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SURVIVAL VERSUS CONSUMPTION*

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We develop an indirect method to estimate utility and willingness to pay (WTP) for reductions in the risk of death at various ages. Using a life-cycle model of consumption, we assume that an individual sets his consumption level each year so as to maximize his expected lifetime utility. Alternative assumptions about opportunities for borrowing and annuities characterize two polar types of societies. In our Robinson Crusoe case, an individual must be entirely self-sufficient, and annuities are not available. In our perfect markets case, an individual can borrow against future earnings and purchase actuarially fair annuities; we show that under these assumptions WTP is the sum of livelihood (discounted expected future earnings) and consumer surplus.

To illustrate our methods, we derive WTP for an average financially independent American man under plausible assumptions. The model is calibrated to 1978 earnings (e.g., $18,000 per year for men aged 45–54 with at least some income). In the Robinson Crusoe case, WTP increases from $500,000 at age 20 to a peak of $1,250,000 at age 40, and declines to $630,000 at age 60. In the perfect markets case, age variations are less pronounced; WTP is $1,050,000 at age 20, peaks at $1,070,000 at age 25, and declines to $600,000 at age 60. These results suggest that individuals value risks to their lives at several times the pro-rata share of their future earnings.

(WILLINGNESS TO PAY; VALUE OF LIFE; LIFE CYCLE; RISK OF DEATH; CALCULUS OF VARIATIONS; UTILITY)

1. Introduction

Many actions—walking a mile, going to the doctor, tightening regulations on air quality—may be viewed as purchases or sales of survival. Most of the important purchases are made by individuals on their own behalf. However, parents frequently buy for their children, and society undertakes a variety of survival-related programs on behalf of all of us.

How should we value survival purchases? Here we address that question in a normative framework, taking information on earning opportunities and preferences for consumption as given. We then employ a lifetime utility model to determine how much an individual should be willing to pay for an increase in survival probability or, more specifically, a reduction in the risk of death at a particular age.

The willingness-to-pay criterion, discussed by Schelling (1968), rests on the principle that living is a generally enjoyable activity for which consumers should be willing to sacrifice other pleasures, such as consumption. Although the approach is conceptually elegant—see overview by Jones-Lee (1976)—it has been fraught with practical difficulties. The better-known pilot surveys (Acton 1973; Fischer and Vaupel 1976; Jones-Lee 1976; Keller 1970) show variability and inconsistencies in the responses; individuals have difficulty responding to complex and disturbing questions.

Several analyses have computed person-years of life as a measure of benefits (Murray and Axtel 1974; Preston, Keyfitz, and Schoen 1972; Cole and Berlin 1977; Neuhauser 1977; Acton 1975), or discounted person-years (Berwick, Keeler, and Cretin 1976). Some studies have adjusted life years to take account of health status (Torrance 1976; Weinstein, Pliskin and Stason 1977) or time preference and health status (Weinstein and Stason 1976; Zeckhauser and Shepard 1976; Pliskin, Shepard and Weinstein 1980).

Our Model of Analysis

In this analysis, we focus on the way important attributes—specifically age, income, and consumption—can be incorporated into a utility function for life. If we wish to prescribe choices relating to survival, we can no longer avoid monetary valuations. (Only if survival purchases involved no monetary consequences or were made out of a fixed budget, would benefit measures alone be sufficient to prescribe the optimal choice.) To derive such valuations, we start with a period utility function with two arguments: alive or not, and consumption in the period. We then add real data on age-related earnings, wealth, and survival probabilities.

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From this we compute actual dollar amounts for willingness to pay for increased survival at various ages. Our analysis is primarily illustrative. In particular, we assume a specific form for the period utility function and posit a discounting formulation for cumulating utility over one's lifetime, rather than developing a utility structure based on information gleaned from experiment or observation. We are reassured that our methods yield realistic estimates of implicit valuations of small changes in survival probability.[1]

The methods we outline for valuing risks rely on the fact that we are concerned only with marginal changes in probability. Most risks to life involve low probabilities, and our abilities to ameliorate those dangers also involve small reductions.

2. The Life-Cycle Model of Consumption

The decision maker in our models devotes his resources to two ends: purchasing survival probability and consuming. (Some goods, such as nutritious food, may serve both functions.) Anyone engaged in lifetime resource allocations must be concerned with what capital and insurance markets are available. For example, can he or she deploy wealth to purchase annuities and thus guarantee a given consumption level over an unexpectedly long life? Contracting problems and information asymmetries plague the operation of fair and efficient markets for capital and risks. In response, society, families, and labor market institutions frequently provide substitute structures, such as free education, loans for house down payments, and long-term labor contracts that spread risks across the business cycle.

To gain tractability and eliminate clutter that might obscure results, we consider two polar models of market availability: one offers no markets, the other perfect markets. They are:

(1) *Robinson Crusoe*: Each individual is entirely self-sufficient. He must support himself entirely from his own wealth and earnings. He has no heirs or dependents. There are no markets on which he can trade.

(2) *Perfect Markets*: Each individual must provide for himself out of his wealth and earnings. Perfect markets are available for trading claims across time periods (i.e., capital markets), and for insuring against variability in length of life (i.e., annuities).

The Consumption-Allocation Model

One heroic assumption underlies our analysis: an *individual's utility over lifespans of different length can be represented as a weighted sum of period utilities, the weights declining geometrically with time*. The model generates a value function for survival probability as a function of age, giving the implicit value of life at different ages, and the tradeoff between improved survival and enhanced consumption. A value function that is a weighted sum of period utility functions is itself a utility function only for small perturbations in the survival function.[2] We believe that most risks to life involve low probabilities, and that we can ordinarily achieve no more than small reductions in risk. Moreover, thinking in terms of lives lost with certainty, rather than risk to life, engulfs the discussion in high tides of emotion. (In an earlier analysis, to highlight concern with marginal changes, Zeckhauser (1979) defined a risk unit or RU as a 0.001 change in probability, and suggested that valuations of changes in survival be discussed in terms of RUs.)

Notation and Assumptions

We denote the rate of consumption at time t by $c(t)$ and the period utility function at t by $u(t)$. The full notation developed in this part is presented in Table 1.

The probability of being alive at time t is given by an actuarial survival function, $l(t)$, where $l(0) = 1$ and $0 < l(t) < 1$. Apart from learning that he is still alive, and his age, we shall assume that the individual does not update $l(t)$ as time moves forward.[3] (Time t need not be the individual's age as ordinarily counted. We can set the time origin arbitrarily as long as we normalize the survival function to $l(0) = 1$ and are consistent.) Let T be the maximum possible survival time, so $l(t) = 0$ for $t > T$.

[1] Arthur (1981) also constructs a lifetime utility function based on U.S. mortality and discounted period utilities. Our model is distinguished by its explicit treatment of the dependence of earnings rates on age, and our analysis of an individual's life-cycle savings and consumption decisions as explicit endogenous variables.

[2] The effects of large changes must be obtained by solving a complex problem in the calculus of variations, and cannot be represented by a simple function (Shepard, Pliskin, and Weinstein 1975). Similar limitations have been discovered in deriving the utility of other attributes from a consumption function, such as return on a risky investment (Meyer 1970) or future wealth (Spence and Zeckhauser 1972). The reason for the limitation is the same in all these examples: decisions must be made before uncertainty is resolved, so the utility of an attribute in one period depends on the probability distribution of the likely amount of that attribute in a future period.

[3] An individual's knowledge of risks associated with his occupation or health characteristics (e.g., whether he is free of life-threatening diseases) provides opportunities for updating survival probabilities. We could allow $l(t)$ to be revised, though that would complicate our model considerably.

TABLE 1

Notation for Model

Notation	Interpretation
τ, t	Time in individual's life, from 0 to T
t_0	Time at which individual first begins to save (accumulate wealth)
$c(t)$	Rate of consumption at time t
$w(t)$	Level of wealth at time t
$w(t) > 0$	Solvency constraint
$m(t)$	Rate of earnings at time t
$y(t)$	Utility of survival at age t of life after age t, *not* conditional on survival at age t
$v(t)$	Value of remaining life at age t, conditional on survival at age t
$R(t)$	Marginal change in optimal $v(t)$ per unit reduction in the force of mortality at time t
$E(t)$	Discounted expected remaining years of life following age t (not conditional on survival at age t)
$\bar{c}$	Constant rate of consumption (in perfect markets case)
H	Scaling factor for utility function
$f(t)$	Net amount received by an annuitant alive at age t
$N(t)$	Discounted expected earnings following age t, discounted to age t and conditional on survival at age t
$J(t)$	Discounted expected consumption following age t, discounted to age t
$G(t)$	Discounted expected consumption following age t, discounted to age t, and conditional on survival at age t

An individual's utility at time t of his remaining life after age t, denoted by $y(t)$, is given by his expected discounted utility of consumption for each year in which he is alive from time t on. It is defined by

$$y(t) = \int_t^T e^{-r(\tau - t)} l(\tau) u(c(\tau))\, d\tau. \tag{1}$$

An important special case, $y(0)$ corresponding to $t = 0$, gives the utility from the initial age onward. Thus (1) is a joint utility function for a consumption trajectory and a survival function.

Three assumptions are implicit in (1). First, death has a period utility of zero relative to the utility $u(c)$ of being alive with consumption c. Since scaling of the utility function can be arbitrary, this restricts death from having a period utility of negative infinity. Our model would not apply if a person would trade off an infinite amount of money for a probabilistic gain of just an instant more of life.

Second, the utility of consumption at one time is independent of past consumption. This assumption, termed the marginality assumption, implies the additive (integral) form for total expected utility. Third, legacies or bequests are not valued—the consumer is indifferent to wealth at the time of death. The third assumption, necessary to keep the mathematics manageable, is plausible under two conditions: (a) the consumer has no economic dependents, or (b) he has purchased paid-up life insurance to satisfy the basic needs of any dependents and is not interested in a further bequest to them (e.g., he feels it might weaken their will to work). The model applies as well if bequests are worth a constant fraction of their value if the individual were alive, or if the availability of perfect insurance and annuity markets assures that bequest levels will be optimal.

The objective is to maximize $y(0)$ in (1), subject to a feasible trajectory on consumption.[4] A feasible trajectory is constrained by the requirement that consumption cannot be negative:

$$c(t) > 0 \qquad \text{for all } t. \tag{2}$$

For the two polar cases, we shall first solve the model to find the optimal lifetime pattern of consumption. Assuming an individual follows that pattern, we will calculate the utility he derives from a reduction in the probability of death at any age and, finally, his willingness to pay, i.e., sacrificed consumption, for improved survival.

[4]Given the stability of the utility function, it will always produce a trajectory that is optimal in a forward-looking framework. Once having survived to a particular age, however, the individual is likely to regret consumption expenditures made in the past, for we attribute no utility to memory or retrospection.

1. *Robinson Crusoe Case.* In this case, the individual is self-reliant. He cannot borrow against future earnings, and no actuarial markets for the purchase of annuities are available. Therefore, he faces a solvency (no debt) constraint on wealth, $w(t)$,

$$w(t) > 0 \qquad \text{for all } t,^5 \tag{3}$$

and an initial condition

$$w(0) = w_0. \tag{4}$$

In general, we assume that when a person is first able to allocate consumption over his lifetime, he has no accumulated wealth and set

$$w_0 = 0. \tag{5}$$

Wealth is related to consumption by the differential equation

$$\dot{w}(t) = rw(t) + m(t) - c(t), \tag{6}$$

where $m(t)$ is the rate of earnings at time t. In this model, the level of earnings depends only on whether the individual is alive at time t. Equation (6) states that the rate of change in wealth is equal to the interest at rate r earned on a risk-free investment (here the rate assumed to equal the discount rate for future utility[6]) plus earnings, less consumption. We begin from a perspective of time zero, so our problem is to maximize $y(0)$ defined by (1), subject to (2) through (6).

Solution. The problem is solved by the calculus of variations. The control variable is $c(t)$, the consumption rate, and the state variable is $w(t)$, wealth. If the utility function on consumption is strongly risk-averse near zero consumption, as will be the case in our analyses, then the consumer's preferences will keep c positive and (2) becomes redundant. We are left considering two cases: those intervals in which the debt constraint (3) is binding (i.e., an exact equality holds) and those in which it is not (a strict inequality applies). The dividing point between these cases is the time t_0 at which solvency ceases to bind, i.e.,

$$t_0 = \min_{(0,T)} \{ t \text{ such that } w(t) > 0 \}. \tag{7}$$

In this analysis we assume, as will be the case with a single-peaked earnings function, that initially (3) is binding but it ceases to be binding at t_0 for the rest of the lifetime. This behavior will prevail whenever period earnings early in life are lower than ideal period consumption based on lifetime income from that point forward. The model could be extended, through the use of inequality constraints, to have multiple transitions between the two cases.

The first case, where (3) is binding, applies for τ between 0 and t_0. To maximize (1), we raise consumption to the limit allowed by the solvency constraint, so that $w(\tau)$ is identically zero over the interval 0 to t_0. In this interval, $w(\tau) = 0$ and $\dot{w}(\tau) = 0$. Substituting into (6) gives

$$c(\tau) = m(\tau) \qquad \text{when (3) is binding.} \tag{8}$$

The second case, where the solvency constraint (3) is not binding, holds for $\tau > t_0$. Solving it is equivalent to maximizing $y(t_0)$ in (1) subject to (6). To proceed, we form the Hamiltonian,

$$H(c, w, \lambda, \tau) = e^{-r(\tau - t_0)} l(\tau) u(c(\tau)) + \lambda(\tau)[rw(\tau) + m(\tau) - c(\tau)]. \tag{9}$$

Using the Euler–Lagrange equations, with the control variable c and the state variable w, we find that $\lambda(\tau)$, the shadow price of wealth at time τ, must satisfy

$$\lambda(\tau) = \lambda(t_0) e^{-r(\tau - t_0)}. \tag{10}$$

In other words, $\lambda(\tau)$ is the marginal value (utility) to the individual at time t_0 of wealth at time τ. Equation (10) indicates that this value is highest at time t_0 and declines with time at the discount rate. Thus the marginal value of a unit of wealth at time t is proportional to its present value at time t.

Most important, the Euler–Lagrange equations also imply that for the nonbinding cases the optimal function $c(\tau)$, denoted by $c^*(\tau)$, must satisfy[7]

$$u'[c^*(\tau)] = \lambda(t_0)/l(\tau). \tag{11}$$

[5]Our analysis is restricted to a financially independent adult. For persons financially supporting or supported by another, the joint survival and utility patterns must be specified.

[6]The assumed equality of the interest and discount rates yields the very reasonable result that the rate of consumption would be constant over time if an individual had positive wealth and faced no risk of death.

[7]Interestingly, this result is independent of the discount rate, r, because the declining present cost of future consumption is offset by its declining contribution to present utility.

Equation (11) shows that the marginal utility of consumption, under the optimal pattern, is inversely proportional to the probability of being alive. As a result, once the individual has positive wealth, his consumption must decline monotonically over time. In the Robinson Crusoe case an individual does not know when he will die, and there is no insurance. Allocating consumption to a later as opposed to an earlier period involves a greater risk of not enjoying it. Therefore, in this case an individual consumes more earlier.

Multiplying both sides of (11) by $l(\tau)$ gives another important result:

$$u'[c^*(\tau)]l(t) = \lambda(t_0). \tag{12}$$

The *expected* marginal utility of consumption (the marginal utility times the probability of being alive) is constant for all ages at which the subject is not on the brink of insolvency.

To solve (11) for the actual level of consumption, we must determine the constant $\lambda(t_0)$. First, we use the initial and solvency constraints on w, (3) through (5), and solve the ordinary linear nonhomogeneous differential equation (6) for wealth at time t, $w(t)$, obtaining:

$$w(t) = e^{rt}\int_{t_0}^{t} e^{-r\tau}[m(\tau) - c^*(\tau)]\,d\tau. \tag{13}$$

Note that $w(t)$ is the future value (at time t) of the difference between earnings and consumption (under the optimal path) from time t_0 (defined by (7)) to time t. Since it cannot be optimal to have wealth remaining at the maximum possible age, T (which may be infinite), we must have

$$w(T) = 0, \quad \text{so} \tag{14}$$

$$\int_{t_0}^{T} e^{r(T-\tau)}[m(\tau) - c^*(\tau)]\,d\tau = 0. \tag{15}$$

The optimal consumption path is the one given by (11) with the value of $\lambda(t_0)$ that satisfies (15). We will present solutions and numerical results for observed values of $l(\tau)$ and $m(\tau)$ and a particular functional form of $u(c)$, in subsequent sections.

2. *Perfect Markets Case.* Insurance annuities offer protection against outliving one's wealth. In return for payments according to some specified schedule during certain years, the insurer promises to pay some stated income beginning at a specified age and continuing indefinitely. Payments and receipts are both conditional on the insured's being alive at the specified age. In the present analysis we assume that annuities are actuarially fair; i.e., for any contract the insurer's expected receipts equal his expected disbursements. Annuities increase the range of consumption allocations available to a consumer and thereby increase his expected utility of living to any age, and of his remaining life beyond any age. With perfect markets, individuals can borrow money. Thus, there will be no need to constrain consumption in early low-earning years as we did in the Robinson Crusoe model.

We shall first define the perfect markets case formally, and then solve to determine the optimal pattern of consumption. The formal model has the same objective, (1), and the same constraints (2) through (5), as the Robinson Crusoe case. The wealth constraint, (6), however, is replaced by

$$\dot{w}(t) = rw(t) + m(t) - c(t) + f(t), \tag{16}$$

where $f(t)$ is the net amount received by the annuitant at age t. (Thus $f(t)$ is negative for net payments by the annuitant (premiums), positive for net receipts, and zero for ages at which the annuity is inactive.) The assumption of actuarial fairness implies

$$\int_{0}^{T} e^{-r\tau} l(\tau) f(\tau)\,d\tau = 0. \tag{17}$$

Since the type of annuity postulated does not require prepayment, it includes borrowing with a life-insured loan; i.e., borrowing against human capital during years with low earnings and no wealth.

Optimal Consumption Pattern. With annuities, it can be shown that the optimal consumption stream is a constant rate of consumption regardless of age, which we will call $\bar{c}$. To derive this result, we maximize $y(0)$ in (1), subject to (2), (3), and (16). We form the Hamiltonian as in (9), but with the right side of (16) substituted for the term in square brackets, and use the Euler–Lagrange equations.

To determine the actual consumption level, note that it cannot be optimal to hold positive wealth at any age, because the chance of death means that wealth would become worthless. It is always better to invest wealth in an annuity, which provides a higher consumption rate in the case of survival. Under perfect markets (actuarially fair annuities and enforceable contracts are available and create no disincentive on work), an individual should exchange his lifetime wealth for a level lifetime annuity. Hence the solvency constraint (3) is binding at every age t, so $w(t) = \dot{w}(t) = 0$ for all t. Solving (16) for $f(t)$ gives

$$f(t) = \bar{c} - m(t). \tag{18}$$

Thus the level of the annuity is the deficit in earnings below the constant level of consumption.

To find the consumption level that the earnings will support on an expected value basis ($\bar{c}$), we develop some notation. We define $E(t)$ as discounted life expectancy (expected remaining years of life) at age t (conditional on survival at age t). Thus

$$E(t) = \frac{1}{l(t)} \int_t^T e^{-r(\tau - t)} l(\tau)\, d\tau. \tag{19}$$

Analogously, we define $N(t)$ as discounted expected earnings following age t, discounted to age t and conditional on survival to age t:

$$N(t) = \frac{1}{l(t)} \int_t^T e^{-r(\tau - t)} l(\tau) m(\tau)\, d\tau. \tag{20}$$

Then, using (17) and (18), we find the optimal consumption level is the constant

$$\bar{c} = N(0)/E(0), \tag{21}$$

which says that the maximum attainable level of consumption is the ratio of discounted lifetime earnings to discounted life expectancy.

Utility of Reductions in the Probability of Death

Value of Remaining Life, $v(t)$. How much utility does a person at age 40 expect to get out of his remaining life? If that person is still alive at age 70, how much does he value his life then? The value of remaining life at age t is needed to assess the utility of reductions in the probability of death at age t. We will show that this value is the same as the utility at age t of remaining life beyond age t conditional on survival at age t for an individual following the probabilistic consumption pattern. Furthermore, we shall show that the value function $v(t)$ based on the optimal consumption pattern behaves like a utility function for small changes in survival probabilities from $l(t)$.

To state these results more precisely, we introduce some additional terminology. To formalize the notion of the utility of remaining life, we recall that $y(t)$ for $t \neq 0$ in (1) is the utility at age t of all expected years of life beyond age t. Because this formulation of expected utility is not conditional on survival at age t, it includes the troublesome possibility that the subject may already have died at age t. For most purposes the more relevant specification is the utility of life following age t, conditional on survival at age t. We define this value, $v(t)$, by dividing $y(t)$ by the probability of survival to age t:

$$v(t) = y(t)/l(t). \tag{22}$$

To calculate the utility of a reduction in the probability of death, we first must introduce the concept of the force of mortality for measuring the risk of death and the meaning of a reduction in it. The force of mortality at any age τ is given by

$$\mu(\tau) = -\frac{d}{d\tau} \log_e l(\tau).$$

It is the probability per unit time of dying immediately after time τ, given that the individual is alive at time τ. Suppose there is a small reduction in the probability of death in the year starting at some age t; i.e., $dq(t) < 0$. It lowers $\mu(\tau)$ by a small amount over an interval beginning at $\tau = t$, but leaves $\mu(\tau)$ unchanged for other values of τ. The perturbation in the probability of death of $q(\tau)$ is equal to the perturbation in the force of mortality, $\mu(\tau)$, times the duration of the perturbation in $\mu(\tau)$. The perturbation changes the survival function from $l(\tau)$ to $l^{\Delta}(\tau)$ where

$$l^{\Delta}(\tau) = \begin{cases} l(\tau) & \text{for} \quad \tau < t, \quad (23) \\ e^{dq(t)} l(\tau) & \text{for} \quad \tau > t. \quad (24) \end{cases}$$

For small $dq(t)$, (24) is approximately

$$[1 + dq(t)] l(\tau). \tag{25}$$

We define $R(t)$ as the marginal utility at time t, conditional on survival at that time, per unit *reduction* in the force of mortality at time t. In other words, for a perturbation $dq(t)$, the change in utility is $R(t) dq(t)$.

1. *Robinson Crusoe Case*. It can be shown that the marginal utility at time t of an immediate decrease dq in the probability of death under the Robinson Crusoe case is equal to dq times the utility of remaining life at t. In other words, in the Robinson Crusoe case at every age t,

$$R(t) = v(t). \tag{26}$$

This result is stated formally and proved in Shepard and Zeckhauser (1982).

The critical element in $v(t)$ is the age (t) at which mortality is changed. For an infinitesimal change, the age at which one learns of this change is not important because marginal readjustments in the consumption rate have only second-order effects, i.e., they go to zero with the square of $dq(t)$, a very small number. For small changes in mortality, $v(t)$ defined by (22) gives the value to the consumer at time t of these changes. Intuitively, this proposition states that the utility that an individual alive at age τ obtains from a reduction in his probability of death at that age is his expected utility after age τ, under the optimal consumption stream $c^*(\tau)$. That is, to evaluate the change in utility associated with a perturbation in an individual's survival function, we do not need to go through all the calculations required to maximize $y(0)$ defined by (1). Instead, we simply multiply the perturbation in survival (the change in force of mortality times the duration of the change) by the utility of remaining life.

2. *Perfect Markets Case.* To find the lifetime expected utility in the perfect markets case, we substitute (19) into (1), getting

$$y(0) = u(c^*) \cdot E(0), \tag{27}$$

where the optimal consumption level $\bar{c}$ is equal to c^*. Shepard and Zeckhauser (1982) show that $R(t)$, the marginal utility of survival probability at age t, is the sum of a financial surplus term and a direct utility-gain effect. Specifically,

$$R(t)_{(\text{perfect markets})} = u'(\bar{c})[N(t) - \bar{c}E(t)] + u(\bar{c})E(t). \tag{28}$$

The first term in (28) measures the financial effect of a reduction in the probability of death: the marginal utility of consumption times expected earnings net of consumption following age t, given survival at that age. The second term measures the pleasure of additional life with quality held fixed.

To rewrite this expression more simply, we define the amount of consumption equivalent to the consumer's surplus from being alive in any year with this consumption level as

$$\bar{c}_s = u(\bar{c})/u'(\bar{c}) - \bar{c}. \tag{29}$$

An equivalent expression to (28) is

$$R(t)_{(\text{perfect markets})} = u'(\bar{c})[N(t) + \bar{c}_s E(t)], \tag{30}$$

which states that the marginal value of saving a life at age t is equal to the product of the marginal utility of consumption times the sum of discounted expected earnings after age t conditional on survival at that age, plus the product of the marginal utility of consumption times the surplus consumption in each year times discounted remaining life expectancy. The term $\bar{c}$ enters in (29) because of the requirement that annuities be actuarially fair. An extension of life must be accompanied by a reduction in the rate of consumption, so the utility of an extension of life with a wealth constraint is the consumer surplus in extra years.

Implicit Willingness to Pay

Willingness to pay (WTP) measures a person's willingness to sacrifice one desired attribute, wealth, for future consumption, in order to obtain another desired attribute, improved survival. It is a theoretically pure, although practically difficult, measure for establishing the consumer demand for improved survival. A major advantage of our model is that it yields estimates of WTP for marginal changes in the probability of death. Furthermore, these estimates can be derived from period and intertemporal preferences on consumption. If these preferences are consistent with the utility functions in our model, then the unreliable, and somewhat anxious, process of trying to assess WTP directly can be avoided.

To calculate WTP, we let dq denote a marginal change in the probability of death over the interval $d\tau$, and dw denote the marginal change in wealth (or dc the marginal change in the rate of consumption) that the person will just accept as compensation to leave his overall conditional utility at age t constant. This WTP applies to an individual alive at age t who offers an immediate payment of dw to reduce his instantaneous risk of death by dq over an interval of short duration. That is, dw or dc is the WTP for a reduction dq in the probability of death.

We first consider the case where solvency is nonbinding and wealth is positive. Then WTP is expressed in terms of dw rather than dc, and[8]

$$\text{WTP} = dw/dq. \tag{31}$$

WTP is determined by the indifference relation that the total differential of $v(t)$ is zero. Thus

$$dv = \frac{\partial v}{\partial q}dq + \frac{\partial v}{\partial w}dw = 0. \tag{32}$$

Willingness-to-pay per year of life gained is an important related concept. It is obtained by dividing WTP at any age, t, by discounted remaining life expectancy at that age, $E(t)$. Willingness-to-pay per year relates the findings of this paper to numerous cost-effectiveness studies of health practices (Zeckhauser and Shepard 1976; Weinstein and Stason 1976; and Berwick, Cretin, and Keeler 1976). Often, these studies summarize the results of a health practice in terms of dollar cost per (discounted) quality-adjusted life year (QALY)

[8]For large purchases of survival, if the solvency constant is not binding, the individual will be willing to pay less out of his present consumption than out of his wealth, for the latter offers opportunities for reallocating the reduction across future time periods.

gained (Zeckhauser and Shepard 1976). Such cost-effectiveness analyses allow decision makers to determine whether one treatment is a more efficient use of medical resources (more cost-effective) than another.

The calculation of willingness-to-pay per year permits another important determination traditionally beyond the scope of cost-effectiveness analysis: Is a given practice better than nothing? If the cost per QALY for that practice is less than the WTP per year, then the practice is worthwhile. For example, the cost per QALY of treating moderate hypertension in a 50-year old male is about $10,000 (Weinstein and Stason 1976). If, for example, WTP per year were $20,000, then this treatment would be worthwhile. For any utility function and market case (Robinson Crusoe or perfect markets), WTP per year can be calculated numerically. In addition, a useful analytic result is derived below for the special case of utility functions on consumption exhibiting constant proportional risk aversion.

1. *Robinson Crusoe Case.* To derive WTP in the Robinson Crusoe Case, we use (26) and a series of substitutions—see Shepard and Zeckhauser (1982)—to show that WTP at age t is

$$\mathrm{WTP} = v(t)/u'(c^*(t)). \tag{33}$$

This important result says that willingness to pay is proportional to $v(t)$, the expected utility of remaining life at age t conditional on survival at that age, and inversely proportional to the marginal utility of consumption at age t.

2. *Perfect Markets Case.* To evaluate WTP with perfect markets, we note that the solvency constraint on wealth is always binding in the sense that one always expends his entire wealth on actuarially fair annuities. Thus the compensating variations that define WTP are given by (32) and WTP itself is given by (33). Substituting (30) into (33) gives

$$\mathrm{WTP} = N(t) + \bar{c}_s E(t). \tag{34}$$

The first term in (34) is discounted expected additional earnings conditional on survival at age t, the human capital or livelihood at age t. The second term is surplus consumption times discounted life expectancy. Thus livelihood provides a lower bound on willingness to pay. There is also a consumer surplus from being alive, which is valued as well.

Comparing the Robinson Crusoe and perfect markets cases, we see that annuities increase the utility both of remaining life and of wealth, which might be offered to purchase such increases. Annuities therefore have an indeterminate effect on willingness to pay for reductions in the chance of death.

3. Results for Constant Proportional Risk Aversion

To obtain some numerical results, we now solve explicitly for an interesting special class of period utility functions on consumption, namely constant proportional risk aversion (CPRA) with constant m.[9] We further assume that the utility functions are time-invariant and scaled such that $u(0) = 0$, which is identical to the utility of death. Under these conditions, an individual's period utility function on consumption, $u(c)$, must satisfy

$$u(c) = Hc^{1-m}, \tag{35}$$

where H is an arbitrary scaling factor of the same sign as the exponent of c.[10] The greater the parameter m, the greater the risk aversion on consumption. This function exhibits constant elasticity of period utility with respect to consumption. The elasticity is $1 - m$, regardless of the level of consumption. This same functional form was used by Arthur (1981).

This function is more general than it first appears under either of two alternative interpretations. Under the first interpretation, the utility function in (35) may be a good approximation for all levels of consumption that actually arise, even though it is not a universal function.

In this interpretation, we consider c to be the absolute level of consumption per period. As c approaches zero, consumption approaches zero. Since part of consumption is for necessities—food, clothing, and shelter—a level of zero consumption is not possible. Values of c that are possible must all exceed some positive threshold. It seems reasonable that the period utility function for consumption levels above this threshold could be approximated by a function in the family $u(c) = c^{1-m}$ for some value of m. Although the approximation may not apply to consumption levels below this threshold, such values would not arise in practice. The approximation works where it is needed.

Under a second interpretation, c represents the excess of consumption above a "minimum amenities level." A year at this minimum amenities level is as bad as a year in which one is not alive at all. Similarly,

[9] CPRA means that the level of consumption (c) times the local risk aversion at that level is the constant m (Keeney and Raiffa 1976).

[10] This function in (35) could be generalized without losing CPRA by adding a constant FH. The constant FH is the pure utility in being alive, regardless of consumption. Thus, if an individual were alive with rate of consumption 0, he would risk odds of 1 to F of being dead in order to raise his period consumption rate to unity if he should live. CPRA is still preserved if the scaling factor H depends on age.

what we term "earnings" in the model represents a flow of purchasing power beyond that required to meet the minimum amenities level in that year. Governmental and private welfare services, not counted elsewhere in this model, tend to provide such a floor on consumption.

If this second interpretation is chosen, then "wealth" represents money wealth above that required for the minimal level. These minimum amenity amounts would have to be subtracted from actual earnings to compute the net earnings function $m(t)$ required in our model. In a numerical calculation, a minimum amenities level needs to be chosen.

Sensible results require the composite range restriction $0 < m < 1$, which applies to the balance of this paper. The first reason for this restriction is that the assumption that the individual is risk-averse on consumption levels requires that $m > 0$.[11] Second, the condition that any level of consumption is preferred to death requires that $m < 1$. The composite restriction means that survival without consumption is no better than death, and utility is finite for all finite levels of consumption. Since the scaling of utility units is arbitrary, without loss of generality, we set $H = 1$ for simplicity. To find optimal consumption paths, we differentiate (35) with respect to c, obtaining

$$u'(c) = (1 - m)c^{-m}. \tag{36}$$

1. *Robinson Crusoe Case*

Optimal Consumption Path. To find $c^*(t)$ in the Robinson Crusoe case where the solvency constraint (3) is not binding, we substitute (36) into (11), obtaining[12]

$$c^*(t) = K[l(t)]^{1/m}. \tag{37}$$

Here K is a constant which spreads earnings over one's lifetime. If we have already found $\lambda(t_0)$ to satisfy the endpoint conditions on $w(t)$, then we can find K by

$$K^m = \frac{(1 - m)}{\lambda(t_0)}, \tag{38}$$

where t_0 is defined by (7). If $\lambda(t_0)$ is not known, we evaluate K directly from the endpoint conditions on $w(t)$, setting $w(t_0) = w(T) = 0$ and $w(t) > 0$ for all t. (See Appendix in Shepard and Zeckhauser 1982.) Thus, as long as the solvency constraint is not binding, optimal consumption is proportional to a power ($1/m > 1$) of survival.

Relations between Discounted Consumption, Utility, and Willingness to Pay. We shall establish some operational relationships between utility, willingness to pay, and discounted consumption. First we define $J(t)$ as the present value at time t of consumption from time t onward under the optimal path, discounted to age t:

$$J(t) = \int_t^T e^{-r(\tau - t)} c^*(\tau)\, d\tau. \tag{39}$$

By (13), $J(t)$ is also equal to a person's total capital at time t: the sum of his monetary wealth at time t, $w(t)$, plus his potential livelihood, the present value of all his future earnings after age t without regard to future risks of death. We define $G(t)$ to be the present value of this consumption, conditional on survival at age t:

$$G(t) = J(t)/l(t). \tag{40}$$

Three propositions follow for the time $t > t_0$ at which the individual is following his optimal consumption path, and has passed age t_0, at which the solvency constraint relaxes. The propositions are proved in Shepard and Zeckhauser (1982). Proposition 1 says that the utility of saving a life at age t (conditional on being alive at age t) is proportional to discounted consumption divided by the probability of survival to age t.

PROPOSITION 1. *Let t_0 be the minimum age at which the solvency constraint* (3) *is not binding, as defined in* (7). *If $u(c)$ is given by* (35), *exhibiting CPRA, and $t > t_0$, then utility at age t conditional on being alive at age t, $v(t)$, satisfies*

$$v(t) = K^{-m}G(t). \tag{41}$$

Utility is proportional to discounted future consumption of a person now alive.

Proposition 2 says that marginal WTP is equal to discounted consumption times the factor $1/(1 - m)$. The factor is the reciprocal of the exponent in the period utility function.

PROPOSITION 2. *If $u(c)$ is given by* (35), *then marginal willingness to pay* (*per unit reduction in the*

[11] If the individual is not risk-averse, the optimal solution degenerates to an infinite consumption rate in the first instant of life.

[12] Even if T is infinite, the solution exists provided life expectancy is finite.

probability of death) for $t > t_0$ is

$$\text{WTP} = \frac{1}{1-m} J(t). \tag{42}$$

This proposition says that WTP is a multiple (greater than one) of discounted future consumption. It is also a multiple of discounted total capital. The more inelastic is the utility function of consumption, the greater is WTP.

Since actual livelihood is less than potential livelihood (the present value of future earnings without regard to risks of death), potential livelihood is less than total capital, and the factor $1/(1-m)$ exceeds one, then we have for $t > t_0$,

$$\text{WTP} > N(t). \tag{43}$$

2. *Perfect Markets Case*

In the perfect markets case, $R(t)$ is given by (30) and WTP is given by (34). Substituting the solutions of the CPRA utility function from (35) and (36) into (29) shows that in CPRA $\bar{c}_s$ is

$$\bar{c}_s = \frac{m}{1-m} \bar{c}. \tag{44}$$

The derivative $u'(\bar{c})$ is given by (36), and the constant $\bar{c}$ is defined by (21). Substituting (44) into (34) gives an alternative expression for WTP,

$$\text{WTP} = N(t) + \left(\frac{m}{1-m} \right)(\bar{c} \cdot E(t)). \tag{45}$$

The factor in brackets in (45) is discounted future consumption for a person alive at age t. Thus, in the perfect markets case, WTP is the sum of livelihood plus a multiple of discounted future consumption. The multiple increases with the degree of risk aversion. In the extreme case where there is no risk aversion, the period utility function is linear in consumption, $m = 0$; hence WTP is equal to livelihood. For positive risk aversion, WTP exceeds livelihood.

We mentioned earlier that WTP per year is a useful criterion for making medical decisions. In the case of CPRA, WTP per year at time 0 has a particularly simple expression. We substitute (21) into (45), obtaining

$$\text{WTP}/E(t)_{(\text{age } 0)} = \left(\frac{1}{1-m} \right) \bar{c}. \tag{46}$$

Thus, WTP per year is a multiple (greater than one) of the constant level of consumption. The more inelastic is the period utility function on consumption, $u(c)$ (i.e., the smaller is $1 - m$), the greater is the multiple $1/(1-m)$.

4. Application: Calculation for Males Age 20 Onward

To illustrate these formulas, we provide numerical calculations for a representative financially self-sufficient individual—defined here as a 20-year old male. We assume that consumption in retirement is supplied only by savings accumulated during years of earnings. Some factors are omitted in this assumption, but may, as an approximation, be treated as cancelling. For example, our earnings measure counts only money earnings, excluding employer-provided fringe benefits (insurance and pension contributions), transfer payments (Social Security benefits), and the value of home production; but work-related expenses (commuting and meals away from home) and taxes (such as income and Social Security taxes) are also excluded.

Assumptions and Data

Calculations require specification of $l(t)$, $m(t)$, r, and m. For $l(t)$, we rescaled a U.S. male life table (National Center for Health Statistics, 1975, Table 5-1) so that for age 20, $l(0) = 1$. The median survival is an additional 54 years (to age 74), so $l(54) = 0.5$. Extrapolating the curve linearly from age 65 to 85, we assumed the probability of survival at age 90 (70 years beyond the initial age) is negligible.

For earnings we used an average profile from a sample of Social Security enrollees and assumed that earnings cease at the system's normal retirement age of 65 (U.S. Senate, 1976, p. 54). In this profile, past earnings were adjusted to constant dollars by an index of wage rates to avoid confounding age effects with general increases in wage levels. Finally, we expressed all earnings relative to earnings during the peak year, which occurred at age 50. The earnings profile is shown in Figure 1.

Because earnings are expressed in constant dollars with the effects of wage increases due to inflation and productivity gains removed, the interest rate r should be a real rate. For this analysis we set $r = 0.05$. As assumed previously, the period utility function is c^{1-m}, which implies constant proportional risk aversion. The value of m must lie between 0 and 1. We assume that any positive consumption level is superior to death, and the utility of zero consumption is set equal to the utility of not being alive. The results for CPRA presented earlier show that this assumption leads to powerful results, and is not unreasonable. This paper's calculations interpret consumption as the excess above a minimum amenities level. This is equivalent, in the

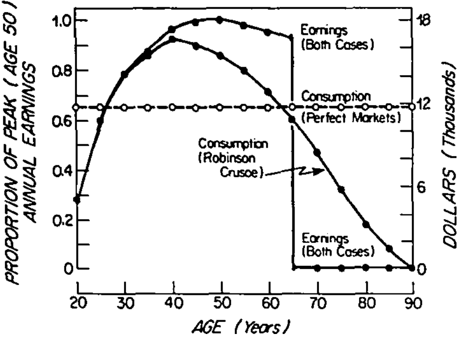

FIGURE 1. Earnings Function, $m(t)$, and Optimal Consumption Functions, $c^*(t)$, in Robinson Crusoe and Perfect Markets Cases for an Average Male at 1978 Wages.

alternative interpretation of constant proportional risk aversion, to setting the minimum amenities level equal to zero.

We set the risk aversion parameter $m = 0.8$, so that utility is proportional to consumption to the 0.2 power. That is, the elasticity of utility with respect to consumption is 0.2. With this value, a consumer faced with a lottery giving equal chances of consumption rates of $10,000 and $20,000 per year would have a certainty equivalent of about $14,300, reflecting a risk premium of $700, or 4.7 percent of mean consumption.

Together, the scaling of utility and the choice of m imply that the level of consumption is relatively unimportant compared with survival itself. To avoid a 1 percent chance of death now, the representative consumer would be willing to cut his consumption over the entire remainder of his life by 5 percent. Note that the units for consumption are the same as those of earnings—peak-year earnings. As we shall see, this choice of m yields relations between national wealth and income that generally agree with observed data, and time-dependent patterns of utility that are roughly similar to survey data.

Analysis of Robinson Crusoe Case

The optimal consumption function is shown by the consumption curve in Figure 1. Notice that it is identical to the earnings curve from age 20 to 35, because increasing earnings make savings not worthwhile. Beyond age 35 savings begin to accumulate as consumption drops below earnings. Reflecting these relationships, the wealth function is identically zero up to age $t_0 = 35$. Then it gradually increases, reaching a maximum of 5.7 times peak annual earnings at age 65 when earnings cease. Thereafter wealth declines as it is depleted by consumption, reaching zero at the terminal age (age 90). Thus, the wealth curve looks like a symmetrical mountain with its peak at the retirement age, age 65.

Figure 2 shows $R(t)$, the utility of reducing the probability of death, for the Robinson Crusoe and perfect markets cases. In a sense, in the Robinson Crusoe $R(t)$ measures the utility of being alive, i.e., the utility of remaining life, $v(t)$. Despite the substantial differences in consumption, $R(t)$ is quite similar in the two cases. Recall that $R(t)$ is the sum of two terms—the utility of additional livelihood (future earnings) and the utility

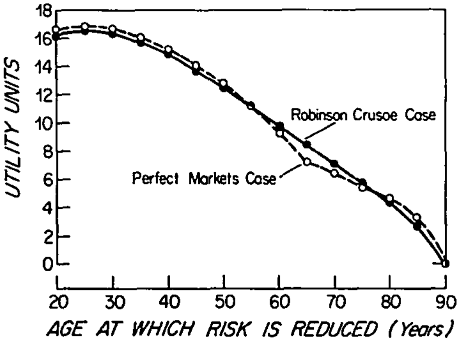

FIGURE 2. Lifetime Utility per Unit Reduction in the Risk of Death, $R(t)$, at Various Ages for an Average Male.

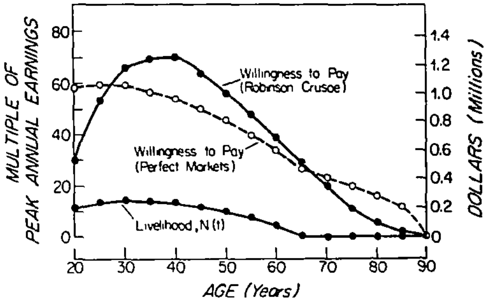

FIGURE 3. Livelihood and Willingness to Pay per Unit Reduction in the Risk of Death as Functions of Age for an Average Male.

of future consumption arising from a decrease in the risk of death. The sharp bend in the utility curve at age 65 is due to the discounting in earnings. From ages 65 through 75 $R(t)$ is lower in the perfect markets case, despite higher consumption rates, because every improvement in survival means a smaller annual annuity.

The remaining lifetime utility function, $v(t)$, is equal to $R(t)$, the value of saving the life of the representative male at age t, according to (26). By Proposition 1, these functions are also equal to $K^{-m}G(t)$. The peak value of $v(t)$ occurs at age 25. It is about 17 times the utility obtained in one year with consumption equal to peak annual earnings. From ages 20 through 50, the function slopes little. During this interval the loss in remaining life expectancy is of relatively little importance: the lost years are far in the future, and hence heavily discounted. Moreover they are years of relatively low quality anyway because of the low level of anticipated consumption. Remaining lifetime utility begins to fall much more steeply at age 50 because the years of life lost come much sooner, and the quality of the more immediate years declines noticeably with age because of lower consumption rates.[13] The utility of remaining life to a 35-year old man is about twice that of a 65-year old.

It is interesting to contrast the utility, $v(t)$, with discounted remaining life expectancy, $E(t)$, a function which treats all years of survival equally, regardless of level of consumption. For our representative male, beyond age 35, both the $v(t)$ and the $E(t)$ curves are roughly approximated by straight lines from their actual values at age 35 to zero at age 95. These straight lines would fit perfectly if life were uniform in quality and free of risk from age 35 to 95, and if death were certain at 95. The departures from linearity are in the direction of upward concavity for utility, because consumption declines with age. They are in the direction of downward concavity for life expectancy, because the possible years of life after age 95 become more important as one's age nears 95.

Figure 3 shows that perfect markets dampen the age variations in WTP. At young (< 30) and older (> 65) ages, WTP is low in the Robinson Crusoe cases because wealth and income are small or zero. Between ages 30 and 65, however, wealth is ample and cannot be shifted efficiently to consumption at other ages in the Robinson Crusoe case. Thus, WTP is relatively high, reaching a peak of $1,250,000 at age 40. Livelihood, or discounted expected future earnings, is one component of WTP in the perfect markets case. Its maximum, reached at age 30, however, is only $260,000. Total WTP is at least four times as great as livelihood alone, reflecting the importance of the other component, consumer surplus. The left scale shows WTP measured in multiples of peak (age 45–54) annual earnings. The right scale shows dollar amounts.

TABLE 2

Valuations of Life at Various Ages Derived from Willingness to Pay for Males With 1978 Average Income Profile[a]

Age	Robinson Crusoe Case	Perfect Markets Case
20	$0.50 million	$1.05 million
40	1.25 million	0.97 million
60	0.63 million	0.60 million
80	0.10 million	0.29 million

[a]Average earnings of 45–54 year-old male (excluding persons with no income) was $18,000 per year.

[13]In this analysis, declining utility with age results solely from consumption patterns, making no assumptions about infirmity affecting utility.

To translate these multiples into dollars, we need actual "earnings" for a recent year. In 1978, the mean income of 45-to-54-year old males with money income was $18,874. We assume that the age profile is similar to that for 1975. Assuming that the distribution of their income by source was similar to the aggregate for all families with income, only 4.4 percent or $830 is attributable to interest, dividends, rents, and royalties (U.S. Bureau of the Census, 1980, pp. 462, 457). The remainder, about $18,000, is from earnings (including self employment) and transfers. Thus, to translate the multiples in Figure 2 into 1978 dollars, we multiply by $18,000. This gives a peak value of WTP for the Robinson Crusoe case of about $1.25 million. Other values are shown in the Robinson Crusoe column of Table 2.

Analysis of Perfect Markets Case

If perfect markets exist (annuities are available), the optimal consumption function is a constant. For the survival and earnings functions of the representative male, we find $E(0) = 17.94$, $N(0) = 11.70$, so by equation (21) $\bar{c} = 0.652$, and by equation (29) $\bar{c}_s = 2.608$.[14] This constant consumption level is graphed as the perfect markets consumption line in Figure 1. The "perfect markets" estimates of willingness to pay per unit reduction in the probability of death in Figure 3 were given by (34). Since the rate of consumption is constant at all ages, the marginal utility per unit of reduction is the constant multiple, $u'(\bar{c})$, times WTP (where $u'(\bar{c}) = 0.142$). The lower curve in Figure 3 is the livelihood component of WTP in the perfect markets case. It is the first term in (34), discounted expected future earnings.

Comparing the Robinson Crusoe and perfect markets curves in Figure 3, we see that annuities make WTP flatter as a function of age. Annuities raise WTP considerably before age 25 and after age 65 but depress WTP somewhat within the high-earnings interval. Using the peak earnings rate of $18,000 per year cited previously, WTP is about $1.06 million for males 20 to 30, and declines with age thereafter. Values for selected ages are given in Table 2. These results show that the availability of perfect markets has different effects on WTP, depending upon age. At very young ages, where consumers would otherwise be limited to their current income, and at advanced ages, when self-reliant consumers would have very little assets left, perfect markets increase WTP. For middle ages, WTP is less because peak consumption is lower.

Using (46), we calculated that under the perfect markets case, our representative male's WTP per year at age 20 is $59,000. Here the multiplicative factor, $1/(1-m)$, is 5, and the constant level of consumption is $11,700 ($0.652 \times$ $18,000). For all feasible CPRA utility functions in our model, WTP per year must be at least $11,700 because the lowest possible value of the multiplicative factor is one. The $59,000 estimate of WTP generated by our model, as well as the lower bound of $11,700, are both higher then the cost per QALY generated by many preventive health measures. For example, we estimated that the cost per QALY of a low cholesterol diet to reduce heart attack deaths was $6,000; the cost per QALY of air bags to reduce motor vehicle deaths was $5,800 (both using 1975 prices) (Zeckhauser and Shepard 1976). Thus, for the preferences and earnings of our representative male, these are both socially desirable programs.

On the average, earnings of women are only 43 percent of those of men (U.S. Bureau of the Census, 1980, p. 462). Although the earnings pattern of women has not been examined in detail in these calculations, it is likely that WTP for females would be slightly less than half of that for males. Thus, peak WTP for females in 1978 prices would be about $500,000.

Validation of Models

We have focused on two polar models, perfect markets and no markets. The real world lies somewhere in between, which suggests that in a rough and ready way, if our models explain behavior for their idealized worlds, behavior in the real world should follow some in-between pattern. Thus, for example, we should observe life-cycle savings patterns, but they should be somewhat less pronounced than the Robinson Crusoe model would suggest.

With departures that do not diminish the validity of the model for our purposes, observed patterns of savings are generally consistent with the life cycle-model. Individuals obviously vary in their survival probabilities and utility structures, and the relationships of our model are certainly nonlinear. Therefore, extrapolations from our calculations for a representative individual should not necessarily match population data. Nevertheless, several comparisons indicate that our model is at least plausible.

Ratio of Wealth to Income. Our first set of testable relationships from our model concerns the ratio of wealth to income. We weighted the age-specific earnings, $m(t)$, and wealth, $w(t)$, by the 1970 age distribution for males in the United States aged 20 and over to give an average earnings rate of 0.68 peak earnings and an average wealth holding of 1.00 peak earnings (the equivalence is coincidental). Thus the ratio of wealth to earnings from our model is 1.47. We compared this ratio with estimates of the actual ratio of personal welath to noninvestment personal income (all personal income except dividends, interest, and rental income) for a "typical" individual. We used two approaches to construct data for this typical individual from aggregate United States macroeconomic data.

As our first approach, we excluded the upper decile of the wealth and income distributions and computed

[14]Shepard and Zeckhauser (1982) miscomputed $\bar{c}_s$, hence WTP, in this case.

arithmetic averages for the remaining population. Using 1972 data for the adult population (males and females), the resulting estimate was 1.97, roughly consistent with our model. As our second approach, we used median wealth, median total income, and the ratio of aggregate total income to noninvestment income. Again using 1972 data, we obtained a ratio of 1.49, quite close to the 1.47 ratio in our model.[15]

Distribution of Wealth by Age. Our second set of relationships concerns the pattern of individual wealth by age. For the Robinson Crusoe case, our model predicts that wealth is zero up to age 35, increases steadily to 5.7 times peak annual earnings at age 65, and declines again thereafter. In the perfect markets case, wealth is identically zero at all ages, as insurance annuities replace individual savings.

Using 1972 U.S. data, we calculated the actual ratios of median household net worth (by age of head of household) to peak noninvestment household income (i.e., income for households headed by a person aged 45–54). Rounded to the nearest decade, these ratios were: age of head < 30, 0.1; age 40, 0.4; age 50, 0.8; age 60, 1.0; age > 70, 0.8.[16]

The inverted-U pattern is qualitatively similar to that predicted for Robinson Crusoe, but the absolute values are smaller—more like those predicted for perfect markets. The low level of wealth is due to the multiple means of support other than return on wealth for retirement income—annuities, pension, Social Security benefits, and support from other household and family members.

Willingness to Pay. Our third test of our model was to compare its complications about willingness to pay with the results of pilot surveys cited earlier. Only the study of Fischer and Vaupel (1976) permits an assessment of the effects of age. In that study subjects were asked to assign utilities to lifespans with specified rates of consumption and specified ages of death. These utilities assumed that an individual was certain to live to exactly age t and consumption would be at a specified constant level c. Assuming the period utility function $u(c)$ is a time-invariant function, lifetime utility $y(0)$ becomes

$$y(0) = \int e^{-rt}u(c)\,dt = 1/r[1 - e^{-rt}]u(c).$$

This function depends on both the specified rate of consumption, c, and the age of death, t. To compare this function with those assessed by Fischer and Vaupel, we rescaled it by a positive linear transformation so as to make the utility zero for the lowest combination of c and t ($4,000 and 30 years, respectively), and the utility unity for the highest combination ($24,000 and 80 years, respectively).[17] Letting $u(c)$ be the function defined by (35) with $m = 0.8$, we determined the lifetime utilities of specified certain lifespans, t, at each of four constant annual levels of consumption, $4,000, $8,000, $12,000, and $24,000.

Holding annual consumption fixed, $y(0)$ is a monotonically increasing convex-upwards function of the specified lifespan, t. The empirical results from Fischer and Vaupel (1976) under these same conditions for their 65 "Type 1" subjects were quite similar in general shape to those from our model. "Type 1" subjects, representing 78 percent of all respondents to their utility survey, were those who rated the shortest lifespan with the lowest annual consumption as the worst of the lifespan scenarios presented. Their responses confirm our model result that utility increases with age at death and consumption level. At a detailed level, it appears that these subjects discounted future utility at a lower rate than the $r = 0.05$ that we used, and were more risk averse on consumption than our parameter of $m = 0.8$ implies. Nevertheless, the model and survey agree on the utilities with the highest consumption level and shortest lifespan (0.2) and with the lowest consumption level and longest lifespan (0.5). It is reassuring that the values of our preference parameters produce willingness-to-pay values that correspond roughly to survey results.

5. Concluding Remarks

Our procedures provide a mechanism for *valuing small changes in risk levels* to individuals of various ages who have particular preferences and earnings opportunities. Under our Robinson Crusoe case, an individual's willingness to pay (WTP) for an increase in survival is a multiple of his discounted future consumption. Under our perfect markets case, WTP is the sum of his consumer surplus from all future consumption plus his net human capital (livelihood) from being alive. The results of our calculations for a representative man are considerably higher than values derived elsewhere based only on discounted earnings (human capital). The maximum implied value of a life is reached at age 40 in the Robinson Crusoe case; it is 70 times peak earnings or $1.25 million as of 1978. For the perfect markets case, a life is valued the highest at age 25, where it is 59 times peak earnings, yielding a 1978 figure of $1.07 million.

[15] Data from U.S. Bureau of the Census, 1973, 1975. For details of calculations, see Shepard and Zeckhauser (1982).

[16] Using data from U.S. Bureau of the Census (1975), the mean noninvestment income for a household headed by a person aged 45–54 was $12,600 in 1972. Other calculations are explained in Shepard and Zeckhauser (1982).

[17] The transformation is to subtract u ($4,000, 30 years) and divide the result by [u ($24,000, 80 years) − u ($4,000, 30 years)].

Our method provides a framework within which the individual can choose between spending on consumption or on survival and can compute the benefits a public program of health or safety offers him. Assuming that the individual's preferences can be calibrated, his implied willingness to pay can also be provided, for example, to courts or policy makers.

Our objective here was to outline an approach to valuing life that recognizes age profiles in earnings and mortality, focuses on available market arrangements, and builds from a traditional utility function for consumption. To introduce additional considerations might add mathematical complexity, but would not require any departure from our basic approach.

A useful first extension would be to include additional sources of income by age—notably fringe benefits, proprietor income, and transfer payments such as Social Security. Such sources, particularly transfer payments, tend to reduce the variability of income with age, weakening that incentive for savings. Thus this income will increase consumption and willingness to pay for survival increments at most ages.

Consideration of individual's preferences and opportunities could be extended to allow for family ties, both emotional and financial, and specifically for bequests. Historically, the importance of bequests has been substantially diminished by the use of life insurance, the advent of pensions and Social Security as important sources of income, and the decline of small-scale enterprises such as the family farm, which often pass at death from one generation to another.

The calculation of specific values for increments in survival will prove controversial. Some will object to the methods because they do not like the policy conclusions they generate for particular areas such as nuclear power regulation or the support of maternal health programs. Others will balk simply because numbers come out too high or too low.

Still others will reject the basic principles on which the calculations rely. Two camps quite antagonistic to each other are likely to voice the common complaint that no valid measure of the worth of survival improvement can be based on individual preferences. The adherents to voluntary contract will be concerned that the numbers generated might be employed to influence some social decision benefiting one party at the expense of another. Certain classes of egalitarians will object that valuations relying on individual preferences favor the rich at the expense of the poor.[18] To the enthusiasts of contract, we would respond that society could not function if all government actions were required to yield Pareto improvements, much less gain explicit agreement. To the "levelers," we would point out that, whatever the merits of an egalitarian society, they do not seem to be recognized by the political process. In any realistic context where the extent of redistribution is limited, requiring that as much be spent to gain an increment in survival for the poor as for the rich is likely to deny the poor more highly valued resources, and thus disadvantage them further.

Valuation differences based on age are likely to prove less controversial, because no distinction is drawn between individuals; rather the same individual is treated differently at different stages in his life. In both the Robinson Crusoe and perfect markets cases, willingness to pay for increased survival follows an inverted-U pattern. At younger ages, it is low because of low earnings and the discounting of middle-aged years of higher earnings. At older ages, willingness to pay is reduced because of short remaining life and, in the Robinson Crusoe case, lower levels of consumption. In both cases, willingness to pay reaches a maximum of about 60 to 70 times peak earnings, at age 25 in the perfect markets case and age 40 in Robinson Crusoe. The peak is earlier with perfect markets because a young adult can borrow against future earnings in purchasing survival. But even in this case, the maximum is only reached after the gap between earnings and consumption narrows, not at the very beginning of the earnings years. Beyond age 65, willingness to pay is also higher in the perfect markets case, because higher consumption increases the utility of advanced ages. Perfect markets allow higher consumption because hoarding to protect against a long life is no longer necessary; moreover there is no chance of wasting money by having it in savings at death.

The objective of this analysis was to demonstrate the feasibility of a methodology and to indicate orders of magnitude, not to generate precise numbers for willingness to pay. If this approach gains acceptance, substantial effort will have to be expended in estimating utility functions. Assessments of data about individual choices, as well as survey work, will be helpful in this task. Empirical research about utilities over lifetimes of different length would also be required to provide ultimate relevance.

In the Robinson Crusoe case, WTP at any age depends on the utility of consumption beyond that age. For the special case of CPRA, WTP is a multiple of total capital (wealth plus potential livelihood), and thus exceeds livelihood. In other words, WTP is the sum of livelihood plus an additional value of being alive.

In the perfect markets case, an individual's calculation of willingness to pay for an increase in survival consists precisely of two terms: his future lifetime utility plus the net contribution survival enables him to

[18] Although we have not dwelt on the fact, our model clearly indicates that willingness to pay rises with income. Indeed, for CPRA utility functions, a doubling of income at all ages would double willingness to pay at every age. For public choices, the technically efficient approach is to accept private valuations. An egalitarian approach would apply the results for a typical income level to all levels of income. Our model can be employed for either formulation, or a mixture between the two.

make his own stock of resources (discounted future earnings less discounted future consumption, the cost of sustaining the individual). In other words, the willingness-to-pay formulation rests on an assessment of the sum of net consumer surplus and net human capital from being alive.[19]

[19]Many results reported here appear in a longer and more technical earlier paper presented at the Conference on Valuation of Life and Safety, sponsored and conducted by the Geneva Association, held in Geneva, Switzerland, March 1981. See Shepard and Zeckhauser (1982) in Jones-Lee (1982).

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