



OXFORD JOURNALS
OXFORD UNIVERSITY PRESS

Assessing Dynamic Efficiency: Theory and Evidence

Author(s): Andrew B. Abel, N. Gregory Mankiw, Lawrence H. Summers and Richard J. Zeckhauser

Source: *The Review of Economic Studies*, Vol. 56, No. 1 (Jan., 1989), pp. 1-19

Published by: Oxford University Press

Stable URL: <https://www.jstor.org/stable/2297746>

Accessed: 25-05-2020 20:43 UTC

JSTOR is a not-for-profit service that helps scholars, researchers, and students discover, use, and build upon a wide range of content in a trusted digital archive. We use information technology and tools to increase productivity and facilitate new forms of scholarship. For more information about JSTOR, please contact support@jstor.org.

Your use of the JSTOR archive indicates your acceptance of the Terms & Conditions of Use, available at <https://about.jstor.org/terms>



JSTOR

Oxford University Press is collaborating with JSTOR to digitize, preserve and extend access to *The Review of Economic Studies*

Assessing Dynamic Efficiency: Theory and Evidence

ANDREW B. ABEL
University of Pennsylvania

N. GREGORY MANKIW
LAWRENCE H. SUMMERS
and
RICHARD J. ZECKHAUSER
Harvard University

First version received May 1987; Final version accepted May 1988 (Eds.)

The issue of dynamic efficiency is central to analyses of capital accumulation and economic growth. Yet the question of what characteristics should be examined to determine whether actual economies are dynamically efficient is unresolved. This paper develops a criterion for determining whether an economy is dynamically efficient. The criterion, which holds for economies in which technological progress and population growth are stochastic, involves a comparison of the cash flows generated by capital with the level of investment. Its application to the United States economy and the economies of other major OECD nations suggests that they are dynamically efficient.

Dynamic efficiency is a central issue in analyses of economic growth, the effects of fiscal policies, and the pricing of capital assets. In a celebrated article, Peter Diamond (1965) shows that a competitive economy can reach a steady state in which there is unambiguously too much capital. In situations where the population growth rate exceeds the steady state marginal product of capital, or equivalently the economy is consistently investing more than it is earning in profit, the economy is said to be dynamically inefficient. In the terminology of Phelps (1961), the capital stock exceeds its Golden Rule level. A Pareto improvement can be achieved in a dynamically inefficient economy by allowing the current generation to devour a portion of the capital stock and then holding constant the consumption of all future generations.

Recent work suggests that dynamic efficiency is crucial for issues of positive as well as normative economic analysis. Weil (1987) argues that dynamic efficiency is a necessary condition for an altruistic bequest motive to be operative and thus for the Ricardian equivalence theorem of Barro (1974) to hold. Tirole (1985) shows that speculative bubbles can arise as rational expectations equilibria of dynamically inefficient deterministic economies, but are ruled out if dynamic efficiency prevails.

Whether actual economies operate above or below the Golden Rule level of capital intensity is a question of substantial importance. Many authors take the position that the United States economy is dynamically efficient. Such a presumption must underlie optimal growth arguments for increased national savings, such as those presented by Tobin (1965), Solow (1970), and Feldstein (1977). These authors base their judgment that the economy is dynamically efficient on estimates of the marginal productivity of

capital derived from observed accounting profit rates. Such calculations (e.g. Feldstein and Summers (1977)) suggest that the marginal productivity of capital is about 10%, far above the economy's growth rate, and thus indicate that the economy is dynamically efficient.

Yet there is an alternative, and on its face equally plausible, procedure for judging whether an economy is dynamically efficient—looking at the safe real interest rate, such as the return on Treasury bills. Unlike the accounting profit rate, the short-term real interest rate can be measured with little ambiguity, and contains at most a negligible risk premium. Feldstein (1976), in arguing against the Ricardian equivalence proposition, explicitly compares the economy's growth rate and safe real interest rate. Numerous other authors in a variety of contexts also note that real interest rates on safe assets are typically less than economic growth rates. Ibbotson (1987) reports that the mean real return on Treasury bills over the 1926–1986 period was 0.3%, well under the economy's average growth rate. Data on other nations, such as those presented by Mishkin (1984), suggest that interest rates are less than growth rates in other countries as well. This comparison of real interest rates and growth rates seems to suggest that mature capitalist economies have overaccumulated capital.

In abstract growth models like Diamond's, there is no difference between the marginal product of capital, which might be inferred from data on profits and the capital stock, and the real interest rate on safe government securities. Nor are there fluctuations in the economy's growth rate or in the relative value of capital goods. Such models therefore give little guidance as to which features of actual economies should be examined in assessing dynamic efficiency. To address this issue, it is necessary to examine models in which profitability, the value of capital, and the growth rate are uncertain, so that the distinction between the marginal productivity of capital and the interest rate on safe government securities is a meaningful one. This paper examines the conditions for dynamic efficiency in such models. Our goal is to ascertain which observable features of actual economies can be used to make inferences about dynamic efficiency and then to assess whether the United States economy is dynamically efficient.

We begin by generalizing the original Golden Rule result of Phelps (1961) and others: an economy is dynamically efficient if it invests less than the return to capital and is inefficient if it invests more than the return to capital. This result applies to a stochastic setting with a very general production technology. In a competitive economy, therefore, the issue of dynamic efficiency can be resolved by comparing the level of investment with the cash flows generated by production after the payment of wages. More metaphorically, the key question is whether the capital sector is on net a spout or a sink. This criterion has the substantial virtue of not requiring any information on capital gains and losses, and not requiring the use of accounting conventions in the measurement of profit. Its application to the United States economy suggests very clearly that it is dynamically efficient. The economy has always (since 1929 at least) consumed a substantial portion of the return to capital. Similar results obtain for other developed capitalist economies as well.

While we argue that the net cash flow criterion is the most natural and general way to think about the issue of dynamic efficiency, much of the previous discussion of the issue centres on comparisons of rates of return and growth rates. We therefore present results characterizing the circumstances under which a comparison of growth rates and rates of return can determine whether an economy is dynamically efficient. Our conclusion is that the appropriate criterion involves evaluations of whether asset returns dominate the growth in wealth valued in terms of consumption goods. That is, dynamic efficiency

cannot be assessed by comparing the safe rate of interest and the average growth rate of the capital stock, output, or any other accounting aggregate. Rather, the safe rate must be compared with the rate of growth of the market value of the capital stock. In actual economies, where the market value of existing capital goods fluctuates widely and declines frequently, this comparison is unlikely to render any verdict about dynamic efficiency.

While these results suggest that economies like that of the United States are dynamically efficient, some troubling questions remain. Why, if physical assets that always yield a positive return are available, is the safe rate of interest so low? If the safe rate of interest is zero or negative, why don't infinitely-lived productive assets have an infinite value? If the safe rate of interest is below the economy's growth rate, can't a pay-as-you-go social security system be Pareto-improving? To address these questions, we present a simple example highlighting the potential importance of capital price risk in determining asset values. We show that in a dynamically efficient economy where output evolves as a geometric random walk with positive drift, the safe rate of interest over all horizons may at all times be negative. This example makes clear that comparisons of the safe rate of interest and average growth rates are misleading for the purpose of assessing dynamic efficiency.

The paper is organized as follows. Section I develops our cash flow criterion for assessing the dynamic efficiency of an economy and applies it to United States data. Section II considers rate of return criteria for dynamic efficiency. Section III highlights the role of capital price risk in accounting for the low observed level of safe interest rates. Section IV concludes the paper by calling attention to a number of issues requiring further research.

I. A GENERAL CRITERION FOR DYNAMIC EFFICIENCY

Here we examine a generalization of the economy studied by Diamond (1965). There are two crucial differences between Diamond's overlapping generations model and ours. First, we allow the rate of population growth and the production technology to be stochastic. Second, our production technology is very general; it allows, as special cases, for costs of adjustment, time-to-build, vintage capital, and embodied and disembodied technological progress. Unlike the technology typically assumed in one-sector growth models, our general production technology allows for fluctuations in the relative price of claims on existing capital goods and consumption. This feature of the model is important, because much of the risk associated with holding capital in actual economies involves uncertainty about capital gains and losses arising from changes in the relative price of capital.

A. The economy

Each individual lives for two periods and maximizes a von Neumann-Morgenstern utility function that, for simplicity, is additively separable over consumption in the two periods. Thus, individuals who are alive in periods t and $t+1$ maximize

$$u(c_t^y) + E_t v(c_{t+1}^o), \quad (1.1)$$

where c_t^y is the consumption of a young person in period t , c_{t+1}^o is the consumption of an old person in period $t+1$, and E_t is the expectation conditional on information available at time t .

There are N_t individuals in the cohort born at time t . The young supply their labour inelastically, and the old do not work, so N_t is also the labour supply in period t . Each young person consumes some of the return to his labour and invests the rest by buying shares in the new and existing firms that compose the market portfolio. In old age, he receives dividends from his shares, sells them ex dividend to the young cohort, and consumes the total proceeds.

An individual therefore faces the following budget constraint:

$$c_t^y = w_t - V_t s_t, \quad (1.2)$$

$$c_{t+1}^o = (D_{t+1} + V_{t+1})s_t, \quad (1.3)$$

where w_t is the wage, s_t is his share of the market portfolio, V_t is the total value of the market portfolio ex dividend, and D_t is the total dividend. Let R_{t+1} denote the gross return on any asset between period t and period $t+1$ —that is, the return inclusive of the return of principal. Then the standard first-order condition for capital asset pricing is

$$E_t \left[\frac{v'(c_{t+1}^o) R_{t+1}}{u'(c_t^y)} \right] = 1. \quad (1.4)$$

Equation (1.4) of course holds also for the return on the market portfolio, for which $R_{t+1} = R_{t+1}^M \equiv (V_{t+1} + D_{t+1})/V_t$.

A few identities are instructive. Aggregate consumption C_t is

$$C_t \equiv N_t c_t^y + N_{t-1} c_t^o, \quad (1.5)$$

and the aggregate share is $N_t s_t = 1$. Equations (1.2) and (1.3) therefore imply that consumption is labour income plus the dividend:

$$C_t \equiv w_t N_t + D_t. \quad (1.6)$$

Let Y_t be gross output, $\pi_t \equiv Y_t - w_t N_t$ be profit, and $I_t \equiv Y_t - C_t$ be investment. Equation (1.6) implies

$$D_t \equiv \pi_t - I_t. \quad (1.7)$$

The dividend equals profit less investment. Equivalently, the dividend equals consumption less labour income.

Note that we are using the term “dividend” in an unusual way. A repurchase of shares by firms is represented here as a dividend payment. Similarly, a new equity issue is a negative dividend. The dividend thus represents the net flow of goods from firms to households (except of course for labour income), and there is no presumption regarding its sign.¹

Firms produce output from capital and labour. We assume that the production technology is:

$$Y_t = F(I_{t-1}, I_{t-2}, \dots, I_{t-n}; N_t, \theta_t), \quad (1.8)$$

where I_t is the gross investment in period t , and θ_t is the state of nature in period t . The vector θ_t includes all current and previous shocks.

The technology expressed in equation (1.8) is very general. Under the common assumption that capital fully depreciates each period, each period's investment equals the next period's capital stock, so $Y_t = F(I_{t-1}; N_t, \theta_t)$. A dependence of output on past

1. If new firms are regularly being started with new equity issues, the aggregate dividend as we define it may always be negative, even when existing firms always pay positive dividends.

investment, however, arises if capital lasts several periods or if there are costs of adjustment of capital. The production function (1.8) is valid even if, as is plausible, capital does not fully depreciate each period and the type of capital built each period varies because of changes in the available technology.

We assume that the technology has constant returns to scale. That is, $F(\cdot)$ is homogeneous of degree one in past investment and current labour supply. The competitive wage is therefore

$$w_t = \partial F(I_{t-1}, I_{t-2}, \dots, I_{t-n}; N_t, \theta_t) / \partial N_t. \tag{1.9}$$

The total return to capital is

$$\pi_t = \sum_{i=1}^{\infty} F^i I_{t-i}, \tag{1.10}$$

where F^i indicates the partial derivative $\partial F(I_{t-1}, I_{t-2}, \dots, I_{t-n}; N_t, \theta_t) / \partial I_{t-i}$. We assume that $F^i \geq 0$. The specification in (1.8) implies that $F^i = 0$ for $i > n$. Equation (1.10) says that profits are composed of return to capital of all vintages.

An equilibrium of this economy is a set of state-contingent allocations and market prices that satisfies all the individual and economy-wide constraints and that also satisfies the first-order condition (1.4). We are interested in characterizing the conditions under which such an equilibrium is dynamically efficient or inefficient.

B. Dynamic efficiency

Let $\Omega(\theta_t)$ be the ex ante utility of generation t given that this generation is born in state of nature θ_t . That is,

$$\Omega(\theta_t) = u(c_t^y(\theta_t)) + E\{v(c_{t+1}^o(\theta_{t+1})) | \theta_t\}. \tag{1.11}$$

We call an initial equilibrium *dynamically inefficient* if it is possible to increase $\Omega(\theta_s)$ for some θ_s without decreasing $\Omega(\theta'_i)$ or any other θ'_i ; if such a Pareto improvement is impossible, an equilibrium is *dynamically efficient*.

Consider a social welfare function

$$W = v(c_0^o) + \sum_{t=0}^{\infty} \sum_{\theta_t} \mu(\theta_t) \Omega(\theta_t) \tag{1.12}$$

where $\mu(\theta_t)$ is the weight the social planner gives to $\Omega(\theta_t)$.

An equilibrium is dynamically efficient if it maximizes social welfare W for some set of positive weights $\mu(\theta_t)$. It is important to stress the nature of the social welfare function in equation (1.12). In the unfettered equilibrium, there are no private markets in which individuals can insure against the state of nature in which they are born. Since our concern in this paper is the issue of overaccumulation, and not intergenerational risk-sharing, we do not call an equilibrium inefficient if the absence of these insurance markets is the only market failure. Formally, we allow the social planner to apply separate social planning weights to the same individual born in different states of nature.

We now turn to the principal result of this paper:

Proposition 1. *If $D_t/V_t \geq \varepsilon > 0$ in all periods and all states of nature, then the equilibrium is dynamically efficient. If $D_t/V_t \leq -\varepsilon < 0$ in all periods and all states of nature, then the equilibrium is dynamically inefficient.*

This proposition states that if goods are on net always flowing out of firms to investors, then the equilibrium is efficient. Conversely, if goods are on net always flowing into firms from investors, then the equilibrium is inefficient. Our proposition is a generalization of the Golden Rule result of Phelps (1961): an economy that invests more than its total profit in steady state is dynamically inefficient.

The familiar condition from the Diamond model for dynamic efficiency is that the marginal product of capital (r) exceeds the growth rate of population (n). Note that the growth rate of population equals the growth rate of the capital stock in Diamond's steady state. Therefore, since rK is the total return on the aggregate capital stock K and nK is new investment, the net flow out of firms (the dividend) is $rK - nK$. Hence, while Proposition 1 is much more general, it is consistent with the traditional criterion in the steady state of the Diamond model.²

The proof of the efficiency condition in Proposition 1 is in the Appendix. The proof of inefficiency follows.

C. Proof of inefficiency

Following Cass (1972), we provide a constructive proof of the inefficiency condition by illustrating a Pareto-improving intervention that is feasible if the inefficiency condition is satisfied. Suppose that in period 1 the consumption of each old consumer is increased by δ and the consumption of the current young consumers and the consumptions of all future generations are left unchanged. Clearly, such a change would be Pareto-improving. If such a change is feasible, then the initial equilibrium was inefficient.

Since aggregate consumption in period 1 is increased by δN_0 , the change in aggregate investment, dI_1 , is equal to $-\delta N_0$. This reduction in I_1 implies that output will be reduced by $F_2^1 dI_1$ in period 2. Since consumption in period 2 is to remain unchanged, investment must fall by the same amount as output

$$dI_2 = F_2^1 dI_1. \quad (1.13)$$

In each subsequent period the reduction in output must be matched by a fall in investment of the same magnitude, so that, differentiating the production function, we obtain

$$dI_t = \sum_{i=1}^n F_t^i dI_{t-i}. \quad (1.14)$$

Now define $\Delta_t \equiv dI_t/I_t$ to be the proportional change in period t investment. Using this definition and dividing both sides of (1.14) by I_t yields a homogeneous linear difference equation in Δ_t ,

$$\Delta_t = \sum_{i=1}^n (F_t^i I_{t-i}/I_t) \Delta_{t-i}. \quad (1.15)$$

If the proportional change in investment Δ_t is always finite and approaches zero as t goes to infinity, then, for an appropriately small δ , the intervention is feasible. Therefore, if the coefficients $F_t^i I_{t-i}/I_t$, $i = 1, \dots, n$, in the difference equation (1.15) are always positive and sum to less than one, then the intervention is feasible.

It remains to show that the condition $D_t/V_t \leq -\varepsilon < 0$ implies that $\sum_{i=1}^n (F_t^i I_{t-i}/I_t) < 1$. We suppose that $D_t/V_t \leq -\varepsilon < 0$ and use (1.7) and (1.10) to obtain

$$[(\sum_{i=1}^n F_t^i I_{t-i}) - I_t]/V_t \leq -\varepsilon < 0. \quad (1.16)$$

2. One might wonder whether there exist interesting stochastic economies in which the dividend always has the same sign, as required for Proposition 1 to be useful. As an example, suppose capital lasts only one period, the production function is Cobb-Douglas, and utility is logarithmic. In this case, one can show that profits, wages, consumption, and investment are all proportional to output. The dividend, which is profits less investment, is thus also proportional to output; the sign of this last factor of proportionality determines whether the economy is dynamically efficient.

Assuming that $I_t/V_t > 0$ and observing that $I_t/V_t \leq 1$, (1.16) implies

$$[(\sum_{i=1}^n F_t^i I_{t-i}) - I_t]/V_t \leq -\varepsilon I_t/V_t < 0. \quad (1.17)$$

Equation (1.17) implies directly that

$$\sum_{i=1}^n F_t^i I_{t-i}/V_t \leq (1 - \varepsilon) I_t/V_t. \quad (1.18)$$

Dividing both sides of (1.18) by I_t/V_t yields

$$\sum_{i=1}^n F_t^i I_{t-i}/I_t \leq 1 - \varepsilon < 1, \quad (1.19)$$

which is the condition that the coefficients in the linear difference equation (1.15) sum to less than one. Therefore, if $D_t/V_t \leq -\varepsilon < 0$, then the proposed Pareto-improving intervention is feasible. \parallel

D. Applying the cash flow criterion

The criterion derived above for determining whether an economy is dynamically efficient can be applied relatively easily to actual economies. Unlike the criteria applied in many previous applications of optimal growth models (e.g. Solow (1970)), our criterion does not depend critically on accounting judgments about depreciation or the measurement of profits in an inflationary environment. It requires only the comparison of the cash flows going into and coming out of an economy's production sector.³ As the preceding discussion makes clear, the applicability of our criterion depends on the assumption that capital receives its marginal product, an assumption that excludes the possibility that capital income includes substantial monopoly profit.

We apply our criterion to the United States economy and to the economies of several other major countries. While the theory developed above is most directly applicable to the economy as a whole, we examine the U.S. nonfinancial corporate sector as well, for two reasons. First, many popular discussions of the desirability of increased capital formation refer to corporate investment rather than investment in housing. Second, in measuring the rental income generated by owner-occupied housing, one must use estimates of imputed rents, rather than direct measurements of market transactions. Fortunately, the calculations for the economy as a whole and the calculations for the corporate sector yield very similar results. Both suggest that our criterion for dynamic efficiency has been satisfied by a wide margin in all years.⁴

The cash flows we examine exclude the returns from, and investment in, human capital and government capital. Therefore, we do not address the question of whether these types of capital are overaccumulated. Yet the exclusion of these types of capital does not affect our conclusions regarding the accumulation of private physical capital, as long as private physical capital earns its marginal product. In essence, we are asking whether private physical capital is overaccumulated, conditional on the path of human capital and government capital.

Table 1 presents information on gross capital income, gross investment, and their difference, all expressed as fractions of gross national product (GNP) for the 1929–1985

3. That it is easier to measure cash flow than economic income has been a major theme in discussions of the consumption tax, such as those presented in Pechman (1983).

4. In applying our criterion to actual economies, the question arises as to how long the length of the period should be. We use annual data, but other frequencies of observation could be used. If there is some partition of time such that $D_t/V_t \geq \varepsilon > 0$ for all t and all states, then the economy is dynamically efficient. Alternatively, if there is some partition of time such that $D_t/V_t \leq -\varepsilon < 0$ for all t and all states, then the economy is dynamically inefficient.

TABLE 1

Gross profit and investment: the United States economy (percent)

Year	Gross profit	Gross investment	$\frac{D}{V}$	$\frac{D}{V}$
	GNP	GNP	GNP	V
1929	32.6	16.1	16.5	—
1930	31.7	11.6	20.1	—
1931	28.6	7.7	20.9	—
1932	26.4	1.9	24.5	—
1933	24.6	2.9	21.8	—
1934	26.1	5.3	20.8	—
1935	27.1	9.1	18.0	—
1936	26.4	10.5	15.9	—
1937	26.9	13.3	13.6	—
1938	26.5	7.8	18.6	—
1939	26.7	10.4	16.3	—
1940	28.2	13.3	14.9	—
1941	29.4	14.6	14.8	—
1942	29.0	6.5	22.6	—
1943	27.2	3.2	24.0	—
1944	25.2	3.6	21.5	—
1945	23.2	5.3	17.9	—
1946	24.0	14.8	9.2	—
1947	25.6	14.9	10.8	—
1948	27.8	18.0	9.8	—
1949	27.3	14.0	13.3	—
1950	28.4	19.1	9.3	—
1951	28.0	18.1	9.9	—
1952	26.9	15.2	11.7	3.9
1953	26.4	14.8	11.6	4.0
1954	26.9	14.5	12.4	4.1
1955	28.2	17.2	11.0	3.7
1956	27.6	17.0	10.6	3.4
1957	27.4	15.8	11.6	3.8
1958	27.0	13.9	13.1	4.1
1959	27.8	16.2	11.6	3.8
1960	27.1	15.2	11.9	3.9
1961	27.1	14.4	12.7	4.2
1962	27.4	15.2	12.1	4.1
1963	27.6	15.3	12.3	4.2
1964	27.7	15.3	12.4	4.3
1965	28.2	16.5	11.8	4.2
1966	27.8	16.7	11.1	4.0
1967	27.2	15.4	11.8	4.2
1968	26.6	15.3	11.3	4.0
1969	25.7	15.9	9.8	3.5
1970	24.6	14.7	9.9	3.5
1971	25.2	15.6	9.6	3.4
1972	25.6	16.7	8.9	3.1
1973	25.6	17.6	8.1	2.8
1974	25.1	16.3	8.8	2.8
1975	26.2	13.7	12.4	3.9
1976	26.4	15.6	10.8	3.4
1977	27.1	17.3	9.9	3.1
1978	27.6	18.5	9.1	2.7
1979	27.6	18.1	9.5	2.8
1980	27.4	16.0	11.4	3.2
1981	28.1	16.9	11.2	3.2
1982	27.7	14.1	13.6	3.9
1983	28.4	14.7	13.7	4.0
1984	28.5	17.6	10.9	3.3
1985	28.0	16.5	11.5	3.6

Source. NIPA. Capital Cash Flow is calculated as national income plus capital consumption allowances less employee compensation and 67% of proprietors' income which is imputed to labour. This imputation is discussed in Christensen (1971).

period. Gross capital income is measured as national income plus capital consumption allowances less: (1) employee compensation and (2) an estimate of the labour income of proprietors. Alternatively, it can be thought of as the sum of profit, rental, and interest income.⁵ Gross investment includes investment in both residential and nonresidential capital along with increases in inventories. The difference between gross capital income and gross investment corresponds to the "dividend", D , paid by firms. For the period 1952–1985, for which an estimate of the market value of the economy's tangible assets is available, the table also presents the ratio D/V , which plays a key role in the preceding analysis.

The data indicate clearly that capital has consistently generated more income than has been invested. Indeed, the difference between capital income and investment has exceeded 8% of GNP in every year since 1929. The economy has never invested much more than two-thirds of its gross capital income. The final column of the table indicates that the dividend generated by capital has consistently exceeded 2½% of total wealth. Given the downward trend in the ratio of the dividend to GNP and the upward trend in the ratio of total wealth to GNP, it is likely that the dividend generated by capital exceeded 2½% during the entire 1929–1985 period.

Table 2 presents calculations paralleling those in Table 1 for the nonfinancial corporate sector. Outflow from the nonfinancial corporate sector (D) is defined as the sum of dividends, interest payments, and corporate tax payments, less net financial investments. Except for a statistical discrepancy, this outflow in turn equals the difference between gross pretax operating profits and gross capital investment. (Again, investment is measured as including both purchases of plant and equipment and increases in inventories.) In all calculations, the denominator is the market value of the corporate sector measured as the sum of the market value of outstanding equity, and net financial liabilities as reported in the *National Balance Sheets*. The results establish that for the 1953–1985 period gross capital income far exceeded gross investment. Indeed, in most years, the disparity is somewhat greater for the corporate sector than it is for the aggregate economy. This finding is not surprising given the heavier tax burdens placed on corporate capital. It appears clear from these calculations that the corporate sector is on net productive, in the sense that net outflows of funds from the corporate sector are positive.

There are two potentially important respects in which the calculations reported in Tables 1 and 2 may be too sanguine about the dynamic efficiency of the American economy. First, the profit figures used in the calculation overstate the return to invested capital because of the inclusion of returns to land.⁶ This point is of some significance. Rhee (1988) reports a number of estimates suggesting that the share of land rents in total income has fluctuated without much trend around a mean about 5% for the American economy over the period 1900–1987. In 1985, the ratio of land value to annual GNP was about 2/3. Even if the real rental rate was 10%, the correction for land would be insufficient to overturn our conclusion that the economy was dynamically efficient.⁷

5. As noted above, rental income includes imputed rent on owner-occupied housing. The imputed rent estimates in the National Income Accounts upon which we rely indicate that the gross rental rate on owner-occupied housing is considerably lower than the rate on other kinds of capital. Thus, excluding owner-occupied housing would reinforce our conclusion.

6. McCallum (1986) has argued that it is impossible for an economy with land to be dynamically inefficient at least in steady state. Rhee (1988) argues that this conclusion is incorrect when allowance is made for a general production technology.

7. The point may however have more significance for Japan where it has been estimated that the land value/GNP ratio has exceeded 3 in recent years. The issue is not clear, however, because the ratio of rents to land values appears to be very low.

TABLE 2

Gross profit and investment: the U.S. nonfinancial corporate sector (percent)

Year	Gross profit	Gross investment	$\frac{D}{V}$
	V	V	V
1953	29.1	14.7	13.6
1954	28.7	13.3	14.3
1955	25.1	13.3	10.2
1956	21.0	12.3	7.0
1957	19.4	10.8	7.5
1958	20.6	9.6	10.0
1959	18.2	9.8	7.6
1960	16.7	9.2	6.3
1961	16.9	8.9	7.5
1962	15.5	8.9	6.9
1963	18.0	9.8	8.0
1964	17.1	9.5	7.3
1965	17.5	10.5	6.9
1966	17.4	11.7	5.9
1967	18.4	11.6	6.9
1968	16.1	9.7	6.2
1969	14.6	9.5	5.7
1970	15.5	9.9	5.6
1971	16.9	10.4	6.8
1972	16.7	10.2	6.5
1973	17.7	11.4	5.7
1974	23.9	15.6	5.5
1975	31.5	15.4	14.4
1976	29.7	17.4	10.3
1977	30.0	17.6	9.6
1978	34.3	21.4	9.9
1979	35.7	21.9	12.4
1980	31.7	18.9	10.2
1981	26.6	16.9	7.1
1982	25.3	15.1	7.9
1983	23.5	14.1	9.3
1984	23.4	17.3	6.7
1985	22.1	15.9	8.5

Second, the closely related assumptions of perfect competition and constant returns maintained in our theoretical calculations may well be inappropriate. One argument might hold that the profits we measure are overstated as indicators of the return to investment because they include monopoly rents. Alternatively, if imperfect competition gives rise to chronic excess capacity our measures may overstate the marginal return to investment. Treating these issues fully is beyond the scope of the current paper.

We are inclined toward the view, however, that our calculation is not badly distorted by monopoly profits. The average value of Tobin's q —the ratio of the market value of firms to their replacement cost—has been about than one in the United States over the past 50 years. If a large part of profits reflected returns to something other than physical capital, one would suspect that the firm's market value would substantially exceed the value of their physical assets.

Table 3 presents values of D/GNP for England, France, Germany, Italy, Canada and Japan for the 1960–1984 period. Examining the dynamic efficiency of economies other than the U.S. economy is of interest because the American savings rate has traditionally been much lower than that of other countries. Japan, in particular, with its high rate of accumulation and tradition of low real interest rates, seems a plausible

TABLE 3
Dynamic efficiency in different countries

Year	Difference of gross profit and investment as a percentage of GNP					
	England	France	Germany	Italy	Canada	Japan
1960	11.8	18.2	12.4	22.1	12.7	17.6
1961	10.6	17.4	10.9	21.0	14.0	9.5
1962	11.5	16.2	10.0	19.9	14.0	14.6
1963	12.1	15.3	10.7	17.8	14.4	13.7
1964	8.8	12.9	9.2	19.1	12.8	12.4
1965	9.4	13.6	8.5	22.9	9.4	15.2
1966	8.7	12.9	9.9	24.1	8.3	15.4
1967	8.3	13.5	13.7	22.5	9.4	14.2
1968	7.7	13.6	13.5	23.1	10.7	12.9
1969	8.3	11.9	10.3	22.1	9.4	12.1
1970	7.5	11.8	7.8	18.9	10.5	11.6
1971	8.9	12.3	7.5	19.1	8.9	11.2
1972	10.4	12.0	2.7	19.9	9.2	11.4
1973	7.8	10.9	7.9	15.4	9.4	8.1
1974	4.4	8.7	10.0	12.2	8.2	5.2
1975	6.0	10.9	12.4	16.6	9.0	6.8
1976	5.5	8.9	11.5	13.5	7.5	7.8
1977	9.1	9.9	11.9	14.6	7.9	7.8
1978	9.6	11.0	12.1	16.4	10.5	8.9
1979	8.4	10.0	9.8	16.7	10.5	7.2
1980	10.1	8.3	8.4	12.9	12.4	7.5
1981	11.9	10.4	10.8	15.1	9.3	7.3
1982	12.9	9.8	13.1	16.5	12.9	8.1
1983	13.2	11.7	13.6	17.1	15.3	9.4
1984	13.9	12.9	13.8	17.3	17.3	9.4

Source. OECD. These data are in some cases not adjusted for the labour income of proprietors. As discussed by Sachs (1979), this approximation is quite innocuous. For the United States, the adjustment is equal to about 4% of GNP.

candidate for inefficiency. Yet the data reveal that our criterion for dynamic efficiency is comfortably satisfied for all countries in all years. Countries like Japan with higher investment rates also have higher rates of profit. In 1984, for example, the U.S. invested 18% of GNP and had gross capital cash flow of 28% of GNP. For Japan, the corresponding figures were 28% and 38%.

The calculations in Tables 1-3 show that our criterion for dynamic efficiency has been very decisively satisfied for the world's major capitalist economies. Yet as Phelps emphasized in his initial development of the Golden Rule, a path cannot be judged as dynamically efficient or dynamically inefficient prior to eternity. That is, dynamic efficiency cannot in principle be judged by observing only a particular segment of time. These calculations do allow us to conclude, however, that if the economy behaves in the future as it has in the past, it will be realizing a dynamically efficient equilibrium.

II. USING RATES OF RETURN TO DIAGNOSE OVERACCUMULATION

In Section I we presented sufficient conditions for dynamic efficiency and dynamic inefficiency expressed in terms of investment and profit. Much of the literature on dynamic efficiency expresses the relevant conditions in terms of rates of return and growth rates. In this section we examine how one can compare rates of return and growth rates to assess dynamic efficiency.

Recall that the rate of return on the market portfolio, R_{t+1}^M , is equal to $(V_{t+1} + D_{t+1})/V_t$. Defining $G_{t+1} \equiv V_{t+1}/V_t$ as the growth rate of the value of the market portfolio we obtain

$$R_{t+1}^M/G_{t+1} \equiv 1 + D_{t+1}/V_{t+1}. \quad (2.1)$$

Equation (2.1) immediately implies the following corollary to Proposition 1.

Corollary 1. *If $R_t^M/G_t \geq 1 + \varepsilon > 1$ in all periods and all states of nature, then the equilibrium is dynamically efficient. If $R_t^M/G_t \leq 1 - \varepsilon < 1$ in all periods and all states of nature, then the equilibrium is dynamically inefficient.*

Corollary 1 states that the rate of return on capital can be used in assessing whether the economy is dynamically efficient. The condition for efficiency in Corollary 1 is satisfied if the dividend is always greater than zero.

A more general result is also available for assessing dynamic efficiency. The competitive rate of return on any other asset can also be potentially useful in determining whether the economy is dynamically efficient or inefficient. Let R_{t+1} be the competitive rate of return between period t and period $t+1$ on an arbitrary asset. In the Appendix we present an extension of the proof of Proposition 1 that proves the following Proposition:

Proposition 2. *If there is some asset with rate of return R_t such that $R_t/G_t \geq 1 + \varepsilon > 1$ in all periods and all states of nature, then the equilibrium is dynamically efficient. If there is some asset with rate of return R_t such that $R_t/G_t \leq 1 - \varepsilon < 1$ in all periods and all states of nature, then the equilibrium is dynamically inefficient.*

This result can be applied in the special case of the safe interest rate. It says that if an economy's safe interest rate is always greater than the growth rate of the market value of the capital stock, the economy is dynamically efficient. If the safe rate is always less than the growth rate of the value of the capital stock, the economy is dynamically inefficient. Neither implication is very helpful in judging the dynamic efficiency of actual economies, where capital gains and losses cause the growth rate of the market value of the capital stock sometimes to exceed and other times to fall short of the safe interest rate. The result here is illuminating primarily in suggesting that comparisons of the safe interest rate with the average growth rate generally are not sufficient to resolve the issue of dynamic efficiency.

III. CAPITAL PRICE RISK AND THE LOW LEVEL OF SAFE INTEREST RATES

The previous section suggests that a safe interest rate below an economy's average growth rate is not sufficient to establish dynamic inefficiency. This conclusion may at first seem surprising. If all the investments an economy is making are productive, how can the safe interest rate be so low? More generally, how can the measured rate of profit suggest that the marginal product of capital is so high at the same time that the real interest rate is so low? To pose the question starkly, can physical assets that always yield a positive dividend have a finite value when the short-term safe real interest rate is always zero or negative?

A basic answer to these questions lies in the notion of capital price risk. The major risks associated with the ownership of capital do not involve the rents it will generate in the succeeding period, but rather involve the possibility of changes in its market value.

Individuals considering the ownership of capital take account of these risks. Even if an asset will always be productive, individuals are exposed to the risk that its price will decline if anticipated future profits fall off, or if the demand for wealth declines. Such risks explain how a low safe rate can coincide permanently with a high rate of profit in a dynamically efficient economy.

We illustrate this point with a simple example based on Lucas's (1978) analysis of asset pricing. In contrast to the preceding sections, we assume here that there is a single representative infinitely-lived consumer. Since the competitive outcome in this case replicates the solution to the social planning problem of maximizing the utility of the representative consumer, we know it is dynamically efficient. (As we shall see, application of our criterion from Section I leads to the same conclusion.) An infinite horizon example is used to highlight the fact that the importance of capital risk in no way depends on the finiteness of individual lifetimes.

Consumers are assumed to maximize the infinite horizon utility function:

$$E_t \sum_{i=0}^{\infty} \beta^i U(C_{t+i}). \quad (3.1)$$

The economy's only productive asset is a "fruit tree" whose output evolves as a geometric random walk with positive drift. It is not possible to produce or cut down the fruit tree. The output of the fruit tree, all of which is consumed in each period, thus evolves as:

$$D_t = D_{t-1}(1 + g + \nu_t), \quad (3.2)$$

where $g > 0$ and ν_t is i.i.d. with zero mean, and $\nu_t > -(1 + g)$. Note that in this example capital income, D_t , is always positive and thus always exceeds the economy's zero level of investment, so our criterion for dynamic efficiency is satisfied.

Assets in this economy are easily priced by making use of the consumer's first-order condition:

$$E_t \{R_{t+1} \beta U'(C_{t+1}) / U'(C_t)\} = 1, \quad (3.3)$$

where R_{t+1} is the gross return on any asset.

Now suppose the utility function is logarithmic, $U(C) = \ln(C)$, and recall that $C_t = D_t$, so that (3.3) can be written as

$$E_t \{R_{t+1} \beta / (1 + g + \nu_{t+1})\} = 1. \quad (3.4)$$

Equation (3.4) can be used to price any asset. In particular, the gross rate of return on a riskless asset, R_{t+1}^F is constant and given by

$$R_{t+1}^F = 1 / E_t \left[\frac{\beta}{1 + g + \nu_{t+1}} \right]. \quad (3.5)$$

If the variance of ν is sufficiently large, R_{t+1}^F can be less than one, so that the net safe interest rate can be negative despite the upward drift in output. Since the safe short-term rate is constant, safe interest rates over any horizon are also negative as long as output is sufficiently volatile.

For example, suppose that $1 + g + \nu_{t+1}$ is lognormally distributed with mean $1 + g$. In this case, $\ln(1 + g + \nu_{t+1})$ is distributed $N(\mu, \sigma^2)$, which implies

$$E(1 + g + \nu_{t+1}) = \exp(\mu + (1/2)\sigma^2). \quad (3.6)$$

$$E[(1 + g + \nu_{t+1})^{-1}] = \exp(-\mu + (1/2)\sigma^2). \quad (3.7)$$

It follows from (3.6) and (3.7) that

$$E[(1 + g + \nu_{t+1})^{-1}] = \exp(\sigma^2) / E(1 + g + \nu_{t+1}) = \exp(\sigma^2) / (1 + g). \quad (3.8)$$

Substituting (3.8) into (3.5) yields

$$R_{t+1}^F = [(1 + g)/\beta] \exp(-\sigma^2). \quad (3.9)$$

For sufficiently large σ^2 , R^F is less than one—that is, the risk-free return is negative.

Equation (3.4) can also be used to determine V_t , the value of the stock of fruit trees. Since the rate of return on a fruit tree is $(V_{t+1} + D_{t+1})/V_t$, equation (3.4) verifies

$$V_t = \left(\frac{\beta}{1 - \beta} \right) D_t. \quad (3.10)$$

Despite the uncertainty in this model economy, the dividend-price ratio is constant. It is straightforward to show that the mean return on the fruit tree is $(1 + g)/\beta$, regardless of the amount of uncertainty. Hence, increases in uncertainty raise the risk premium solely by lowering the risk-free rate.

This example makes clear that because of asset-price risk, permanently productive assets can exist in equilibrium with a negative real rate of interest. Furthermore, such a situation can easily be efficient. A low safe interest rate, even a negative rate in perpetuity, need not imply that the economy is overcapitalized.

The example suggests the limited validity of arguments based on comparisons of the safe rate of return and the average growth rate of output. For simplicity, suppose the safe rate is negative and the mean growth rate is positive. A pay-as-you-go social security scheme that transferred one dollar from young to old in each period would be Pareto improving if it were feasible. But as long as output can conceivably fall in each period, the feasibility of such a scheme is not guaranteed. The fact that output has an arbitrarily high probability of rising as the horizon is lengthened is not sufficient to assure feasibility. Internally, the argument here parallels the well-known fallacy regarding the appropriateness of investing only in high-yielding assets over long horizons, discussed monosyllabically by Samuelson (1979). In both cases, “almost surely” is not certain enough.

Our “fruit tree” example shows that a low safe rate of interest need not imply dynamic inefficiency. Yet the results of Mehra and Prescott (1985) suggest that additional considerations, such as market imperfections, may be necessary to explain the observed low safe rate. The interaction between these imperfections and dynamic efficiency may be important. For example, some individuals may be liquidity constrained, so that the relevant interest rate for evaluating a hypothetical Social Security scheme exceeds the safe rate of interest. Considering dynamic efficiency in “distorted” economies is a topic for future research.

IV. CONCLUSION

In the literature on optimal accumulation and dynamic efficiency, substantial intellectual capital has been put into place since the early 1960s. Yet despite this investment and the growing policy concern over capital formation, no clear answer has emerged as to whether actual economies are dynamically efficient. To address the issue of dynamic efficiency in practice, it is imperative to recognize the impact of uncertainty on production, investment, and growth. The goal of this paper has therefore been to extend dynamic efficiency results to an uncertain world.

Our central finding is simply expressed. If the capital sector is regularly contributing to the level of consumption, then the economy is dynamically efficient. If it is a continual drain, then the economy is inefficient. In the United States, profit has exceeded investment in every year since 1929. This finding leads us to conclude that the United States economy is dynamically efficient.

Established tradition suggests that dynamic efficiency is to be evaluated by comparing growth rates and rates of return on assets. In an uncertain world, there is no obvious metric for economic growth; nor is there a single rate of return. The appropriate indicator of dynamic efficiency is the rate of growth of the value of the capital stock, as measured in consumption goods. If the rate of return on any asset dominates this rate, the economy is dynamically efficient. If the return on any asset is dominated by this rate, the economy is dynamically inefficient. While this rate-of-return criterion is theoretically valid, we believe it is less useful in practice than the cash flow criterion for assessing dynamic efficiency.

We started this analysis with the troubling observation that real interest rates are startlingly low. We reassured ourselves by demonstrating that low, or even negative, safe rates, even in perpetuity, need not indicate dynamic inefficiency. Our fruit-tree example suggests that asset price risk could in principle explain the low safe rate, but we have made no attempt to measure this risk.⁸

Our conclusion that the United States economy is dynamically efficient has several important corollaries. First, Tirole (1985) shows that rational speculative bubbles are ruled out in dynamically efficient economies.⁹ We suspect, but have not proven, that Tirole's result generalizes to economies with uncertainty. Our empirical results thus call into doubt the existence of rational bubbles of the sort Tirole examines.¹⁰

Second, Weil (1987) shows that an interior bequest motive, of the sort on which Barro (1974) relies, can be ruled out in dynamically inefficient economies. Again, we suspect that this result generalizes to uncertain settings. Our finding of dynamic efficiency, however, suggests that such a bequest motive cannot be ruled out. In contrast to the conclusion of Feldstein (1976), the low safe rate of interest may not preclude Ricardian equivalence.

While it is comforting that our cash flow criterion leads unambiguously to the conclusion that our economy is dynamically efficient, several questions remain open for future research. The most important direction for future research is the evaluation of alternative dynamic paths using stronger criteria than the dynamic efficiency criterion. Our criterion is the dynamic analogue of the standard Pareto criterion. The dynamic efficiency test is therefore relatively weak—passing it does not imply that a path is desirable for any very attractive social welfare function. Results characterizing the social welfare functions necessary to justify particular paths would aid in evaluating and ranking them.

The importance of going beyond the Pareto criterion is greater in the context of dynamic efficiency than in the normal static context. As we have already noted, a path cannot be judged as dynamically inefficient prior to eternity, because there is always the possibility that "excess" capital will be consumed later. Using social welfare functions would permit, at least sometimes, judgments that regardless of what happened in the indefinite future, certain policies maintained over finite intervals would be undesirable. Putting the point differently, the use of social welfare functions would make possible the evaluation of alternative social decision rules for determining the level of investment.

The importance of this issue may be highlighted by assessing the current position of the American economy in the following way. While risk-free assets are not available,

8. The results of Mehra and Prescott (1985) suggest that observed consumption variability cannot itself explain the spread between the return on capital and the safe rate of interest. Our point is that a low safe rate is not, in itself, inconsistent with dynamic efficiency.

9. Tirole shows that bubbles can drive an economy to the Golden Rule. Our finding of a strictly positive dividend indicates that the capital stock is strictly below the Golden Rule, in which case bubbles cannot exist.

10. Of course, our results do not speak to the possibility of irrational bubbles, or fads, such as those discussed by Shiller (1984) and Summers (1986).

the historical behaviour of treasury bill real returns suggests that the riskless rate of return over the next 30 years will be close to zero. If the economy is on an optimal trajectory, this implies that the expected social marginal utility of consumption in 30 years equals the marginal utility of consumption today. Rationalizing this implication would seem to require substantial pessimism about the mean or variance of economic growth over the next 30 years, very substantial risk aversion, or a very high planning weight on the utility of future consumers.

Risk-sharing considerations, which we have systematically excluded from our analysis, provide yet another reason for introducing a social welfare function. Programmes such as social security have the ability to spread risks across generations. We dodged the possibility of ex ante risk-spreading bargains by treating individuals born at the same time under different realizations of the world as being on different, separately evaluated paths. But given that social security or tax and expenditure policy can promote ex ante risk spreading, such risk spreading could be considered in studies that follow.

APPENDIX

Proof of Proposition 1 (Efficiency). The social planner maximizes social welfare (1.12) subject to the set of constraints

$$N_t c_t^y + N_{t-1} c_t^o + I_t = F(I_{t-1}, I_{t-2}, \dots, I_{t-n}; N_t, \theta_t) \tag{A.1}$$

for all θ_t . The Lagrangian for the social planner is

$$\begin{aligned} L = & v(c_0^o) + \sum_{t=0}^{\infty} \sum_{\theta_t} \mu(\theta_t) \{ u(c_t^y(\theta_t)) + \sum_{\theta_{t+1}} p(\theta_{t+1} | \theta_t) v[c_{t+1}^o(\theta_{t+1})] \} \\ & - \sum_{t=0}^{\infty} \sum_{\theta_t} \lambda(\theta_t) \{ N_t(\theta_t) c_t^y(\theta_t) + N_{t-1}(\theta_{t-1}) c_t^o(\theta_t) \\ & + I_t(\theta_t) - F(I_{t-1}, I_{t-2}, \dots, I_{t-n}; N_t, \theta_t) \} \end{aligned} \tag{A.2}$$

where $p(\theta_{t+1} | \theta_t)$ denotes the probability of θ_{t+1} given that θ_t occurs. The variable θ_t is a vector indexing the state of nature in $t+1$ including history up to $t+1$.

The first-order conditions are

$$v'(c_0^o) = \lambda(\theta_0) N_{-1}, \tag{A.3}$$

$$\mu(\theta_t) u'[c_t^y(\theta_t)] = \lambda(\theta_t) N_t(\theta_t), \tag{A.4}$$

$$\mu(\theta_{t-1}) p(\theta_t | \theta_{t-1}) v'[c_t^o(\theta_t)] = \lambda(\theta_t) N_{t-1}(\theta_{t-1}), \text{ for } \theta_t | \theta_{t-1}, \tag{A.5}$$

$$\sum_{j=1}^{\infty} \sum_{\theta_{t+j} | \theta_t} \lambda(\theta_{t+j}) F_{t+j}^j = \lambda(\theta_t). \tag{A.6}$$

The expression $\theta_{t+j} | \theta_t$ denotes those states of nature in $t+j$ that can follow from θ_t . The transversality condition requires

$$\lim_{t \rightarrow \infty} \sum_{\theta_t} \lambda(\theta_t) V(\theta_t) = 0, \tag{A.7}$$

where

$$\sum_{\theta_t} \lambda(\theta_t) V(\theta_t) = \sum_{j=0}^{\infty} \sum_{i=1}^{\infty} \sum_{\theta_{t+i+j}} \lambda(\theta_{t+i+j}) F_{t+i+j}^{j+i}(\theta_{t+i+j}) I_{t+1-i}(\theta_{t+1-i}). \tag{A.8}$$

If the initial equilibrium satisfies these conditions for some set of positive weights $\mu(\theta_t)$ and multipliers $\lambda(\theta_t)$, the equilibrium is efficient.

Equation (A.3) defines the initial Lagrangian multiplier λ_0 . Equations (A.4) and (A.5) then uniquely define all multipliers $\lambda(\theta_t)$ and social planning weights $\mu(\theta_t)$, given the initial equilibrium. Thus, to show that the equilibrium solves a planning problem, we need only show that (A.6) and (A.7) are satisfied.

The first-order condition (A.6) follows from the capital market equilibrium condition (1.4). To see this, let q_{t+j}^i be the price (relative to consumption) at the end of period $t+j$ of a unit of investment installed in period t . Observe $q_t^0 = 1$ and $q_t^n = 0$. Let R_{t+j+1}^{t+1} be the rate of return between period $t+j$ and period $t+j+1$ on a unit of capital installed in period t . Therefore,

$$R_{t+j+1}^{t+1}(\theta_{t+j+1}) = [F_{t+j+1}^{j+1}(\theta_{t+j+1}) + q_{t+j+1}^{j+1}(\theta_{t+j+1})] / q_{t+j}^j(\theta_{t+j}). \tag{A.9}$$

Using (A.9), the capital market equilibrium condition (1.4) implies

$$q_{t+j}^j(\theta_{t+j}) = \sum_{\theta_{t+j+1}} [F_{t+j+1}^{j+1}(\theta_{t+j+1}) + q_{t+j+1}^{j+1}(\theta_{t+j+1})] \times p(\theta_{t+j+1} | \theta_{t+j}) v'[c_{t+j+1}^0(\theta_{t+j+1})] / u'[c_t^y(\theta_{t+j})]. \tag{A.10}$$

Now observe from (A.4) and (A.5) that

$$p(\theta_{t+j+1} | \theta_{t+j}) v'[c_{t+j+1}^0(\theta_{t+j+1})] / u'[c_{t+j}^y(\theta_{t+j})] = \lambda(\theta_{t+j+1}) / \lambda(\theta_{t+j}), \tag{A.11}$$

for $\theta_{t+j+1} | \theta_{t+j}$. Substituting (A.11) into (A.10) yields

$$q_{t+j}^j(\theta_{t+j}) = \sum_{\theta_{t+j+1} | \theta_{t+j}} [F_{t+j+1}^{j+1}(\theta_{t+j+1}) + q_{t+j+1}^{j+1}(\theta_{t+j+1})] \times \lambda(\theta_{t+j+1}) / \lambda(\theta_{t+j}). \tag{A.12}$$

Now recall that $q_t^0 = 1$ and solve (A.12) recursively forward to obtain

$$\lambda(\theta_t) = \sum_{j=0}^{\infty} \sum_{\theta_{t+j+1} | \theta_t} F_{t+j+1}^{j+1}(\theta_{t+j+1}) \lambda(\theta_{t+j+1}), \tag{A.13}$$

which is equivalent to (A.6).

To establish efficiency, we still need to show that the transversality condition (A.7) is satisfied. From equation (A.8), we know that

$$\sum_{\theta_t} \lambda(\theta_t) V(\theta_t) = \sum_{\theta_{t+1}} \sum_{i=1}^{\infty} \lambda(\theta_{t+1}) F_{t+1}^i I_{t+1-i} + \sum_{j=1}^{\infty} \sum_{i=1}^{\infty} \sum_{\theta_{t+j+1}} \lambda(\theta_{t+j+1}) F_{t+j+1}^{j+i} I_{t+1-i}. \tag{A.14}$$

(Some arguments specifying the state of nature are omitted where there is no ambiguity.) The first term on the right-hand side of this inequality equals $\sum_{\theta_{t+1}} \lambda(\theta_{t+1}) \pi_{t+1}$. The second term can be rewritten with j running from zero and i running from two:

$$\sum_{\theta_t} \lambda(\theta_t) V(\theta_t) = \sum_{\theta_{t+1}} \lambda(\theta_{t+1}) \pi_{t+1} + \sum_{j=0}^{\infty} \sum_{i=2}^{\infty} \sum_{\theta_{t+j+2}} \lambda(\theta_{t+j+2}) F_{t+j+2}^{j+i} I_{t+2-i}. \tag{A.15}$$

The second term can now be written using (A.8) as the sum of $\lambda(\theta_{t+1}) V(\theta_{t+1})$ and a remainder, producing

$$\sum_{\theta_t} \lambda(\theta_t) V(\theta_t) = \sum_{\theta_{t+1}} \lambda(\theta_{t+1}) \pi_{t+1} + \sum_{\theta_{t+1}} \lambda(\theta_{t+1}) V(\theta_{t+1}) - \sum_{j=0}^{\infty} \sum_{\theta_{t+j+2}} \lambda(\theta_{t+j+2}) F_{t+j+2}^{j+1} I_{t+1}. \tag{A.16}$$

The third term of (A.16) can be rewritten as

$$\sum_{j=0}^{\infty} \sum_{\theta_{t+j+2}} \lambda(\theta_{t+j+2}) F_{t+j+2}^{j+1} I_{t+1} = \sum_{j=1}^{\infty} \sum_{\theta_{t+j+1}} \lambda(\theta_{t+j+1}) F_{t+j+1}^j I_{t+1} \tag{A.17}$$

by beginning the index j at one rather than zero. This expression is now identically

$$\sum_{j=0}^{\infty} \sum_{\theta_{t+j+2}} \lambda(\theta_{t+j+2}) F_{t+j+2}^{j+1} I_{t+1} = \sum_{j=1}^{\infty} \sum_{\theta_{t+1}} \sum_{\theta_{t+j+1} | \theta_{t+1}} \lambda(\theta_{t+j+1}) F_{t+j+1}^j I_{t+1}. \tag{A.18}$$

Equation (A.6) now implies

$$\sum_{j=0}^{\infty} \sum_{\theta_{t+j+2}} \lambda(\theta_{t+j+2}) F_{t+j+2}^{j+1} I_{t+1} = \sum_{\theta_{t+1}} \lambda(\theta_{t+1}) I_{t+1}. \tag{A.19}$$

Therefore, returning to (A.16),

$$\sum_{\theta_t} \lambda(\theta_t) V(\theta_t) = \sum_{\theta_{t+1}} \lambda(\theta_{t+1}) \pi_{t+1} + \sum_{\theta_{t+1}} \lambda(\theta_{t+1}) V(\theta_{t+1}) - \sum_{\theta_{t+1}} \lambda(\theta_{t+1}) I_{t+1}. \tag{A.20}$$

Noting that $D_{t+1} = \pi_{t+1} - I_{t+1}$, we obtain

$$\sum_{\theta_t} \lambda(\theta_t) V(\theta_t) = \sum_{\theta_{t+1}} (1 + D_{t+1} / V_{t+1}) \lambda(\theta_{t+1}) V(\theta_{t+1}). \tag{A.21}$$

By hypothesis, $D_{t+1} / V_{t+1} \geq \varepsilon > 0$, which implies

$$\sum_{\theta_t} \lambda(\theta_t) V(\theta_t) \geq (1 + \varepsilon) \sum_{\theta_{t+1}} \lambda(\theta_{t+1}) V(\theta_{t+1}). \tag{A.22}$$

Recursive substitution shows

$$\sum_{\theta_t} \lambda(\theta_t) V(\theta_t) \leq (1 + \varepsilon)^{-1} \lambda_0 V_0. \tag{A.23}$$

Since $\varepsilon > 0$, (A.23) implies the transversality condition is satisfied.

In this case, the equilibrium can be represented as the optimum of the social planning problem defined above. ||

Proof of Proposition 2. The capital market equilibrium condition (1.4) implies that

$$\sum_{\theta_{t+1} | \theta_t} R(\theta_{t+1}) p(\theta_{t+1} | \theta_t) v'(c_{t+1}^0(\theta_{t+1})) / u'(c_t^y(\theta_t)) = 1. \tag{A.24}$$

Setting j equal to zero in (A.11) and substituting into (A.24) yields

$$\lambda(\theta_t) = \sum_{\theta_{t+1} | \theta_t} R(\theta_{t+1}) \lambda(\theta_{t+1}), \tag{A.25}$$

from which it follows

$$\lambda(\theta_t)V(\theta_t) = \sum_{\theta_{t+1}|\theta_t} R(\theta_{t+1})\lambda(\theta_{t+1})V(\theta_{t+1})[V(\theta_t)/V(\theta_{t+1})]. \quad (\text{A.26})$$

Recalling that $G(\theta_{t+1}) = V(\theta_{t+1})/V(\theta_t)$, equation (A.26) implies

$$\lambda(\theta_t)V(\theta_t) = \sum_{\theta_{t+1}|\theta_t} [R(\theta_{t+1})/G(\theta_{t+1})]\lambda(\theta_{t+1})V(\theta_{t+1}). \quad (\text{A.27})$$

Now sum both sides of (A.27) over θ_t to obtain

$$\sum_{\theta_t} \lambda(\theta_t)V(\theta_t) = \sum_{\theta_{t+1}} [R(\theta_{t+1})/G(\theta_{t+1})]\lambda(\theta_{t+1})V(\theta_{t+1}). \quad (\text{A.28})$$

Therefore, if $R(\theta_{t+1})/G(\theta_{t+1}) \geq 1 + \varepsilon > 1$, then (A.22) and (A.23) hold, which verifies the transversality condition. In this case, the equilibrium can be represented as the optimum of the social planning problem defined above.

If $R(\theta_{t+1})/G(\theta_{t+1}) \leq 1 - \varepsilon < 1$, then

$$\sum_{\theta_t} \lambda(\theta_t)V(\theta_t) \leq (1 - \varepsilon) \sum_{\theta_{t+1}} \lambda(\theta_{t+1})V(\theta_{t+1}). \quad (\text{A.29})$$

Recursive substitution yields

$$\sum_{\theta_t} \lambda(\theta_t)V(\theta_t) \leq (1 - \varepsilon)^{-1} \lambda_0 V_0. \quad (\text{A.30})$$

(A.30) implies the transversality condition cannot be satisfied. Hence, the equilibrium cannot maximize any social welfare function of the form in equation (1.12).

Note that the social welfare function in (1.12) is linear in $\Omega(\theta_t)$. This proof can be applied to a general social welfare function that is non-linear in $\Omega(\theta_t)$. In this case, $\mu(\theta_t)$ can be interpreted as the derivative of social welfare with respect to $\Omega(\theta_t)$ evaluated at the equilibrium. \parallel

Acknowledgement. We are grateful to David Bradford, Stanley Fischer, Philippe Weil, and the referees for comments, and to the National Science Foundation, the Amoco Foundation, and the Sloan Foundation for financial support. The research reported here is part of the NBER's research programmes in Economic Fluctuations and Productivity. Any opinions expressed are those of the authors and not those of the National Bureau of Economic Research.

REFERENCES

- BARRO, R. (1974), "Are Government Bonds Net Wealth?", *Journal of Political Economy*, **82**, 1095-1117.
- CASS, D. (1972), "On Capital Overaccumulation in the Aggregative Neoclassical Model of Economic Growth: A Complete Characterization", *Journal of Economic Theory*, **4**, 200-223.
- CHRISTENSEN, L. (1971), "Entrepreneurial Income: How Does it Measure Up?", *American Economic Review*, **61**, 575-585.
- DIAMOND, P. (1965), "National Debt in a Neoclassical Growth Model", *American Economic Review*, **55**, 1126-1150.
- FELDSTEIN, M. S. (1976), "Perceived Wealth in Bonds and Social Security. A Comment", *Journal of Political Economy*, **84**, 331-336.
- FELDSTEIN, M. S. (1977), "Does the United States Save Too Little?", *American Economic Review*, **67**, 116-121.
- FELDSTEIN, M. S. and SUMMERS, L. (1977), "Is the Rate of Profit Falling?", *Brookings Papers in Economic Activity*, 211-227.
- IBBOTSON, R. G. (1987), *Stocks, Bonds, Bills, and Inflation: Market Results for 1926-1986. 1987 Yearbook*. (Chicago: R. G. Ibbotson Associates, Inc.)
- LUCAS, R. E. (1978), "Asset Prices in an Exchange Economy", *Econometrica*, **46**, 1429-1445.
- MCCALLUM, B. T. (1986), "The Optimal Inflation Rate in an Overlapping Generation Economy with Land" (NBER Working Paper No. 1892).
- MEHRA, R. and PRESCOTT, E. (1985), "The Equity Premium: A Puzzle", *Journal of Monetary Economics*, **15**, 145-161.
- MISHKIN, F. S. (1984), "The Real Interest Rate: A Multi-Country Empirical Study", *Canadian Journal of Economics*, **17**, 283-311.
- PECHMAN, J. (ed.) (1983), *What Should Be Taxed: Income or Expenditure?* (Washington, DC: The Brookings Institution).
- PHELPS, E. S. (1961), "The Golden Rule of Accumulation: A Fable for Growthmen", *American Economic Review*, **51**, 638-643.
- PHELPS, E. S. (1966), *Golden Rules of Economic Growth* (New York: Norton and Amsterdam: North Holland).
- RHEE, C. (1987), "Dynamic Efficiency in an Economy With Land" (unpublished mimeo, Harvard University).
- SACHS, J. (1979), "Wages, Profits, and Macroeconomic Adjustment: A Comparative Study", *Brookings Papers on Economic Activity*, 269-332.

- SAMUELSON, P. (1979), "Why We Should Not Make Mean Log of Wealth Big Though Years to Act Are Long", *Journal of Banking and Finance*, **3**, 305-307.
- SHILLER, R. (1984), "Stock Prices and Social Dynamics", *Brookings Papers on Economic Activity*, 457-498.
- SOLOW, R. (1970), *Growth Theory: An Exposition* (New York and Oxford: Oxford University Press).
- SUMMERS, L. (1986), "Does the Stock Market Rationally Reflect Fundamental Values?", *Journal of Finance*, **41**, 591-601.
- TIROLE, J. (1985), "Asset Bubbles and Overlapping Generations", *Econometrica*, **53**, 1499-1528.
- TOBIN, J. (1965), "Economic Growth as an Objective of Government Policy", in *Essays in Economics*, Volume 1; Macroeconomics (Amsterdam: North-Holland Publishing Company), 174-194.
- WEIL, P. (1987), "Love Thy Children: Reflections on the Barro Debt-Neutrality Theorem", *Journal of Monetary Economics*, **19**, 377-391.