TAXES IN FANTASY, OR MOST ANY TAX ON LABOR CAN TURN OUT TO HELP THE LABORERS

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with an EPILOGUE by

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This fable is designed to demonstrate that merely knowing the structure of two taxes that raise equal amounts of revenue will rarely be sufficient to determine which tax will prove preferable for a specific reference group. Most particularly, it shows that any of the traditional forms of taxation on labor may prove beneficial to the laborers themselves.

The taxes considered are lump sum levies collected as money or labor, and proportional and nonproportional taxes on wage earnings. The analysis takes the form of a series of examples yielding counter-intuitive results. An epilogue discusses simulations and unstable equilibria.

1. Introduction

Computing the incidence of a tax in a partial equilibrium situation is a reasonable assignment for a beginning student of economics. Identifying the burden of a tax in a general equilibrium system may be an impossible task for even the most sophisticated economist or policy analyst. Those who proclaim that a real tax enacted by a real legislature will have specific ultimate effects are likely to be proved correct only in a world of fantasy.

It is in the small country Fantasy that this fable takes place. The fable is designed to demonstrate that merely knowing the structure of two taxes that raise equal amounts of revenue will rarely be sufficient to determine which tax will prove preferable for a specific reference group. Most particularly, it shows that any of the traditional forms of taxation on labor may prove beneficial to the laborers themselves. The analysis takes the form of a series of examples with counterintuitive results.[1]

[1]This paper was stimulated in part by Martin Feldstein's (1975) artful analysis that shows a lump sum tax on land, a factor in inelastic supply, can induce such significant changes in the supply of complementary factors that both the per period net rental and sale price of land can increase.

Fantasy, having matured to the post-agricultural state, produces and consumes the single product cellulose. The production function for this product is

$$f[K, L] = [0.5K^{-3} + 0.5L^{-3}]^{-1/3},$$

a production function with an elasticity of substitution of 0.25. Capital in this system consists of machines owned by the capitalists. The machines pass by inheritance from the capitalists of one period to those of the next.

In this system, labor of a single type is furnished by two groups. The peasants work to live. The capitalists can survive without working, living on the income earned by their machines, but under certain circumstances they may choose to work, and will earn the same hourly wage as peasants. To date they have chosen not to work. To keep things simple, we shall assume that both peasants and capitalists live for but a single period, consuming all the cellulose that is produced by the labor and machines they provide to the productive process. The nonoverlapping generations succeed each other. The economic link between generations is the passing forward of capital. The intellectual link is the *Little Red Textbook*, a treatise on the dismal discipline. The opening quotation of the treatise is from Ben Franklin:

> Our constitution is in actual operation; everything appears to promise that it will last, but in this world nothing is certain but death and taxes.

The utility functions for each of the two classes in Fantasy have after-tax income, Y, and leisure hours, H, as their arguments. Each argument is computed on a weekly basis, leisure being what remains out of the 100 hours conceivably available for working. The utility function for each individual peasant is

$$U_P(H, Y) = 100[\alpha - e^{-\beta H} - e^{-\gamma Y}].$$

The utility function for each capitalist is

$$U_C(H, Y) = \delta H - (\varepsilon - Y)^2,$$

for $Y < \varepsilon$. Both of these utility functions have convex indifference curves. The values of the parameters for the utility functions are

$$\alpha = 1.2, \quad \beta = 0.01015, \quad \gamma = 0.2, \quad \delta = 0.1, \quad \varepsilon = 30.05.[2]$$

There are 99 capitalists, each of whom owns 27 machines. There are 100 peasants. These numbers are sufficiently large that an individual supplying labor or machines assumes that he has no influence on factor prices. Fantasy is a strictly competitive free enterprise system, with factors paid according to their marginal products.

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[2]In this paper, as in the real world, the parameters defining preferences do not always have convenient mnemonic values.

As those who have observed capitalist states at work might have suspected, the system was constructed to burden the peasants. Indeed, the constitution of Fantasy, written in feudal times, permits only two types of taxes. The taxation section reads as follows:

(1) Taxes to promote the welfare of the general populace shall be raised only by proportional levies on the earnings of a citizen providing labor to the production process.
(2) In the special circumstances where the security of the nation is at stake, a head tax on each member of the citizenry may be imposed in lieu of the aforementioned proportional tax on earnings.

The first stipulation was originally designed to exempt land from taxation. Although land is no longer a vital element in the production process, the stipulation still serves the interests of the upper classes. It exempts capital from taxation for ordinary state purposes; the result is that the non-laboring capitalists will pay taxes only in the special circumstance when security is at stake, a circumstance that has not yet arisen.

The tax system offers only one sop to the peasants. They are guaranteed that they will not be taxed to the point where their after-tax incomes fall below the level of the survival stipend, S, the subsistence level of income and a very meagre level indeed. S was specified to be 2.3, an amount that is only about 2/3 of the peasants' current not very exalted level of income. This guarantee for S is set forth in a constitutional amendment that was hastily passed as an instrument to defuse the peasant uprising during the Peasants' Famine Revolt of 1956. (To achieve the 3/4 plurality required for amendments, at least 50 of the frightened capitalists had to cross the aisle and vote along with the peasants.)

The survival stipend amendment states explicitly that the first 2.3 dollars of a citizen's income is excluded from taxation, whether the tax imposed is a lump sum head tax or a proportional tax. Hence S is equivalent to the deductibility provision of a variety of traditional income taxes. (A capitalist who decides to work thus finds his entire wage income subject to tax, for his rental income far exceeds 2.3.)

There are no public goods in Fantasy. What is more impressive from the capitalist point of view, however, is the fact that the tax system effectively discourages any efforts to institute redistributional programs, for the state can acquire cellulose for redistribution only from the peasants themselves. Thus, there have been no taxes in Fantasy. The world there is one of an undisturbed competitive equilibrium.

Matters now change for the worse, unfortunately. Fantasy's neighboring nation, Nightmare, becomes militarily aggressive, and Fantasy must protect herself. The most economical way to do this is by purchase from abroad, specifically through participation in NOGO, a military alliance. The charge

for joining NOGO will be 50 units of cellulose per year, to be paid directly
to Fantasy's allies, with no return flow of funds or increased demand for
Fantasy's products. To collect these 50 units of cellulose for protection, Fantasy
for the first time must levy taxes.

Because this is a fable, there is no harm in assuming that despite its economic
structure Fantasy holds itself out to be a democracy. Alongside the many
inequities built into its constitutional structure are provisions requiring Fantasy
to follow majority rule in making social choices. The situation at hand is clearly
one where 'the security of the nation is at stake,' so that a head tax on all
citizens becomes a possibility. The peasants, who outnumber the capitalists
100 to 99, can decide whether they prefer a proportional tax on labor income
only, or a head tax on all citizens. Admittedly this is an oppressive situation,
for the peasants at best will bear a substantial tax burden, but less oppressive
than it would have been had the peasants had no voice or no choice.

Some of the peasants have been studying the *Little Red Textbook* with
great diligence. A passage within that volume suggests that machines are
nothing more than congealed labor. If the interpretation that machines are in
fact 'providing labor to the production process' could be presented to and
accepted by the Supreme Court, a most attractive new possibility would be
made available. Following the explicit language of the taxation section of the
constitution, the earnings of machines could then be subjected to taxation.
Moreover, because machines were in fixed supply, this tax on their earnings
would have the same effect as a lump sum tax, and indeed could be imposed as
if it were a lump sum tax. Finally, peasant earnings would need not be taxed at
all. The peasants' hopes are not high, however. Although the Supreme Court
decisions of recent years have tilted more toward the peasants, it seems unlikely
that the Court will subscribe to an interpretation that in effect admits to a labor
theory of value, thereby threatening the whole capitalist system of the nation.

To sum up, there were three conceivable tax schemes for Fantasy:

(1) A lump sum tax on machines, provided the Supreme Court concurs, or
(2) A lump sum (or head) tax on all citizens, both capitalists and peasants, or
(3) A proportional tax on labor earnings, in excess of the survival stipend.

2. The history

To see how things worked out, we must trace the history of Fantasy year by
year over a five-year period. The following notation will speed our story. The
peasants work P hours, the capitalists C, both at wage W. The capitalist will
receive a rental R for each of his machines. Let j represent a lump sum tax on
machines, k a head tax, and t a proportional tax. The survival stipend, i.e. the

deductible amount before the proportional tax applies, is S. Each peasant will have an after-tax income of

$$Y_P = WP,$$

or
$$Y_P = WP - k,$$

$$Y_P = WP - (WP - S)t,$$

depending on whether he is subjected to no tax, a head tax, or a proportional tax, respectively. The capitalist receives a rental for each of his N machines and wage income if he works, less any taxes. A capitalist's income will be

$$Y_C = NR + WC,$$

$$Y_C = NR + WC - k,$$

$$Y_C = N(R - j) + WC,$$

or
$$Y_C = NR + WC - WCt, \qquad \text{all assuming } NR > S,$$

depending on whether capitalists are subject to no tax, a head tax, a lump sum tax per machine, or a proportional tax on wage income respectively.

One point should be made clear. The laws of Fantasy expressly require that individuals allow the free market to operate. In making all decisions (as on hours of work, for example) they must optimize with respect to prevailing market prices. Indeed, collusion to restrict the supply of peasant labor is a capital offense in Fantasy.

Let P^* be the optimum number of hours for a peasant to work. That is, P^* is the maximizer of $100[1.2 - e^{-0.015(100-P)} - e^{-0.2Y}]$. Similarly, the optimum number of hours of work for the capitalist is given by C^*, where C^* is the maximizer of $0.1(100 - C) - (30.05 - Y)^2$. It is required that $0 \leq P^*$, $C^* \leq 100$. The total supply of labor in a weekly period is

$$L = 100P^* + 99C^*.$$

The wage is equal to the marginal product of labor,

$$W = \frac{\partial f(K, L)}{\partial L} = 0.5[0.5K^{-3} + 0.5L^{-3}]^{-4/3}L^{-4},$$

and R is defined in parallel fashion as the marginal product of capital.

Year 1: The initial peace, with no taxes.   The first year in our chronicle was the peaceful one before Nightmare developed its aggressive intentions. The equilibrium situation was then as follows:

$$W = 0.0637, \quad P^* = 54.28, \quad C^* = 0, \quad U_P = 7.070,^3$$

where U_P represents the utility of each peasant. It is only the peasants' utility that is of interest, for they are the voting majority.

Year 2: Lump sum tax on machines.   The second year was the first year in which national defense expenditures were necessary. The new generation of peasants petitioned the Supreme Court to permit a tax on machines. To their surprise they won. (The capitalists were not at all surprised. For years they had deplored the waxing radicalism of the Court.) Now all three taxes were viable possibilities. It did not take the peasants long to decide that obviously a lump sum tax borne by the capitalists should prove preferable to any tax that would be imposed partially or completely on the peasants. When the equilibrium played itself out, however, the peasants were in for another surprise. Subjected to the lump sum tax on their earnings from capital, the capitalists were attracted into the labor force. With more labor supplied, the price of labor fell significantly. The situation at the new equilibrium was as follows:

$$W = 0.0586, \quad P^* = 53.00, \quad C^* = 2.587, \quad U_P = 4.228.$$

Indeed, when the peasants examined this new equilibrium, it was evident that they would end up better off if they could forget the tax on machines, return to the old equilibrium, and then simply impose a head tax of 0.2513 units of cellulose on each citizen. As they figured it, each peasant's disposable income would then be 3.209. The utility value for $P = 54.28$, $Y = 3.209$ is $U_P = 4.490$, greater than the equilibrium U_P value that results when a lump sum tax is imposed on machines. The peasants, mindful of the interests of the coming generation of peasants, composed an appendix to the *Little Red Textbook* detailing their calculations and advising their successors that a lump sum tax on machines did not seem to be working out so well.

Year 3: Lump sum (or head) tax on all citizens on both capitalists and peasants· Recognizing that their fathers had been unpleasantly surprised, the next generation of peasants decided to pay more attention to their analysis. Those among them who had studied a bit of economics were well acquainted with the method of analysis that looks at the representative individual, and deemed this

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[3]Computed outputs in this report are rounded to four digits.

method suitable for their situation, for all the peasants have identical preferences and utility functions. What is best for one peasant is clearly best for all.

The line of reasoning then proceeded as follows. There are two basic differences between the head tax and the proportional tax on labor income. First, with the head tax a peasant ends up paying only 100/199 as much tax, capitalists pay 99/199 of the tax. Second, the proportional tax, being inefficient, will impose a welfare loss beyond the tax revenues raised. Since the capitalists would have nothing to do with the proportional tax, the peasants, through the distortion of their labor supply, will bear all of this excess burden. It seemed that on both grounds, less tax to pay and absence of additional welfare loss to be borne by the peasants, the head tax was preferable. There was only one possible catch: the capitalists might be induced to compete in the labor market. The peasants checked this out with a capitalist econometrician, whose speciality was the labor supply functions of rentier classes; he assured them (quite correctly) that the capitalists would not work at all if the head tax were imposed.

Therefore in the third year the peasants, taking the warnings of the *Little Red Textbook* appendix to heart, imposed a head tax on all citizens, themselves and the capitalists. Alas, they had not foreseen that the peasants themselves would choose to work more because of the tax they were now required to pay. The situation at the new equilibrium was as follows:

$$W = 0.0591, \quad P^* = 55.43, \quad C^* = 0, \quad U_P = 1.799.$$

The peasants wound up worse off than they would have been if they had merely allowed the original equilibrium to be achieved, and then each paid a 0.5 unit tax to raise the total imposition of 50 units of cellulose. (That procedure would yield a U_P of 1.805.) What was most frustratingly evident was that this lump sum tax 'imposing no excess burden' was in fact substantially burdening the peasants. And most galling of all, it worked to the equal and opposite benefit of the capitalists. It was only the net effect of the tax that produced no excess. The capitalists, it turned out, were even better off than they had been at the initial peacetime equilibrium when there were no taxes. Some of the peasants were heard to mutter in their mead (the liquid form of cellulose) that this was a typical capitalist outcome. Expenditures for defense, even though flowing overseas, were aiding the capitalists. Other peasants with more concern for the future carefully noted the results of their fiscal experiment and recorded them in a second appendix to the *Little Red Textbook*.

Year 4: Proportional tax on labor earnings. The fourth year, it is sad to relate, a militant peasant brought a new case before the Supreme Court. He argued that Nightmare was in fact no danger to Fantasy, that Nightmare's bellicose anticapitalistic propaganda was intended only for its own domestic consumption. Moreover, though some might claim that the security of the nation was at stake,

that is a fact not demonstrable within a court of law. On those narrow grounds, he alleged that the head tax is unconstitutional. No doubt this militant was stirred by the devastating outcome of the previous year. Still, he (like most militants) had given little thought to the alternatives. The case was brought, tried, and won. A proportional tax on labor earnings remained the only means of paying the NOGO dues.

With the exception of a radical fringe, neither capitalists nor peasants were willing to withdraw from the alliance just yet. There was some thought, however, of attempting to normalize relations with Nightmare, which was showing signs of more reasonable behavior. The decision was made to wait for a year to see if the international situation stabilized. In the meantime, the 50 units of cellulose payable for alliance participation would be raised through a proportional tax on labor income in excess of the survival stipend. The situation at the resulting new equilibrium was as follows:

$$W = 0.0753, \quad P^* = 43.46, \quad C^* = 8.412, \quad U_\mathrm{p} = 8.441.$$

The required tax rate was 31.25%.

Wonder of wonders, matters had improved. Indeed, U_p exceeded its value in Year 1, when there had been no defense expenditures, and consequently no taxes. In celebration, the peasants issued a special edition of the *Little Red Textbook*, with three appendices. The third appendix explained how a proportional tax on labor income can actually help the laborers if it encourages them strongly to cultivate their own pleasure rather than the capitalists' cellulose, that is, it gives a solid push towards greater leisure. The authors of the appendix failed to observe that this had happened in this instance only because earnings up to the survival stipend were excluded from taxes, with the result that the average tax rate was below the marginal rate. 'Not taxable till viable' was a constitutional guarantee, and an issue hardly to be raised in a text on economics, a discipline long known to have little to say on rights and entitlements.

Year 5 and thereafter: Proportional tax on labor earnings.   In the fourth year, the long suffering citizens of Nightmare revolted, overthrew that nation's ruling military junta, and changed its name to Euphoria. Relations between Fantasy and Nightmare improved rapidly, but by now the peasants of Fantasy (having studied the three appendices to the *Little Red Textbook*) were understandably cautious about changes in the structure of taxes. The equilibrium that they had finally achieved was far more to their liking than any of its predecessors. Turning a deaf ear to the neopacifism of the capitalists, who demanded withdrawal from NOGO and the consequent repeal of all taxes, the peasants voted unanimously to renew NOGO membership and to continue the proportional tax on labor income. They foresaw that peasants could live happily ever after.

Lest future peasants go astray, they summarized in still a fourth appendix their accumulated knowledge about the impact of various tax systems on the welfare of peasants. They found their message could be more easily communicated with the aid of some illustrations. For the first time, they embraced the diagrammatic techniques of neoclassical economics, a discipline that no longer seemed so antagonistic to the class interests of the peasants. First they selected the labor supply curves of an individual peasant and an individual capitalist. Next they developed a market supply curve for each of the tax possibilities by summing horizontally the supply curves of 100 peasants and 99 capitalists. These curves and the resulting equilibria are reproduced below. (The labor supply curves for the capitalists, displayed for the two tax situations in which they are working at the equilibrium, depend on the return to their capital. The curves drawn assume the capitalists receive the return on capital that is achieved at the equilibrium.)

The peasants swiftly rejected a proposal that peasant welfare across situations be compared using the geometry of these diagrams. This would be accomplished by subtracting areas under supply curves from areas representing incomes net of tax. This procedure for comparing welfares rightly was regarded as being much too proximate, particularly since it could be readily demonstrated that for Fantasian peasants the marginal utility of income was far from constant. Moreover, the straightforward numerical calculations of peasant utility were already available. The peasants recorded the results of those calculations in the appendix. With the production function, capitalist and peasant utility functions and population proportions of Fantasy, the peasants' preference ordering among different ways to raise a given amount of money was:

(1) A proportional tax on labor earnings (in excess of the survival stipend),
(2) A lump sum tax on machines,
(3) A lump sum (or head) tax on both capitalists and peasants.

Moreover, the proportional tax on labor earnings was so attractive that it was even better than no taxes at all!

3. Revisionism

In the sixth year, with their own house in order, the peasants' thoughts turned to others in future generations and faraway lands. It was time to spread the gospel. The scholar peasants pointed out that the *Little Red Textbook* by now had acquired four appendices each of which to some extent contradicted findings found in the body of the text. A committee was appointed to systematize the findings of the past five years and to rewrite the *Little Red Textbook* accordingly. The committee summarized the new knowledge as follows:

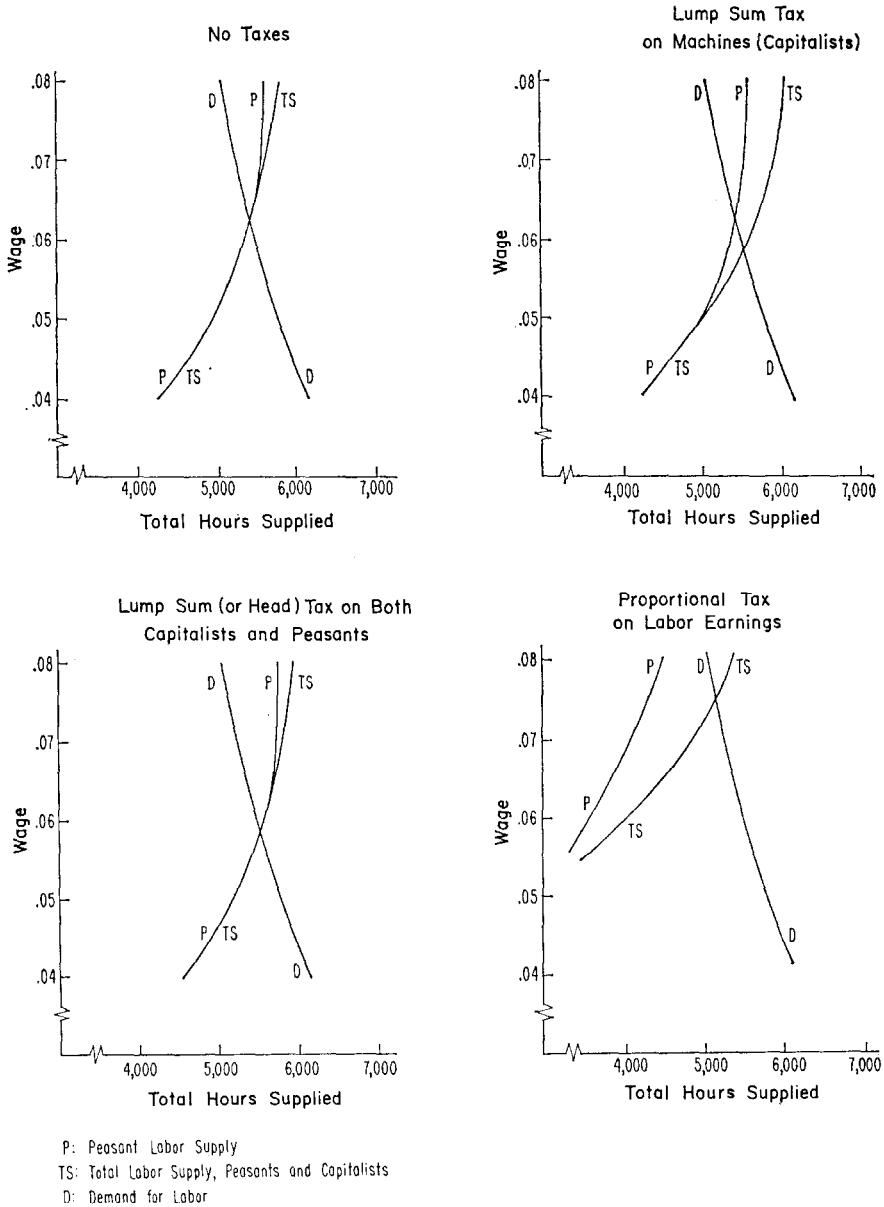(1) Partial equilibrium analyses of tax incidence are misleading.

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Fig. 1. Labor market equilibria with various tax structures.

(2) Even in those situations where the owners of the taxed factor own no other factor, or can not vary their supply of any other factor, partial equilibrium analyses of incidence can be highly misleading. Indeed, in situations where partial equilibrium analysis suggests that the full costs imposed on the factor will exceed the taxes it pays, the divergence between the general and partial equilibrium analyses is likely to be the greatest. The owners of the factor might even benefit from being taxed.

(3) A lump sum tax on an inelastically supplied factor can alter factor prices if those owning the factor respond to the income effect by altering their provision of some other factor which is not in inelastic supply.

(4) A lump sum tax on an inelastically supplied factor can create an excess burden if the resulting income effect induces changes in the supply of other factors that are subject to tax.

(5) The owners of factors may prefer a tax on the factor's earnings to a lump sum tax that garners the same amount of revenue. The substitution effect exerted by the earnings tax will restrict the supply of the factor, driving up its per unit price. Alternative collusive mechanisms designed to restrict supply may be illegal or unenforceable.

(6) Depending on its effect on factor supply, the incidence of a tax can fall more than 100% on the owners of the taxed factor, can actually increase the well-being of those owners, or can produce an intermediate result.

These were most instructive lessons. The most compelling conclusion seemed to be that nothing was as unsure as the incidence of taxes. One of the scholar peasants observed, however, that there may be particular circumstances, applying in other lands though not in Fantasy, that would make prediction certain. With this in mind the committee writing the revised text turned to scholarly inquiry.

4. Scholarly research

The committee identified simple situations that would be relevant for other states where peasants were a voting majority, though their choices too might be severely constrained by a constitution. In the situations considered, all taxes would be levied on labor.

The only factor in variable supply was labor. The production function and population proportions were those of Fantasy. Recognizing that the particular preferences of the peasants of Fantasy might be peculiar to them, and not being the most sophisticated methodologists in the world, the committee decided to test various propositions by computing equilibria with the peasant utility function having a variety of parameter values.

The capitalist utility function was assumed to be identical to that of a Fantasian

capitalist, for the committee believed that rentier classes must exhibit similar preferences whenever and wherever they might exist.

The first case the committee considered was a tax collected in units of labor; elsewhere this would be called conscription. Army work was neither more nor less attractive than traditional paid labor. Hence it seemed evident that a lump sum tax of this form would reduce the utility of the peasants. By way of demonstration, it computed the equilibrium under the assumption that each peasant would have to give three uncompensated hours per week for military service. The leisure argument of the utility function then became

$$H = 100 - P - 3.$$

For the utility function

$$U_P(H, Y) = 10,000(1 - e^{-0.25H} - e^{0.0001Y}),$$

the committee discovered that subjecting the peasants to conscription actually improved their well-being over the initial situation in which there were no taxes of any kind. The two equilibria were as follows: no tax,

$$W = 0.0535, \quad P^* = 56.99, \quad C^* = 0, \quad U_P = 2.834;$$

and conscription tax,

$$W = 0.0624, \quad P^* = 54.61, \quad C^* = 0, \quad U_P = 3.157.$$

This result was initially puzzling. Then one of the scholars recalled a historical situation described in a bourgeois text in which the pig farmers of a nation found it desirable to agree that each would destroy a certain number of piglets. (The constitution did not prohibit such collusion among capitalists.)

If you can learn one thing from a bourgeois text you can probably learn another, the committee members reasoned. What sorts of taxes would necessarily prove to be detrimental? Their readings suggested that Fantasy's proportional tax on labor income had worked out to the peasant's benefit only because it was really not a proportional tax at all, but rather a progressive one. In effect the peasants were being allowed to keep some of the revenues a straightforward proportional tax would have raised, an amount equal to the tax rate times the survival stipend. But most peasants unfortunately did not live in capitalist states that had had uprisings sufficiently disruptive to secure a constitutional guarantee excluding some minimal amount of income from taxation. Without such a guarantee it would seem that because labor is but a commodity, a proportional tax on labor income must hurt the peasants, just as a straight sales tax hurts any seller. The peasants could keep none of the tax; the effect of the

tax would be to drive down their after-tax wage. A lowered peasant utility would be the necessary result. In the limiting case, demand is completely inelastic and the peasants would not lose, but they certainly would not gain.

Once again the scholars tried a variety of parameter values for the utility function. One that gave them trouble was

$$U_P(H, Y) = 1{,}000[1 - e^{-0.0004H} - 0.1 e^{-0.79Y}].$$

The initial situation in which there would be no taxes produced the following equilibrium: no tax,

$$W = 0.0510, \quad P^* = 57.74, \quad C^* = 0, \quad U_P = 7.005.$$

They then imposed a straightforward tax of 5% on all labor income. The equilibrium with this flat tax rate turned out to be: flat tax on labor earnings,

$$W = 0.0564, \quad P^* = 56.17, \quad C^* = 0, \quad U_P = 8.085.$$

Welfare had improved. Ignoring some radical efforts to dismiss this result as a capitalist trick (the capitalists owned the computer), the peasant scholars delved further and discovered possible relationships between backward bending supply curves for labor and the incidence of per unit or proportional taxes. (Admittedly, another less attractive equilibrium would be stable, and this equilibrium was not. But there was no reason for the peasants to defect from this preferable, if unstable, equilibrium.) Mutterings about diminished exploitation and the like were heard, but the straightforward neoclassical explanation seemed sufficient. The scholars were a little shaken by these results. Their initial preconceptions, perhaps deceitfully informed by bourgeois theory, had led them astray.

There was one matter of which they were certain, however. In the circumstances just described (only peasants work and supplies of nonlabor factors are completely inelastic), a straightforward lump sum cellulose tax on peasants can not improve their welfare. This much seemed obvious. Once again they turned to simulation, their favorite methodology. They decided to carry out their computer runs with a lump sum tax of 0.1583 units of cellulose on each of the peasants. This lump sum imposition would give a total tax yield just equal to the amount provided by the proportional tax on wage income. The first run, naturally, was made with the utility function that had just been employed. The committee members were immediately surprised. Employing the no-tax equilibrium as the reference point, the utility of the peasants was increased by subjecting them to a lump sum tax. The after-tax equilibrium was: lump sum tax on peasants,

$$W = 0.0599, \quad P^* = 55.23, \quad C^* = 0, \quad U_P = 9.446.$$

Not only did the peasants enjoy a higher utility once taxed, but both their after-tax income and their leisure were greater than they had been before the tax was imposed. The explanation was quickly discovered. The peasants displayed backward bending supply curves for labor. These curves summed horizontally to a market supply curve for labor that was significantly flatter than the relatively inelastic demand curve for labor. The tax led the peasants to behavior that shifted their individual supply curves outward, and the market
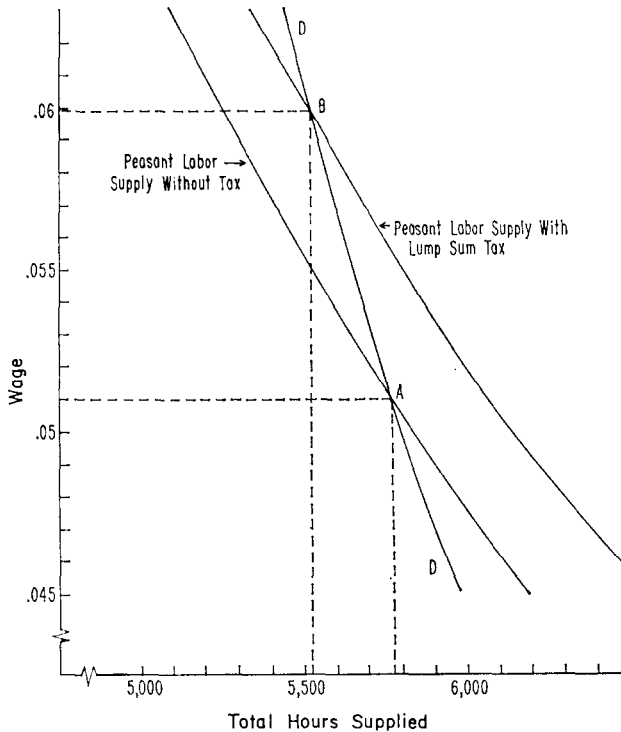

Fig. 2. The labor market effects of a lump sum tax, with a backward bending labor supply curve.

supply curve shifted out correspondingly. An equilibrium (also unstable) with a higher wage, more income, and less work was the consequence. The committee provided the above illustration of the workings of the tax.

The lump sum cellulose tax pushed the equilibrium from A back up the demand curve to B.

Observing this unexpected result which followed right on the heels of some other surprises, the committee decided that further research was not warranted. Such research could only produce more negative results in a field of study that was already well known as the dismal science.

5. The new economics

One of the scholars suggested that the revised version of the *Little Red Textbook* should include a few extra chapters on the class theory of leisure. A counter-suggestion carried the day. Only the title page, with the Franklin quotation remained unchanged. It was followed by one hundred blank pages. The last page read:

Economics is a tricky discipline, so it is no surprise that the Franklin quotation is only half right. Though death at the end of the year is inevitable, nothing is so uncertain as the incidence of taxes.

Epilogue[4]

It turned out quite by chance that the peasants of the neighboring state of Euphoria had a utility function that was precisely the same as the hypothetical utility function the Fantasian peasant scholars had employed in their second and third simulations. The Euphorian peasants, pleasantly surprised at this exact coincidence, concluded that they could take guidance from the findings of those simulations. They voted unanimously to submit themselves to a lump sum tax.

The Euphorian peasants then appointed a commission to discover why the result, quite unexpected, was not an increase but rather a substantial decline in welfare. The peasants were working more and enjoying it less. Income as well as wage rates had fallen, not risen.

The commission noticed that the favorite analytical technique of the Fantasian peasant-scholars had been something known as 'simulation'. This was a technique by which the aggregate behaviors of large numbers of market-responding individuals could allegedly be predicted by computing their individual behaviors from assigned utility functions. Computers were regularly used for the work. The members of the commission investigated some of the properties of that historically famous technique.

First they double checked the utility functions, supposing perhaps that they had misestimated a parameter or two. But their estimates were precise; the problem wasn't there. Next they checked the relative quantities of machinery and potential labor. Again, identity. The problem wasn't there.

Finally, they ran the simulation. *And it gave them a drastic reduction in wages with an actual increase in hours worked.*

What had they done differently?

Eventually they discovered that the Fantasians had not actually run the simulation. Instead they had 'solved' their model. And 'solving' it meant

[4]By Thomas Schelling.

finding the incremental change in the location of an intersection of two curves, upon the shift of one of the curves. They then remembered something, possibly hinted at in Samuelson's *Foundations*, about the apparently perverse, or counter-intuitive, shift in a 'solution' that corresponds to an unstable equilibrium when one of the functions shifts.

What the Euphorians had done, and the Fantasians had not done, was to simulate the behavior of the peasants upon imposition of the tax. The peasants,
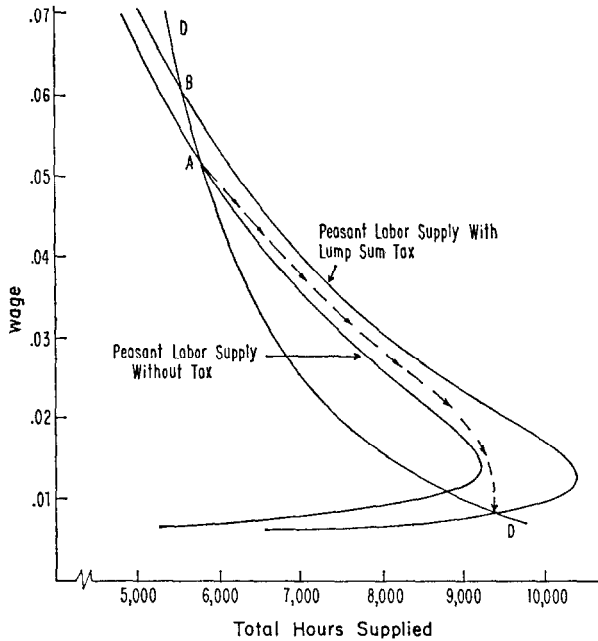

Fig. 3. Middle and lower-wage labor market equilibria in Euphoria with and without lump sum tax.

having less income but the same marginal exchange rate between work and leisure, offered to work longer hours, and undercut each other in the market. As the demand curve sloped downward, they had to accept lower wages to get the extra work. But demand was inelastic, and the more they worked, the less they earned. Their labor offerings got larger and larger as wages got lower and lower. Eventually wages were so low that, despite their unaccustomed poverty, the trading rate of work for wages was so poor that they offered no more labor as the wage rate fell. Eventually some of them withdrew some of their available hours from the labor market.

To communicate their analysis, the members of the commission took a leaf out of the old *Little Red Textbook*, in which a truncated diagram showed the

wage rate being 'pushed back' from *A* to *B*. They completed the diagram and plotted the dynamics of the market, taken from their own computer printout. The diagram contained a dotted line showing, by directed arrows, the path of the wage rate and the hours offered during the interval of market disequilibrium.

Their conclusion of course went almost without saying, but they said it anyway. *Don't rely on the capitalists' computer if you want a simulation for the peasants.*

Addendum to the Epilogue

Later the Euphorian capitalists got a copy of the Euphorian Commission's report. They then regretted that their peasants no longer played the lump sum tax game and had gone back to that old untaxed, unstable equilibrium. They decided, however, that all was not lost. The capitalists cooked up a scheme to tax themselves without arousing suspicion. Alleging that some capitalists were unfairly competing by violating regulations that had been imposed for the peasants' safety, they volunteered to tax themselves to cover the cost of more rigorous enforcement of the safety regulations. The peasants, not on their toes intellectually, let an excise tax go through. The demand for labor shifted slightly leftward, an excess supply of labor occurred, and Euphorian capitalists enjoyed euphoria.

Still, scholars wondered how that initial 'unstable equilibrium' had ever come about much less been re-established, and how it weathered the little storms of occasional absenteeism, Christmas demand for extra income, shutdowns for overhaul, and all those small perturbations that are alleged in the textbooks to make such an equilibrium not viable. It took real sleuthing, both contemporary and historical, to get the answer.

Before universal suffrage and competition were guaranteed in the Euphorian constitution – Nightmarian constitution, that is – the peasants had developed a collusive arrangement. It kept the supply of labor at the higher, unstable, intersection of demand and supply. (They felt they had to forego the luxury of letting the third, spectacularly higher, intersection occur. They feared that it would provoke a capitalistic amendment that would require competition on the part of the peasants, thereby outlawing this practice. Moreover, the amendment would undoubtedly specify that where a stable low-wage equilibrium was available, a high-wage equilibrium was demonstrably anticompetitive.) The peasants had actually arranged for a few 'market stabilizers' to stay home, or work extra, to compensate small day-to-day shifts in labor supply. After the constitution was adopted, those somewhat left-wing market stabilizers kept up their little conspiratorial task, unknown to capitalists and peasants alike. They had been manipulating the market successfully all that time, although clearly they had never had a large perturbation to compensate or it all would have spiralled off in one direction or the other.

The leftist conspirators had the last laugh, though. Taking their cue from that capitalistic hoax about safety regulations, they launched a campaign of sabotage against all the safety devices in the capitalists' machinery, and peasant after peasant lost a finger or a toe and became unable to work. With the deficient supply of labor, wages rose along the inelastic, backward-bending supply curve, all the way up to that old equilibrium level and a little beyond, where, with no further need to sacrifice fingers and toes, the wage rate spiralled up to that third and highest equilibrium. (The capitalists were working as well at that low-rent, high-wage equilibrium.) The workers supported the disabled idlers, and a favorite topic of conversation among peasants was whether it was better to be whole and have to work or to lack a member and be supported by others.

Reference

Feldstein, M., 1975, The surprising incidence of a tax on pure rent: A new answer to an old question, Journal of Political Economy 85, no. 2, 349–360.