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THE FAIR AND EFFICIENT DIVISION OF THE WINSOR FAMILY SILVER

JOHN WINSOR PRATT AND RICHARD JAY ZECKHAUSER*
Harvard Business School, Harvard University, Boston, Massachusetts 02163
John F. Kennedy School of Government, Harvard University, 79 John F. Kennedy Street, Cambridge, Massachusetts 02138

This is the true story of the actual use of a formal, decentralized division procedure to allocate silver heirlooms among eight grandchildren fairly and efficiently without distasteful direct monetary payments. Each grandchild's stated preferences for objects in contention were roughly represented by a von Neumann-Morgenstern utility function. Allocations were made as they would be in a market for probability shares in the objects, assuming each grandchild had a fixed amount of an artificial currency and made optimal purchases. The market-clearing equilibrium prices were chosen as in a second-price auction to reward honest reporting. Although the procedure was decentralized and most participants did not fully understand it or the preference information desired, it handled all major considerations well and was regarded as equitable.
(FAIR DIVISION, INCENTIVE COMPATIBILITY, PROBABILITY SHARES, EFFICIENT ALLOCATION, PSEUDO-MARKET, PREFERENCE REVELATION)

1. Introduction

When Mary Anna Lee Paine Winsor died at the age of 93, her estate included two trunks of silver, left undivided. Here follows the true story of the division of that silver among eight of her grandchildren. Unlike many other accounts of divvying up possessions, this is a happy tale. It revolves around the successful use of a decentralized, fair and efficient allocation procedure, of a type more congenial to *Management Science* readers than, say, estate planners. The problem presented some formidable difficulties: preferences were unknown, goods were indivisible, an external medium of exchange was ruled out, and markets were thin. Yet the outcome was accepted by the participants and was efficient, given the constraints on transfers. Moreover, it was fair in the sense that no grandchild would have preferred another's claims.

An efficient and fair method to divide resources is needed in many situations to avoid conflict and inefficiency. The most celebrated and most maligned mechanism for allocating resources is, of course, the market. Its unhindered operation assures efficiency, and also guarantees that no individual envies another who started with the same endowments of resources and capabilities. Charges of unfairness, for the most part, are related to (1) original endowments (such as parents, tastes, and talents), which in effect predate the mechanism; (2) assertions that the method has not worked, for example because of racial discrimination; or (3) dislike of the outcomes when it does work (investment bankers earn too much, poets too little). Auctions, whose properties have been much studied—see McAfee and McMillan (1987) for an excellent survey—are also employed to allocate resources, albeit in a much more limited domain. Apart from auctions and the market, formal mechanisms have rarely been employed to allocate significant resources among independent parties with conflicting interests. Part of the explanation, no doubt, is psychological. People may not understand such mechanisms, and may suspect that the outcomes will be rigged against them. A nontransparent mechanism may lack the legitimacy of a good old-fashioned struggle where each party bargains for himself. Finally,

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people may be reluctant to turn their birthrights over to a mechanism, and may back out of its results if they find the outcome unacceptable in some way.

Some formal mechanisms have been proposed for resource division problems, nevertheless. Fair division (Knaster and Steinhaus 1946, Steinhaus, 1948, 1949) has been a classic problem in allocating indivisible resources among competing claimants. As originally formulated, the central concern is to find a Pareto efficient outcome, respecting the distributional objectives of the donor or authority. In the simplest version of the problem—with known preferences, divisible goods, an external medium of exchange, and sufficiently thick markets (many participants interested in each good)—there are ready solutions.

But few resource allocation problems will possess these characteristics. Unknown preferences, by themselves, raise no complications. Distributional objectives can be met by specifying endowments, and the market can then assure efficient allocation. If indivisibilities are added, some variant of second-price auction—the highest bidder gets the object at the price bid by the second highest bidder—can induce the honest revelation of preferences. Thin markets, in the absence of known preferences, create the possibility of strategic behavior against the market, such as monopoly pricing, and of collusion. When no external medium of exchange is available, efficiency can only be assured if preferences are known. To illustrate, suppose individuals A and B each prefer item X to item Y, but A would pay \$10,000 for the preferred outcome while B would pay only \$1,000. Without an external medium of exchange, there is no way to extract the information needed about the intensity of preferences for the alternative items, assuming the participants correctly understand how the information will be used.

We do not wish to suggest that the mechanism outlined here will be immediately applicable to drafting entitlements or profit shares under the Law of the Sea Treaty, reaching a labor agreement between General Motors and the United Auto Workers, or formulating a United States-Soviet Union arms control treaty. But we do believe that this method for dividing an estate—a frequently contentious issue—has implications in broader arenas. Any fair division problem—and most disputes involve an element of fair division—would be amenable to our methods. The ultimate applicability in practice of any formal method for identifying desirable divisions of resources will be revealed only with time.

2. Background

Let us return to the two trunks of silver, which Mary Winsor left undivided to her surviving children, Dorothy Winsor Bisbee and Theresa Winsor Pratt. One trunk, containing the more valuable items, including a coffee pot made by Paul Revere, had been in a bank for many years, and most family members had not seen the contents. The other trunk contained the deceased's everyday silver and was stored "temporarily" in the most convenient family basement, which happened to be that of a grandchild, John Winsor Pratt. When the bank was preparing to move some twelve years later, it requested that the trustees make some disposition of the first trunk. The trustees readily decided to dispose of both, especially in view of the increase in silver prices and the ages of the joint owners, who had both passed 75 and lost whatever interest they might once have had in the silver.

It thus fell to the five Bisbee and three Pratt grandchildren to decide how to proceed. The possibility of donating at least the museum-quality pieces to a suitable institution was discussed but given up because no one volunteered to arrange it, the grandchildren had unequal financial resources (several being schoolteachers), and the trustees preferred not to complicate matters or ruffle the tax authorities. It was decided, with the advice and concurrence of the trustees, that Dorothy and Theresa would each keep any silver she wanted and give the rest of her share to her children in equal shares. The grandchildren would then keep what they wanted and the remainder would be sold at auction.

The silver was delivered to Sotheby Parke-Bernet for safekeeping, appraisal (after grouping of related pieces into lots, called items hereafter), viewing by the family, and eventual auction of the items not kept. In the end, Dorothy and Theresa chose to keep nothing; thus each Bisbee grandchild was entitled to one-tenth of the silver and each Pratt grandchild to one-sixth. It appeared that, for tax purposes, the value of the gift would be the current appraisal, and the grandchildren would be liable for the realized capital gains above the date-of-death appraisal.

Most of the grandchildren indicated that they wanted a few keepsakes. Indeed, most hoped to see some of the pieces with the greatest sentimental value kept within the family. Despite this externality, however, it was decided to reduce each grandchild's share of the proceeds by the "full value" of any item kept. This decision proved significant since some grandchildren turned out to want much more than a few keepsakes (one had a child of his own about to get married), while one wanted nothing. "Full value" was defined as the average of the auctioneer's low and high estimates of the sale price, without adjustment for commissions, catalogue fee, insurance, or the tripling of metal-silver prices between appraisals of the two trunks. Mary Winsor's two other grandchildren were invited to purchase on the same terms any items not spoken for by Bisbees or Pratts. Hard upon these decisions, the auctioneer scheduled the silver viewing for Monday week, with the delivery of any items kept by the grandchildren for the following Saturday. Time was short.

3. The Mechanism

The preferences of the grandchildren for the items were unknown. It was believed that a substantial portion of the value of the silver to the grandchildren was sentimental. It was felt to be distasteful and inappropriate to charge more for an item than its fair market value as assessed by the auctioneer. Running a market was not possible because not all grandchildren would attend simultaneously, and not desirable because they wished to remain on speaking terms, and had generally less enthusiasm than the authors for market processes. Drawing lots to take turns choosing was proposed, but presented similar problems and threatened serious inefficiencies. It seemed to be up to John Pratt—who, though a student of decision theory, was trusted by his relatives—to find a mechanism that was feasible and that would appear to be fair and understandable. He wanted it also to be truly fair and Pareto efficient within the world where sentiment was not allowed to be sold. Pratt consulted Richard Zeckhauser, knowing that he had worked on similar problems. A mechanism developed by Hylland and Zeckhauser (1979), hereafter HZ, proved relevant. HZ were concerned with allocating indivisible items, such as jobs or dormitory assignments, at most one to an individual. Using money to bid for such items is frequently infeasible or unpalatable. An auction would be complex even with money, since an individual can only buy one item and needs probabilistic predictions about all prices to formulate optimal bids. Individual preferences were unknown, thus ruling out traditional job assignment algorithms, and creating the potential for gain through misrepresentation. Furthermore, individuals must be treated equally or fairly in some sense, and each must receive at most one item.

HZ solve these problems by creating an implicit pseudo-market where individuals have equal (or other externally specified) endowments in an artificial currency used to buy probability shares in the items. The individuals submit von Neumann-Morgenstern utilities, possibly nontruthfully. These are needed only for the items individually, since at most one is bought. (Identical items are allowed.) Each individual is assigned probability shares so as to maximize his expected utility subject to the constraints that he spend no more than his budget and purchase a total of 100% for all items together. Prices are set that clear the market. They are guaranteed to exist by a fixed-point theorem and obtained by the Scarf algorithm. (In general there are multiple equilibria. HZ select the one where individuals minimize expenditure when their optimum purchase is not unique.) The equilibrium purchases are both efficient and envy-free with respect to the announced utilities. HZ show that ordinarily—in particular, when each item appeals to many individuals—misrepresentation has negative expected gain. (As in a regular market, it loses through suboptimal purchase and gains only by a sufficiently favorable influence on prices that would be unlikely and hard to predict.) The lottery assigning items to individuals in accordance with the probability shares purchased can be carried out so that each item and individual is assigned at most once by a form of stepping-stone algorithm. In similar slot-assignment problems with other constraints, implementation of the market-clearing probabilities subject to constraints is not always possible.

We incorporated the spirit of this approach when addressing the Winsor silver problem. Here an individual can receive more than one item. If utilities are additive in the objects, market-clearing prices exist and all implementations of the market-clearing probabilities are allowable and *ex ante* equivalent. Otherwise complications abound. We assumed (correctly) that any nonadditivities would be amenable to ad hoc treatment.

Ostensibly, a second difference from the HZ approach is that money played some role, since the appraised value of the objects one bid for and won were deducted from one's auction proceeds. This difference, however, is not real. In effect, the items the grandchildren were bidding for were composites of a silver object and the negative of its appraised value, for example, "serving ladle, −$200." This involves no more complication than if two silver objects were linked together.

Fortunately Pratt had *de facto* authority to make all necessary decisions and interpretations. One decision was that the eight grandchildren would be given equal endowments of the artificial currency, in contrast to their one-tenth and one-sixth monetary shares. Pratt felt that this was more in accord with the family wishes; as a one-sixth-share recipient he could make this decision with a clear conscience. The grandchildren were invited to request as much silver as they wished with the stipulation that it was not for resale.

Three steps remained: eliciting utilities after the viewing of the silver, solving mathematically for the equilibrium, and implementing the randomization.

4. Implementation

A. *Elicitation of Utilities*

Instructions were carefully prepared and disseminated at the viewing. They explained the terms of purchase by family members and the division of the total cash proceeds, and sought information from which utilities might be deduced.

The viewing was heavily attended. The instructions went mostly unread and totally unheeded. Each grandchild merely left with the auctioneer a list of items wanted. Under these circumstances, the possible conflicts were identified and those grandchildren in conflict over more than one item were interviewed by telephone (by John Pratt) about their utilities for these items. Ideally their tradeoff rates among chances of receiving items would have been elicited, and any exceptions to additivity explored. The rough-and-tumble of reality required less reputable scaling procedures, as disclosed in the Appendix. Table 1 shows the utilities used in the analysis and the only important exceptions to additivity identified.

We believe that there are more general lessons here for formal allocation mechanisms. First, people may not immediately understand even very simple systems. However, if they trust the organizer, they are likely to respond straightforwardly, though quite possibly not in the form requested.

TABLE 1

Utilities and [Probability Shares] Purchased to Clear the Market at Prices Shown

	Bryant	Ethan	Joyce	John	Frederick	Charles	Prices
A					10 [1/2]	6 [1/2]	1
B					5 [0]	4 [1]	.5
C	4 [3/8]	20 [5/8]					1.6
D	2 [5/8]		0 + [3/8]				0+
E		1 [0]	10 [1]				.08
F			9 [92/192]	1 [100/192]			1.92
G					8 [1]	3 [0]	.5
H	1 [1]					2.4 [0]	.4

Additional constraints:

Bryant does not want both trays (N22 and N23).

Charles wants at most two items.

Assignment of random numbers to outcomes:

A to Charles: 000–499; to Frederick: 500–999

C to Bryant: 000–374; to Ethan: 375–999

D to Joyce: 000–374; to Bryant: 375–999

F to John: 000–520; to Joyce: 521–999

Identification of items

A. 10 fiddle pattern teaspoons, W. Moulton, Newburyport, c. 1830.

B. Stuffing spoon, Joseph Moulton, Newburyport, c. 1800.

C. Presentation tray, "Frederick Winsor from his Faculty," 12″ diameter, Gorham, c. 1938.

D. Presentation tray, "To Mary Winsor," $14\frac{1}{4}$″ diameter, J. E. Caldwell, c. 1938.

E. Pair of Sheffield plated chalices, $7\frac{1}{2}$″ high, c. 1815.

F. Sheffield plated egg warmer, 8″ high, c. 1850.

G. 12 dessert spoons, "MSL," J. Bard, Phila., c. 1800.

H. Tablespoon, "CEC" + "Bryant," American maker "J.S.," c. 1780.

B. *Finding the Equilibrium*

The next step was to solve for equilibrium prices. Call the unit of simulated currency a winsor. Items desired by only one grandchild were to go to those who wanted them without charge to their winsor accounts, for the equilibrium price of these goods in a market would be 0. This reduced consideration to the items in contention, and to the six of the eight grandchildren who were contenders. The special nature of Charles's preferences (see Appendix) permitted a further reduction to the eight items—some involving multiple pieces—shown in Table 1. Utilities were needed only for those items and only for the five grandchildren wanting more than one item; the budget of a grandchild wanting only one contended item would obviously be spent entirely on that item.

A variety of solution algorithms was available. To begin, the "see if you can work it out" method was employed. Fortunately, in part because no more than two parties were interested in any one item, and because the degree of intertwining across the valuation matrix was limited, this classic method succeeded. Indeed, it succeeded within an hour.

Following the HZ procedure, if two individuals both value something positively, but only one ends up purchasing probability shares in the equilibrium configuration, we charged him or her precisely the price in winsors that the second bidder would be willing to pay. We therefore started by trying to assign 100% interests in particular pieces to different individuals, which enabled us to compute a number of prices. We always had to be careful not to exhaust budget constraints, of course. With a bit of higgle and wiggle, the equilibrium prices shown in the table tumbled out.

More specifically, comparing Frederick and Charles, it appears that both will buy shares of A, that Charles will buy B at a price making Frederick indifferent between B and A, and that Frederick will buy G at Charles's $G - A$ indifference price. This determines the relative price of A, B, and G. The price of D is 0+, since only Bryant wants it more than marginally. Since Charles wants only two items from a longer list, try having Bryant buy H at Charles's $H - A$ indifference price. Then Frederick and Charles must buy equal shares of A, and the prices of A, B, G, and H are determined relative to the endowments. Bryant spends the rest of his endowment and Ethan all of his on C, which must be priced to make the shares they purchase total 1. E is priced to make Ethan indifferent between it and C, and is bought by Joyce. Joyce spends the rest of her endowment and John all of his on F, priced to make their shares total 1. Bryant wants D only if he does not get C; otherwise D goes to Joyce. The prices and probability shares work out as shown and are indeed an equilibrium. The extra constraints are satisfied as well.

A range of other equilibria were possible, charging winners any amount from the value of the next highest bidder up to their own reservation price. Two advantages of the procedure we invoked, which in many ways resembles a second-price auction, are that it reflects what happens in a normal competitive market, and that it makes truth telling a near dominant strategy. (With a thin market and few players, a knowledgeable participant might theoretically be able to drain the resources of others by overbidding on objects where he could expect to be the second highest bidder.)

C. *Running the Lottery*

Independent lotteries could have been conducted for each contending item. Instead we ran one combined lottery correlating each individual's outcomes as negatively as possible, thus minimizing the variance of utility received. This procedure has several advantages: it probably accords with desires, although additive utilities do not reflect this; it is fairer and hence less questionable *ex post*; and it can be accomplished with a single random number in an easily understood way. In general there could be questions about what criterion of variability to use for each individual and how to aggregate individuals. Here, however, each individual faced at most two uncertainties, and we were able to minimize the number of eggs in one basket for all individuals simultaneously. Specifically, if Bryant won C he lost D (a constraint he imposed), and if Joyce won D she lost F. Table 1 shows the full randomization.

To run the lottery in a convincing decentralized manner, John Pratt and Joyce Bisbee Andrews agreed by telephone on delivery day to use the last three digits of the Massachusetts lottery number printed in the *Boston Globe*. The number found was 160. This meant that Bryant got the tray so strongly desired by Ethan. The system had not allowed for absolute intensities of preference. But since they were known, should they not be taken into account, at least in this extreme case? John told Joyce about this problem betwixt Bisbees and explained that, in principle, Ethan should buy the tray from Bryant in an after-market, though this would be problematical in practice. John was in a quandary, but it was no quandary to Joyce. She declared Ethan the winner and so it was done. This is an example of a difference between ivory tower and real-world residents.

5. Discussion

The title of this paper suggests that the division was fair and efficient, and so it was perceived by the participants. But was it fair in a sense that would satisfy an analyst? Certainly the procedure was designed to treat all participants' interests equally. But like a second-price auction, this system works by charging the purchaser of each object the amount that the next highest bidder would pay (in the relevant currency, winsors). Say that A and B respectively value items 1 and 3 intensely, but A gets his item for free because no one else likes it enough to pay its assessed monetary price, whereas B must take a substantial reduction in his winsor assets because C likes object 2 almost as much as he does. Some might argue that this is unfair. Their strongest argument might be that B is charged for his sentimental (or other) value just because others attach sentimental value as well.[1] But the same phenomenon applies to hedonic values in ordinary markets. A second perception of unfairness could relate to the chance outcomes of the lotteries. Given the indivisibility of the silver objects, and the desire not to introduce money as a grand equilibrator, such situations could not be avoided. The probabilities were needed to introduce continuity into a discrete situation. Before the lotteries were run, however, allocations were fair in the sense that no grandchild would envy another's lottery tickets.

The general presumption was that an individual's preferences were additive in the different goods he might receive. Two individuals expressed exceptions to this presumption, both subadditive. Charles restricted himself to purchasing no more than two out of three of the contended items, Bryant to one out of two trays. Subadditivity of preferences can be readily accommodated within our scheme in theory, and proved amenable in practice. If worst came to worst, we could have run a contingent claims market offering for sale such items as "stuffing spoon if Ethan gets tray 1." Superadditivity would be less readily handled, but fortunately no one expressed superadditive preferences.

The outcome was only efficient within the constrained world where winsors and dollars were not tradable. Deducting the assessed value of the objects from a purchaser's auction proceeds brought the outcome closer to the full optimum that could be achieved with an external currency. Unconstrained full efficiency using money as the measuring rod presumably could have been achieved with some form of second-price auction employing dollars alone.

Some logical-seeming alternatives would not have worked. For example, if the procedure had been employed with an after-market for trading goods and dollars, strategic behavior and the accompanying inefficiency would have been almost unavoidable. Selling winsors for dollars in advance would not have overcome these problems. A promising procedure would determine simultaneously the relative prices of winsors and dollars, replicating many features of a market where ration coupons are sold.

The procedure that was actually employed to divide the Winsor family silver, though imperfectly understood by the participants, was accepted by them, was fair in the sense of offering equal endowments and symmetric opportunities, and was efficient subject to the constraint of no external currency.

6. Conclusion

The Winsor family silver division shows that formal mechanisms can be successfully employed in resolving real-world negotiations over resources. Admittedly, the value of the resources involved was not enormous, and passions were more moderate than in many disputes. But formalized procedures may well have their greatest comparative advantage over the tug and haul of traditional negotiations when the issues are of paramount importance and passions would otherwise be great. Raiffa (1982, p. 299) makes

[1] Whether or not it is efficient to pay for sentiment depends on the structure of the utility function and the sources of differences among individuals. Assume that the objective is to maximize total utility when everyone has the same utility function. Assume that everyone has the same utility function, $U(s, w)$, where s is sentimental value and w is wealth. If it is of the form $U(s, w) = v(s + w)$, then charging for sentimental value transfers wealth to its highest marginal use. (We are leaving aside indivisibility problems.)

By contrast, if the utility function is $U(s, w) = s + Y(w)$, then if initial wealths are equal, charging for sentiment reduces total utility by in effect transferring wealth from a higher to a lower marginal utility state (individual). (This result builds on the principle that it may not be desirable to insure irreplaceable commodities at fair actuarial cost. See Cook and Graham 1977 and Zeckhauser 1973.) However, even with this utility function, if wealths differ, it might be efficient to charge for sentiment. Transfers will be toward a higher marginal utility state if the primary difference among individuals is wealth rather than the sentimental value they attach to an object, and vice versa.

If individuals differ in the forms of their utility functions, little can be said, but it should be obvious that in such circumstances it may be inappropriate to treat money (or sentiment) as the numeraire good with presumed equal value across individuals.

a complementary observation about fair division problems: "This allocation is usually accomplished by means of some sort of negotiation. The more parties that are involved, the more intricate the dynamics of unstructured negotiations become, and the more desirable it becomes to adopt a formalized procedure."

Our experience shows that: (1) For a general class of resource allocation problems there is a formal procedure that rewards individuals for honest reporting of their preferences for alternative allocations. (2) Formal mechanisms for resolving bargaining situations can be implemented in practice, even among individuals who do not fully understand the principles on which they operate. (3) Outside commercial dealings, real-world bargaining and negotiation situations rarely offer money as a freely available medium of exchange; despite this limitation, efficiency can still be sought and achieved. (4) Probability shares and pseudo-markets can be key concepts in the process. (5) Decentralized, formal procedures may avoid hostilities or feelings of unfairness that could arise in unstructured face-to-face bargaining.[2]

[2] Support from the S. S. Huebner foundation and from the Decision, Risk, and Management Sciences Division of the National Science Foundation is gratefully acknowledged.

Appendix

Elicitation of Utilities

The instructions distributed at the viewing employed the following language to elicit the required information about utilities:

Indicating choices. The following information is needed, in writing, in order to allocate items wanted by more than one person as fairly as possible, taking into account individual priorities and possible conflicting desires. Some allocations may have to be made by flipping a coin.

1. Please list *all* items you *might* wish to keep (under the conditions stated above: roughly, each item you keep will reduce your cash share by the auctioneer's appraisal of the item).
2. Beside each item on your list, put a number indicating its relative importance to you. In other words, if one item is twice as important to you, give it twice as high a number. (Only the ratios of your numbers will be used in the allocation process, so you may use any scale you like.)
3. If you have alternative choices, please so state. For example, you might want any six matching teaspoons, or one of the presentation trays but not both.
4. You may request only a portion of an item. (The value will be prorated.)
5. You may place a limit on the cash value of the items you receive, while listing more in case you do not receive your top choices.
6. You may state a reservation price at which you would be willing to buy an item if it does not bring more at auction.

Dorothy and Theresa, three Bisbee grandchildren and a representative of a fourth, two Pratt grandchildren, and assorted other family members and spouses viewed the silver, which was on display in a small room for five hours. Most grandchildren did not read the instructions and none followed them. Instead they merely left with the auctioneer a list of the items they wanted.

Charles saw only the auctioneer's appraisal list but provided an ordered list of eight items, of which he wanted the first two available except that he did not want both his third and sixth choices. One Bisbee abstained. At the viewing, John and Frederick (actually their wives) found that they were each interested in part of a set of flatware listed as one item and apparently not sought by others, and they agreed on how they wished to divide this set if it should be available to them, as it turned out to be.

The utilities that are displayed in Table 1 were arrived at as follows. Bryant had spoken for one of two trays, to be sure it stayed in the family. He mentioned to the interviewer that he knew that the other had been requested by someone and thought that he therefore could not request it, although he would have preferred it. Since the procedure was clearly predicated on allowing conflicts of interest to be expressed, rather than being first-come first-served, the interviewer decided to elicit Bryant's utilities, and Zeckhauser, knowing neither party involved, later agreed that this was appropriate. Because Bryant was, after John, perhaps the most mathematical grandchild, by both training and experience, probabilities were used in eliciting his utilities. It appeared that his preference was in the ratio of 2 to 1, but when the interviewer suggested that this meant he would regard a 50% chance at the preferred item as equivalent to a 100% chance at the other, he said "Shouldn't it be the other way?" The interviewer never did convince him, and in the heat of the moment failed to get explicit information about a third item, except that it was of less interest. The analysts (Pratt and Zeckhauser) arbitrarily used another factor of 2.

Ethan had tremendous interest in one tray (because one of his sons was named after his grandfather, to whom the tray was inscribed). He had a slight interest in another item also sought by someone else. When probabilities were brought into the picture, he and his wife both thought in terms of buying raffle tickets and felt that they should put some of their money on each item, but much more on the tray, so that they would have some chance of getting both. They found the concept that they were to choose between a small probability of the tray and a large probability of a much less desirable item uncongenial, unfair, and not to be seriously countenanced. The interviewer gave up on the idea and the analysts decided that 20-to-1 was quite extreme, like significance at the 5% level.

Joyce was nearly indifferent between two items in conflict and had included one tray on her list to be sure it stayed in the family, but had essentially no personal interest in it. The analysts set her utilities at 10, 9, and .1.

All of Frederick's choices were valued, on the basis of such judgments as $A \prec B \sim C \prec D$, $B + C$ slightly preferred to $A + D$, $E \sim F \sim A$, $A + F \sim D$. A number of revisions were made as discrepancies between tentative sets of values and such judgments emerged. In particular, the originally assessed array of values was found to have too narrow a range. There was little or no discussion of probability. "Slightly" was somewhat arbitrarily set at 1 point on a scale assigning 10 points to the most desired item. The whole array was reconsidered several times. The final array was arrived at without too much strain, and both interviewer and interviewee felt fairly comfortable with it. This may bespeak an advantage of including enough items to provide a natural numeraire without probability, provided utility can be taken as additive. Frederick also had a slight interest in one piece of an item requested by someone else. This was left for a possible after-market.

It appeared, once all choices were known, that it would suffice to value Charles's first four preferences. Using only the knowledge that each item was definitely preferred to the next, and some feelings about what this might mean in a list of eight items, and what it had meant for others, the analysts decided to set the values of successive items proportional to $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, etc., giving successive ratios $\frac{2}{3}$, $\frac{3}{4}$, etc.

John was involved in only one conflict and Alice in none; therefore their utilities were not needed, though John's entered the calculation.

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