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Economic Contests: Comparative Reward Schemes

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Contests are situations in which an individual's reward depends on his performance relative to others. Students are graded on a curve; the candidate with the most votes gets the political office; the underling who performs best is promoted to the executive position. Contests are useful in dealing with indivisible rewards, reducing monitoring costs, and minimizing risks from common uncertainties. They are employed to sort potential participants and, once they have entered, to induce appropriate effort from them. With monitoring precision and prize spreads as potential choice variables, optimal contest structures are derived for fair and unfair contests among equal and unequal participants. The converse problems of climbing—low-ability individuals enter the contest designed for high-ability candidates—and slumming are shown to be manageable.

I. Introduction

In the classical model of employment, a worker's wage depends on his own performance; rewards are individualistic. Many real-world situations, by contrast, offer rewards that depend on an individual's perfor-

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mance relative to others.[1] We call such situations economic contests. Economic contests are found in a great variety of arenas. The aircraft manufacturer that submits the most attractive design/price proposal gets the contract. Students are graded on a curve. Some firms reward their highest-volume salesmen in sales contests. The baseball players with the best minor league records get the chance to play in the major leagues. Pillsbury runs a contest for the best recipes using Pillsbury products. The politician with the most votes is elected.

Contests serve three main functions. First, people derive utility from participating in or watching some contests (e.g., beauty pageants, sports contests, and the Pillsbury Bakeoffs). The contest is enjoyed in its own right. Second, many contests select the most appropriate individual (or firm) for a given function. This category includes auditions for a part in a play, free-agent tryouts, a bidding competition among potential suppliers for a contract, or a policy of promoting the managers with the best track records. Third, many contests play a useful role as incentive mechanisms. Most sales contests, competitions for the best-run Dunkin' Donuts store in a region, and promotion contests in many bureaucracies provide incentives.

Our focus here is on contests that are organized to encourage certain patterns of behavior. Leaving aside the joy-of-competition factor, why use contests rather than individualistic reward schemes (e.g., quotas, standards, piece rates)? One important reason has to do with the indivisibility of rewards. Only occasionally are rewards indivisible in and of themselves. They are usually made so to deal with other problems. There is only one chief executive in a corporation because having a single decision maker helps keep patterns of responsibility and information flow from being confused. Similarly, the numbers of management positions at lower levels may be limited by factors such as historical precedent or the need for one manager in each geographic territory. The structure of civil service career ladders is usually tightly constrained by the legislative branch, lest the executive branch exert "unjustified" authority. To prevent an arms-race type of phenomenon, professional sports leagues have found it desirable to limit the size of rosters.

When rewards are indivisible, incentives are maintained by awarding individuals probabilistic chances of winning. Random lotteries give no incentives, but contests can ensure that an individual's likelihood of being rewarded depends on his performance relative to others.

Contests do more than hand out indivisible rewards. In contrast with other forms of remuneration, such as piece rates, they may involve lower

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[1] In some situations, greater performance by others actually enhances one's expected reward. In teams, it may be difficult to monitor who has contributed what. Thus, the expected reward for an engineer working on a particular project may increase as his co-workers' performance improves.

information costs. It is generally cheaper to monitor the rank order of performance levels than to monitor absolute performance levels. This is particularly true if there are to be only one or two top prizes. To select the best English essay or cereal jingle, it is not essential to read all of the also-rans with great care. Potential employers frequently sort through résumés at considerable speed, evaluating only the very top candidates carefully. Even for those contestants who must be reviewed, the contest format may be efficient, since relatively crude, and thus perhaps inexpensive, measurements may be adequate to distinguish among candidates. In some cases, such as footraces, cardinal measurements may be completely unnecessary.

Lazear and Rosen (1981) have examined the situation in which some risks are common to all contestants. In this case, the contest may serve as a form of insurance for contestants. Both the employer and the employees may be uncertain about the relationship between effort and output. For example, when a new firm starts up, its owners may have a good idea of the level of compensation required to attract junior executives but may not know what level of output should be expected from them. Using a piece rate or a quota system may impose a good deal of risk on both the owners and the employees: the owners may end up paying a great deal more than necessary if the piece rate turns out to be too high or the quota is set too low, while the employees may end up getting paid a great deal less than their wage in an alternative job if the piece rate turns out to be too low or the quota too high. When the owners use a contest, they know exactly what employee compensation costs will be, while the range of possible outcomes for an individual employee is bounded above and below by the top and bottom prizes.

In sum, we have identified four reasons to employ contests rather than piece rates or other individualistic reward schemes (two or more may apply in any given situation):
1. utility of contest itself;
2. dealing with fixed, indivisible rewards;
3. reduced monitoring costs; and
4. reduced risk from common uncertainties.

Though contests offer numerous advantages in a variety of contexts, they may entail significant dangers of two types: eliciting the wrong level of effort, and enticing the wrong people to participate in a contest. "Wrong effort" and "wrong people" are the traditional economic problems of moral hazard and adverse selection.

If the prize spread is substantial and if the contest result is sensitive to increased effort, workers may exert excessive effort—that is, the value of the additional output to society is less than the cost that is imposed on the contestant. The resulting efficiency loss will be shared between contestants and those who run the contest, depending on elasticities of supply

and demand.[2] We see metaphoric evidence of this phenomenon in sports figures who exert themselves to the breaking point or in junior lawyers who burn the midnight oil hoping to make partner. Undergraduates competing for medical school may fit this pattern as well.

Insufficient effort is also a possibility. If the bottom prize in a contest is relatively high, contestants may choose to coast rather than compete. Problems of insufficient effort are likely to be particularly severe in contests where individuals are of unequal ability.

This analysis addresses two major questions: (1) In situations where contestants are of unequal ability, how should we get the right contestants to compete? (2) How can we elicit an appropriate level of effort from each contestant?

Most of our illustrations involve contests to produce the greatest quantity of a single homogeneous commodity. The analysis generalizes immediately to multiple dimensions: our quantitative indicator of output becomes a vector. For example, in many areas of economic competition, a contest will be conducted on dimensions involving both price and quality. Thus, two firms may be competing to get the contract to construct a building. Each will propose its own construction techniques and price. Though our models are equally applicable to contracting, industrial organization, and sports, we follow the tradition established in the literature on tournaments by focusing on illustrations drawn from labor economics.

In comparison with earlier analyses of tournaments, particularly those of Lazear and Rosen (1981), Green and Stokey (1982), Holmstrom (1982), Nalebuff 1982, and Nalebuff and Stiglitz (1983), we shall be less concerned with conditions that make a contest (possibly) superior to piece rates or quotas.[3] We take the contest form as a given, for one or more of the four reasons given above, and ask, What properties should we expect when we observe a contest? How can contests be conducted to achieve the best feasible outcomes?

II. Even Contests

Even contests are those between individuals who are identical in ability. We assume throughout Section II that each player knows his own and the other player's ability level. Section IIA deals with "fair" contests between two or more players and section IIB deals with "unfair" contests.

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[2] Of course, in perfect competition, such inefficient contests will be driven out of existence. Glenn MacDonald (personal communication 1983) has reminded us that destructive competition is like rent seeking, in that resources are used up in the process of trying to redistribute them.

[3] Green and Stokey (1982), e.g., found that contests may be superior to individualistic reward schemes if common elements of risk across workers are more important than idiosyncratic risks to different workers; Holmstrom (1982) found that more general forms of comparative reward schemes may dominate contests among teams in this case.

A. Fair Contests

We define fair contests to be those that are symmetric with respect to permutations of the contestants. Each contestant faces the same payoff function. Let z_i be the effort of player i and assume that the contest is even, so that players have the same ability. Then, if we define $p(z_1, z_2)$ and $q(z_1, z_2)$ to be the probabilities that players 1 and 2 will win the top prize, respectively, a fair contest has the property that $p(a, b) = q(b, a)$ for all values of a and b. (We could generalize the definition of a fair contest to include the case of uneven contests by defining permutations of the contestants to include permutations of their abilities as well as of their identities.)

1. Fair Contests between Two Contestants

Consider a situation in which two identical workers will be rewarded on the basis of their relative outputs (as observed by their employer). In effect they are participating in a rank-order tournament, such as that considered by Lazear and Rosen (1981). The firm awards two prizes—a top prize, M, and a bottom prize, m, where $M > m$. The firm running the contest must decide on the magnitude of these prizes as well as the precision with which to monitor employees. Each worker selects his work effort, z, which together with the effort of the other worker and some chance elements determines the prize he will receive.[4]

If the firm could monitor worker effort costlessly, the link between effort and rewards could be quite direct: greater effort could assure the top prize. However, here we will show that even with costless monitoring, the optimal contest requires that effort be monitored imperfectly.[5] Moreover, as was suggested above, although monitoring costs may be a sufficient reason, they are hardly necessary. If, for example, rewards are indivisible, a contest will be desirable, even if monitoring is costless.

Consider a particular prize spread, $M - m$. It may be dictated by external conditions. (For example, M may be getting the vacant tenured position while m is being denied tenure.) If the probability of receiving the top prize is very sensitive to worker effort (i.e., random factors are relatively unimportant), the reward structure may generate destructive competition.[6] Even if workers choose to compete, the levels of effort will be above the efficient amount. An employer who runs such a contest will

[4] Throughout this analysis, we will be treating effort as a scalar variable, but our results also apply if effort is a vector.

[5] It is intuitively clear that with costless monitoring of effort and risk-averse workers, even the optimal contest would be dominated by a contract depending solely on worker output (Holmstrom 1982, theorem 7). Unfortunately, indivisible rewards may make such contracts infeasible.

[6] Destructive competition has emerged in a variety of models. See, e.g., Akerlof (1976) and Mortensen (1981). The notion that randomness may be desirable also has antecedents in the social science literature. Skinner (1953) observed that vari-

find it difficult to attract employees and will have to pay more in order
to attract them. One way to eliminate the incentives for inefficiently high
effort would be to narrow the prize spread. However, it may not be
possible to reduce the prize spread sufficiently to give the correct marginal
incentives for efficient effort without violating the global incentive con-
dition. That is, if the prize spread is sufficiently narrow, workers may
choose to exert no effort at all and collect the bottom prize. Thus the
only way to be sure that the contest gives appropriate marginal and global
incentives may be to increase the importance of random factors in the
contest. This is not to say that exogenous random noise should be in-
troduced, but rather that monitoring should not be so precise as to di-
minish the impact of one or a few random occurrences. For example, the
employer might choose to make spot checks occasionally rather than
frequently.

Figure 1 illustrates the effect of differences in monitoring precision in
contests between workers with equal ability. Let $p(z_1, z_2)$ be the proba-
bility that worker 1 is awarded the top prize if his effort is z_1 and that
of worker 2 is z_2. With a perfect screen, $p(z_1, z_2)$ is zero if z_1 is below
z_2, .5 if the efforts are equal, and 1 if z_1 exceeds z_2. If the monitoring of
worker output is imprecise, the value of $p(z_1, z_2)$ will increase as z_1 rises
relative to z_2, but it will not jump from 0 to 1 when the performances
are equal. Since this section deals with fair contests between equally able

FIG. 1

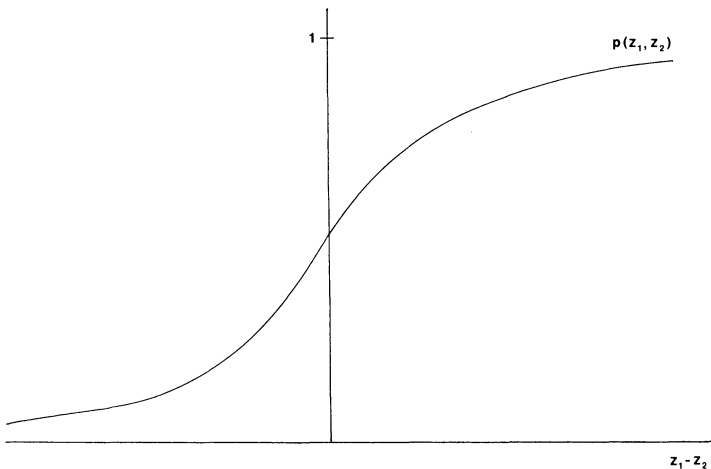able-interval reinforcement made pigeons peck harder, in part because this sched-
ule avoided the fatigue associated with reinforcement at regular intervals. Skinner
noted the potential desirability in the labor market of random penalties (e.g.,
occasional criticism of tardy workers by the supervisor) and the debilitating effect
of incentive mechanisms that elicit too much effort, such as some piecework
systems.

workers, p must have the property that $p(a, b) = 1 - p(b, a)$, however imprecise the monitoring may be. The partial derivative of p with respect to an individual's effort at $z_1 = z_2$ reflects the precision of a fair contest, since this derivative measures the responsiveness of the reward systems to increases in effort. In the case of perfectly precise monitoring, this derivative would be infinite.

Whereas in Lazear and Rosen's model the degree of precision was determined exogenously, here we will give the employer discretion over the degree of randomness in his rewards system.[7] The employer might choose to make fewer spot checks, to use more impressionistic evaluations, or simply to pay less attention to subordinate performance. Random elements often become formalized within institutional procedures governing promotions, including the degree of emphasis on seniority, which is sometimes asserted to be inconsistent with the traditional marginal productivity frameworks for wage determination (see, e.g., Doeringer and Piore 1972). Seniority systems decrease the link between individual effort and the probability of promotion. (These systems may, however, reduce other kinds of uncertainty, such as the risk of unequal treatment.)

Risk-neutral workers are the focus of our analysis here. We assume that each worker has an identical utility function of the form:

$$U(y, z) = y - Z(z),$$

where y is money income and z is effort. For these risk-neutral workers, the function Z converts effort into monetary equivalents, where Z' and Z'' are positive for positive z.

We will assume that the labor and product markets are competitive.[8] Competition (with unbiased assessments of the probabilities) requires that the firm's expected profits be zero in equilibrium and that the contest design maximize the utility of the workers subject to that constraint. Thus the contest must have the following three properties.

Property 1: The contest must be designed to elicit the efficient amount of effort.

[7] One could superimpose our model on theirs, enabling the employer to increase or decrease the inherent randomness in the effort-monitoring process. Since most of our results require only that the employer be able to increase the random component, this complication could be handled quite easily.

[8] It is possible to extend this analysis to the case of an employer with a monopoly in the product market and/or a monopsony in the labor market. Contests have the same advantages for monopolists as they do for competitive firms. However, contests may have an additional advantage (compared with a uniform piece rate) for monopsonists, in that they may allow the monopsonist to extract some or all of the employee's surplus on inframarginal effort units. Some of the authors of the present paper are working on further analysis of this problem of contests in situations with market power.

Property 2: The contest must give global incentives for effort (as well as the appropriate marginal incentives of property 1). That is, the contract must be designed so that the worker does not choose to set his effort level at zero and collect the bottom prize.

Property 3: The firm should make zero expected profits in equilibrium.

Property 1 means that the contest must give appropriate marginal incentives for workers to exert the efficient effort level, z^*. By definition of efficiency, z^* maximizes the total surplus, $vz - Z(z)$, so that z^* satisfies

$$Z'(z^*) = v \qquad (1)$$

if the price of output is v.

Assume that worker 2's effort is at the efficient level of z^* and consider worker 1's incentives. For simplicity, we drop subscripts and state his problem as choosing z to maximize

$$EU = p(z, z^*)[M - Z(z)] + [1 - p(z, z^*)][m - Z(z)]$$
$$= p(z, z^*)(M - m) + m - Z(z).$$

Thus, his choice of z will satisfy the first-order condition

$$Z'(z) = p_1(M - m), \qquad (2)$$

where p_1 is the derivative of p with respect to z_1.

Combining equations (1) and (2), we see that property 1 requires that the contest satisfy

$$p_1(M - m) = v \qquad (3)$$

in order to give worker 1 the appropriate marginal incentives. Because we are dealing with fair (symmetric) contests in this section, satisfying equation (3) also guarantees that worker 2 will have the appropriate marginal incentives.

Equation (3) shows that one can give the appropriate marginal incentives for efficient effort through a range of combinations of rewards and monitoring systems. As the prize gap increases, the precision of the optimal monitoring system declines. Small prize gaps, such as minor differences in pay among typists, must be coupled with a system that measures output quite closely (as is now being done in some typing pools with word processors); otherwise sufficient incentives will not be provided. Conversely, if the prize spread is quite large, as in the case of a promotion to an important corporate position, a large random element is required to prevent the participants from working excessively.

Note that if both workers exert the efficient effort in equilibrium, each receives an expected prize of $.5(M + m)$ since the contest is symmetric.

Property 3 requires that the firm must make zero profits in competitive equilibrium or

$$vz^* = .5(M + m). \tag{4}$$

Combining (3) and (4), we have

$$M = vz^* + \frac{v}{2p_1} \tag{5}$$

and

$$m = vz^* - \frac{v}{2p_1}. \tag{6}$$

Together, equations (5) and (6) define a competitive equilibrium contest structure for a local effort optimum, that is, one satisfying properties 1 and 3.

As Lazear and Rosen's analysis indicated, employer discretion with regard to the precision of the monitoring system is not needed for rank-order tournaments to satisfy these local efficiency conditions. In practice, however, the task of eliciting worker effort must also meet a more stringent global constaint. Rather than compete at all, workers may simply choose to set their effort level at the minimum and collect the bottom prize.

This minimum level is not necessarily the minimum physically possible effort, that is, the absence of all effort. It may be a minimum acceptable level of effort specified by the employer as a condition for staying in the contest, that is, for keeping the job. For example, the employer might require that employees come to work each day or make a certain number of sales calls a week. For convenience, we define the effort variable z so that $z = 0$ specifies the minimum acceptable effort level.

Thus, to avoid making shirking the worker's best choice, the prize structure must have property 2, which we can write as[9]

[9] Note that we are implicitly assuming that a worker's probability of winning is zero if he exerts no effort at all, i.e., $p(0, z^*) = 0$. In many situations, this is plausible—a salesman who calls no customers is not likely to win any sales contests, even if monitoring is very imprecise. In other situations, this assumption may be less accurate, particularly when we consider that "zero" effort may not be defined as a physiological minimum but rather as a minimum effort standard set by the firm. However, the assumption that $p(0) = 0$ is not necessary; our results are easily generalized as long as $p(0)$ is bounded above by a number independent of the precision of p at (z^*, z^*).

$$U(m, 0) = m - Z(0) < EU(z^*) = .5(M + m) - Z(z^*).$$

Note that the higher the minimum acceptable effort level, the less the employer need be concerned about the global incentives problem, since this inequality is easier to satisfy when $Z(0)$ is larger. (Of course, if he could set a minimum effort level of z^*, he would not need to run a contest in the first place. In many cases, however, the employer cannot observe effort well enough to set a minimum effort level of z^*, but he can observe effort well enough to set some lower minimum acceptable effort level.) If we measure utility so that $Z(0) = 0$, then property 2 just requires that:

$$Z(z^*) < .5(M - m). \tag{7}$$

To meet this inequality constraint as well as the three conditions cited above, the employer needs additional discretion over the rewards structure, which he receives in the manipulability of p_1. The efficient reward system is not uniquely defined.[10] Although (7) gives an upper bound on the permissible bottom prize, the extent of the prize spread can be varied so long as increases in the spread are coupled with reductions in the precision of the monitoring system. Note, however, that the precision of the monitoring system can be reduced and the prize spread increased, with impunity, only if workers are risk neutral, as assumed here.

If workers are risk averse, it is clear that a first-best optimum is not obtainable with a contest, since a minimum prize spread is always required to motivate workers to satisfy the global effort property.

Even if workers are risk neutral, there may be morale problems if there are very large and obvious random elements in the evaluation process. Employers who have recently begun using computers to monitor output have observed that "many workers prefer an objective measure of their output to the subjective measurements that foremen or middle managers often make." On the other hand, "bad health effects" have been attributed to computer monitoring ("Monitoring Workers by Computer" 1982). Apparently, relentlessly precise monitoring is stressful for some workers.

The fundamental importance of the global constraint and discretion

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[10] The employer has three contest parameters under his control, M, m, and p_1. Equations (5) and (6) and the inequality in (7) are the restrictions these parameters must satisfy in order to elicit the efficient level of effort from the contestants. Because (7) is an inequality, this system does not specify a unique set of parameters. At first glance, it might look as if these parameters are overrestricted by eqq. (1)–(6) plus the inequality in (7), but this is not the case. Equation (1) merely defines the efficient level of effort, z^*, while eq. (3) is equivalent to eq. (2) plus the definitional eq. (1); finally, eqq. (3) and (4) are jointly equivalent to eqq. (5) and (6). Note that our global constraint, (7), in effect, deals with the difficulty Lazear and Rosen (1981, p. 845) describe with second-order conditions.

over p_1 can be illustrated with the following example. Let utility functions take the form $U(y, z) = y - z^2$ and let v equal 2.

Equation (1) for efficient effort becomes

$$z^* = \frac{v}{2} = 1, \tag{8}$$

while the prize structure must satisfy

$$M = v\left(z^* + \frac{1}{2p_1}\right) = 2 + \frac{1}{p_1} \tag{9}$$

and

$$m = v\left(z^* - \frac{1}{2p_1}\right) = 2 - \frac{1}{p_1}, \tag{10}$$

(using [5] and [6]). To ensure that the global effort constraint is met, these values must satisfy inequality (7), which implies that $0 < p_1 < 1$. If the precision of the effort-monitoring system is exogenously determined and is not below 1, no efficient contest exists that satisfies the global effort conditions. In other words, if we monitor too precisely, we may destroy the viability of the contest. Some random elements must be allowed to remain.[11] With risk-neutral workers, it is never necessary to reduce exogenously determined errors to provide appropriate incentives, since the prize spread can always be increased to accommodate any arbitrarily small value of p_1.

To illustrate the flexibility afforded to the firm, consider three different efficient tournaments. If the firm sets p_1 at the midpoint of the acceptable range, the optimal prize pair (m, M) from equations (9) and (10) is $(0, 4)$. The narrowest efficient spread is obtained by letting p_1 become arbitrarily close to 1, in which case the optimal prize spread approaches 2 from above. This minimal spread is a consequence of the global effort constraint. Since the efficient effort level is 1 and the chance of winning the top prize is .5, the increase in a player's rewards from winning the contest must be at least 2 in order to offset the disutility of greater effort. Finally, the maximal prize spread is unbounded as we can see from (9) and (10) if we let p_1 approach 0.

[11] Of course, there is no pure-strategy Nash equilibrium in a contest with perfect monitoring (if effort is a continuous variable). However, Nalebuff and Stiglitz (1983) show that there is a mixed strategy equilibrium, even in this case.

To summarize this section, we state the following.

PROPOSITION II.1: In an even, fair contest between two contestants in a world with competitive markets, the following three conditions must hold:

$$p_1(M - m) = v; \tag{3}$$

that is, the monitoring precision and the prize spread must be inversely proportional so that the local incentives will be appropriate;

$$Z(z^*) < .5(M - m); \tag{7}$$

that is, a minimum prize spread is required to provide global incentives; and

$$vz^* = .5(M + m); \tag{4}$$

that is, the value of the total product is equal to the expected prize so that the zero-profit condition is satisfied.

Throughout this section, we have been assuming that the contest awards a top prize and a bottom prize with certainty, that is, that if worker 1's probability of winning the top prize is p, then worker 2's probability of winning the top prize is $1 - p$. This assumption is unnecessarily restrictive. Let worker 2's probability of winning be $q(z_1, z_2)$. It is straightforward to generalize our results to the case where $p(z_1, z_2) + q(z_1, z_2) = k$, where k need not be 1. Thus, for example, if there is only a 50% probability that a promotion slot will open up (so that $p + q = 0.5$), it is easy to show that a contest can give efficient incentives. For risk-neutral workers, this contest would be equivalent to a contest with $p + q = 1$ and a prize spread half as large. Similarly, a contest can also work if one promotion slot is sure to open up but there is only a 30% chance that a second slot will open ($p + q = 1.3$). For the remainder of this paper, however, we will continue to assume for convenience that $p + q = 1$.

2. Fair Contests among N Contestants

It is straightforward to extend the contest to N risk neutral workers. Here, we carry out this extension for symmetric ("fair") contests among identical contestants. We continue to assume that there are two prize levels, M and m. We define $p(z_1, z_2, \ldots z_N)$ to be the probability that worker 1 wins a top prize, M, when his effort is z_1 and the effort levels of the others are $z_2, \ldots z_N$.

The contest structure might provide for $N - 1$ prizes of M and one prize of m, or for one prize of M and $N - 1$ prizes of m, or for $N/2$ prizes of M and $N/2$ prizes of m, or any of the combinations in between. Let the number of top prizes in the contest structure be k (assumed to

be an integer with $1 \leq k \leq N - 1$). We denote the proportion of top prizes by $\alpha = k/N$, so the proportion of bottom prizes is $1 - \alpha$.

The condition for local incentives for efficient effort is

$$(M - m) \frac{\partial p}{\partial z_i}_{(z^*, z^*, \ldots, z^*)} = v.$$

In a symmetric contest, each worker gets an expected prize of $\alpha M + (1 - \alpha)m$ if he exerts the equilibrium effort level of z^*. Thus, the global no-shirking condition is $m - Z(0) < \alpha M + (1 - \alpha)m - Z(z^*)$ or $\alpha(M - m) > Z(z^*) - Z(0)$.

As the proportion of top prizes, α, decreases, the minimum prize spread required by the global no-shirking constraint increases. This could be one reason why the difference between the salary of the president of a company and the average salary of those on the next rung down the ladder often seems to be very large relative to differences between salaries on the lower rungs of the ladder. An alternative explanation of this hierarchical wage structure is given by Rosen (1981). If a firm's organizational structure requires that half of the assistant vice presidents become vice presidents, the minimum salary differential required to motivate effort at this level is less than the minimum salary differential required to motivate effort if only one of the 10 vice presidents can become president. Note that local incentives are not the problem here: arbitrarily small prize spreads can give local incentives for efficient effort if effort is monitored sufficiently precisely. Of course, the output and effort of executives may be very difficult, if not impossible, to monitor precisely. In that case the local incentives condition may also require large prize spreads.

Finally, competition requires that workers receive an expected prize equal to the value of their product, so that $\alpha M + (1 - \alpha)m = vz^*$ must hold.

For the remainder of this paper, we shall deal only with two-person contests. Generalization to N-person contests is straightforward, as we have demonstrated above for fair, even contests, as long as contestants are risk neutral.

B. Unfair Contests

Employers often run unfair contests. Discrimination, affirmative action, or nepotism may yield contests in which two equally able contestants who work equally hard have unequal chances of being promoted.[12]

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[12] Such contests do not exist in classically competitive markets, but regulation or other market imperfections can lead to such contests. In addition, if the firm's owner has tastes for discrimination, unfair contests can arise as a form of owner consumption. Nepotism is an even more obvious example of owner consumption.

Contests that are held over a period of time also may become asymmetric. Consider a sales competition that lasts for 2 quarters, in which intermediate standings are posted at the end of the first quarter. Even if the contest was fair (i.e., symmetric) at the outset, it becomes asymmetric from the point of view of the players once the intermediate results are posted.

In one form or another, every teacher has confronted this type of question: "Why should I keep working now that I have ruined my record and can no longer receive an A?" Similarly, a firm's struggle for market share is frequently an attempt to get into an asymmetric contest in which others will no longer make an effort. A firm whose location may give it a geographic advantage is engaged in such a contest.

Some asymmetric contests spontaneously occur in society. We should attempt to understand how they operate. In other instances we, like the teacher running a grading system over a semester, must run contests that are likely to be asymmetric. How should they be conducted?

Since we continue to assume that both workers have identical abilities and utility functions, we can define unfair contests as those characterized by a function $p(z_1, z_2)$ that does not satisfy $p(a, b) = 1 - p(b, a)$. In particular, $p(a, a)$ is not equal to .5. Even if both players work equally hard, one is more likely to win than the other, despite their equal abilities.

The symmetric contests we discussed in Section IIA had two claims to being called fair. First, symmetric contests are ex ante fair, by definition. Furthermore, although with imprecise monitoring the contest could end up being ex post quite unfair (i.e., if the top prize is awarded to the employee who exerted strictly less effort), in equilibrium this does not happen. As we saw in IIA, in equilibrium each worker exerts the same amount of effort and each has a 50% chance of winning the top prize. (Of course, any contest with a random component necessarily involves some degree of ex post unfairness.)

Our question now is, Can an asymmetric contest still be designed to give workers the appropriate incentives? The answer is yes, but not as easily as with a symmetric contest. In an unfair contest, it turns out, giving contestants the correct marginal incentives is quite straightforward but satisfying the global incentives property is more difficult. The contestant who is disadvantaged by the contest will have greater incentives to shirk and collect the bottom prize than he would in a fair contest. Thus, the minimum possible prize spread in an unfair contest will be greater than the minimum possible prize spread in the corresponding fair contest. Another problem is that it will be more difficult to attract workers to work for a firm whose contests are biased against them. Unless the worker against whom the contest is biased has less attractive alternatives (e.g., if all employers discriminate or practice affirmative action), an unfair contest will have to offer higher prizes to attract those workers against

whom the contest discriminates. Thus, since an unfair contest must make the disfavored contestant at least as well off as his alternative opportunities, it essentially redistributes money from profits to the favored contestant.

Since we continue to assume that both workers are identical in ability and utility functions, we still want to elicit the same efficient level of effort from each worker. Thus the efficient effort condition remains:

$$Z'(z^*) = v. \tag{11}$$

Worker 1 takes worker 2's effort level as given at some level, z_2, and chooses z_1 to maximize

$$EU_1 = p(z_1, z_2)M + [1 - p(z_1, z_2)]m - Z(z_1).$$

Thus worker 1's choice of z_1 satisfies

$$p_1(M - m) = Z'(z_1), \tag{12}$$

where p_1 is the partial derivative of p with respect to its first argument evaluated at the worker's chosen effort levels (z_1, z_2). Similarly, worker 2's choice of z_2 satisfies

$$q_2(M - m) = Z'(z_2^*), \tag{13}$$

where $q(z_1, z_2)$ gives the probability that worker 2 will win the top prize if his own effort is z_2 and worker 1's effort is z_1. Analogously to p_1, q_2 is defined as the partial derivative of q with respect to its second argument, evaluated at (z_1, z_2).

Combining (11), (12) and (13), we find that the contest characterized by the functions p and q must satisfy

$$p_1(M - m) = q_2(M - m) = v \tag{14}$$

in order to provide local incentives for efficient effort.

Note that equation (14) merely constrains the derivatives of p and q to be equal. Thus, appropriate incentives do not require that p and q themselves be equal when effort is equal. For example, suppose that p is of the form:

$$p(x, y) = p(x - y). \tag{15}$$

Since, by definition, $q(x, y) = 1 - p(x, y)$, q will also have the form $q(x - y)$. When (15) holds, then, any scheme in which p is chosen to

give player 1 the correct incentives will automatically give player 2 the correct local incentives. This will be true whether the contest is symmetric (in the case $p[0] = .5$) or asymmetric ($p[0] \neq .5$).

All of the analysis above assumed that we were dealing with an interior solution; that is, we were concerned only with marginal incentives. To guarantee an interior solution, the contest must also provide global incentives; otherwise one or both players might choose to exert no effort at all. There will be two global no-shirking conditions (which are analogous to [7] in the last sec.):

$$Z(z^* < p^*(M - m), \tag{16}$$

and

$$Z(z^*) < q^*(M - m), \tag{17}$$

where $p^* = p(z^*, z^*)$ and $q^* = q(z^*, z^*)$. Only one of these constraints will be binding. If the contest is tilted in favor of player 2, then (16) will be binding since $p^* < q^*$. Since $p^* < .5$, providing global incentives in an asymmetric contest will require a larger prize spread ($M - m$) as well as correspondingly less precise monitoring than in a symmetric contest.

To sum up: An owner can discriminate in favor of anyone he wants to (or is required to) in setting up his contest and still maintain incentives for the other workers to exert themselves efficiently, but he will have to set higher prize levels and a larger prize spread than would otherwise be necessary. No worker suffers from an unfair contest, but there is a redistribution of income from owners of firms to favored employees. This is true for a single unfair contest only; if, for example, the government requires all employers to run unfair contests, some workers clearly suffer.

III. Climbing and Slumming: Heterogeneous Contestants

Most contests in this world are among unequal contestants. Some firms have better distribution networks or better name recognition than others, workers have different abilities, politicians have different amounts of charisma, and so on.

In Sections III and IV, we drop the assumption that players are identical in abilities. Now it can be more difficult to maintain appropriate incentives.

If no one knows the workers' abilities during the contest, that is, if contestants are ex ante identical, contests similar to those in Section II will work as well as any alternative if workers are risk neutral. Ideally, in such a case, the more able should be induced to work harder than the less able, but since no one (including the workers themselves) has information on abilities, no scheme can achieve this result. Note that in this case, a contest with precise monitoring of relative output levels and a

substantial prize spread may provide appropriate incentives: uncertainty about one's own and one's opponent's ability levels can play the same role that imprecise monitoring played in Section II. Thus, a contest among heterogeneous but ex ante identical contestants can provide incentives, while identifying (with precise output monitoring) the most able contestant as the winner. This may be important if the prize is promotion to a position that the employer wishes to fill with the most able candidate.

At the other extreme, if everyone (including the employer) knows the ability levels of the contestants, it should be possible to devise a system of handicaps so that appropriate incentives are preserved for all. These might involve different prizes or probability of reward functions tailored to each worker. In the risk-neutral case particularly, there are several degrees of freedom available for this purpose.

For example, assume that the probability function in figure 2a represents an efficient scheme when player 1 and player 2 are equally able. Now, suppose that player 1 is more able than player 2 so that 2's efficient effort level remains at z_2^* but 1's efficient level is now $z_2^* + T$. Is there a way to construct a contest that elicits the efficient effort level from each player? The answer is yes, and the solution is straightforward. If we translate the function in figure 2b to the right by T units, we have a
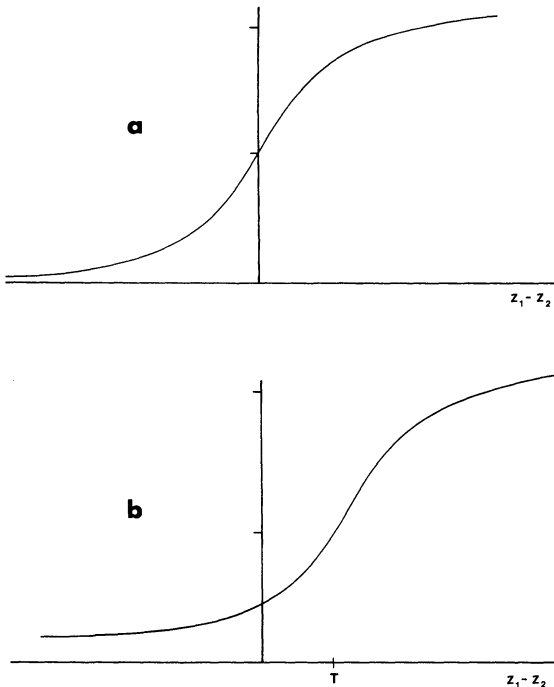

Fig. 2

probability function that represents an efficient scheme for players with unequal abilities.[13] Of course, to attract the more productive player 1 into the contest, we may wish to move the function upward as well. Section II*B* showed that we can do this in the neighborhood of the equilibrium without disturbing incentives as long as we preserve the derivative of the probability function at the desired equilibrium and keep the prize spread large enough to satisfy the appropriate global no-shirking constraints.

Another way to attract player 1 into this contest would be to have a separate, higher set of prizes for him. This strategy is essentially equivalent to paying him a fixed fee for entering. Many marathon directors pay "expenses" (sometimes using a very generous definition of expenses) to world class runners to induce them to participate in their races.[14]

Another alternative for an employer who knows employee ability levels is to segregate workers so that each competes only against others of equal ability.

The real difficulty arises when workers' abilities are known to themselves but not to their employers. In this section we assume that the employer has no direct information about employees' ability levels but he sets up separate contests, with one labeled a high-ability contest and the other labeled a low-ability contest. We assume that a worker knows his own ability level and assumes that the other contestant in a high-ability contest will have a high ability and that the other contestant in a low-ability contest will have a low ability. We show that, under these circumstances, the employer may be able to structure the contests so that workers self-select into the appropriate contest *and* each contest gives its contestants appropriate incentives. In Section IV, we will discuss the case in which employers attempt to run a single type of contest designed for all workers while each employee knows only his own ability level.

When Lazear and Rosen considered the problem of heterogeneous contestants, with monitoring precision exogenously determined, the outlook was especially bleak. For any given effort level, the low-ability workers always preferred to compete against their more able counterparts in the larger prize contest designed for the high-ability workers. Thus Lazear and Rosen's rank-order tournaments were always undermined by

[13] There is a difference between the translation that maintains incentives in an unfair contest and the translation that gives incentives in an uneven contest. Suppose that the function in fig. 2*a* gives the appropriate incentives in an even, fair contest. If we now make the contest unfair, we have to translate the function vertically at the equilibrium, so that we preserve the slope of the function at z^*. If instead, we make the contest uneven, we must translate the function horizontally, preserving the slope at $p = .5$. Once we have done that, we are free to translate the function upward or downward, as may be necessary to attract entrants.

[14] Alternatively, the employer could use a handicapping system that simply adjusted the prizes separately for each contestant without changing the probability functions. This adjustment would generally increase the prize spread for both contestants.

the inability to structure contests as both a self-selection mechanism and an incentive device.

In our model, monitoring precision is a choice variable. Low-ability workers' attempts to infiltrate contests designed for higher-ability workers—a phenomenon we refer to as "climbing"—can potentially be eliminated by having the employer monitor less precisely.

The converse phenomenon of "slumming" occurs when high-ability workers try to infiltrate the contest designed for the low-ability workers, where they seek to collect the top prize with a low level of effort. Such incursion of high-ability workers could clearly reduce the low-ability workers' incentives to compete. In general, slumming will not be a problem if the prize spread in the contest designed for low-ability workers is sufficiently small. When the prize spread is small, the top prize will not be very far above the expected output of a low-ability worker. In this case, a worker with significantly higher ability can generally do better if he stays in the contest designed for workers like himself, where his expected prize will be equal to the expected output of a high-ability worker. To maintain incentives with a small prize spread, we must increase the precision of the low ability-contest.[15]

The following proposition is proved in the Appendix.

PROPOSITION III.1: By suitably decreasing the prize spread in the contest designed for low-ability workers (along with an appropriate adjustment of the monitoring precision of the contest), the firm can simultaneously (*a*) maintain marginal incentives for the low-ability workers, and (*b*) induce high-ability workers to self-select into their own contest.

Interestingly, it turns out that the way to prevent climbing is to increase the prize spread in the high-ability contest; the bottom prize must go down and the top prize must go up. Since a climber is more likely to receive the bottom prize than the top prize, increasing the prize spread makes the high-ability contest less attractive to him without destroying its appeal for high-ability contestants, who can continue to anticipate the same expected prize. Of course, the monitoring precision at the high-ability equilibrium will have to be decreased to maintain appropriate incentives for the high-ability contestants.[16]

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[15] Unfortunately, if the worker is not much higher in ability than the low-ability workers, the required increase in the precision of the low-ability contest may violate the global incentives condition.

[16] There is an analogy between designing statistical procedures so as to reduce the probability of type 1 or type 2 error and designing contests so as to deter slumming or climbing. However, while it is often the case that a statistical procedure requires that we trade off higher probabilities of one type of error for lower probabilities of the other type, we have two independent instruments to use in deterring slumming and climbing. That is, decreasing the prize spread in the low-ability contest deters slummers and increasing the prize spread in the high-ability contest deters climbers.

The following proposition is also proved in the Appendix.

PROPOSITION III.2: By suitably increasing the prize spread in the contest designed for high-ability workers (and adjusting its precision appropriately), the firm can simultaneously (a) maintain incentives for the high-ability workers, and (b) induce the low-ability workers to self-select into their own contest.

As a practical matter, there are limits on the amount of self-selection we can induce by increasing the prize spread of the contest. We have already alluded to morale problems and to the fact that our assumption of risk neutrality becomes less plausible (and a less realistic approximation of reality) with the larger prize spreads. A further problem is that as we increase the prize spread, $M - m$, we must keep the expected prize, $0.5(M + m)$ constant. A substantial decrease in the precision of the contest would require that m be a large negative number; legal and institutional constraints prevent implementation of such contests.

Before stating propositions III.1 and III.2, we made the assumption that each worker believes that his opponent in the high-ability contest would be a high-ability worker and that his opponent in the low-ability contest would be a low-ability worker. This assumption might seem unwarranted. Since we assume that the employer cannot observe his employees' ability levels directly, why should we assume that a worker thinks he knows the ability level of his opponent? One argument is that such a belief is rational if the contests are structured (along the lines of propositions III.1 and III.2) to prevent slumming and climbing. Furthermore, in some situations workers know more about each other's ability than does the employer, for example, when the workers have a skill (such as bricklaying) that is not shared by their boss. Finally, it should be noted that this assumption is not required in order to prevent slumming, since the low-ability contest will be relatively less attractive and the high-ability contests will be relatively more attractive if there are impostors in either. However, for these same reasons, this assumption is critical for preventing climbing.

IV. Contests in Which Contestants Know Only Their Own Abilities

Up to this point, we have assumed contestants know both their own ability levels and those of their competitors. If all workers are identical (as in Sec. II), it may be reasonable to assume that they know that they are all equally able. If workers have different abilities, however, they may not know the abilities of their co-workers.

In this section, we examine a major source of the uncertainty in most contests—that is, each contestant's uncertainty about the other's ability. We do not propose to investigate the optimal contest under these conditions, only to show that for some distributions of ability, contests with precise output monitoring necessarily elicit less effort from the high-ability worker than from the low-ability worker. This is clearly inefficient,

since an efficient reward scheme should induce the more able to work harder.

We assume that workers have identical risk-neutral utility functions but that identical efforts exerted by different workers produce different amounts of output. The utility function takes the simple form: $y - Z(z)$. Output, Q, is the product of effort and an ability factor, A, so that $Q = A \cdot z$. A worker knows his own ability level but has only a probability distribution function, F, describing his opponent's ability. We assume that the lower and upper limits of the ability distribution are 1 and 2, respectively.

We can describe the contest very succinctly: it awards the top prize, M, to the contestant who produces the greater output and the bottom prize, m, to the other worker. The only uncertainty in this contest is each contestant's uncertainty about his opponent's ability level. In particular, the employer can observe output perfectly and there is a deterministic relation between effort and output for any worker.

In equilibrium, there will be functions, $z(A)$ and $Q(A)$, that describe the effort exerted and the output produced, respectively, by a worker with ability level A. Of course, $Q(A)$ satisfies $Q(A) = A \cdot z(A)$.

Let us examine the problem from the point of view of a worker who assumes that his opponent is using the equilibrium strategy functions, $z(A)$ and $Q(A)$. His problem is then to choose some output level Q^* that maximizes his utility. For any Q^* he might choose, there is some ability level, call it A^*, such that $Q^* = A^* \cdot z(A^*)$. Since we will show below that $Q(A)$ is monotonically increasing, his choice of Q^* is equivalent to a decision to beat all potential opponents with ability less than A^*. Thus we can think of his problem as the choice of A^*, the most able opponent he chooses to beat, given his own ability level, A. His problem can then be stated:

choose A^* to maximize $EU(A,A^*)$

$$= \int_1^{A^*} (M - m)f(\xi)d\xi + m - Z\left[\frac{A^* \cdot z(A^*)}{A}\right].$$

The first-order condition is then:

$$\partial EU(A, A^*)/\partial A^* = (M - m)f(A^*)$$
$$- Z'\left[\frac{A^*z(A^*)}{A}\right]\frac{[z(A^*) + A^*z'(A^*)]}{A} = 0.$$

In equilibrium, however, A^* must equal A, so the first-order condition becomes

$$\frac{(M - m)f(A)}{Z'[z(A)]} = \frac{z(A)}{A} + z'(A),$$

or

$$z'(A) = \frac{(M - m)f(A)}{Z'[z(A)]} - \frac{z(A)}{A}. \tag{18}$$

Note that since $Q(A) = A \cdot z(A)$,

$$Q'(A) = A \cdot z'(A) + z(A) = \frac{(M - m)f(A)}{Z'[z(A)]} > 0$$

for all A in the support of f, so that in equilibrium output is indeed an increasing function of ability.

In general, the equilibrium effort function will be determined by the distribution of abilities, as we can see from equation (18). The optimal effort function, however, is independent of the distribution. Unless the employer has control over the distribution of abilities among his workers (or, more precisely, the workers' perception of that distribution), he will be unable, in general, to design a contest to give the appropriate incentives to workers, no matter what levels he chooses for M and m. All of the analysis above assumed that the employer was monitoring output precisely; if we now allow him to choose the degree of precision with which to monitor output, we give him another way to manipulate incentives. Appropriately imprecise monitoring may be particularly helpful in motivating the most able workers.[17]

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[17] Variable rewards may also be useful to maintain rewards in an uneven contest. Consider a many-person contest in which a few people are far superior to others. If the top half of the performers will get prizes, then it may not pay the ablest contestants to work very hard. However, if the number of prizes is variable, marginally better performance by the very ablest could be critical to them. Large numbers of different prize levels could accomplish the same outcome. A related phenomenon is line drawing, a common means to reduce monitoring costs. Anyone who gets across the line wins; for example, any score above a certain threshold on the bar exam will qualify the contestant to practice law. Difficulties arise, however, if the line is visible, and if individuals can calibrate how certain efforts and natural abilities will relate to performance. Thus, we might expect that most eggs will weigh just about enough to get over one grade line or another. To avoid the stingy passing problem, either individuals must be highly uncertain about their performance, or the location of the line must be a variable. With a bar exam, virtually all candidates study hard because, never having taken the exam before, they find it difficult to predict how they will do. If individuals were able to predict accurately, and if it were essential for the test administrators to motivate high levels of effort among all contestants, they might choose to make the passing ratio (i.e., the ratio of those who pass to those who take the exam) variable from test to test. This would be a contest with an uncertain number of prizes.

Since we could not get a closed-form expression for $z(A)$ from (18), we have integrated (18) using numerical methods for several special cases to indicate some of the possibilities. The results for two of these cases are given in tables 1 and 2.

In both of these cases we assumed a prize spread $(M - m)$ of 100 and assumed that $Z(z) = z^2$. We also specified the initial condition that a worker with ability 1 (the lowest level) would exert an effort of 1. Since the lowest-ability worker cannot expect to beat anyone in equilibrium anyway, it is reasonable to assume that he will exert a minimal level of effort. This initial condition can (implicitly) be specified by the employer because we have assumed that he can observe output. Thus, if the employer sets a minimum acceptable output of 1, the worker with ability level 1 will be required to exert an effort of 1.

In case 1, we assumed that ability had the uniform distribution, while in case 2, we assumed that the distribution of ability was triangular, that is, that $f(A) = 4 - 2A$ for all A in [1, 2].

Note that effort is monotonically increasing with ability in case 1, while

Table 1
Results of Integrating (18): Case 1,
Uniform Distribution of Abilities

Ability (A)	Effort [Z(A)]	Output [Q(A) = A·Z(A)]
1.0	1.00	1.00
1.1	3.16	3.48
1.2	4.20	5.04
1.3	4.93	6.40
1.4	5.50	7.70
1.5	5.97	8.96
1.6	6.38	10.21
1.7	6.75	11.47
1.8	7.08	12.74
1.9	7.38	14.02
2.0	7.66	15.31

Table 2
Results of Integrating (18): Case 2,
Triangular Distribution of Ability

Ability (A)	Effort [Z(A)]	Output [Q(A) = A·Z(A)]
1.0	1.00	1.00
1.1	4.27	4.70
1.2	5.57	6.68
1.3	6.35	8.25
1.4	6.84	9.58
1.5	7.14	10.72
1.6	7.30	11.68
1.7	7.33	12.47
1.8	7.26	13.07
1.9	7.08	13.45
2.0	6.80	13.59

effort first increases and then decreases with ability in case 2. For a heuristic explanation of this, examine figures 3a, b.

In figure 3a, with a uniform distribution, a worker at A_1 and a worker at A_2 each have the same number (i.e., density) of workers just ahead of them, loosely speaking. Assume that initially everyone exerts the same amount of effort. In this case, each worker would derive equal marginal benefits from passing the people who are just a bit ahead of him while the worker at A_2 has a lower cost of passing; thus we would expect him to work harder in equilibrium.

In figure 3b, we see that worker 2 has fewer people just ahead of him than does worker 1. If worker 2 were to expend more effort, he might pass fewer people than worker 1 would. Thus, over some range with sufficiently high abilities, effort may decrease rather than increase with ability. Note that output, of course, still increases monotonically with ability. Efficiency, however, clearly requires that effort increase with ability, so the situation in the contest in case 2 could not be optimal.

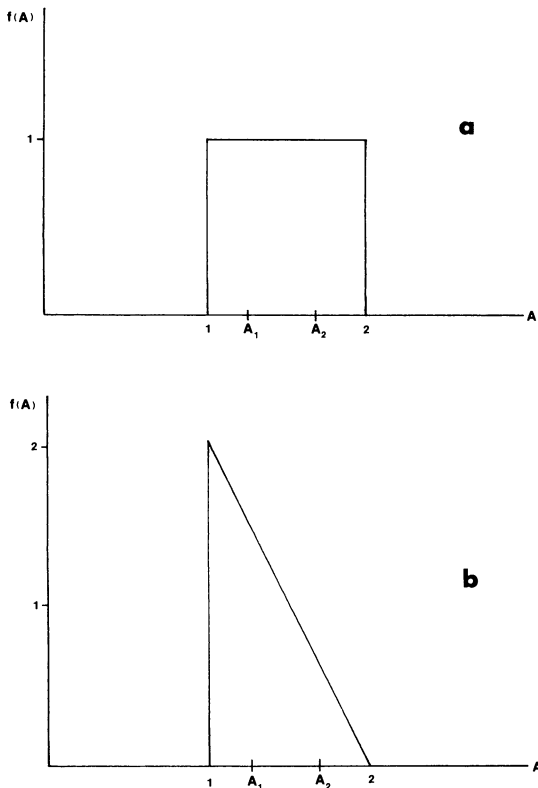This suggests that for many distributions of workers' abilities, there

FIG. 3

may be no contest that induces efficient effort. For example, if the value of output is v, then the function z^* that specifies the efficient level of effort for a worker of ability level A is $z^*(A) = vA/2$. If we substitute this into (18), we find that only if the (perceived) distribution of workers' abilities has a density function of the form $f(A) = kA$ for some $k > 0$ will a contest induce each worker to work at his efficient level of effort.

V. Conclusion

Contests are a pervasive phenomenon in our society. Apart from the joys they bring in and of themselves, most widely observed on the playing fields, they serve important economic functions. Past work on contests has been predominantly in the field of labor economics. Most of our examples follow in this tradition. But the principles are quite general. They apply to bidding for contracts among construction companies, patent races, and perhaps even contests for the heart.

Previous work showed that contests may be desirable when an unobservable parameter, which represents a common risk, affects the output of all contestants. A contest avoids problems of excess or insufficient payment in such situations.

Indivisible rewards, such as a promotion to a single position or the awarding of a contract to a firm, create the need for probabilistic payment. At first glance, we might think we could reward each agent separately. Firm A would get a $1/3$ chance for the contract and firm B a $2/3$ chance. This approach encounters two difficulties: (1) The levels of performance may be such that the total probability of reward is greater or less than the number of prizes. This is particularly likely if there is some uncertainty about the abilities (or preferences, which turns out to be the same thing) of the agents. (2) Even if the probabilities sum appropriately, it is important to have negative correlations on the outcomes; that is, when A wins, B should not. A contest neatly solves both of these problems arising from the indivisibility of prizes.

Contests may be valuable in a third domain, enticing the right people to "play the game." In traditional gaming contests, this is often done by imposing an entry fee. In general, prize structures can be employed to deter individuals of inappropriate quality from entering contests not intended for them, whether by climbing or slumming.

Microeconomic theory advances in waves. First, it elegantly demonstrated the accomplishments of free markets when certain conditions were fulfilled. A second wave, in response, detailed the innumerable situations in which these conditions—most particularly the unimpeded and costless flow of information—were not satisfied. A third wave may now be on us. Economists increasingly are discovering that real-world actors are immensely inventive in designing private contractual arrangements that produce satisfactory outcomes when the precise conditions for the prin-

cipal theorems of welfare economics are not met. The use of contests to deal with indivisibilities and asymmetric information is but one example.

Appendix
Proofs of Propositions III.1 and III.2

We start by setting up some notation that we will use in both proofs. We index the high-ability worker by $=$ and the low-ability by $-$. A high-ability worker has a utility function, $\bar{\bar{U}}(y, z) = y - \bar{\bar{Z}}(z)$, and a low-ability worker has a utility function, $\bar{U}(y, z) = y - \bar{Z}(z)$. We assume that $\bar{\bar{Z}}(z) < \bar{Z}(z)$ and $\bar{\bar{Z}}'(z) < \bar{Z}'(z)$ for all positive z. Thus, we are defining high-ability and low-ability workers by their respective disutilities for effort.

We also use $=$ and $-$ to index the contests designed for the high- and low-ability workers, respectively. For example, $\bar{\bar{M}}$ is the top prize in the high-ability contest, $\bar{M}$ is the top prize in the low-ability contest, $\bar{\bar{p}}_1$ is the derivative of $\bar{\bar{p}}$ at $(\bar{\bar{z}}^*, \bar{z}^*)$ in the high-ability context, $\bar{p}_1$ is the derivative of $\bar{p}$ at $(\bar{z}^*, \bar{z}^*)$ in the low-ability contest, and so on.

PROPOSITION III.1: By suitably decreasing the prize spread in the contest designed for low-ability workers (along with an appropriate adjustment of the monitoring precision of the contest), the firm can simultaneously (a) maintain marginal incentives for the low-ability workers, and (b) induce high-ability workers to self-select into their own contest.

PROOF: Equation (5) gives us that $\bar{M} = v\bar{z}^* + v/2\bar{p}_1$. As we increase $\bar{p}_1$ we can reduce the prize spread so that $\bar{M}$ approaches $v\bar{z}^*$. Thus, by increasing the monitoring precision sufficiently, we can ensure that $\bar{M} < v\bar{z}^* + \epsilon$, for arbitrarily small ϵ. In this case, the utility for a high-ability worker who attempts to infiltrate the low-ability contest and walk off with the top prize, $E\hat{U}$ must satisfy

$$E\hat{U} < v\bar{z}^* + \epsilon - \bar{\bar{Z}}(\bar{z}^*),$$

since the high-ability worker must work harder than $\bar{z}^*$ in order to be sure of the top prize. If the high-ability person stays in his own contest (or works at the appropriate piece rate), his utility would be

$$E\bar{\bar{U}} = v\bar{\bar{z}}^* - \bar{\bar{Z}}(\bar{\bar{z}}^*),$$

where $\bar{\bar{z}}^*$ maximizes $vz - \bar{\bar{Z}}(z)$. Thus, for sufficiently small ϵ, the high-ability person is better off in his own contest.

Unfortunately, there may be problems with achieving this no-slumming equilibrium if the required prize spread becomes so small that the global

effort constraint for low-ability individuals is violated and the low-ability individuals choose to shirk. (Note that shirking will not be a problem for any would-be high-ability infiltrators until after it is a problem for the low-ability contestants, because high-ability workers have a lower cost of effort.)

PROPOSITION III.2: By suitably increasing the prize spread in the contest designed for high-ability workers (and appropriately adjusting its precision), the firm can simultaneously (*a*) maintain incentives for the high-ability workers, and (*b*) induce the low-ability workers to self-select into their own contest.

PROOF: We will refer to the low-ability worker as the climber. The climber's expected utility, $E\hat{\bar{U}}$, if he enters the high-ability tournament is given by

$$\max_{z} E\hat{\bar{U}}(z) = \bar{m} + \bar{p}(z, \bar{\bar{z}}^*)(\bar{\bar{M}} - \bar{m}) - \bar{Z}(z)$$

$$= v\left[\bar{\bar{z}}^* - \frac{1}{2\bar{\bar{p}}_1} + \frac{\bar{\bar{p}}(z, \bar{\bar{z}}^*)}{\bar{\bar{p}}_1}\right] - \bar{Z}(z),$$

where $\bar{\bar{z}}^*$ is the prevailing optimal effort level of high-ability workers, and $\bar{p}$, $\bar{m}$, and $\bar{M}$ define the high-ability contest's prize structure. The value of $\bar{p}_1$ is evaluated at the optimal effort pair for high-ability workers, $(\bar{\bar{z}}^*, \bar{\bar{z}}^*)$. The second equality follows from equations (3) and (4).

Competing in the low-ability contest, the climber would receive an expected utility, $E\bar{U}$, of

$$E\bar{U} = v\bar{z}^* - \bar{Z}(\bar{z}^*),$$

where $\bar{z}^*$ is the efficient effort level for the low-ability worker.

At a minimum, the high-ability contest must be designed so that

$$\bar{m} < v\bar{z}^* - \bar{Z}(\bar{z}^*).$$

Otherwise, the low-ability worker would prefer to enter the high-ability contest, exert no effort, and collect the bottom prize rather than stay in the low-ability contest. This constraint is stronger than the global constraint that keeps high-ability workers from shirking, which can be satisfied simply by increasing the prize spread and decreasing the precision in the high-ability contest so as to maintain incentives for the high-ability workers.

Now, since $\bar{z}^*$ maximizes $vz - \bar{Z}(z)$, it must be that

$$E\bar{U}(\bar{z}^*) = v\bar{z}^* - \bar{Z}(\bar{z}^*) < v\bar{\bar{z}}^* - \bar{Z}(\bar{\bar{z}}^*).$$

Thus, we can design the high-ability contest so that $\bar{\bar{m}}$ satisfies

$$v\bar{\bar{z}}^* - \bar{Z}(\bar{\bar{z}}^*) < \bar{\bar{m}} < v\bar{z}^* - \bar{Z}(\bar{z}^*). \tag{A1}$$

We now assume that the contest has already been modified so that (A1) is satisfied. Next we will show that by adjusting $\bar{p}$ away from the high-ability optimum, we can guarantee that the best a climber can do for himself in the high-ability contest is to exert no effort and collect $\bar{\bar{m}}$, the bottom prize. Then, by (A1), we will be done.

Consider the contest in figure 4. At $\bar{\bar{z}}^*$, the climber certainly does better than he would do at $z = 0$. Suppose we modify the contest, changing $\bar{\bar{p}}$ but only for values of z away from the high-ability optimum, in the domain $[0, \bar{\bar{z}}^* - \epsilon]$ for some $\epsilon > 0$, so that it looks like figure 5. Then the climber would choose to set his effort at zero in the high-ability contest. But (A1) already guarantees that he would rather compete in the low-ability contest than collect the bottom prize with no effort in the high-ability contest.

We now justify what we have just done and state the condition a bit more rigorously. At $\bar{\bar{z}}^*$, we have that

$$(\bar{\bar{M}} - \bar{\bar{m}})\left[\left.\frac{\partial\bar{\bar{p}}}{\partial z_1}\right|_{(\bar{\bar{z}}^*, \ \bar{\bar{z}}^*)}\right] = \bar{\bar{Z}}'(\bar{\bar{z}}^*) < \bar{Z}'(\bar{\bar{z}}^*).$$
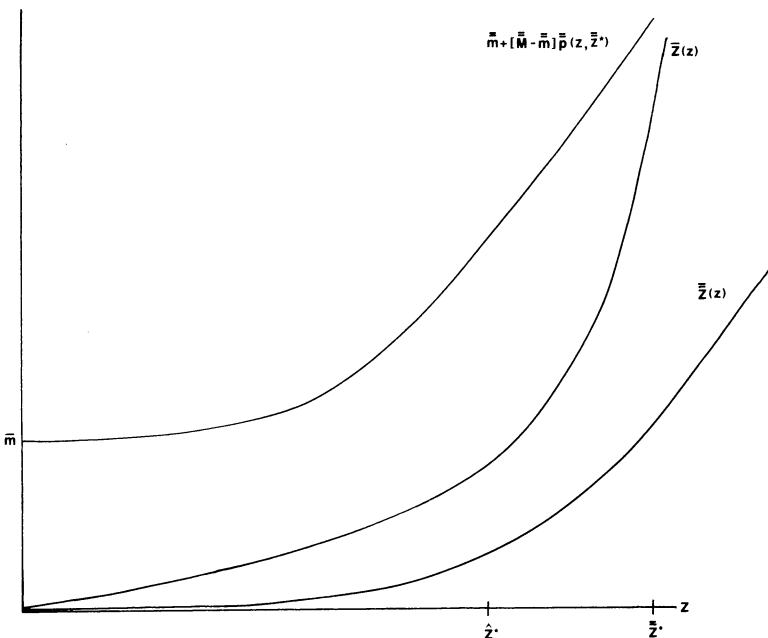
FIG. 4

In fact, by continuity,

$$(\bar{\bar{M}} - \bar{\bar{m}})\left[\frac{\partial \bar{\bar{p}}}{\partial z_1}\bigg|_{(z,\,\bar{\bar{z}}^*)}\right] < \bar{Z}'(z), \tag{A2}$$

for all z in some neighborhood $(\bar{\bar{z}}^* - \epsilon, \bar{\bar{z}}^*)$. What we need to do is modify the derivative of $\bar{p}$ for z less than $\bar{\bar{z}}^* - \epsilon$ so that (A2) will be satisfied for all $z \leq \bar{\bar{z}}^*$. When (A2) holds for all $z \leq \bar{\bar{z}}^*$, the best that the climber can do is to set his effort level at zero. Note that we can modify $\bar{\bar{p}}$ so that (A2) is satisfied for the appropriate z's without affecting the value of $\bar{\bar{p}}$ or its derivatives in the neighborhood of the high-ability equilibrium, and without affecting the prizes, $\bar{M}$ and $\bar{m}$. Thus global and local incentives for the high-ability workers are preserved. (The first inequality in (A1) guarantees that it is possible for (A2) to be satisfied for all z in $[0, \bar{\bar{z}}^*]$ without changing the value of $\bar{m}$.)

To discourage climbing by inducing low-ability workers to self-select into their own contests thus requires that (A1) and (A2) be satisfied. A contest designed for high-ability workers can give appropriate incentives to high-ability workers and also satisfy (A1) and (A2), thereby deterring low-ability workers from climbing, if the prize spread is enlarged and the monitoring function is appropriately adjusted.
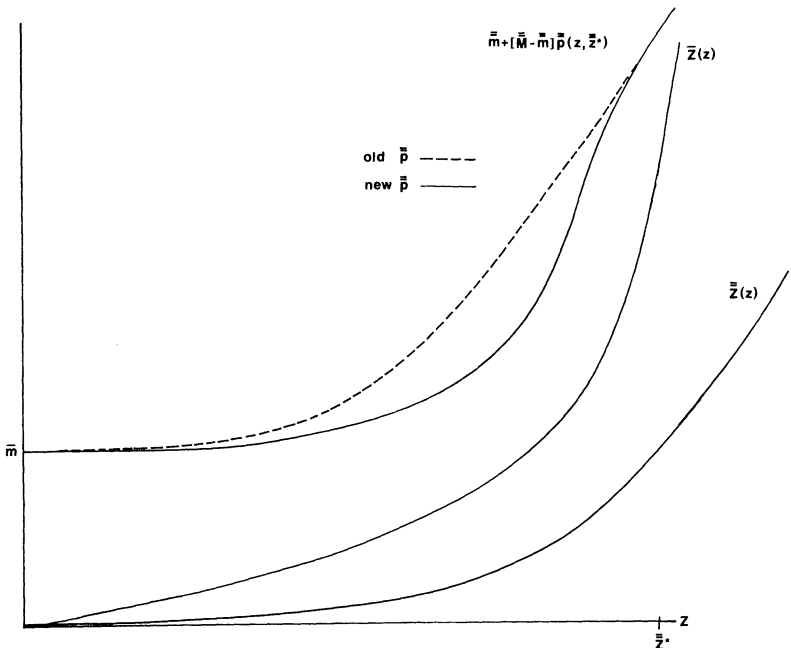

FIG. 5

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