Appendix B to Maurice Obstfeld and Kenneth Rogoff, "Global Current Account Imbalances and Exchange Rate Adjustments," in William Brainard and George Perry (eds.), *Brookings Papers on Economic Activity*, Spring 2005.

Normalization for Simulation Analysis

To conduct our simulation analysis using the model of the text, it is helpful to normalize the model so that, for example we express all current accounts and net foreign asset positions in terms of ratios to the tradable component of U.S. GDP. In this Appendix section, we illustrate how to do this.

Let $ca^U = CA^U / \left(P_U Y_T^U\right)$, $f^U = F^U / \left(P_U Y_T^U\right)$, $ca^E = CA^E / \left(P_U Y_T^U\right)$, and $f^E = F^E / \left(P_U Y_T^U\right)$. Also, it is helpful to define $\sigma_{U/E} = Y_T^U / Y_T^E$, $\sigma_{U/A} = Y_T^U / Y_T^A$, $\sigma_{N/U} = Y_N^U / Y_T^U$, $\sigma_{N/E} = Y_N^E / Y_T^E$, and $\sigma_{N/A} = Y_N^A / Y_T^A$. Finally, let the relative price indices for traded and nontraded goods in each country be given as $x^U = P_N^U / P_T^U$, $x^E = P_N^E / P_T^E$, and $x^A = P_N^A / P_T^A$. With these normalizations, can write eqs. (2)-(6) as:

$$1 = \alpha \frac{1}{\left[\alpha + (\beta-\alpha)\tau_{U,E}^{1-\eta} + (1-\beta)\tau_{U,A}^{1-\eta}\right]}\left(1 + rf^U - ca^U\right)$$

$$+ (\beta-\alpha)\frac{1}{\left[\alpha\tau_{U,E}^{1-\eta} + (\beta-\alpha) + (1-\beta)\tau_{U,A}^{1-\eta}\right]}\left(\frac{\tau_{U,E}}{\sigma_{U/E}} + rf^E - ca^E\right)$$

$$+ \left(\frac{1-\delta}{2}\right)\frac{1}{\left[\delta\tau_{U,A}^{1-\eta} + \left(\frac{1-\delta}{2}\right) + \left(\frac{1-\delta}{2}\right)\tau_{U,E}^{1-\eta}\right]}\left[\frac{\tau_{U,A}}{\sigma_{U/A}} - r\left(f^U + f^E\right) + ca^U + ca^E\right],$$

$$1 = (\beta-\alpha)\frac{\tau_{U,E}^{1-\eta}}{\left[\alpha + (\beta-\alpha)\tau_{U,E}^{1-\eta} + (1-\beta)\tau_{U,A}^{1-\eta}\right]}\left[\frac{\sigma_{U/E}}{\tau_{U,E}}\left(1 + rf^U - ca^U\right)\right]$$

$$+ \alpha\frac{\tau_{U,E}^{1-\eta}}{\left[\alpha\tau_{U,E}^{1-\eta} + (\beta-\alpha) + (1-\beta)\tau_{U,A}^{1-\eta}\right]}\left[1 + \frac{\sigma_{U/E}}{\tau_{U,E}}\left(rf^E - ca^E\right)\right]$$

$$+ \left(\frac{1-\delta}{2}\right)\frac{\tau_{U,E}^{1-\eta}}{\left[\delta\tau_{U,A}^{1-\eta} + \left(\frac{1-\delta}{2}\right) + \left(\frac{1-\delta}{2}\right)\tau_{U,E}^{1-\eta}\right]}\left\{\frac{\sigma_{U/E}}{\tau_{U,E}}\left[\frac{\tau_{U,A}}{\sigma_{U/A}} - r(f^U + f^E) + ca^U + ca^E\right]\right\},$$

$$\sigma_{N/U} = \left(\frac{1-\gamma}{\gamma}\right)\left(x^U\right)^{-\theta}\left[\alpha + (\beta-\alpha)\tau_{U,E}^{1-\eta} + (1-\beta)\tau_{U,A}^{1-\eta}\right]^{-\frac{1}{1-\eta}}\left(1 + rf^U - ca^U\right),$$

$$\sigma_{N/E} = \left(\frac{1-\gamma}{\gamma}\right)\left(x^{E}\right)^{-\theta}\left[\alpha + (\beta-\alpha)\tau_{U,E}^{-(1-\eta)} + (1-\beta)\tau_{E,A}^{1-\eta}\right]^{-\frac{1}{1-\eta}}\left[1+\frac{\sigma_{U/E}}{\tau_{U,E}}\left(rf^{E} - ca^{E}\right)\right],$$

$$\sigma_{N/A} = \left(\frac{1-\gamma}{\gamma}\right)\left(x^{A}\right)^{-\theta}\left[\delta + \left(\frac{1-\delta}{2}\right)\tau_{U,A}^{-(1-\eta)} + \left(\frac{1-\delta}{2}\right)\tau_{E,A}^{-(1-\eta)}\right]^{-\frac{1}{1-\eta}}$$

$$\times\left\{1-\frac{\sigma_{U/A}}{\tau_{U,A}}\left[r\left(f^{E}+f^{U}\right) - ca^{E} - ca^{U}\right]\right\}.$$

The preceding five equations are the core of the simulation model, though we also make use of the real exchange rate definitions above, the definitions of the x's (the relative price of traded and nontraded goods), and the relations $q_{E,A} = \frac{q_{U,A}}{q_{U,E}}$, $\tau_{E,A} = \frac{\tau_{U,A}}{\tau_{U,E}}$.

Nominal Exchange Rates

Most of our text discussion concerns real exchange rates, but of course many readers will be interested in nominal exchange rates. Here we show the assumptions underlying our calculations in the text's tables.

If we assume central banks stabilize CPIs (as in our earlier discussions in Obstfeld and Rogoff 2000a, 2004), then real and nominal exchange rate changes coincide in this model. Here, however, we show how to extend the analysis to the case in which central banks instead stabilize GDP deflators. (Because many readers will be interested in our results for nominal exchange rates, this is an important detail. However, we find in our text simulations that the differences between nominal and real exchange rate changes are uniformly small.) Unlike the case of CPIs, in which utility maximization dictated the appropriate weighting scheme, it is not clear *a priori* what the appropriate definition of the GDP deflator should be. A plausible case is one in which central banks stabilize geometric averages of the prices of tradable and nontradable domestic output. Thus, in the U.S., Europe, and Asia, respectively, we have

$$P_{U}^{\gamma}\left(P_{N}^{U}\right)^{1-\gamma} = 1,$$

$$P_{E}^{*\gamma}\left(P_{N}^{E*}\right)^{1-\gamma} = 1,$$

$$P_{A}^{*\gamma}\left(P_{N}^{A*}\right)^{1-\gamma} = 1,$$

where the asterisk signifies that the nominal price is measured in the *local* currency. Notice that, for calculating log changes, setting these indexes equal to unity is a simple normalization that does not affect the answers. Also, the parameter γ above is the same one that enters the total consumption index.

To solve for nominal exchange rates, rewrite the above equations as

$$P_U^{\frac{\gamma}{\gamma-1}} = P_N^U ,$$

$$\left(P_E^*\right)^{\frac{\gamma}{\gamma-1}} = P_N^{E*},$$

$$\left(P_A^*\right)^{\frac{\gamma}{\gamma-1}} = P_N^{A*},$$

and then make use of the five solutions, derived in the last subsection, for the terms of trade and relative prices of nontradables in terms of tradables.

By definition of the dollar/euro real exchange rate, the dollar price of the euro — the nominal dollar/euro exchange rate — is:

$$E_{U,E} = q_{U,E} \times \frac{P_C^U}{P_C^{E*}}$$

$$= q_{U,E} \times \frac{\left(P_T^U\right)^{\gamma}\left(P_N^U\right)^{1-\gamma}}{\left(P_T^{E*}\right)^{\gamma}\left(P_N^{E*}\right)^{1-\gamma}}$$

$$= q_{U,E} \times \frac{\left(P_T^U / P_U\right)^{\gamma}}{\left(P_T^{E*}/P_E^*\right)^{\gamma}}$$

$$= q_{U,E} \times \frac{\left[\alpha + (\beta-\alpha)\tau_{U,E}^{1-\eta} + (1-\beta)\tau_{U,A}^{1-\eta}\right]^{\frac{\gamma}{1-\eta}}}{\left[\alpha + (\beta-\alpha)\tau_{U,E}^{-(1-\eta)} + (1-\beta)\tau_{U,A}^{-(1-\eta)}\right]^{\frac{\gamma}{1-\eta}}}.$$

The equation makes it clear that in response to U.S. current account adjustment, the nominal exchange rate movement is greater than the real exchange rate movement. This result follows from the positive covariance between real exchange rates and terms of trade. Similarly

$$E_{U,A} = q_{U,A} \times \frac{\left[\alpha + (\beta-\alpha)\tau_{U,E}^{1-\eta} + (1-\beta)\tau_{U,A}^{1-\eta}\right]^{\frac{\gamma}{1-\eta}}}{\left[\delta + \left(\frac{1-\delta}{2}\right)\tau_{U,A}^{-(1-\eta)} + \left(\frac{1-\delta}{2}\right)\tau_{U,E}^{-(1-\eta)}\right]^{\frac{\gamma}{1-\eta}}}.$$

Of course,

$$E_{E,A} = E_{U,A} / E_{U,E}.$$

Using the preceding solutions, we define nominal effective exchange rates by:

$$E^U = \left(E_{U,E}\right)^{\frac{\beta-\alpha}{1-\alpha}}\left(E_{U,A}\right)^{\frac{1-\beta}{1-\alpha}},$$

$$E^E = \left(1/E_{U,E}\right)^{\frac{\beta-\alpha}{1-\alpha}}\left(E_{E,A}\right)^{\frac{1-\beta}{1-\alpha}},$$

$$E^A = \left(1/E_{E,A}\right)^{1/2}\left(1/E_{U,A}\right)^{1/2}.$$

These expressions use the same weightings as our measures of effective real exchange rates.