## The Puzzle of War

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- Review of game theory
- Review of the graphical model
- Game theoretic version of the model ... with math! - $^{2}$


## WHY WAR?

## Naive explanations

- "They hate each other"
- "They're crazy"
- They chose to go to war
- Create a model in which war does not occur
- Find the conditions that will break this result


## GRAPHICAL MODEL

## Graphical model

## Graphical model



## Graphical model



## Graphical model



## Graphical model



## Graphical model



## Graphical model



## GAME-THEORETIC MODEL

## War as a costly lottery

- Outcomes
- Lose
- Win
- Probability of each outcome
- $\operatorname{Pr}($ Lose $)=0.98$
- $\operatorname{Pr}($ Win $)=0.02$
- Payoffs
- $\operatorname{Win}=100,000$
- Lose $=0$
- Value of the ticket?
- $0.98 \times 0+0.02 \times 100,000=2,000$


## Elements of a game

- Players
- Actions
- Preferences


## Model

- Players: Country A, Country B
- Outcome: Divisions of a piece of territory of size $=1$
- Country A gets $x$
- Country B gets $1-x$
- Game: Country A offers a division: $\{x, 1-x\}$
- Country B: accepts or rejects
- If Country B accepts, territory is divided
- If Country B rejects, they go to war
- Country A wins entire territory with probability $p_{A}$
- Country $B$ wins entire territory with probability $1-p_{A}$
- Country A pays cost $c_{A}$, Country B pays cost $c_{B}$


## Game tree



## Country B's dilemma

- $E U_{B}($ Accept $)=1-x$
- $E U_{B}($ Reject $=$ War $)=\left(1-p_{A}\right) \times\left(1-C_{B}\right)+p_{A} \times\left(-c_{B}\right)=1-p_{A}-c_{B}$
- Accept if $E U_{B}($ Accept $) \geq E U_{B}($ Reject $)$
- $1-x \geq 1-p_{A}-c_{B}$
- $x \leq p_{A}+c_{B}$
- Player 1 offers:
- $x^{*}=p_{A}+c_{B}$ THE MINIMUM OFFER


## Country A's dilemma

- $E U_{A}($ Offer $)=x^{*}=p_{A}+c_{B}$
- $E U_{A}($ War $)=\left(p_{A}\right) \times\left(1-c_{A}\right)+\left(1-p_{A}\right) \times\left(-c_{A}\right)$
- Accept if $E U_{A}($ Offer $) \geq E U_{A}$ (War)
- $p_{A}+c_{B} \geq p_{A}-c_{A}$
- $c_{A}+c_{B} \geq 0$ ALWAYS TRUE
- There exists an equilibrium such that Country A makes an offer and Country B accepts. No war.
- Solution (NE) is: $\{x, 1-x\}=\left\{p_{A}+c_{B}, 1-p_{A}-c_{B}\right\}$


## Comparative statics (predictions)

- How does $x^{*}$ change as..?
- $p_{A}$ increases
- $x^{*}$ increases
- $c_{B}$ increases
- $x^{*}$ increases
- $c_{A}$ increases
- $x^{*}$ stays the same


## Why war?

1. Asymmetric information (+ incentives to misrepresent)
2. Commitment problems
3. Indivisibility

## Mistakes?



## What have we learned?

- Review of the graphical model
- Game-theoretic model
- Why war? (key concepts)

