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Stockpiling strategies and cartel prices

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Consuming nations can stockpile cartelized commodities to suppress prices in future periods. This analysis employs a multiperiod framework and simple concepts of game theory to assess stockpiling strategies by the government(s) of consuming nation(s) and pricing strategies by a cartel. Ultimate consumers are active, though nonstrategic, players in the game. The paper examines outcomes when discount rates, time horizons, resource constraints, and storage and production costs vary; when consuming nations do not cooperate fully; and when a consuming nation or a cartel can issue threats and promises. Both producers and consumers realize economic benefits from stockpiling in most of the cases that we investigate. Depletable resources are not considered except in an appendix. The net benefits of stockpiling constrained resources are problematical.

1. Introduction

■ Commodity stockpiling is currently the subject of serious policy discussion. In the aftermath of the OPEC oil embargo, it has been proposed that the United States develop reserves to protect against future supply disruptions.¹ Recognizing that cartels are also being considered by countries producing commodities, such as coffee and bauxite, for which imports comprise a substantially greater proportion of U.S. consumption than they do for oil, Congress has created a National Commission on Supplies and Shortages. This analysis examines stockpiling as a strategy that a consuming nation might employ to suppress future prices.²

Stockpiling has been employed in the past as a policy instrument, though not primarily to influence prices. Since the Second World

¹ The Energy Policy Act of 1975 sets a target oil reserve of one billion barrels, of which one-half is to be purchased by the end of 1982. Recently, authorization has been requested to spend \$871 million for building storage facilities, purchasing oil, and conducting further studies (Cowan, 1976).

² Special cases of price suppression include actions that reduce the probability of an embargo (what is in effect an infinite import price), or discourage monopolistic behavior by extant or potential cartels. Bergsten (1973), discussing the threat of OPEC-type pricing actions by Third World producers, notes that in addition to oil, cartels exist in bananas, bauxite, coffee, copper, iron ore, mercury, phosphates, and tin.

War, the United States has maintained substantial stockpiles of various strategic materials.³ Our most significant national stockpiles have been our large grain reserves, held until several years ago, which developed primarily as byproducts of price support programs. More recently, prospects of future famines, brought into prominence by experiences in the Sahel and the Indian subcontinent, together with widespread price variations, have generated proposals to stockpile grains. At present, the developed and developing nations are discussing the creation of raw material stockpiles, the avowed purpose being to smooth out price fluctuations that threaten economies concentrated in one or a few primary products.

Most previous analytic work on stockpiling is related to its capability for smoothing, not suppressing, prices, specifically to storing and releasing grain to level fluctuations in crop availability due to nature. These studies typically take as given historically based probability distributions on harvest size, and apply recursive optimization techniques to maximize the expected present value of the sum of producers' and consumers' surpluses.⁴ Analyses of natural resource stockpiles follow in the same spirit, taking as given the probabilities of supply disruptions or other events that may lead to greatly increased prices.⁵

□ **The analysis that follows.** The stockpiling situations we shall consider are fundamentally different in that the main determinants of supply conditions are the decisions of economic actors, the producing nation(s), and not nature or chance. Although by the nature of the production processes, markets for many commodities could be highly competitive, in practice supply decisions are significantly influenced by marketing organizations of the type found with diamonds, cartels such as OPEC, and various national and international efforts to intervene in the market. Our analyses posit a rational unified actor on the production side, whether an international cartel or a single firm possessing a monopoly. That actor is called "the cartel." Extraeconomic political considerations are suppressed. The cartel is assumed to maximize its economic welfare, subject to the levels of demand it will face from the consuming nation(s).

Section 2 of the paper introduces the basic models; they assume that there is a unified consuming nation as well. Section 3 extends the analysis to allow for multiple consuming nations, of varying sizes and strategic configurations. Section 4 discusses several alternative assumptions regarding the ability of participants to cooperate and to commit themselves to strategies. The analysis employs the elementary concepts of *n*-person game theory. The players are the producer (or cartel) and the consuming nation(s). Only in Appendix 2 do we

³ See Snyder (1966) for a history of U.S. strategic stockpiling from before the Second World War through the mid-1960s.

⁴ See, for example, Gustafson (1958) and Danin, Sumner, and Johnson (1975). Walker and Sharples (1975) review a considerable number of such studies.

⁵ See, for example, U.S. Department of the Interior (1975). That report briefly considers the case where additions to the stockpile reduce the probability of an embargo. Nichols and Zeckhauser (1976, pp. 4-6) demonstrate that the stockpile becomes a public good in such a situation, and that the competitively determined stockpile will be below the socially optimal level. In effect, of course, the embargo situation is but a special case of the more general models considered here, where the stockpile influences future prices.

consider the case of depletable resources. The results there vary significantly from those obtained in the text, which is concerned with nondepletable resources entirely.

We shall find that in most circumstances the consuming nation benefits substantially from the pursuit of a stockpiling strategy designed to influence the actions of the producers. At the expense of increased early-period purchases, and the interest costs thereon, it will be able to induce substantial reductions in later-period prices. Stockpiling can also generate a strong positive efficiency effect by giving increased early profits to the producer—something akin to a lump-sum payment—in return for future prices that are closer to marginal cost. Although gains to the consuming nation outweigh those to the cartel in most of the models we consider, the producer must always benefit from the consuming nation's ability to stockpile. (Surprisingly, the consumer need not, as two later examples will show.) Costs of production and storage, the discount rates of the producer and consumer, and the length of the time horizon are the parameters that determine the magnitudes of the gains to the two parties.

□ **The role of government in stockpiling.** Why might the government consider playing a role in stockpiling? Why not rely on speculators? The great risks inherent in such speculation, particularly if economies of scale make small stockpiles infeasible, might deter private parties from stockpiling sufficiently. Moreover, speculators might anticipate that the government would adopt penalizing strategies, such as excess profits taxes or price controls, should any of the situations develop where speculators would otherwise make great profits. A somewhat different argument, frequently overlooked, would suggest that if the government stockpiles a commodity subject to substantial supply and hence price fluctuations, risk-averse producers will be induced to direct efficient levels of resources to these commodities. Arguments of this sort provide the only justifications for government stockpiles in areas such as grains, where the avowed objective is to smooth out price fluctuations.

We shall be concerned with a quite different consideration that is central when there is a cartel or monopoly on the production side. Some form of governmental strategy may help to influence prices, or the probabilities of events such as embargoes that will affect prices, in a manner that favors the nation's consumers. No consumer would be willing to alter his purchases on an individual basis, because he would receive only a small portion of the benefits should prices be influenced in a favorable direction. In fact, in most of the models considered, the socially optimal strategy requires that the stockpiler buy high and sell low, hardly an inducement to private stockpiling.

□ **Basic assumptions.** To keep the analysis manageable, we make critical simplifying assumptions in three areas: valuation, production and stockpiling costs, and permissible strategies. We do not believe that the qualitative nature of our results would be changed significantly if any of these assumptions were elaborated to correspond more closely to a detailed reality.

The cartel is assumed to maximize its discounted revenues net of production costs, i.e., producer's surplus. For our initial models,

production costs of the commodity in question are assumed to be constant, perhaps zero. (In Appendix 2 we consider cases where there is a constraint on the total amount of the resource produced.) *The objective of the consuming nation is to maximize the present value of what we shall refer to as net consumers' surplus.* It consists of consumers' surplus, as ordinarily defined, less the costs associated with the stockpile. The stockpile costs are purchase costs plus costs of storage, less resale receipts. Following the analyses of Kalyon (1975) and Kennedy (1974), aggregate demand curves of a consuming nation are linear; they have the same form in each of the periods of the analysis.⁶

The government of a consuming nation cannot intervene in the market directly to regulate its citizens' consumption demands. The only policy option available is to stockpile. Storage costs are constant per unit of stockpile; in most cases they are zero.

All transactions between producers and consumers are at arm's length; they are conducted through the market. No side deals between the cartel and consumer(s) are possible; nor can there be any form of package deal involving a lump-sum payment and a lowered marginal price—an arrangement that would achieve increased efficiency through a degree of price discrimination.

■ In the basic situation of this analysis, a single producer or a perfectly cohesive cartel of producers sets the price in each period, but permits consumers and the stockpiling government to purchase whatever quantities they desire at the price fixed for that period, p_t . In each period, consumption, C_t , is determined by the linear demand curve

$$C_t = K - \alpha p_t. \quad (1)$$

The simplest case in which stockpiling can be analyzed is a two-period model with a single consuming nation, and with production and storage both costless activities. Although a two-period time horizon is obviously too short for a full evaluation of stockpiling strategies, it yields insights into the processes by which the model operates, and has the added virtue of analytic tractability. Beyond two periods, a strictly analytic approach quickly becomes unwieldy, even with linear demand curves. We developed a computer algorithm to solve the many-period version of the model; most of the results presented are based on a ten-period horizon. Appendix 1 deals with an infinite time horizon.

Our models assume that the players engage in self-interested maximizing behavior. There are no institutional arrangements external to the models through which they can threaten or promise each other, or in some other way forgo their noncooperative strategies. Despite the fact that both players follow their noncooperative strate-

2. Models with unified consumer stockpiling

⁶ Given the multiperiod interactive gaming context in which the analysis is conducted, little insight can be gained from working with very general demand functions. Other specific functional forms were considered, particularly those exhibiting constant price elasticity. The difficulty with a constant elasticity demand function is that, given costless production, the optimal monopoly price is either infinite, if the elasticity is less than unity, or arbitrarily small, if the elasticity is greater than unity.

gies, we shall see that stockpiling allows for significant efficiency gains over the period-by-period monopolistic outcome.

□ **A two-period model.** We assume that the consuming nation enters the first period with no stockpile, and that any stockpile acquired in that period is released in the second. In the first period the cartel sets a fixed price, p_1 , at which it will sell whatever quantities are demanded. Consumers in the importing nation will then demand C_1 units for current consumption, as defined by equation (1). The government in the importing nation may choose to purchase units of the commodity, the level of the stockpile being denoted S . Thus the total amount demanded from the cartel in period 1 will be:

$$D_1 = C_1 + S. \quad (2)$$

In period 2, the process is repeated, with the cartel setting p_2 and consumers demanding C_2 units, again in accordance with equation (1). However, the importing nation will also release its stockpile, so the total amount demanded from the cartel in period 2 will be

$$D_2 = C_2 - S. \quad (3)$$

The goal of the cartel is to maximize the present value of its profits in the two periods. With marginal costs of production at 0, the objective of the cartel in effect is to maximize discounted revenue:⁷

$$\begin{aligned} Y &= Y_1 + \beta_p Y_2 \\ &= p_1 D_1 + \beta_p p_2 D_2, \end{aligned} \quad (4)$$

where $\beta_p = 1/(1+r_p)$ is the cartel's one-period discount factor, and r_p is its discount rate.

The total value, V , reaped by the consuming nation has two components: (1) the consumers' surplus its citizens derive and (2) the government's revenues, net of purchase costs. The second component is expected to be negative, but must be more than offset by the gain in consumers' surplus. We aggregate these two components on a period-by-period basis to get net consumers' surplus. This sum in period 1 will be

$$V_1 = \frac{1}{2} \left(\frac{K}{\alpha} - p_1 \right) C_1 - p_1 S, \quad (5)$$

where the first term is consumers' surplus, the conventional triangle under the demand curve but above the price line, and the second term is the government's first-period loss due to purchasing a stockpile. Net consumers' surplus in the second period will be

$$V_2 = \frac{1}{2} \left(\frac{K}{\alpha} - p_2 \right) C_2 + p_2 S, \quad (6)$$

⁷ Alternative versions of all of the models presented in this paper have been developed using positive marginal costs of both production and storage. The results, only illustrative examples of which are presented in this paper, are relatively insensitive to variations in the level of production costs and storage costs, for most commodities are very small relative to monopoly prices. The U.S. Federal Energy Administration (1974), for example, estimates oil storage costs at about \$1 per barrel capital costs (if salt domes are used) and less than \$0.01 per barrel annual charges. In an analysis of policies with regard to aluminum, chromium, platinum, and palladium, the U.S. Department of the Interior (1975) found the storage costs of those materials to be low enough to justify exclusion from their calculations.

where the first term has the same interpretation as before, and the second term represents the government's revenues when it supplies S units of demand from the stockpile.

The two-period objective function of the consuming nation can then be written

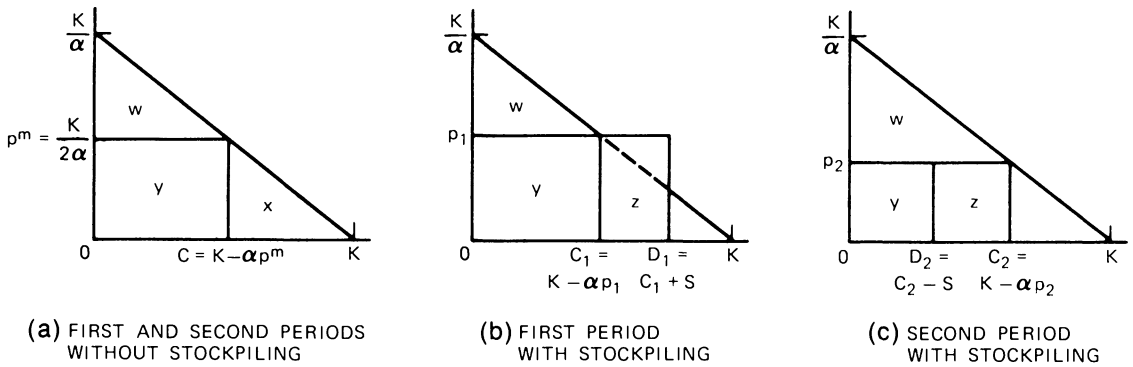
$$V = V_1 + \beta_c V_2, \quad (7)$$

where $\beta_c = 1/(1+r_c)$ is the consuming nation's one-period discount factor.

The cartel's and consuming nation's value functions are shown geometrically in Figure 1. Figure 1a illustrates the case where no stockpiling is carried out. The area of triangle w represents consumers' surplus, while the area of rectangle y is producers' surplus. The area of triangle x is the net loss in total surplus due to monopoly pricing; the Pareto optimum, given zero marginal cost, would call for consumption of K units and some allocation of the total area under the demand curve between producers and consumers. In the absence of stockpiling, the results in periods 1 and 2 will be identical. Note that the areas of triangles w and x are equal, and that their sum is equal to the area of y . Thus, the efficiency loss without stockpiling is equal to consumers' surplus, or to one-half of the cartel's revenues.

Figure 1b shows the results for the first period when stockpiling is

FIGURE 1
GEOMETRIC REPRESENTATIONS OF CONSUMERS' AND PRODUCERS' SURPLUSES



carried out. Net consumers' surplus is the area of triangle w minus the area of rectangle z . Cartel revenues are the sum of rectangles y and z . In Figure 1c, second-period net consumers' surplus is the sum of the areas of triangle w and rectangle z . The area of rectangle y represents cartel profits.

We assume that both the cartel and the consuming nation's government are farsighted, that they choose their actions in each period in light of future consequences. There are three actions in each period: (1) the cartel sets the price, p_t ; (2) consumers purchase an amount for consumption, C_t ; and (3) the stockpiling government selects a stockpile level, S_t . The two-period model thus involves six sequential actions or decisions:

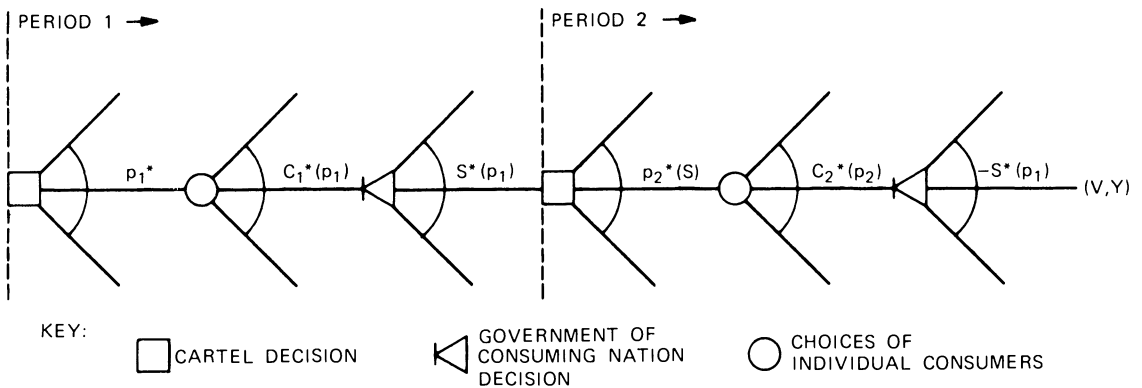
- (1) Cartel selects p_1 .
- (2) Consumers demand $C_1 = K - \alpha p_1$ units for consumption.

- (3) Consuming nation's government selects S . Total purchases from the cartel are thus $D_1 = C_1 + S$.
- (4) Cartel selects p_2 .
- (5) Consumers demand $C_2 = K - \alpha p_2$ units for consumption.
- (6) Consuming nation's government releases stockpile S . Total purchases from the cartel are thus $D_2 = C_2 - S$.

The decision tree for this problem is shown in Figure 2.⁸ This is

FIGURE 2

DECISION TREE FOR TWO-PERIOD MODEL



the traditional formulation of a game in extensive form. Each player will fold back to his decision nodes in turn. Note that the level of consumption, C_t , is *not* a decision variable for the consumer nation. As noted earlier, we assume that the consuming nation's government does not intervene to set quantities consumed directly, nor to alter the demand curve by imposing tariffs or taxes. Thus the last decision point in the model is that at which the cartel sets p_2 to maximize second-period income:

$$\begin{aligned}
 Y_2(p_2, S) &= p_2 D_2 & (8) \\
 &= p_2 (K - \alpha p_2 - S).
 \end{aligned}$$

Setting the derivative of $Y_2(p_2, S)$ with respect to p_2 equal to zero yields the optimal p_2 as a function of S :

⁸ This model has some of the features of bilateral monopoly situations where the seller sets the price in each period and the buyer then chooses how much to purchase. In those cases, the monopsonist secures no return for his market power, barring some credible external means by which he can commit himself to respond other than by choosing his welfare-maximizing quantity, given the price set. There is a substantial literature on the nature of "solutions" when this game is played many times, but all of them break down if the players are unable to commit themselves to strategies. In the final period there will be no incentive for the buyer to cooperate; working backwards, each successive period then becomes the last one so far as cooperation is concerned.

This formulation has an analogue to traditional von Stackelberg leader-follower models in oligopoly. In his price choice for the last period, the producer acts as the leader, selecting the optimal price given the known reaction of the buyer (as given by his demand curve and stockpile level). This behavior by the producer is taken as the given reaction curve when the consuming nation decides how much to stockpile in the second to last period. The pair of equilibrium strategies then represents a process of alternating leadership to optimize against the other participant's predictable response. In contrast to oligopoly models, leadership here is dictated by timing, not tradition or any calculation of strategic advantage.

$$p^*_2(S) = \frac{K-S}{2\alpha}. \quad (9)$$

Given $p^*_2(S)$, C_2 can also be written in terms of S :

$$\begin{aligned} C^*_2(S) &= K - \alpha p^*_2(S) \\ &= \frac{K+S}{2}. \end{aligned} \quad (10)$$

The next-to-last decision is the consuming nation's choice of S in period 1 to maximize its value function:

$$\begin{aligned} V(p_1, S) &= \left[\frac{1}{2} \left(\frac{K}{\alpha} - p_1 \right) C_1 - p_1 S \right] \\ &+ \beta_c \left[\frac{1}{2} \left(\frac{K}{\alpha} - p^*_2(S) \right) C^*_2(S) + p^*_2(S) S \right]. \end{aligned} \quad (11)$$

Differentiating $V(p_1, S)$ with respect to S , setting the result equal to 0, and making the appropriate substitutions yield the optimal S as a function of p_1 :

$$S^*(p_1) = K - \frac{4\alpha}{3\beta_c} p_1. \quad (12)$$

Substituting this expression into equations (9) and (10), we obtain p_2 and C_2 in terms of p_1 , which we denote as $p^{**}_2(p_1)$ and $C^{**}_2(p_1)$, respectively.

The final step in the solution process takes us back to the first decision, the cartel's selection of p_1 to maximize discounted profits:

$$\begin{aligned} Y(p_1) &= p_1 [C_1(p_1) + S^*(p_1)] \\ &+ \beta_p [p^{**}_2(p_1) \cdot (C^{**}_2(p_1) - S^*(p_1))]. \end{aligned} \quad (13)$$

Solving for the optimal value of p_1 , we obtain:

$$p^*_1 = \frac{K}{\alpha} \left[\frac{9\beta_c^2}{9\beta_c^2 + 12\beta_c - 4\beta_p} \right]. \quad (14)$$

Through substitution, the values of all the variables in both periods can be expressed in terms of the basic parameters, K , α , β_c , and β_p .

Comparison with no-stockpiling case. Let us now compare the results just derived with the base case illustrated in Figure 1a, where no stockpiling is possible. There the cartel will charge the same price in both periods, $p_1 = p_2 = \frac{K}{2\alpha} = p^m$, the one-period monopoly price.

The level of consumption is then the same in each period, as are the level of demand, the cartel's revenues, and consumers' surplus.

In the absence of data on the parameters of the demand curve, K and α , it would be meaningless to present absolute figures comparing the two cases. It can be shown, however, that for both cases the prices will be proportional to $\frac{K}{\alpha}$, producers' and consumers' surpluses will be proportional to $\frac{K^2}{\alpha}$, and, in the stockpiling case, the optimal stockpile level will be proportional to K .⁹ Thus, the effects of stockpiling can be assessed on a percentage basis independent of the values of either K or α . If, for example, the discount rates are 0.05 for both the

⁹ Changes in K and α may be viewed simply as changes in the units of measurement. Viewed in that way, it becomes obvious that the proportional effects will be invariant with respect to K and α .

cartel and the consuming nation ($r_c = r_p = 0.05$), then the following results are observed: The first-period price, p_1 , is 3.4 percent higher than the one-period monopoly price and the second-period price is 27.6 percent lower. The optimal stockpile level is substantial, 27.6 percent of K , the amount demanded for consumption at a price of 0. The present value of net consumers' surplus in the two periods rises by 7.7 percent, and the cartel's revenues also rise, by 6.0 percent. Almost 20 percent of the original deadweight loss, shown as triangle x in Figure 1a, is eliminated. In all cases, the proportional gain in efficiency will be equal to the gain in net consumers' surplus plus twice the gain in cartel revenues.¹⁰ Thus, stockpiling aids both consumers and the cartel.

The producer must always gain if the consuming nation stockpiles at all, as becomes obvious upon reflection. A possible strategy for the producer is to charge the monopoly price in each period. Whatever strategy the government of the consuming nation chooses in response, its citizen consumers will be consuming the monopoly amount in each period. The producer, the only source of supply, cannot be held below the discounted sum of his monopoly profits. Should the consuming nation stockpile, at the very least the producer would be getting some of his profits sooner. If he went further and changed his pricing policy, that would imply that he was doing better still.

□ **A many-period model.** The results of the two-period model are suggestive, and indeed foreshadow the direction of the results that follow. But a longer time horizon is needed to evaluate more fully the potential benefits of stockpiling. For some materials, an infinite horizon may be most appropriate, but for many technological progress or the growth of a domestic production capability may be expected to eliminate the monopolistic power of the producers of the stockpiled community within a finite number of periods.

In the many-period model, the sequence of moves within each period is exactly the same as before. The solution proceeds in a similar manner. It begins with the last period, period T , assuming that $S_T = 0$. The cartel selects its final price, p_T , to maximize Y_T , given the level of the incoming stockpile S_{T-1} . Both Y_T and V_T can then be expressed as functions of S_{T-1} . Moving backwards to period $T-1$, the consumer selects S_{T-1} to maximize discounted net consumers' surplus in the two remaining periods, $V_{T-1} + \beta_c V_T$, given p_{T-1} and S_{T-2} . All of the variables in periods $T-1$ and T can then be expressed in terms of p_{T-1} and S_{T-2} . The cartel then selects p_{T-1} to maximize profits in the two periods, $Y_{T-1} + \beta_p Y_T$, given S_{T-2} . The solution process continues in that manner back to the first decision, the cartel's choice of the optimal initial price, p_1 .

Though intractable analytically, this model can be solved for specific parameter values using the computer.¹¹ We focus on the time paths of price and stockpile level, and the distribution of the gains from stockpiling. Fortunately, the results exhibit the same proportionality observed in the two-period case. Thus, as before, the effects

¹⁰ The relationship holds only when the cartel and the consuming nation employ a common discount rate. When the discount rates differ, an indexing problem arises; the gain in efficiency will depend on which discount rate is applied.

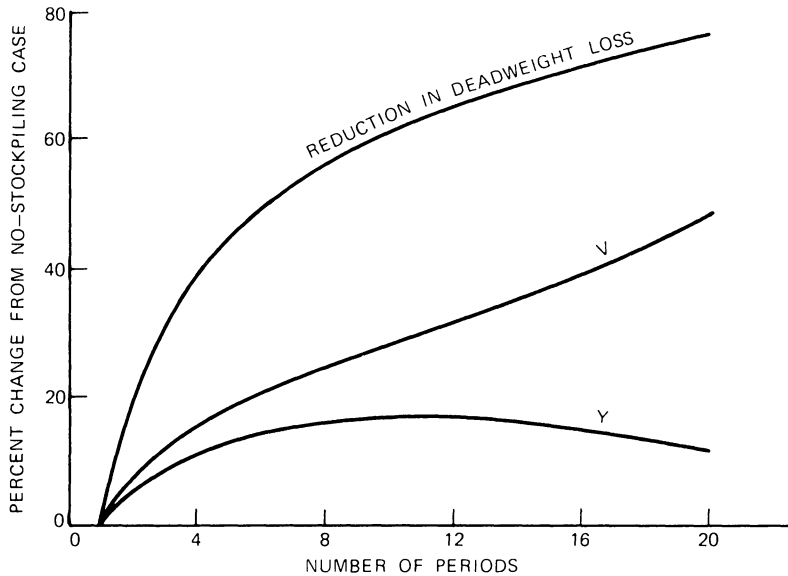
¹¹ We wish to thank Surender Gulati for developing and programming the algorithm used to solve the many-period model.

of stockpiling can be assessed on a percentage basis, independent of the value of either K or α .

The effects of lengthening the time horizon are shown in Figure 3,

FIGURE 3

GAINS FROM STOCKPILING AS THE TIME HORIZON LENGTHENS

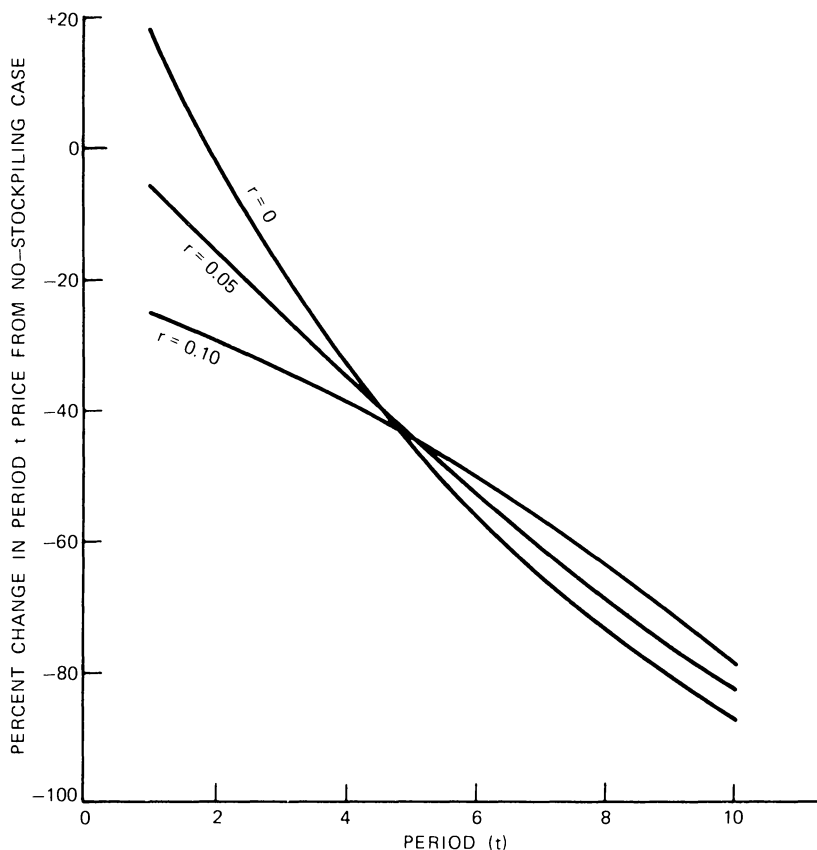


which plots improvements over the monopoly price, no-stockpiling case, for $r_c = r_p = 0.05$. The gain in consumers' surplus is substantial as the number of periods increases, rising from 7.7 percent in the two-period case to 48.5 percent with a twenty-period model. The improvement in cartel profits rises from 6.0 percent with a two-period horizon to a peak of 16.8 percent with a twelve-period model, and then falls, reaching 13.9 percent at twenty periods. The gain in total efficiency, measured as the reduction in deadweight loss, increases from 19.6 to 76.2 percent. As the time horizon stretches to infinity, as in the model of Appendix 1, the reduction in deadweight loss reaches 83 percent. As might have been expected, the two-period model, given its limited time for stockpiling, understates the possible value of stockpiling strategies, which are likely to extend over a considerable number of years.

Figure 4 plots the time paths of prices in the ten-period model for three different pairs of discount rates ($r_c = r_p$): 0, 0.05, and 0.10. With a discount rate of 0, the price starts 18.0 percent above the one-period monopoly price, but falls rapidly thereafter, until it is 87.4 percent lower in period 10. At discount rates of 0.05 or 0.10, prices in all periods are below the one-period monopoly price, but they fall less swiftly.

The time paths of the stockpile levels, shown in Figure 5, yield considerable insight into the cartel's pricing strategy. Note that for each of the three discount rates, the stockpile is built up through period 4 or 5, and then drawn down steadily. In every period the stockpile level varies inversely with the discount rate. The higher the consuming nation's discount rate, the more rapidly its demand for

FIGURE 4
TIME PATHS OF PRICES IN A 10-PERIOD MODEL

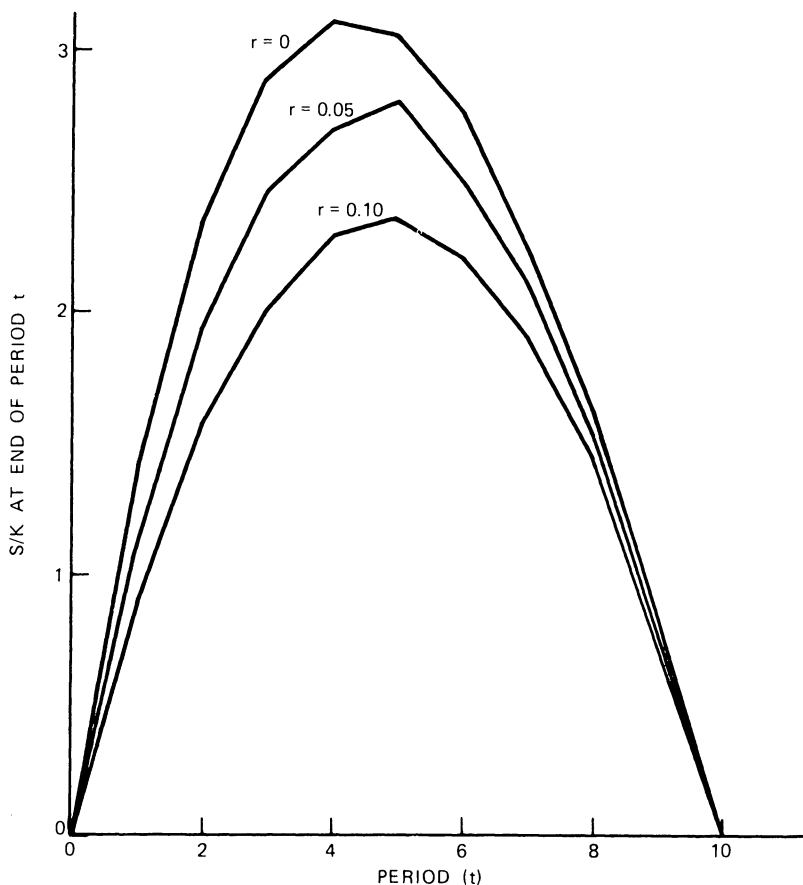


additional units for the stockpile diminishes. Thus, in the earlier periods, when the stockpile is being accumulated, the optimal cartel strategy calls for setting a price that varies inversely with the consumers' discount rate. The price is not lowered enough, however, to make the consuming nation stockpile as much as it would at lower discount rates. Thus, during the later periods, when the stockpile is being drawn down, the consuming nation with a higher discount rate has fewer stockpiled units to release to drive down prices.

The present values of consumers' and producer's surpluses in a ten-period model are plotted as functions of the discount rate in Figure 6. The improvement in net consumers' surplus over the no-stockpiling case rises rapidly with the discount rate; gains for cartel profits fall.

The gains from stockpiling, moreover, will be affected by discount rate differences between producing and consuming nations. How might this divergence affect the gains from stockpiling? Stockpiling causes the consuming nation to take losses in the early periods, when the stockpile is being acquired, in order to achieve a higher level of consumers' surplus later. For the producers, the situation is reversed; the stockpile's acquisition drives up revenues in the early periods, but its release lowers income in the future. Thus, intuition suggests that the total gains from stockpiling will decrease with the consuming nation's discount rate, and increase with the cartel's. Sample calcula-

FIGURE 5
 TIME PATHS OF STOCKPILE LEVELS IN A 10-PERIOD MODEL



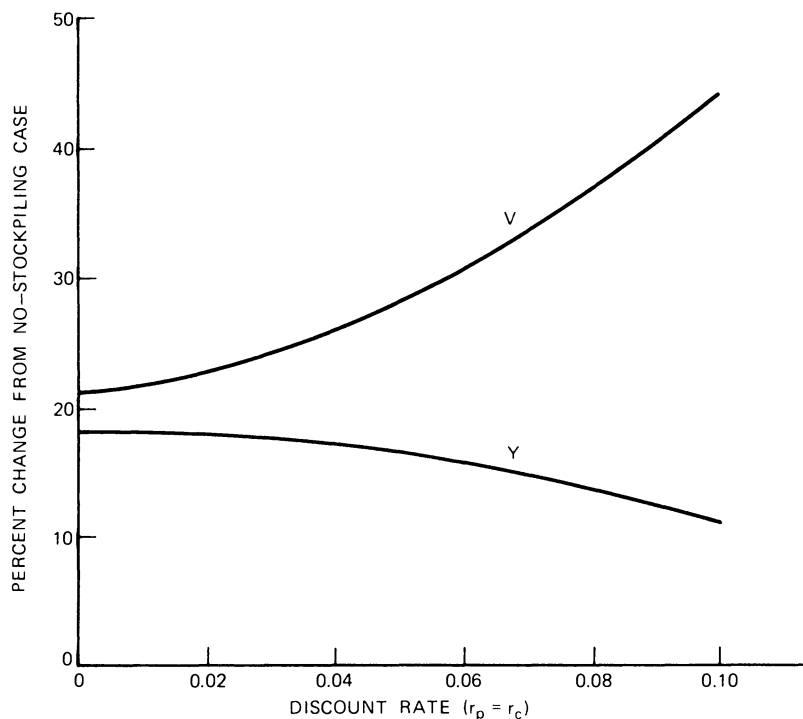
tions from the ten-period model confirm this pattern, as Figure 7 shows.

Most natural resource cartels or potential cartels are composed of developing countries, where interest rates are often considerably higher than in developed countries. (A few OPEC members, such as Saudi Arabia, may offer notable exceptions.) By stockpiling, the developed nations would transfer current income to the developing cartel nations, in exchange for which they would gain the power to hold down future prices. The divergence in interest rates magnifies the benefits to both parties.

Positive production and storage costs. For ease of exposition, most of the models presented in this paper assume that production and storage are costless activities. A simple example based loosely on OPEC illustrates the effects of dropping this assumption. Kalyon (1975) has estimated a linear demand curve for OPEC exports in 1975, with $K = 22.4$ billion barrels and $\alpha = 1.5$. He also estimates the costs of production, again in 1975, at just under \$0.20 per barrel. The U.S. Federal Energy Administration (1974) has estimated the cost of storing crude oil in salt domes at \$1.00 per barrel for capital costs and \$0.01 per barrel for annual costs; for simplicity, we assume a per barrel annual cost of \$0.10. Consistent with our earlier examples, we

FIGURE 6

SENSITIVITY ANALYSIS OF DISCOUNT RATE IN A 10-PERIOD MODEL



assume that both the consuming nation and the cartel employ a discount rate of 0.05.

Using the parameter values assumed above, the gain to the consuming nation from stockpiling is 22.7 percent, down only slightly from 23.0 percent when production and storage costs are zero. For the cartel, the gains from the consumer's stockpiling are 12.3 percent, as opposed to 12.5 percent before. In every period, the stockpile level with positive production and storage costs is lower than before; its peak level is roughly two-thirds as large.

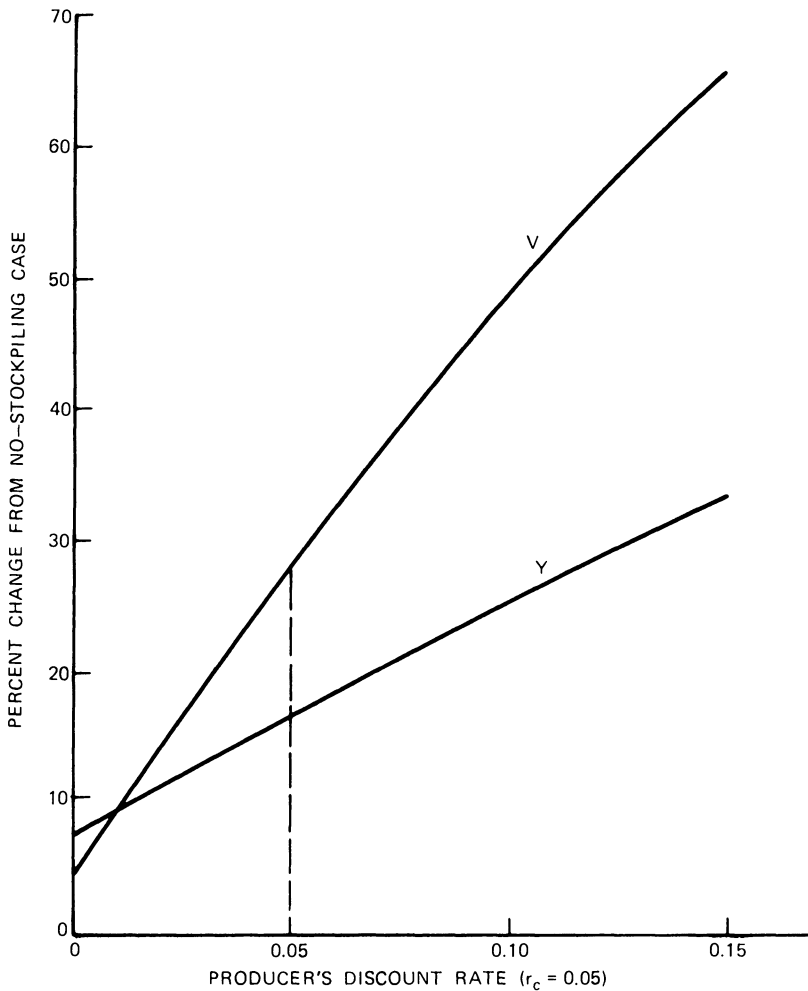
3. Models with two or more consuming nations

■ Our analysis thus far has assumed that stockpiling is carried out by a single benevolent actor whose objective is to maximize the surplus of all consumers. Such an assumption may be realistic if all of the consumers of a particular product reside in a single country, or if the national governments of all consumers form a perfectly cohesive alliance. For most materials of policy interest, however, consumers reside in many nations, and the prospects for an alliance of all consuming nations are exceedingly dim. This section explores what happens when coordination is imperfect, unity less than complete.

In a world where consumers are distributed among different nations—the governments of which are concerned primarily, if not exclusively, with the welfare of their own citizens—stockpiling is an international public good. Its benefits, in the form of reduced prices, are available to consumers in all nations, regardless of their own government's stockpiling efforts. Four cases illustrate possible patterns of strategic interaction when there is more than one consuming

FIGURE 7

SENSITIVITY ANALYSIS OF CARTEL'S DISCOUNT RATE IN A 10-PERIOD MODEL



nation: (1) many identical consuming nations; (2) a partial alliance of consuming nations; (3) two consuming nations of different sizes; and (4) two consuming nations of different sizes, with one acting as leader. In all four cases we shall continue to assume that the producers are joined in a perfectly cohesive, joint-profit-maximizing cartel. To facilitate comparison, total consumer demand per period in each case will be $C = K - \alpha p$. For brevity, graphs are employed in lieu of equations. The relevant supporting equations appear in Nichols and Zeckhauser (1976).

□ **Many identical consuming nations.** In this first model involving more than one consuming nation, we assume that there are N identical consuming countries which split total consumer demand equally so that

$$C_i = \frac{1}{N}(K - \alpha p), i = 1, 2, \dots, N. \quad (15)$$

We assume that the cartel is restricted to setting a uniform price for consumers in all nations. Thus, the period 2 price will depend only

on the total size of the stockpile, and not on how that total is divided among the consuming nations. If all consuming nations apply a uniform discount factor, β_c , the optimal stockpile for the i th country, S^*_i , is a function of p_1 and of the total stockpile held by the other consuming countries, $\bar{S}_i = \sum_{j \neq i} S_j$. Following the same solution techniques applied earlier, the optimal stockpile for the i th nation is then

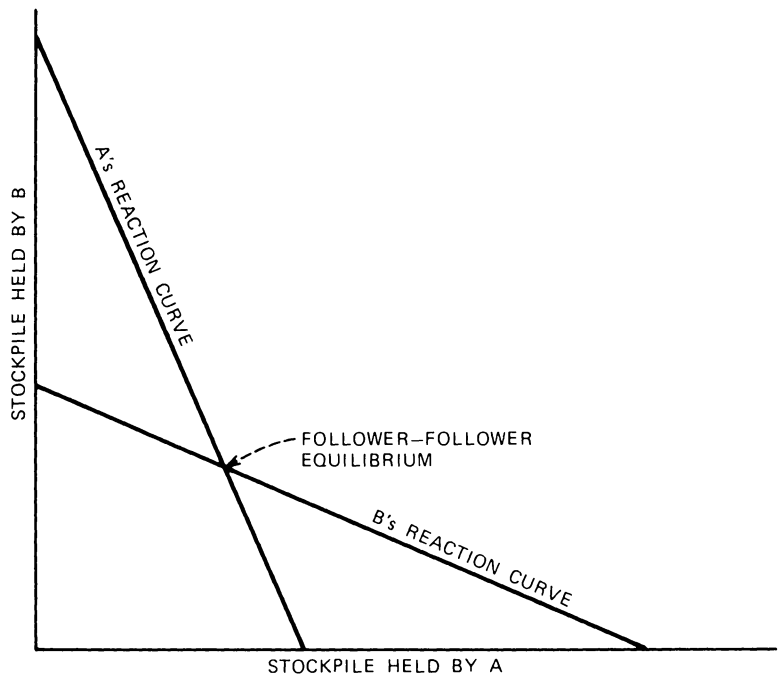
$$S^*_i(p_1, \bar{S}_i) = \frac{K\beta_c(2N+1) - 4N\alpha p_1}{\beta_c(4N-1)} - \bar{S}_i \left(\frac{2N-1}{4N-1} \right),$$

$$\text{for } \bar{S}_i \leq \frac{K\beta_c(2N+1) - 4N\alpha p_1}{\beta_c(2N-1)}, \quad (16)$$

and 0 otherwise. Thus for any fixed p_1 , each country has a linear reaction curve in N -space. For each country, the optimal stockpile level declines with the total stockpile of other nations. Why should it be on a less than a one-for-one basis? Because stockpiling reduces the revenues returning to previously stockpiled amounts, it produces a public disbenefit that is proportionate to a nation's holding of the total stockpile.

Many outcomes are possible, depending on the nature of the interaction among consuming nations. If each country takes the others' stockpiles as given, the equilibrium will be defined by the intersection of the reaction curves as shown in Figure 8. The situation is

FIGURE 8
TYPICAL REACTION FUNCTIONS FOR TWO CONSUMING NATIONS



perfectly analogous to the follower-follower outcome of the Cournot oligopoly model. With N equal-size nations behaving in identical fashion, self-interested maximization will lead to individual nation stockpiles

$$S^*_i(p_1) = \frac{K}{N} - \frac{4\alpha p_1}{\beta_c(2N+1)}, \quad (17)$$

giving a total stockpile

$$\begin{aligned} S^*(p_1) &= \sum_{i=1}^N S^*_i(p_1) \\ &= K - \frac{4N\alpha p_1}{\beta_c(2N+1)}, \\ \text{for } p_1 &\leq \frac{K\beta_c(2N+1)}{4N\alpha}, \end{aligned} \quad (18)$$

and 0 otherwise. Note that for any fixed positive value of p_1 , the size of the total stockpile decreases as N increases.

The final step in the solution process is to find the first-period price that maximizes the cartel's discounted revenues, given $S^*(p_1)$. In general, the maximizing p_1 will decline as N increases until some critical number of consuming nations, N^+ , is reached. Beyond that number, the cartel reverts to the one-period monopoly price; stockpiling is driven to zero. This two-part strategy results from the constraint $S^*(p_1) \geq 0$.¹² Figure 9a illustrates the nature of the equilibrium for $r_c = r_p = 0.05$, with the first-period price scaled in terms of the percentage change from the one-period monopoly price. Note, for example, that despite the decrease in p^*_1 , the combined stockpile of two independent consuming nations is less than 60 percent of the amount that would be held if there were but a single nation.

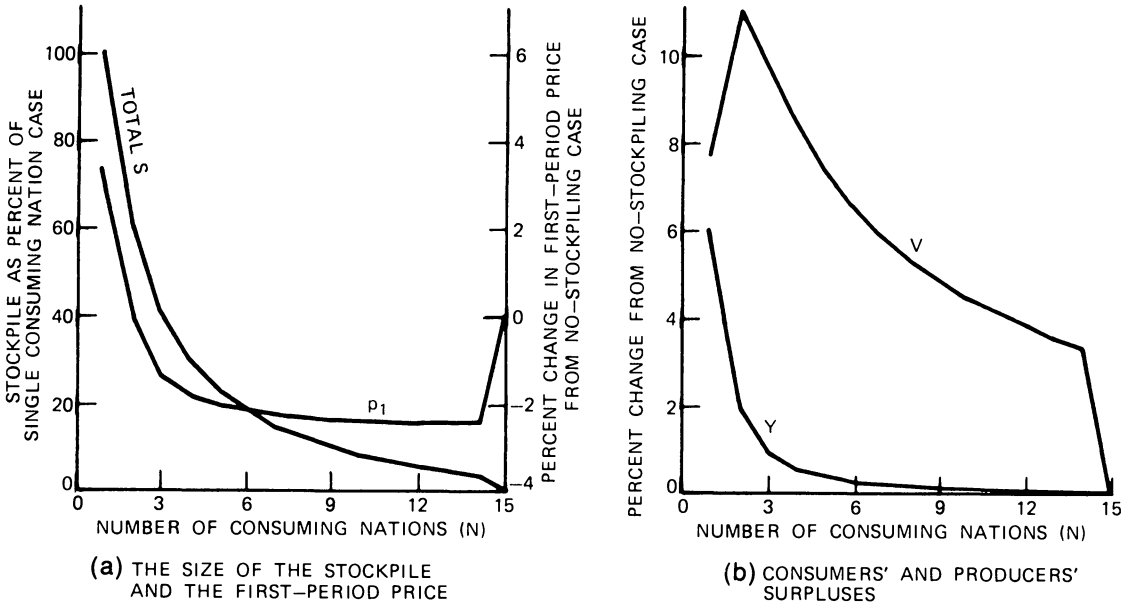
The relationship between consumers' and producer's surpluses and the number of consuming nations is shown in Figure 9b. The cartel's revenues fall steadily throughout. The level of consumers' surplus follows a more complicated path, rising from $N = 1$ to $N = 2$, and then falling steadily, until $N = N^+$, when the cartel abandons efforts to induce stockpiling and reverts to the one-period monopoly price. The intuitive explanation for this result is straightforward. Two conflicting forces are at work. As the number of consuming nations increases, the total level of the stockpile for any given first-period price becomes increasingly smaller and suboptimal. Yet, at the same time, the cartel's optimal first-period price also declines—a factor working to the benefit of consumers. In a very real sense, the “weakness” of the consuming nations in the case $N = 2$ is a virtue; it forces the cartel to lower its first-period price to induce stockpiling.

□ **A partial alliance of consuming nations.** Even with many consuming nations, some coordination of stockpiling efforts could be achieved through multilateral arrangements or alliances. Consider a situation where some but not all consuming nations form an alliance to manage their stockpiles, which in effect become a joint stockpile, for the benefit of their citizens. Alliance members account for the fraction δ of total demand. Each nonmember is assumed to be so small that it will never stockpile, though its citizens will certainly benefit from the alliance's stockpiling efforts. The solution techniques applied earlier

¹² One could interpret a negative stockpile as the consuming nation selling short. However, as the cartel is, by assumption, the only supplier, it could extract an infinite price in the second period from any country which had stockpiled negatively. Thus, even if negative stockpiling is not ruled out *a priori*, it will never be in the consuming nation's interest to do so.

FIGURE 9

MANY IDENTICAL CONSUMING NATIONS



yield the alliance's optimal stockpile as a function of p_1 and δ :

$$S^*_A(p_1) = \frac{K\beta_c(\delta+2)-4\alpha p_1}{\beta_c(4-\delta)},$$

$$\text{for } p_1 \leq \frac{K\beta_c(\delta+2)}{4\alpha}, \quad (19)$$

and 0 otherwise. Note that if $\delta = 1.0$, the alliance is a coalition of all consumers and the expression for $S^*_A(p_1)$ reduces to (12), the formula derived for the case of a single consuming nation. The cartel charges the monopoly price when the stockpilers' alliance has a very small share of the market. At some point, δ^+ , however, the alliance becomes large enough to lead the cartel to lower the first-period price to induce stockpiling. As Figure 10a illustrates, using the same discount rates as before, the optimal first-period price then rises with δ .

Figure 10b plots the percentage gain in consumers' surplus, both for the stockpilers and the free-riding nonmember nations, and in producer's revenues as functions of δ . For $\delta > \delta^+$ the percentage gains rise as δ increases. The curves well illustrate the benefits of nonmembership; at every point the nonstockpilers reap proportionately greater benefits than do the stockpilers. There is every incentive to defect from or refuse to enter into a stockpiling agreement, an incentive that becomes more substantial as the market share of existing participants increases.

□ **Two consuming nations of unequal size: follower-follower case.** Consider now a situation where stockpiling nations are of unequal size; for simplicity assume that there are but two of them. The bigger country has a market share γ , with $1-\gamma$ left for the smaller nation. Each nation will stockpile an amount that depends on the first-period price and on the size of the stockpile held by the other consumer. Assume that both nations take the other's stockpile levels as a given;

FIGURE 10

A PARTIAL ALLIANCE OF CONSUMING NATIONS

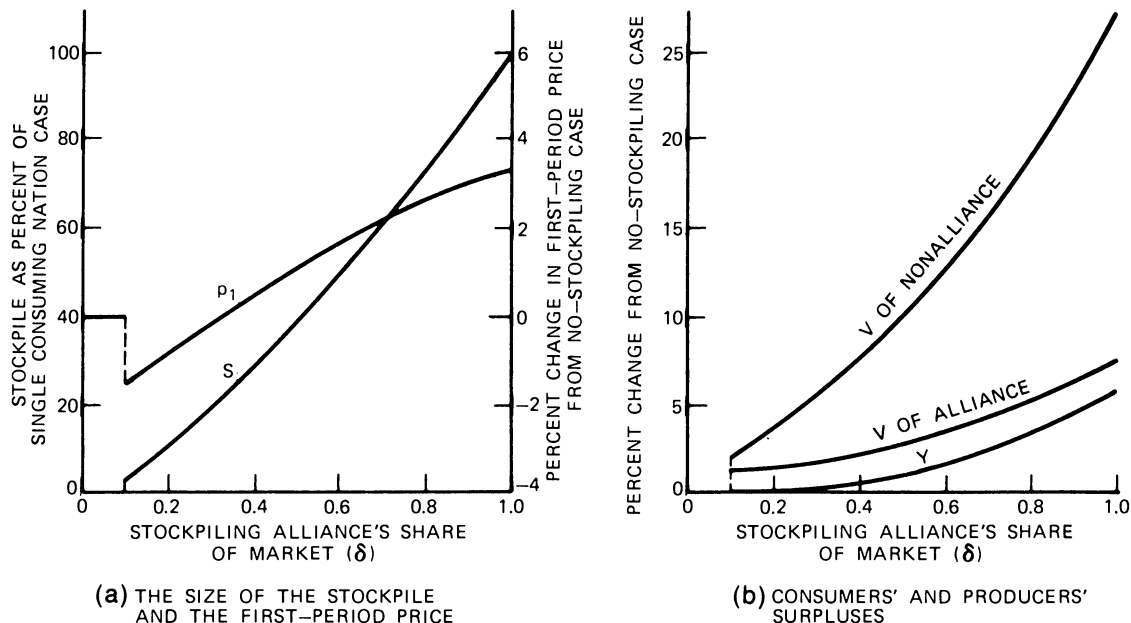
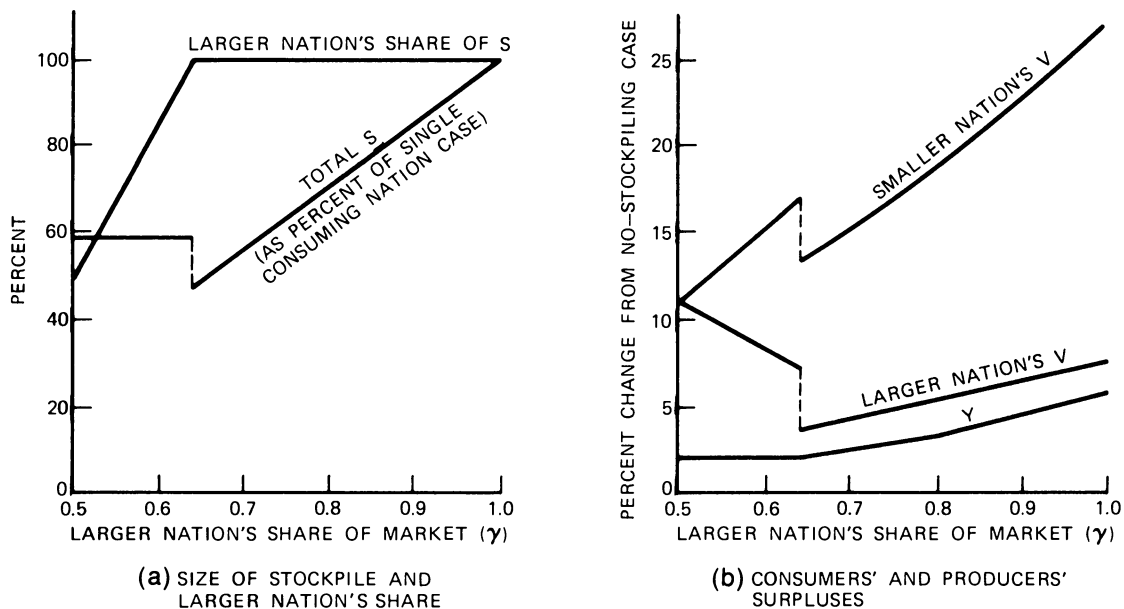


FIGURE 11

TWO CONSUMING NATIONS OF UNEQUAL SIZE: FOLLOWER-FOLLOWER CASE



that is they act as Cournot followers. With market shares relatively balanced, both nations will stockpile. But once γ exceeds some critical value γ^+ , only the big nation stockpiles. Once that happens, the models for nations of unequal size and for a stockpiling alliance become identical. Figure 11 tells the story.

It should be stressed that the smaller consuming nation's higher level of consumers' surplus does *not* presuppose that it exploits the larger country in any way. These are the natural results from a follower-follower model. They are a special case of a more general proposition in the economic theory of alliances: larger countries are likely to provide a disproportionately large share of public or semipublic goods (Olson and Zeckhauser, 1966).

□ **Two consuming nations of unequal size: leader-follower case.** One stockpiling nation may act as a leader, recognizing the effect its own stockpile has on the other's behavior. It may be able to assume this role because it can commit itself to a stockpile level before the other consuming nation can act, or simply because it alone recognizes and chooses to exploit the interaction between the amounts that the two stockpile. We assess this case not only because we suspect it may possess empirical validity in particular circumstances, but also because it leads to interesting general insights into strategic situations.

Intuitively, one might expect a nation to benefit from assuming a leadership role. For any given first-period price, a consuming nation's surplus will increase if it becomes the leader. The situation is complicated, however, by the fact that the cartel need not follow the same pricing strategy in the leader-follower and follower-follower cases. As we shall show below, in many cases a consuming nation is actually made worse off by assuming a leadership role.

Let λ be the market share of the leader. The follower's reaction function will, as before, be a function of the first-period price and the stockpile held by the leader. The task of the leader-nation is to select its stockpile level to maximize the consumers' surplus of its own citizens, given the reaction function of the follower. The cartel's optimal second-period price will, as before, depend only on the total stockpile held by both nations. Solving the leader's maximization problem yields a three-part strategy:

$$\text{Leader's optimal stockpile} = \begin{cases} 0 & , \text{ for } p_1 \geq \hat{p}_1 \\ \lambda K - \frac{\alpha(\lambda^2 + 6\lambda + 3)}{\beta_c(6 + \lambda)} p_1 & , \text{ for } p_1 < \hat{p}_1 \text{ and } p_1 < p_1^+(\lambda) \\ \frac{K\beta_c(2 + \lambda) - 4\alpha p_1}{\beta_c(4 - \lambda)} & , \text{ for } \hat{p}_1 > p_1 \geq p_1^+(\lambda), \end{cases} \quad (20)$$

where

$$\hat{p}_1 = \frac{\lambda K \beta_c (6 + \lambda)}{\alpha(\lambda^2 + 6\lambda + 3)}.$$

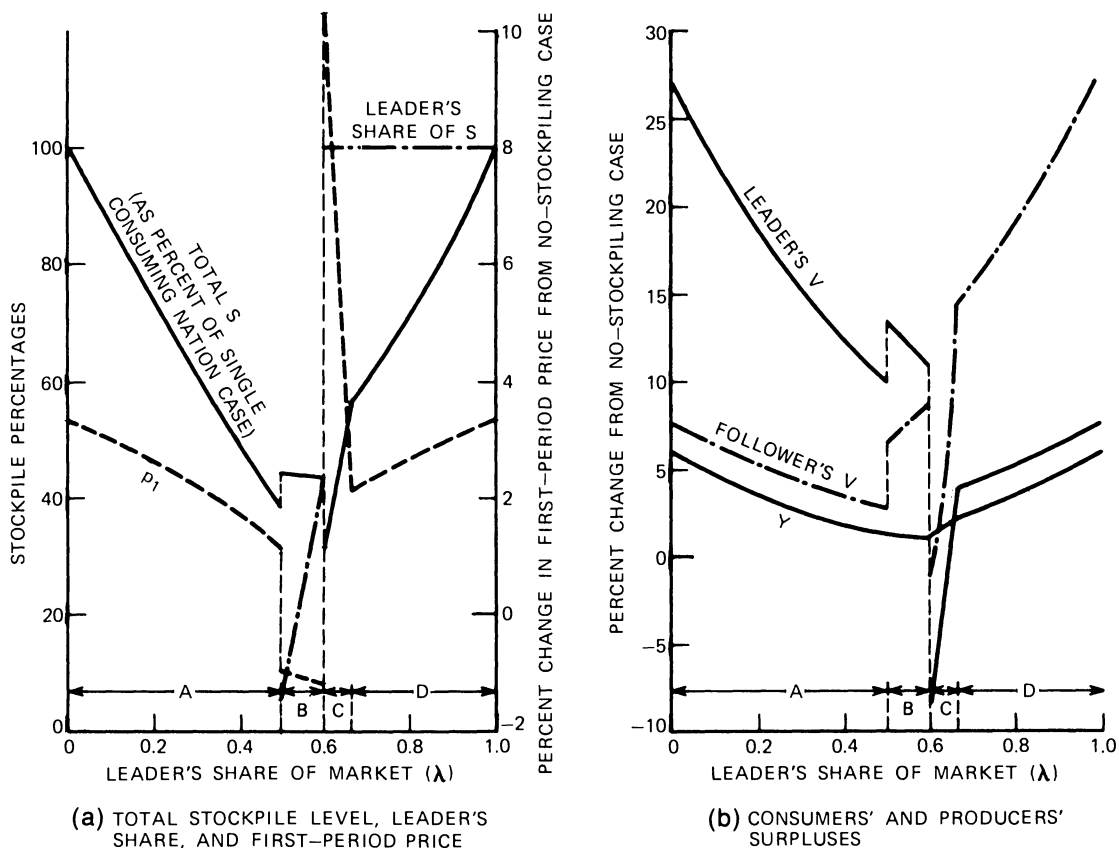
In other words, if p_1 is high enough relative to the leader's share of the market, the leader will not stockpile at all. If p_1 is low enough, again relative to λ , the second expression above applies, and the optimal strategy for the leader is to stockpile a positive amount, but not enough to eliminate stockpiling by the follower-nation. Finally, if p_1 is greater than some critical value of p_1 , which is a function of λ and is written above as $p_1^+(\lambda)$, the optimal strategy for the leader is to make its stockpile large enough to suppress all stockpiling by the follower. As λ , the leader's share of the market, increases, the price necessary to induce this shift declines; i.e., $dp_1^+(\lambda)/d\lambda < 0$.

The total stockpile is the leader's stockpile, a function of p_1 , plus the follower's stockpile, a function of both p_1 and the leader's stockpile selection. Corresponding to the conditions of the leader's three-part strategy, respectively, as λ increases, first only the follower stockpiles, then both leader and follower stockpile, and finally only the leader stockpiles.

There are four possibilities for the cartel's optimal strategy. If the leader controls a relatively small share of the market, the alliance model pertains, and the cartel's optimal strategy is to charge the alliance price, where $1-\lambda = \delta$. When the leader has a somewhat larger share of the market, a lower price that induces stockpiling by both the leader and the follower is optimal. Then, as λ increases over a range, the cartel should raise its first-period price to $p_1^+(\lambda)$, the minimum price needed to make the leader stockpile sufficiently to drive out stockpiling by the follower. Finally, when the leader is large enough, the alliance price for $\lambda = \delta$ is higher than $p_1^+(\lambda)$. The cartel should shift once again to its alliance pricing strategy, only this time the leader is the stockpiler. The four critical ranges of market-shares are indicated as A, B, C, and D in Figure 12a, which plots the optimal

FIGURE 12

TWO CONSUMING NATIONS OF UNEQUAL SIZE: LEADER-FOLLOWER CASE



first-period price and equilibrium stockpile level as functions of the leader's market share.

As noted earlier, it is not always advantageous to a consuming nation to assume the role of leader. Consumers' surpluses for both the leader and follower nations and the cartel's profits are graphed in Figure 12b. Only over the narrow range where both leader and follower stockpile, *B*, does the leader nation achieve a higher level of consumers' surplus than it would in the follower-follower case with the same share of the market. The welfare of its citizens is actually lower over some of *A* and all of *C*; at some points it falls below the level that would be achieved if no stockpiling were possible. For the follower-nation and the cartel, the results are no better than in the follower-follower case for all values of λ , and are worse for part of *A* and all of *B* and *C*.

The results in the leader-follower model at first appear counterintuitive: it would seem that a nation's position ought to improve if it recognizes the full impact of its actions and acts accordingly. The difficulty, however, is that if the cartel is aware of the leader's role, it must follow the pricing strategy shown in Figure 12a in order to avoid exploitation by the leader nation. Thus, over the range $1 - \delta^+ \leq \lambda < \delta^+$ it must charge a higher price in the leader-follower situation than it would if both consuming nations acted as followers. Thinking of this as the traditional Cournot leader-follower duopoly model overlooks the strategic role played by the third party involved, the cartel. Here, in contrast to the duopoly model, leadership can prove detrimental.¹³

The leader-follower model and the other models considered in this section illustrate the wide range of outcomes possible when stockpiling is available in a world with two or more consuming nations. Stockpiling is a public good for the consuming nations; as traditional theory would predict, private provision results in a suboptimal level of stockpile. However, some surprising results arise from the nature of the interaction of these potential stockpilers with the cartel. No longer do we find the passive marketplace of classic models of oligopoly or public goods provision. Instead, the cartel plays an active role, setting prices that take account of the strategies the consuming nations will employ. As a consequence, consuming nations may benefit from disunity, and be hurt by their capability to assume leadership roles.

4. Alternative models of strategic interaction

■ Our preceding analyses assume a simple form of interaction in the market. In each period, the producer specifies a price; the government(s) of the consuming nation(s) purchases or releases a quantity—the net change in its stockpile. More elaborate strategies could be considered. Producers might, for example, offer price-quantity schedules rather than mere prices.¹⁴ Either side might make commitments,

¹³ Schelling (1963, p. 37) describes the complement of this result, where "weakness" improves a player's position: "When a person—or a country—has lost the power to help himself, or the power to avert mutual damage, the other interested party has no choice but to assume the cost or responsibility."

¹⁴ If price-quantity schedules were possible, the cartel could extract all consumers' surplus by quoting as the total price for each quantity the area under the demand curve to that point.

If the producer were a single nation, we would be much more likely to observe such schedules. The need to coordinate the various members of a cartel and to avoid issues that could lead to its breakdown, however, makes it much more likely that

either explicitly, perhaps by a threat or a promise, or tacitly, for example through the choice of a nondominant strategy in a multiplayer game.

The richness of available models is limited only by the imaginative capabilities of the players on the two sides of the market, and the possibilities for institutional arrangements in the real world. To illustrate, we consider four classes of models: (1) side payments are possible, (2) full cooperation is possible, but no side payments, (3) the producer has the ability to make binding commitments, and (4) the consuming nation has the ability to make binding commitments. For each we consider a numerical example in the two-period context.

If side payments between the two players were available, full efficiency could be achieved. In return for a lump-sum payment, the producer would set the price each period equal to the marginal cost of production, zero.

A full-cooperation-no-side-payments model would make the stockpile level and price in each period control variables. The objective would be to achieve an efficient outcome, one that maximizes discounted benefits to the producer, Y , given any level of discounted benefits to the consumer, V . Different values of the control variables would correspond to each particular V .

The ranges of available outcomes in a two-period context are shown in Figure 13, for the linear demand curve $C = K - \alpha p$, and zero production and storage costs. The dashed line represents the efficiency locus with side payments. The unbroken curve shows the feasible points if there is full cooperation, but no side payments are available. The outcomes represented along this frontier have $p_2 = 0$ and $S = K$, i.e., complete stockpiling for second period demand;¹⁵ all variation is taken through p_1 . Point M shows the outcome if stockpiling is unavailable. Point S shows the original outcome with stockpiling.

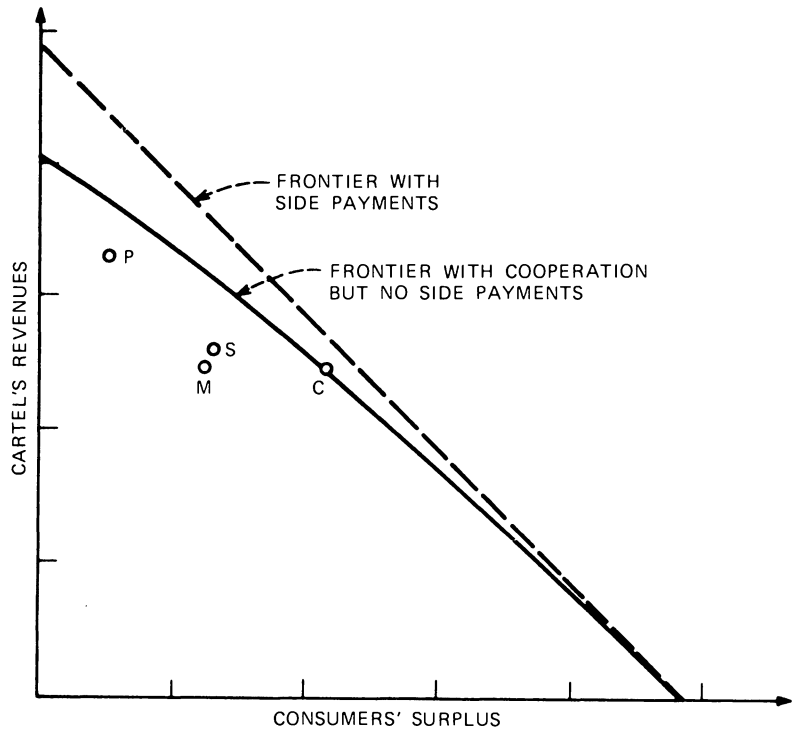
If the producer can make a binding commitment, he should combine a threat with a promise. His maximal threat is to withhold all goods from the market in the second period—i.e., to set a price of infinity—unless the consumer engages in a particular mode of stockpiling behavior. In formulating his strategy, the producer is subject to the binding constraint that the consuming nation be no worse off stockpiling the demanded amount than it would be if it responded optimally to the threatened pricing strategy, (p_1, ∞) . The role of the producer's promise is to improve the consumer's payoff should he comply. The optimal package for the producer employs both his maximal threat and his maximal promise. He sets $p_1 = (K/\alpha)[2\beta/(1+2\beta)]$, and demands that the consumer purchase a stockpile, $S = K[2\beta/(1+2\beta)] - \epsilon$, where β is the common discount factor, and ϵ is an arbitrarily small quantity designed to tip the consumer to comply. The producer promises to set $p_2 = 0$ if the consumer complies; otherwise he will set it at ∞ . The outcome if this strategy is complied with is at point P . Note that when the producer has the

market offers will be expressed solely in terms of price, as they are for the most part with OPEC.

¹⁵ We could set p_1 arbitrarily close to 0, make S exceedingly large, and in effect achieve a system with lump-sum side payments. To avoid this anomaly, which arises only in the case of zero production and storage costs, we constrain S to be no larger than K , the amount demanded in the second period if $p_2 = 0$.

FIGURE 13

OUTCOMES WITH ALTERNATIVE MODELS OF STRATEGIC INTERACTION



ability to commit himself, the consuming nation loses substantially, even in comparison with the no-stockpiling outcome, point *M*.

If the consuming nation is the participant that can make a commitment, it will detail the stockpile schedule it will follow depending upon first-period price. Its maximal threat is to stockpile zero; its maximal promise is to stockpile *K*. No threat is possible with regard to the second period price, for in the second period responses in the consuming nation are dictated. The optimal commitment strategy by the consuming nation employs its maximal threat and promise. It promises to stockpile *K* if $p_1 \leq (K/2\alpha)[2 - (3 - \beta)^{1/2}] - \epsilon$; otherwise it will stockpile 0. Inclusion of the arbitrarily small quantity ϵ will make the cartel comply rather than revert to the strategy of setting its monopoly price in each period. The outcome that is achieved is given by the point *C* in the diagram. The result is not parallel to the one achieved when the producer can make a commitment. The availability of commitment here gets the players to the full-cooperation, no-side-payments frontier. Moreover, the recipient of the commitment, the producer, reaches the same welfare as he would without stockpiling.

Stockpiling models, like most models of strategic interaction, lend themselves to a wide range of possible outcomes, depending in large part on the abilities of the two parties to communicate and to commit themselves to strategies.

5. Conclusions

■ Stockpiling can be an effective tool for a consuming nation purchasing a commodity from a cartel.¹⁶ Interestingly, the cartel will

¹⁶ Stockpiling may also serve as a useful strategy for a commodity that is competi-

always realize economic benefits from this activity, which suggests that stockpiling should be viewed as a mutually beneficial economic policy instrument, and not solely as a weapon for the consuming nations.¹⁷ The gains to the two participants depend on the length of the time horizon, the discount rates for the cartel and the consuming nation, and the costs of production and storage. Section 2 of this paper explored the roles of these factors through a series of sensitivity analyses. Under most circumstances, the longer the time horizon, the lower the consumer discount rate, the higher the cartel discount rate, and the lower production and storage costs, the greater the benefits to the consuming nation (measured in relation to consumers' surplus in the absence of stockpiling).

Section 3 assessed stockpiling in a variety of strategic situations where there are two or more consuming nations. In the absence of a united alliance, as would be expected, the incentives for stockpiling are for the most part diminished. Less obvious are the results regarding the patterns of payoffs and the rapid shifts in the cartel's and the consuming nations' optimal strategies as the relationships among consuming nations change. The models in Section 4 incorporate the possibility of commitment strategies on the part of cartel or consuming nation, and of cooperative outcomes. Appendix 1 presents our basic model when the time horizon is infinite. Appendix 2 examines the implications of a finite supply for the stockpiled commodity.

A wide range of important models remains to be explored. Models incorporating various types of strategic interaction among producers merit particular attention. One such class of models has been explored implicitly already. Even if there is less than perfect unity among producers, the results of this paper apply immediately if there is a price leader, if market shares are independent of price, and if marginal costs of production are equal across all producers for any level of total output.

Extensive empirical studies would be of immense help in guiding the formulation of strategies in any particular context. Unfortunately for economic science, though not social welfare, our recent experience with cartels, though significant, is limited. Theoretical investigations of the type presented here may be the only mechanism available to evaluate the benefits of alternative consuming nation strategies.

Stockpiling traditionally has been considered as a means of coping with supply uncertainties or as a mechanism to achieve military or political objectives. This paper examined stockpiling in a different context. Prices are determined by the cartel's strategic decisions, not by nature's uncertainties; the sole objective of stockpiling is to enhance economic welfare. Our results suggest that if only for economic benefits, consuming nations should seriously consider stockpiling as a strategy for dealing with cartels.

tively supplied. The stockpiling central government exercises some degree of monopoly power through its indirect influence on its citizens' purchases. If stockpiling is to prove desirable, the competitive supply curve must increase in elasticity over some range of output. The increase in elasticity that is sufficient to warrant stockpiling depends on the demand curve and the discount rate.

¹⁷ In this paper we have assumed that each nation considers only the welfare of its own citizens. In reality, governments usually attach some weight, occasionally negative but more often positive, to the welfare of citizens of other nations. The models presented here could easily be extended to such cases.

Appendix 1

An infinite horizon model

■ An infinite horizon model¹⁸ differs from its finite counterparts in that the stockpile need not be drawn down to reduce prices. Its existence and the threat of its release serve to suppress prices. An equilibrium occurs when each player's strategy is optimal over an infinite time horizon and the other's strategy is taken as fixed.

The cartel selects a price as a function of the existing stockpile. The consuming nation's decision variable is the level of the stockpile it will maintain as a function of the cartel's price and the incoming stockpile. The purpose of stockpiling for the consuming nation in this context is to induce the cartel to lower its price. At any given price, p , we may ask: What is the maximum increment in the stockpile that the cartel could demand in exchange for reducing p slightly, and what is the minimum amount by which the consuming nation would have to decrease the stockpile in order to stop the cartel from raising the price slightly?

We answer first the question as to the maximum change in S for a given change in p . Recall that the one-period expression for consumers' surplus, disregarding stockpiling, may be written:

$$V = \frac{1}{2} \left(\frac{K}{\alpha} - p \right) (K - \alpha p). \quad (\text{A1})$$

Total differentiation yields:

$$dV = -(K - \alpha p) dp. \quad (\text{A2})$$

The present value of the benefits from such a change, starting one period hence, and continuing forever, will be:

$$\sum_{t=1}^{\infty} \frac{dV}{(1+r_c)^t} = - \frac{(K - \alpha p) dp}{r_c}. \quad (\text{A3})$$

Against this benefit must be set the cost of increasing the stockpile, which will be the current price times the increment to the stockpile, or $p dS$. Equating that cost with the present value of the benefits leads to an expression for the maximum value, in absolute terms, of $\frac{dS}{dp}$:

$$\frac{dS}{dp} = - \frac{(K - \alpha p)}{r_c p}. \quad (\text{A4})$$

Calculation of the minimum value of $\left| \frac{dS}{dp} \right|$ proceeds in a similar fashion to yield

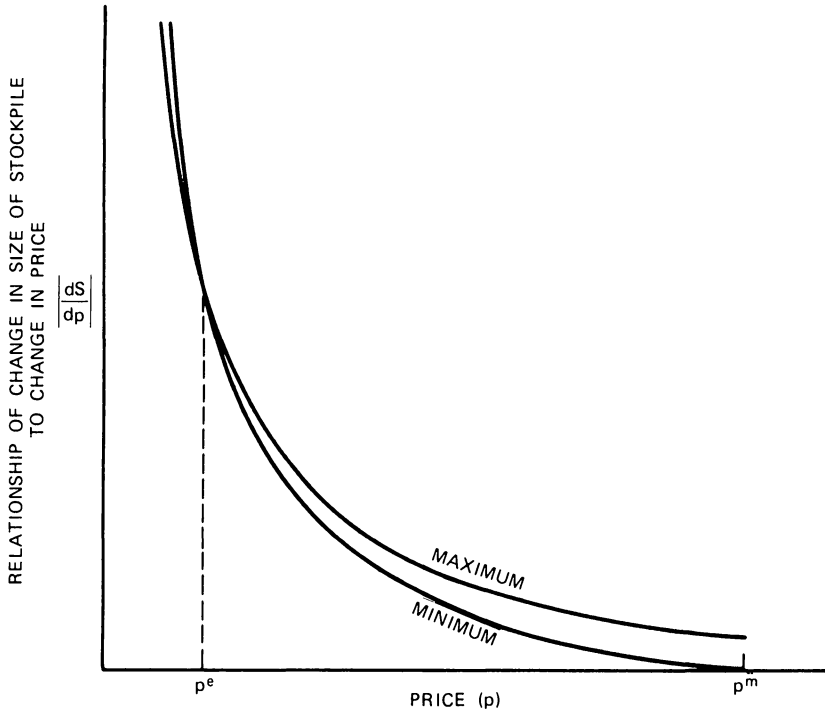
$$\frac{dS}{dp} = - \frac{(1 + r_p)(K - 2\alpha p)}{r_p p}. \quad (\text{A5})$$

Equilibrium price. The minimum and maximum values of $\left| \frac{dS}{dp} \right|$ are plotted as functions of p in Figure A1 for representative parameter values. Note that at $p = p^m = \frac{K}{2\alpha}$, the one-period monopoly price, the minimum value of $\left| \frac{dS}{dp} \right|$ is 0, as the cartel has no incentive to charge a higher price, even in the absence of stockpiling. As p de-

¹⁸ For a more detailed derivation of the infinite-horizon model, see Nichols and Zeckhauser (1976, pp. 25–38).

FIGURE A-1

DETERMINATION OF THE EQUILIBRIUM PRICE IN THE INFINITE HORIZON MODEL



creases from p^m , the maximum value of $\left|\frac{dS}{dp}\right|$ is initially above the minimum, but following their intersection at $p = p^e$, the order is reversed. That is, for $p < p^e$, the rate at which the consuming nation will increase its stockpile in order to secure a decrease in price is less than the amount needed to ensure that the cartel will not raise its price. The natural interpretation is that p^e is the equilibrium price.

In order to solve for p^e , we set the maximum and minimum expressions for $\left|\frac{dS}{dp}\right|$ equal to one another, which yields:

$$p^e = \frac{K}{\alpha} \left[\frac{(1 + r_p)r_c - r_p}{2(1 + r_p)r_c - r_p} \right]. \quad (\text{A6})$$

For some discount rates, $r_c \leq \frac{r_p}{1 + r_p}$, this expression will not yield a stable equilibrium price. In those cases, as p approaches 0, the maximum value of $\left|\frac{dS}{dp}\right|$ remains greater than the minimum, and both grow without bound.

□ **Price-and-increment-to-stockpile path to the equilibrium price.** The approach outlined above defines an equilibrium price, as well as equilibrium conditions for stockpiling and pricing behavior in the neighborhood of the equilibrium price. Note that the meeting of the maximum and minimum curves leaves little latitude for variations in the nature of equilibrium strategies in the neighborhood of p^e .

Starting far away from p^e , however, there can be a variety of strategies that are in equilibrium with each other. Associated with

each pair of equilibrium strategies, beginning at any point, there will be a unique time path of price and stockpile behavior. We have identified one constraint on possible paths; they cannot lead either player to take steps that would be outside the region defined by the maximum and minimum curves.

To find the minimum stockpile level, $h(\bar{p})$, needed to maintain any given price, \bar{p} , multiply the expression for the minimum value of $\left| \frac{dS}{dp} \right|$ needed to discourage the cartel from raising its price by dp and integrate over the range p^m to \bar{p} .

Let the consumer's strategy be of the following form:

$$S_T = \begin{cases} h(p) & \text{if } p > h^{-1}(S_{T-1}) \\ S_{T-1} + \Lambda, \Lambda \geq 0, & \text{if } p \leq h^{-1}(S_{T-1}). \end{cases} \quad (\text{A7})$$

That is, if the cartel fails to lower its price to $h^{-1}(S_{T-1})$, the consuming nation, simply following its prescribed strategy, will reduce its stockpile to the minimum stockpile level corresponding to the new price, to $h(p)$. Given the way $h(p)$ is calculated, that reduction will at least offset any gain to the producer from raising the price.

Given the consumer's strategy, it will never be optimal for the cartel to set a price in excess of $h^{-1}(S_{T-1})$. We consider the equilibrium pair where the cartel's strategy is:

$$p = h^{-1}(S_{T-1}). \quad (\text{A8})$$

In response to that pricing strategy, the consuming nation's optimal stockpile acquisition strategy will be of the form given in (A7). The values of Λ can be calculated by using discrete approximations and an iterative process of dynamic programming. The resulting strategies are an equilibrium pair in the sense that each is optimal against the other. The strategies differ from those derived in the finite-period models, however, in that we have permitted the consuming nation to commit itself to a contingent strategy.¹⁹ The equilibrium price and stockpile are approached asymptotically, as shown in Figure A2, for the case $r_p = r_c = 0.05$. The ultimate equilibrium stockpile is more than 30K. By the end of the tenth period, approximately 31 percent of the stockpile is acquired, and almost 83 percent is in place by the end of 50 years. The price follows a similar path, changing most rapidly in the earlier periods. At equilibrium, it is about 91 percent lower than the one-period monopoly price, p^m .

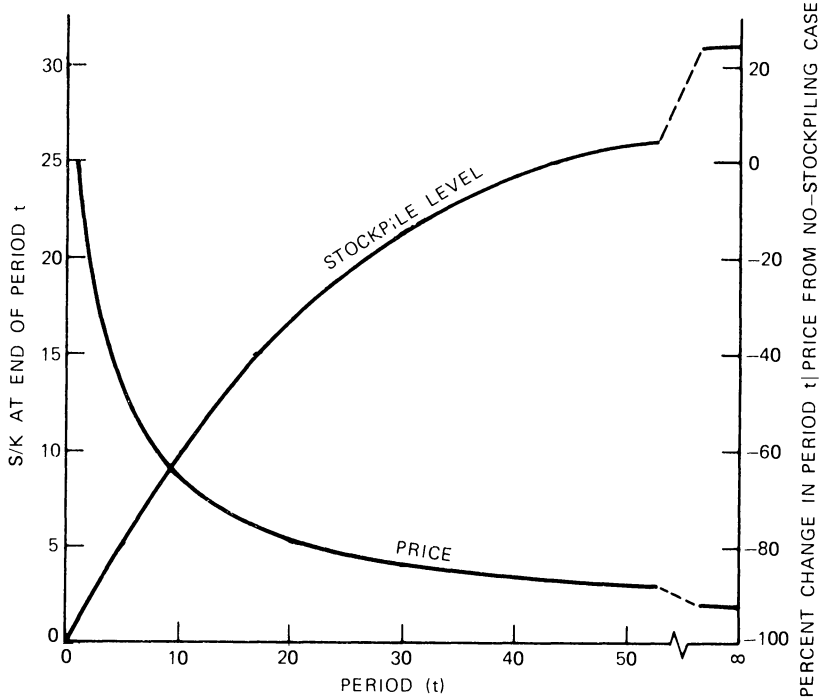
The gain to consumers from stockpiling in the infinite horizon

¹⁹ An alternative approach to solving for equilibria would be to consider only discrete values of S and p . All of the possible consumer and cartel strategies could then be arrayed against one another in a payoff matrix. The difficulty is that the number of possible strategies very quickly gets out of hand. If there are M possible stockpile levels and N possible prices, the number of cartel strategies will be N^M . The number of possible consumer nation strategies will be far greater, M^{NM} , as the choice of stockpile levels is contingent on both the incoming stockpile and the price. For example, if there are just three possible stockpile levels and three prices, the cartel will have $3^3 = 27$ strategies available and the consumer $3^{3 \cdot 3} = 19,683$. Thus the payoff matrix will have 531,441 cells, and over 1 million entries.

model is dramatic, 71.8 percent. The cartel gains only 5.6 percent. Stockpiling eliminates about 83 percent of the original deadweight loss. Higher discount rates shrink the stockpile and the consumers' gains, but raise the cartel's revenues.

FIGURE A--2

PRICES AND STOCKPILE LEVELS IN THE INFINITE HORIZON MODEL



Results with production and storage costs. The infinite horizon model can be extended to permit the inclusion of production and storage costs. The proportionality observed earlier is no longer present, so that particular parameter values must be specified. We rely on Kalymon's (1975) oil estimates once again. The demand curve is $D = 22.4 - 1.5p$; production costs are \$0.20 per barrel. It is easy to treat capital costs explicitly in the infinite-horizon model. Following the U.S. Federal Energy Administration (1974), we estimate capital costs of creating storage capacity at \$1.00 per barrel, with annual operating costs of \$0.01 per barrel. For $r_c = r_p = 0.05$, the equilibrium price with stockpiling is \$3.76, slightly less than one-half the one-period monopoly price of \$7.57. The corresponding stockpile level is equal to 98, or about 4.4K. Note that with positive production and storage costs, the equilibrium involves a much higher price and a much lower stockpile. Gains over the no-stockpiling case are 49.5 percent in consumers' surplus, 2.3 percent in cartel profits, and 54 percent in the elimination of deadweight loss. Thus, the qualitative results of the model continue to hold when production and storage costs are introduced; both consumers and producers are likely to benefit from the use of stockpiling.

Appendix 2

Stockpiling a constrained resource

■ Many of the commodities being considered for stockpiling are depletable resources, the prices of which may be affected by constraints on the cumulative supply. In this appendix we consider the impact of a resource constraint on cartel and consuming-nation strategies in a two-period model. The constraint adds considerable complications to the stockpiling model. Even for two periods there are four different expressions for $S^*(p_1)$ and five for p_1^* ; each is accompanied by rather complicated boundary conditions.

The model we consider is identical to the one developed in Section 2, except for the addition of a constraint on total consumption (production):

$$C_1 + C_2 \leq R. \quad (\text{A9})$$

We assume $r_c = r_p = 0.05$; the common discount rate eliminates some of the possibilities for $S^*(p_1)$ and p_1^* .

If R is sufficiently large, the resource constraint will play no role. Let \bar{R} be the minimum resource constraint which is not binding on the original solution. As intuition might suggest, if $R \geq \bar{R}$, the unconstrained solution derived earlier—represented by equations (9), (12), and (14)—still applies. At the opposite extreme, for $R < R^+$, the constraint will be so tight that no stockpiling takes place and the cartel charges the p_1 that would be charged if stockpiling were unavailable. The range in between, $R^+ \leq R < \bar{R}$, still has stockpiling, although the resource constraint is binding.

Over that range, the critical complication is that p_2 can never be lower than the price that would lead consumers to demand all of the remaining resource in the second period. This constraint is

$$K - \alpha p_2 \leq R - C_1, \quad (\text{A10})$$

or

$$p_2 \geq \frac{2K - R - \alpha p_1}{\alpha}. \quad (\text{A11})$$

Hence there is no benefit to the consumer from stockpiling beyond the amount needed to drive p_2 to this lower limit. Note that this lower limit on p_2 will decrease as p_1 increases. Thus, the size of the optimal stockpile actually increases over the relevant range of p_1 :

$$S^*(p_1) = \begin{cases} 2R - 3K + 2\alpha p_1 & \text{for } \frac{3\beta(2K-R)}{\alpha(3\beta+2)} \geq p_1 \geq \frac{3K-2R}{2\alpha} \\ 0 & \text{for } p_1 < \frac{3K-2R}{2\alpha} \end{cases} \quad (\text{A12})$$

The optimal first-period price given this stockpiling strategy is:

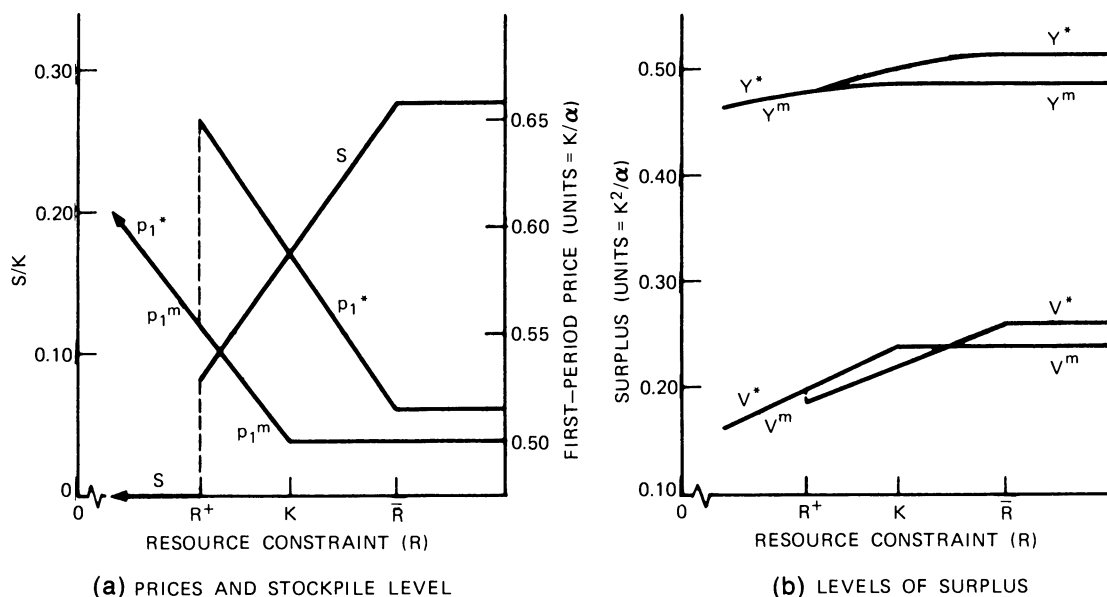
$$p_1^* = \begin{cases} \frac{3\beta(2K-R)}{\alpha(3\beta+2)} & \text{for } R^+ \leq R < \bar{R} \\ \frac{K(1+3\beta) - 2\beta R}{2\alpha(1+\beta)} & \text{for } R < R^+. \end{cases} \quad (\text{A13})$$

It is at the extreme of the range if there will be stockpiling.

Figure A3a shows the effect of a resource constraint on the

FIGURE A-3

STOCKPILING A CONSTRAINED RESOURCE



stockpile level, S^* , and on the first period price, p_1^* . The curve p_1^m shows the first-period price that would prevail if stockpiling were not available. Note that for $R \geq \bar{R}$, neither the prices nor the stockpile level depend on R . As the constraint is tightened below \bar{R} , p_1^* rises and S^* falls; p_1^m is unaffected until $R < K$, when the constraint becomes binding even if stockpiling is not possible. When R reaches R^+ , the cartel shifts pricing strategies; for $R < R^+$, $p_1^* = p_1^m$ and $S^* = 0$.

The impact of a resource constraint on the levels of consumers' and producers' surpluses is shown in Figure A3b. Levels of consumers' surplus with and without the availability of stockpiling are denoted V^* and V^m , respectively. The corresponding amounts for the cartel are Y^* and Y^m . As before, none of the quantities is affected by the constraint so long as $R \geq \bar{R}$. For $\bar{R} > R \geq K$, V^* and Y^* fall as the constraint is tightened; V^m and Y^m are unaffected, as the constraint is not binding on the outcome when stockpiling is not available. For $K > R \geq R^+$, all four levels of surplus fell with R . Finally, for $R < R^+$, the availability of stockpiling has no impact on the outcome: $V^* = V^m$ and $Y^* = Y^m$.

It is interesting to note in Figure A3b that over a considerable range of R , $V^* < V^m$; the availability of stockpiling harms the consumer. The type of strategic interaction which leads to this somewhat counterintuitive result is illustrated by the simple payoff matrix shown in Figure A4. In this game, the cartel selects the column (p_1), then the consumer picks the row (S). The cell entries are the payoffs to the cartel (upper right-hand corner) and to the consumer (lower left-hand corner). We consider the situation where $R = 1.03K$, $r = 0.05$, and $K = \alpha = 1.0$. We consider just discrete choices; a continuous model would exhibit the same behavior. The two prices available to the cartel are the price it would charge if stockpiling were unavailable, $p_1^m = 0.5$, and the price that is optimal for the cartel when the

FIGURE A-4

AN ILLUSTRATIVE
PAYOFF MATRIX

		CARTEL STRATEGY (p_1)	
		0.500	0.571
CONSUMER STRATEGY (S)	0.0	0.488	0.483
	0.201	0.244	0.211
	0.530	0.498	0.512
		0.248	0.226
		0.515	0.574
		0.231	0.163

constraint is at this level and stockpiling is available, $p_1^* = 0.571$. The consumer has three possible strategies: no stockpiling, $S = 0$; its optimal stockpiling response to p_1^* , $S = S^* = 0.201$; or a large stockpile, $S = 0.530$. The strategy pair ($p_1 = 0.5, S = 0$) would be the outcome if it were not possible to stockpile. If stockpiling is available, the cartel's optimal strategy is $p_1 = p_1^* = 0.571$; the consumer's optimal response is $S = S^* = 0.201$, yielding the payoff pair (0.512, 0.226) in the middle cell of the right-hand column. This result is worse than the no-stockpiling outcome for the consumer, but better for the cartel. Note that if cooperation were possible, both would prefer to shift to the ($p_1 = 0.5, S = 0.530$) strategy pair, which yields the payoffs (0.515, 0.231). That outcome can be achieved, however, only if the consumer can make a binding commitment to choose $S = 0.530$ rather than $S = 0.201$, which is its optimal strategy in response to $p_1 = 0.5$.

That the consuming nation actually loses from having a stockpiling capability in this constrained-resource example is somewhat disturbing. Most of the commodities being considered for stockpiling are constrained. The levels of resources, discount rates and overall strategic situation may be such that stockpiling actually hurts. Any practical situation merits detailed study to see what types of commitments might be made to improve the outcome for one or both parties.

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