pr(H_{min}\text{ accepts }w_{M2})\cdot w_{M2} + \Pr(H_{min}\text{ accepts }w_R)\cdot E(w_{R_k}^{H_{min}})}{\Pr(H_{maj}\text{ accepts }w_{M2})\cdot w_{M2} + \Pr(H_{maj}\text{ accepts }w_R)\cdot E(w_{R_k}^{H_{maj}})}$$
$$= 1 - \frac{e^{H_{min}}\cdot w_{M2} + (1-e^{H_{min}})\cdot E(w_{R_k}^{H_{min}})}{e^{H_{maj}}\cdot w_{M2} + (1-e^{H_{maj}})\cdot E(w_{R_k}^{H_{maj}})}$$

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27. The PDF for a uniform distribution is $f_Y(y) = n\cdot y^{n-1}$. This leads to $E[Y] = \int_{y\in A} y f_Y(y)dy = \int_0^1 yny^{n-1}dy = \frac{n}{n+1}$.