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PECUNIARY EXTERNALITIES DO MATTER WHEN CONTINGENT CLAIMS MARKETS ARE INCOMPLETE

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That pecuniary externalities do not impede efficiency is a principle central to the effective operation of a market economy, as Scitovsky [1954] demonstrated more than a quarter of a century ago. That is, if A's increased production in a market reduces the price of the good sold there, and if that in turn affects the welfare of producers and consumers within the market, the workings of the competitive system are such that if all individuals act as price takers, an efficient outcome is reached. Complete markets and production opportunities and preferences that are well behaved are of course assumed.

Here we shall ask what happens to this result if markets are incomplete—specifically if some markets for contingent claims do not exist. Our central question is as follows: Given the absence of complete markets for sharing risks, will individuals in general undertake production decisions that are optimal from the standpoint of society as a whole? The answer is negative, as we demonstrate with the aid of a numerical example.

The logical next questions is as follows: Can the direction of the inefficiency be predicted? In particular, will decisions in general be excessively cautious? A second numerical example reveals that the answer is no. Overly risky decisions on the part of all individuals are a possibility.

Our focus on contingent claims markets is justified, we believe, because their nonexistence is so common. If there were a measure of performance for market failures that was in some way equivalent to a batting average, contingent claims markets might well lead the league. That is, the ratio of non-established contingent claims markets to all desirable contingent claims markets is high in relation to, say, the ratio of public goods relative to all goods, or goods generating nontrivial externalities relative to private goods. Admittedly, we have markets for many types of insurance. But many of the most desirable markets for insurance are simply so infeasible that we hardly miss them. We do not insure people on their earning abilities, nor do we offer businesses insurance against loss of profits. Some social policies, such as the welfare and unemployment insurance system, and the range of progressivity within the corporate tax system might be thought of as efforts to compensate for the lack of contingent claims markets, but they are hardly such markets in and of themselves.

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In a different vein, various macroeconomic policies are sometimes justified on the basis that they direct resources to areas where they might normally go if contingent claims markets were established. The rationale might go roughly as follows: Since we do not have extensive contingent claims markets, and since investors fear recession, they will invest less. The consequence may be a recession. Therefore, some form of government incentive policy, either now to stimulate protection against recession, or as a guarantee in case of recession, may improve matters for all. In some sense, protecting ourselves against an unfortunate future is a public good. The government has a variety of ways to make such a future less likely or less bad.

The first best solution, of course, would be merely to create desirable contingent claims markets where they do not now exist. If this is impossible, and their nonexistence provides evidence to that effect, then we must do as well as we can in a second-best situation. The pattern of investment or decision should not then in general be what would have pertained in a world with contingent claims markets; rather it should be the best we can do, given that such markets will not exist.

THE QUESTION

Assume that the only market imperfection is that contingent claims markets do not exist. Thus, producers and consumers will make their decisions now, knowing only the probability distribution on future states of the world. Once a state of the world is reached, all goods will be traded on competitive markets. Thus, on an ex post basis we can be assured of reaching a Pareto optimum. If we rephrase our central question in this context, the critical issue is whether we can be assured of reaching a Pareto optimum ex ante; i.e., whether we can be assured of reaching a constrained maximum in terms of the participants' expected utilities.

Against what set of outcomes should we judge Pareto optimality? Any comparison that allowed for additional risk-sharing mechanisms, e.g., state-dependent subsidies or taxes, must be ruled out. We shall simply ask whether there exists another set of pre-lottery production decisions that on an ex ante basis would be preferred by all parties. (In effect, this is asking whether the use of taxes and subsidies that were not state-dependent could produce a Pareto superior outcome.)

Relation to the Literature

The issues we address here are related to the more general question of the attractiveness of competitive outcomes when not all markets exist. The aspiration level is constrained efficiency—Pareto optimal relative to the set of allocations that can be achieved through the existing market structure—a concept developed by Diamond [1967] and others. A number of negative and positive results have already been produced. Restricting himself to a one-good, two-period economy, Diamond showed that a competitive equilibrium will be constrained Pareto optimal. Hart [1975] showed that this result does not hold in more general cases, when there are two or more goods, or three or more periods. An equilibrium need not exist. More than one equilibrium may exist, in which case one may be Pareto dominated by another. Starrett [1973] showed that even in a one-good, two-period economy, if there are transactions costs involved in futures trades, the outcome will be inefficient.

Our example is most similar to those given by Hart [1975] and Stiglitz [forthcoming]. Though ours is cast in terms of alternative states of the world in one period, we can interpret it directly as referring to a two-period economy without uncertainty, in which expected values are replaced by present values and futures markets do not exist. Thus, equally probable states of the world correspond to a zero discount rate.

The difference between Hart's model and ours is that individuals in Hart's sequential economy have fixed but state-dependent endowment streams, and rational expectations of prices, whereas in our example individuals determine their endowment stream by choosing which technology to use in producing goods. Stiglitz deals with individuals who invest their wealth in a stock portfolio, and aim to maximize their expected utility from this investment. They can invest in several firms producing different goods, all of which have constant stochastic returns to scale. Each good is produced by only one firm, using a fixed technology. Stiglitz shows that in general portfolio holdings will be inefficient. Since the firms in question have constant returns to scale, this example can be reinterpreted to refer to individuals who decide what to produce themselves, rather than individuals who invest in stock. This makes Stiglitz's example directly comparable to ours. The essential remaining difference then is that we are explicitly concerned with individuals who have a choice of alternative technologies for producing the same good, and we show that under conditions of uncertainty they will often make inefficient choices.

THE EXAMPLE

Suppose that there are two equal sized classes of people, A and B. We shall call representative members of A and B, α and β, respectively. There are two equally probable states of the world, S_1 and S_2. There are two goods, X and Y, which everyone values. Thus, α's utility $U(x,y)$ is a function of his consumption of these two goods, and so is β's utility function $V(x,y)$. Here α produces X, and β produces Y. In general, their output depends on the production strategy they choose, and the state of the world that prevails. After the state is revealed, everyone trades X and Y, each person acting as a price taker in the market.

The strategy α employs to produce X can be any linear combination of two extreme cases. In one, his output is a_1 if S_1 happens, and nothing if S_2 happens. In the other, he produces nothing if S_1 happens, and a_2 if S_2 happens. We may describe his strategy by his outputs (x_1,x_2) in both states. (x_1,x_2) will satisfy $\Sigma(x_i/a_i) = 1$. Similarly, β's strategy (y_1,y_2) for producing Y satisfies $\Sigma(y_i/b_i) = 1$.

Each person chooses his strategy to maximize his expected utility. We shall use specific utility functions $U(x,y) = V(x,y) = \sqrt{x} + \sqrt{y}$ in our example, to show that the equilibrium outcome can be inefficient. With much effort, we can prove that for arbitrary U and V the outcome will in general be inefficient, though we shall not discuss the general case in this paper. We define the following notation:

U_i	α's utility in S_i,
V_i	β's utility in S_i,
$EU = (U_1 + U_2)/2$	α's expected utility,
$EV = (V_1 + V_2)/2$	β's expected utility, and
p_i	the price of Y relative to X in S_i.

Given p_i and S_i, if α's output is x_i, he will trade $x_i/(p_i + 1)$ of X for $x_i/p_i(p_i + 1)$ of Y. After trading, he will possess $p_i x_i/(p_i + 1)$ of X, $x_i/p_i(p_i + 1)$ of Y, and $U_i = \sqrt{(1 + 1/p_i)x_i}$. For α will arrange his final holdings (x,y) of X and Y to maximize $U(x,y) = \sqrt{x} + \sqrt{y}$, subject to his budget constraint $x + p_i y = x_i$. The maximum occurs at the stated point. Similarly, if β's output is y_i, he will trade $p_i y_i/(p_i + 1)$ of Y for $p_i^2 y_i/(p_i + 1)$ of X. After trading, β will possess $p_i^2 y_i/(p_i + 1)$ of X, $y_i/(p_i + 1)$ of Y, and $V_i = \sqrt{(1 + p_i)y_i}$.

If p_i is to be the market-clearing price, the amounts that the α's and β's wish to trade must agree. Since there are equal numbers of α's and β's, the condition is

$$x_i/p_i(p_i + 1) = p_i y_i/(p_i + 1), \qquad \text{or} \qquad p_i = \sqrt{x_i/y_i}.$$

So

$$U_i = \sqrt{(1 + 1/p_i)x_i} = \sqrt{x_i + \sqrt{(x_i y_i)}},$$
$$V_i = \sqrt{(1 + p_i)y_i} = \sqrt{y_i + \sqrt{(x_i y_i)}}.$$

This is the condition for market equilibrium in one state, after the state is known. To derive the condition for general equilibrium, suppose that p_1 and p_2 are given. Then if α chooses the strategy (x_1,x_2), his expected utility after trading will be

$$EU = (\Sigma\sqrt{(1 + 1/p_i)x_i})/2.$$

α seeks to maximize this, subject to his production constraint $\Sigma x_i/a_i = 1$. His optimal strategy is $x_i = a_i^2(1 + 1/p_i)/\Sigma a_j(1 + 1/p_j)$. Under the same conditions, β's optimal strategy is $y_i = b_i^2(1 + p_i)/\Sigma b_j(1 + p_j)$. In equilibrium, the market-clearing prices will be precisely p_i. So $p_i^2 = x_i/y_i$, which yields on substitution for x_i and y_i

$$p_i^2 = \frac{a_i^2(1 + 1/p_i)}{b_i^2(1 + p_i)} \frac{\Sigma b_j(1 + p_j)}{\Sigma a_j(1 + 1/p_j)} = \frac{a_i^2}{b_i^2} \frac{1}{p_i} \frac{\Sigma b_j(1 + p_j)}{\Sigma a_j(1 + 1/p_j)}.$$

If we define $p_i = (a_i/b_i)^{2/3}t$, these conditions are satisfied, provided that

$$t^2 = \frac{\Sigma(1 + (a_j/b_j)^{2/3}t)b_j}{\Sigma((b_j/a_j)^{2/3} + t)a_j}.$$

The question is whether everyone could be better off if each chose a non-equilibrium strategy. We may plot indifference curves of α and β on a plane whose coordinates (x_1,y_1) correspond to their respective strategies. Note that though these are indifference curves of α and β, α does not control the horizontal coordinate, nor β the vertical one. Since α is only one of many members of A, he cannot shift x_1 appreciably. Many people must concert their actions to do so, and such collusion has been explicitly ruled out. Similarly for β. Hence α's indifference curve will not necessarily be horizontal, nor β's vertical, at the equilibrium E.

Near E, α's expected utility is given by the first-order approximation,

$$EU \simeq EU_0 + EU_{x_1}\Delta x_1 + EU_{y_1}\Delta y_1,$$

where

$$EU_0 = \text{expected utility at } E,$$

$$\Delta x_1;\Delta y_1 = \text{shift in } x_1,y_1 \text{ from equilibrium value},$$

and the partial derivatives EU_{x_1},EU_{y_1} are evaluated at E. Likewise,

$$EV \simeq EV_0 + EV_{x_1}\Delta x_1 + EV_{y_1}\Delta y_1.$$

Now $EU = \sqrt{x_i + \sqrt{x_i y_i}}/2$ so that

$$\begin{aligned} EU_{x_1} &= \frac{1}{4}\frac{1}{\sqrt{x_1+\sqrt{x_1y_1}}}\left(1+\frac{1}{2}\sqrt{\frac{y_1}{x_1}}\right) \\ &\quad + \frac{1}{4}\frac{1}{\sqrt{x_2+\sqrt{x_2y_2}}}\left(1+\frac{1}{2}\sqrt{\frac{y_2}{x_2}}\right)\frac{dx_2}{dx_1} \\ &= \frac{1}{4}\left\{\frac{1}{\sqrt{x_1+\sqrt{x_1y_1}}}\left(1+\frac{1}{2}\sqrt{\frac{y_1}{x_1}}\right)\right. \\ &\quad \left. - \frac{1}{\sqrt{x_2+\sqrt{x_2y_2}}}\left(1+\frac{1}{2}\sqrt{\frac{y_2}{x_2}}\right)\frac{a_2}{a_1}\right\}, \end{aligned}$$

and similar expressions can be computed for EU_{y_1}, EV_{x_1}, EV_{y_1}.

Provided that $EU_{x_1}/EU_{y_1} \neq EV_{x_1}/EV_{y_1}$, the indifference curves through E will not be tangent there, and we can find small Δx_1, Δy_1, such that

$$EU_{x_1}\Delta x_1 + EU_{y_1}\Delta y_1 > 0,$$
$$EV_{x_1}\Delta x_1 + EV_{y_1}\Delta y_1 > 0,$$

and this $(\Delta x_1,\Delta y_1)$ will represent a point superior to E.

By substituting numerical values for the coefficients a_b, b_i, we can compute the equilibrium conditions. For example, suppose that $a_1 = 2$, $a_2 = 1$, $b_1 = 1$, $b_2 = 2$. Then $t = 1$. The equilibrium may be summarized as follows:

S_1: $p_1 = 2^{2/3} = 1.5874$, $x_1 = 1.1150$, $y_1 = 0.4425$,

$U_1 = 1.3481$, $V_1 = 1.0700$.

S_2: $p_2 = 2^{-2/3} = 0.6300$, $x_2 = 0.4425$, $y_2 = 1.1150$,

$U_2 = 1.0700$, $V_2 = 1.3481$.

$EU = EV = 1.2091$.

The tangents to the indifference curves at the equilibrium E are shown in Figure I. The tangent for α is vertical, while β's is horizontal. This suggests that moving in a southeasterly direction from point E will increase the welfare of both participants. More generally, all points within the shaded cigar-shaped region defined by α's and β's indifference curves through point E are Pareto superior to that competitive equilibrium. Thus, at F, $x_1 = 1.2035$, $x_2 = 0.3983$, $y_1 =$

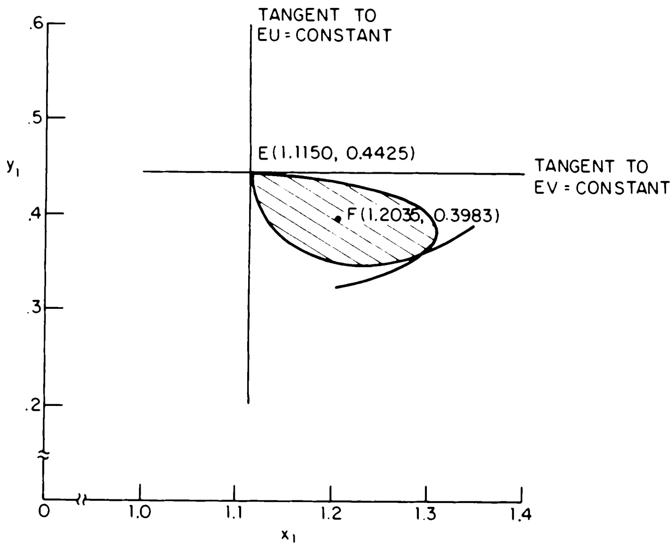

FIGURE I
Equilibrium is Too Safe

0.3983, y_2 = 1.2035, $EU = EV$ = 1.2106. This happens to be Pareto optimal. In this example, Pareto improvements occur if all of the α's and β's choose more risky strategies than they would in equilibrium.

The following example illustrates the case where choosing less risky strategies than in the equilibrium gives a Pareto improvement. Suppose that $a_1 = a_2 = 1$, $b_1 = 8$, $b_2 = 1$. Then $p_1 = t/4$, $p_2 = tt^2 = 3(3 + t)/(5 + 2t)$, $t = 1.3028$. In equilibrium,

$$S_1: p_1 = 0.3257, \quad x_1 = 0.6972, \quad y_1 = 6.5728$$
$$S_2: p_2 = 1.3028, \quad x_2 = 0.3028, \quad y_2 = 0.1784$$
$$EU = 1.2081, \quad EV = 1.7964.$$

Figure II shows the tangents to the indifference curves through the equilibrium point. Points in the shaded cigar-shaped region represent Pareto improvements. F, a Pareto optimum that dominates E, corresponds to a case where both the α's and the β's are less risky.

Here α has a comparative advantage over β in producing in S_2. Thus, α need only reduce his output in S_1 by 1 to increase his output in S_2 by 1, whereas β must reduce his output in S_1 by 8 to increase his output in S_2 by 1. Everyone gains if all the α's collectively put more resources in S_2 than they would in equilibrium.

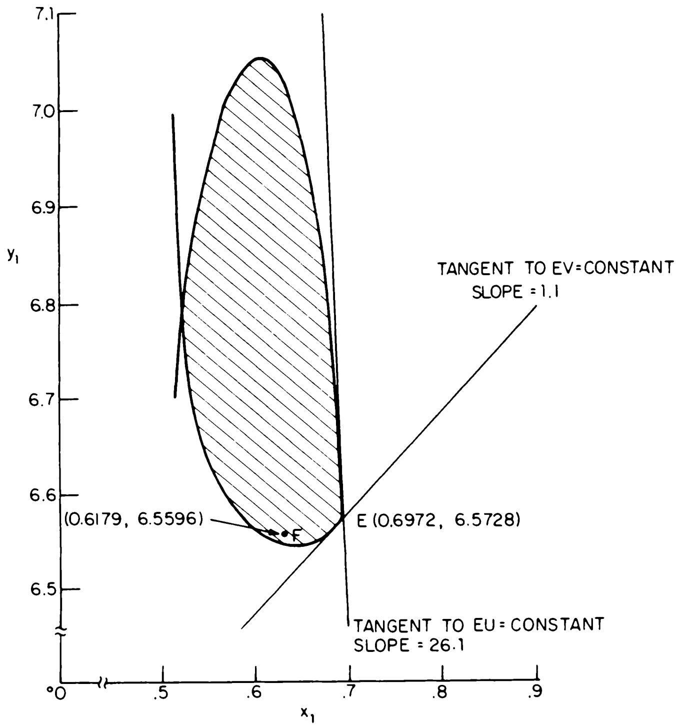

FIGURE II
Equilibrium is Dominated by More Risky Point, F

CONCLUSION

In the absence of contingent claims markets, pecuniary externalities matter because individuals have different rates of tradeoff for a resource between different states of the world. Thus, α might be willing to give up 1 unit of X in S_1 for 1 unit in S_2, whereas β would be willing to give up 2 units in S_1 for an additional unit in S_2. Both can be made better off if α directs relatively more resources toward S_2, and β directs relatively more resources toward S_1.

For a class of situations, which might be called "all in the same boat," counterexamples of the type shown above will not exist. All-in-the-same-boat situations arise if all individuals confront the same opportunity set, as they do, for example, in representative-man-type analyses. This suggests that the frequent assertion that individuals in like circumstances might all be made better off if they all directed more resources toward the poor outcome state; e.g., a recession, does not hold, or at least not for the reasons cited below. (Individuals may also be in the same boat if even though they produce different goods, there is symmetry in their preferences and perfect correlation in their fortunes.)

The problem that we cited in the text is likely to apply most strongly when individuals confront personal fates unrelated to such general conditions as the state of the economy, weather, or war. In the usual case, in the absence of contingent claims markets, individuals will direct too many resources to their state of relative deprivation. There may be an implication for policy inherent in this observation. Policies, perhaps a strongly progressive income tax, may exaggerate individuals' (and corporations') natural propensities toward risk aversion, which leads them to direct resources toward poor outcome states. This analysis suggests that in the absence of contingent claims markets, pecuniary externalities may make an opposite (or at least a countervailing) policy preferable.

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