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Joint control of production, remanufacturing, and disposal activities in a hybrid manufacturing–remanufacturing system

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ABSTRACT

To generate insights into how production of new items and remanufacturing and disposal of returned products can be effectively coordinated, we develop a model of a hybrid manufacturing–remanufacturing system. Formulating the model as a Markov decision process, we investigate the structure of the optimal policy that jointly controls production, remanufacturing, and disposal decisions. Considering the average profit maximization criterion, we show that the joint optimal policy can be characterized by three monotone switching curves. Moreover, we show that there exist serviceable (i.e., as-new) and remanufacturing (i.e., returned) inventory thresholds beyond which production cannot be optimal but disposal is always optimal. We also identify conditions under which idling and disposal actions are always optimal when the system is empty. Using numerical comparisons between models with and without remanufacturing and disposal options, we generate insights into the benefit of utilizing these options. To effectively coordinate production, remanufacturing, and disposal activities, we propose a simple, implementable, and yet effective heuristic policy. Our extensive numerical results suggest that the proposed heuristic can greatly help firms to effectively coordinate their production, remanufacturing, and disposal activities and thereby reduce their operational costs.

1. Introduction

Over the past decade there has been quickly growing interest in the recovery of used products in different industries. As seen, for instance, in the case of van der Laan and Teunter (2006) in the automotive industry, remanufacturing is not only an important step toward *sustainability* and other environmental considerations, but is also economically beneficial for firms. This economic benefit is mainly due to two reasons: (1) remanufactured products can be usually sold with the same warranty as new products and (2) remanufacturing processes are usually cheaper than manufacturing ones. For instance, Samsung Electronics (SEC) collects used electronic products through 1560 branch retail stores and 24 local distribution centers in Korea. In 2010, SEC reused 27,466, 13,054, and 12,158 tons of parts and materials in production of refrigerators, displays, and washing machines, respectively (Samsung Electronics, 2011). SEC is also operating STAR (Samsung Takeback and Recycling Programme) to recycle copy or print cartridges in 21 countries and operating more than 2000 centers in 61 countries to collect used mobile phones (Samsung Electronics, 2011).

The study of Tang and Teunter (2006) provides a good example of a concrete application of the hybrid manufacturing–remanufacturing system. Their research was motivated by a company which produces and remanufactures car parts. Both manufacturing and remanufacturing operations are performed on the same production line. Workers are faced to a large variety of products: diesel engines, petrol engines, water pumps, cylinder heads, etc. For the water pumps, the remanufactured products represented approximately 30% of annual sales. Another example on an application of a hybrid system is found in Zhou, Tao, and Chao (2011) which considered an energy company that provides service on meters and transformers for private houses and commercial buildings. The meters and transformers are owned by the energy company and all failed meters/transformers are shipped to the warehouse/distribution center, diagnosed in operational conditions, and stored in inventory. Returned items with very bad conditions are disposed. The company's inventory control system manages the replenishment of stock levels. However, the system did not take the returned products into consideration, which account for over a third of the company's total business. The problem the company faces was how to make remanufacturing decisions on the various types of returns jointly with replenishment decisions, so as to minimize total cost.

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This research is a step toward gaining fundamental insights into how production of new products and remanufacturing and disposal of returned items can effectively be coordinated. Such coordination provides financial incentives for firms to move toward sustainable operations. We consider a hybrid production system with manufacturing and remanufacturing in continuous time where a single type of product is stocked in order to meet demand from customers who may also return products after usage. In our model, demands arrive randomly and are possibly satisfied from on-hand inventory of serviceable products (i.e., manufactured or remanufactured finished goods that are ready for sale). We consider the case where unmet demand is lost, which better represents the current competitive market in many industries in which customers will switch to a competitor under a shortage. Product returns also arrive randomly in our setting and each returned item can be disposed of or accepted for recovery. Accepted returned items are converted into serviceable products. Therefore, in our model, serviceable inventory can be replenished through either manufacturing new items or remanufacturing returned products.

The primary goal of this paper is to (i) develop an appropriate model to jointly optimize production, remanufacturing, and disposal activities in hybrid manufacturing–remanufacturing systems, (ii) investigate the structure of the jointly optimal policy, (iii) develop a simple heuristic to effectively coordinate disposal, remanufacturing, and production activities, and (iv) generate insights into the benefits of considering the remanufacturing and disposal options. Our results provide insights into how firms can increase sustainability and still increase their profit.

Compared to traditional production systems, a hybrid production system with manufacturing and remanufacturing poses important challenges with respect to inventory management. In a hybrid system, product returns generate a new source of material flow which indirectly affect the inventory level of serviceable products. Thus, the two alternative sources for replenishing the serviceable inventory (i.e., production and recovery) should be coordinated and jointly controlled. Many studies have discussed production planning and inventory control problems with consideration of the product return process. Detailed literature reviews can be found in Fleischmann et al. (1997), Junior and Filho (2012), Mahadevan, Pyke, and Fleischmann (2003). To clarify our contributions to the literature, we classify the references related to our study using the characteristics listed below:

- Inventory review: periodic or continuous.
- Production and remanufacturing lead time: stochastic or constant.
- Production capacity: limited or unlimited.
- Fixed costs for production and remanufacturing: included or not included.
- Types of unit variable costs considered: holding serviceable inventory, holding remanufacturable inventory, production, remanufacturing, disposal.
- Stockout treatment: backorder or lost sales.
- Types of control considered: production, remanufacturing, disposal.

With respect to this classification, our paper possesses continuous inventory review, stochastic production and remanufacturing lead times, limited production capacity, no fixed costs for production and remanufacturing, all five above-mentioned types of unit variable costs, lost sales, and control of production, remanufacturing, and disposal processes. Table 1 summarizes the literature review by comparing the related studies. Since we study the joint control of production, remanufacturing, and disposal in this paper, we will restrict the detailed review in the remainder of this section to those papers, and point out our contributions.

Simpson (1978) studies a periodic review inventory system where unmet demand is backlogged and neither fixed costs nor lead times are considered for purchase and recovery processes. By trading-off purchasing cost, recovery cost, and holding cost of both inventories, the optimality of a three-parameter policy that controls the purchase, recovery, and disposal decisions is shown. Inderfurth (Inderfurth, 1997) studies the effect of both replenishment and remanufacturing deterministic lead times on the optimal inventory control policy. He shows that the policy identified in Simpson (1978) is also optimal for identical deterministic lead times. Kiesmüller and Scherer (2003) presented a method for the exact computation of the parameters which determine the optimal policy for the model of Simpson (1978) and Inderfurth (1997). DeCroix (2006) extended Inderfurth (Inderfurth, 1997) by considering the multi-echelon system with remanufacturing and identifying the structure of the optimal remanufacturing/ ordering/ disposal policy.

van der Laan and Salomon (1997) considered a $(s_m, Q_m, s_r, S_r, s_d)$ policy that can be described as follows: remanufacturing starts whenever the inventory position (on-hand serviceable inventory minus backorders plus outstanding (re)manufacturing orders) is at or below s_r and the remanufacturable inventory contains sufficient products to raise the inventory position level to S_r, manufacturing starts with batch size Q_m whenever the inventory position drops to $s_m(<s_r)$, and incoming returns are disposed of whenever the remanufacturable inventory equals s_d. They compared the performance of this policy with a (s_m, Q_m, Q_r, s_d) policy under which remanufacturing starts whenever the remanufacturable inventory contains exactly Q_r products, manufacturing starts with batch size Q_m whenever the inventory position reaches s_m, and incoming returns are disposed of if the inventory position is at or above s_d.

Inderfurth and van der Laan (2001) considered a (s_m, Q_m, Q_r, S_d) policy where manufacturing of Q_m products starts whenever the serviceable inventory position (serviceable inventory plus outstanding (re)manufacturing orders minus backorders) drops to the level s_m, remanufacturing starts whenever a batch of returned products of size Q_m is available, and incoming returns are disposed of whenever the inventory position equals or exceeds S_d. They showed that by using the remanufacturing leadtime as a decision variable (i.e. by changing the definition of the inventory position), the performance of (s_m, Q_m, Q_r, S_d) policy can be improved considerably. Wei, Li, and Cai (2009) provided a linear programming model and analyzed the impact of the key parameters on the solutions.

Our model contributes to this stream of research in several ways. First, we generate insights into coordinating decisions of when to produce a new product, when to remanufacture a returned item, and when to dispose of a product return. In our framework, manufacturing, remanufacturing, and disposal decisions are dynamically controlled based on inventory levels of both serviceable (i.e., "as-new") and remanufacturable (i.e., returned and not disposed of) items. Second, we characterize the jointly optimal production, remanufacturing, and disposal policy in a continuous review inventory control environment. Previous product recovery studies with continuous inventory control have mainly tried to find optimal parameters for given policies rather than characterizing the structure of the optimal policy. This is partially due to the complexity of the underlying control problem in such settings, especially when (1) production and recovery lead times are stochastic and non-zero and (2) the unmet demand is lost (rather than backlogged). Third, we show the effectiveness of two simple heuristic control policies. Our proposed heuristics provide firms with a simple tool that effectively helps them to achieve higher profits while encouraging the use of sustainable operations. Fourth, we provide

Table 1
Summary of literature survey.

Articles	Inventory review		Lead time				Prod'n capacity		Fixed cost		Unit cost					Stockout treatment		Types of control		
			Stoch.		Const.															
	Pi	Ci	P	R	P	R	U	L	P	R	Hs	Hr	P	R	D	B	L	P	R	D
DeCroix (2006)	+				+	+	+				+	+	+	+	+	+		+	+	+
Fleischmann and Kuik (2003)	+						+		+		+					+		+		
Fleischmann et al. (2002)		+			+		+		+		+					+		+		
Heyman (1977)		+					+				+		+		+			+		+
Inderfurth and van der Laan (2001)		+			+	+	+		+	+	+	+	+	+	+	+		+	+	+
Inderfurth (1997)	+				+	+	+				+	+	+	+	+	+		+	+	+
Kiesmüller and Scherer (2003)	+				+	+	+				+	+	+	+	+	+		+	+	+
Mahadevan et al. (2003)		+			+	+	+				+	+				+		+	+	
Muckstadt and Isaac (1981)		+			+		+		+		+					+		+		
Simpson (1978)	+						+				+	+	+	+		+		+	+	+
Teunter and Vlachos (2002)	+				+	+	+		+	+	+	+	+	+	+	+		+	+	
Teunter et al. (2004)		+			+	+	+		+	+	+	+				+		+	+	
Teunter et al. (2006)	+						+		+	+	+	+						+	+	
van der Laan and Teunter (2006)		+			+	+	+		+	+	+	+				+		+	+	
van der Laan et al. (1996)		+		+	+		+		+		+	+	+	+	+	+		+		+
van der Laan et al. (1996)		+		+	+		+		+		+					+		+		+
van der Laan and Salomon (1997)		+			+	+	+		+	+	+	+	+	+	+	+		+	+	+
van der Laan et al. (1999)		+	+	+			+		+	+	+	+	+	+		+		+	+	
van der Laan et al. (1999)		+			+	+	+		+	+	+	+	+	+		+		+	+	
Wei et al. (2009)	+				+	+	+				+	+	+	+	+	+		+	+	+
Our model		+	+	+				+			+	+	+	+	+		+	+	+	+

Pi = Periodic review; Ci = Continuous review; P = Production; R = Remanufacturing; L = Limited; U = Unlimited; D = Disposal; Hs = Holding serviceable inventory; Hr = Holding recoverable inventory; B = Backorder; L = Lost sales.
References are identified with first author and number.

insights into the value of remanufacturing and disposal options. We find that there is a *synergy* between these two options: the value of having both options together is higher than the sum of their individual values. Our results also show that the disposal option is on average more valuable than that of remanufacturing, which sheds more light on the importance of acceptance/rejection decisions regarding the returned products.

Although our research focus here is on production planning and inventory control problems in a remanufacturing setting, other research efforts have considered remanufacturing from a more strategic perspective. The optimal acquisition price of used products and the selling price of remanufactured products are studied in Bakal and Akcali (2006), Guide, Teunter, and Van Wassenhove (2003), Karakayali, Emir-Farinas, and Akcali (2007). Kaya (2010) studies the joint decisions of the acquisition price with the remanufacturing and manufacturing quantities, and Li, Li, and Saghafian (2013) extend (Kaya, 2010) to the case where both remanufacturing yield and used product acquisition are random. Another important characteristic of products that are considered for remanufacturing is a portfolio of new and remanufactured products because the remanufactured product reduces the sales of the new product when sold on the same market and the sales of remanufactured products are constrained by the availability of used products that are generated by past sales of new products (Debo, Toktay, & Van Wassenhove, 2006). The issue of product portfolios with new and remanufactured products is studied in Debo, Toktay, and Van Wassenhove (2005), Debo et al. (2006), Ferrer and Swaminathan (2006, 2010) and Ferguson and Toktay (2006).

The remainder of the paper is organized as follows. In the next section, we present our model. Analysis of the optimal policy is given in Section 3. In Section 4, we numerically implement performance comparisons between models with and without remanufacturing and disposal options. Section 5 presents our proposed heuristic policies. Finally, we conclude in Section 6.

2. The model

Demands for a single product arrive according to a Poisson process with rate λ_1. If demand is satisfied from on-hand inventory, a revenue of R_1 is generated. Hereafter, we refer to the on-hand inventory as serviceable inventory representing the as-new finished goods that can satisfy the demand. We assume unmet demand is lost. While some settings are best modeled with backorders, lost sales may be more representative to model stockouts when the firm is in a competitive market and customers can easily turn to a competitor during a stockout. As shown in Table 1, the research dealing with inventory control for a hybrid production system has the prevalent assumption that complete backlogging of orders is allowed in case of stockouts, and the number of models dealing with lost sales is limited. Since each lost demand implies the loss of opportunity of generating a revenue of R_1, there is an implicit penalty for each lost sale and therefore, we do not need to include an additional lost sales cost. However, the extension with an additional lost sales cost is straightforward and our main results will still hold.

The time required to produce an item (i.e., the production leadtime) is exponentially distributed with mean μ_1^{-1}. Returned products arrive according to a Poisson process with rate λ_2. A returned item can be either disposed of or accepted for recovery. The time required to perform the recovery process and transform the returned item into a serviceable one is exponentially distributed with mean μ_2^{-1}. Similar to the production process, we assume recovery is performed item by item (i.e., no batching). Hence, the recovery process in our model can be viewed as an $M/M/1$ queueing system with admission control. Assume manufacturing a new item costs c_M and disposing of a returned product incurs a cost of, $c_D \in (-\infty, \infty)$, where $c_D < 0$ implies a salvage value. Holding costs are assessed at rate h_1 and h_2 for each unit in serviceable and recoverable inventory, respectively. Whenever an item undergoes the recovery process, a fixed cost of c_r is incurred. Fig. 1 illustrates a schematic representation of the model under consideration.

For tractability, we assume the demand and return processes are independent; that is, product returns are an exogenous process. It may be more natural to assume that there is correlation between returns and past sales, since the actual return process may be a function of many factors including the previous sales, the product's service life, incentives provided for returning products, and several market factors causing customers to switch from using the product in favor of a competing (e.g., newly introduced) product. The most obvious type of correlation between returns and past sales is a positive one, where increased demand generates a greater number of items in the field. When the life cycle is sufficiently long and a large number of products are already in the market, then the correlation between current demand and failed items returned to the manufacturer is typically not significant. Our model with an exogenous process of product returns can also serve as an approximation of scenarios with positive correlation as long as it is parameterized to match the current operational environment. Furthermore, our framework yields a simple and easily implementable control policy. Fleischmann (2000) (see pages 144–145) provided a detailed justification of the assumption of the independence between returns and past sales.

We further assume that the quality of a recovered item is the same as the quality of a newly produced one. This setting is observed in various industries. For example, in the case of the energy company in Zhou et al. (2011) described in the previous section, customers are indifferent between a new product and a remanufactured product. Another example can be seen in the case of Caterpillar which builds all engines according to factory specifications by trained personnel that follow the company's own strict remanufactured engine procedures. Production of printer cartridges and single-use cameras are other practices in which consumers do not distinguish between a new and a remanufactured product.

The set of decision epochs in our model corresponds to a demand arrival, a return arrival, a production (manufacturing) completion, or a recovery (remanufacturing) completion. At each decision epoch, a control policy specifies whether or not to produce an item and whether or not to remanufacturer a returned item from the remanufacturing inventory. Furthermore, at epochs corresponding to a return arrival, a decision must be made regarding whether to dispose of or accept the returned item. By modeling the production and recovery activities as queueing processes, we model finite capacities, stochastic production and recovery lead times, and we allow for a more realistic cost model compared to most studies in the literature.

We seek to find a *joint* production, remanufacturing, and disposal control policy that maximizes the long-run average profit. The optimal control problem can be formulated as a discrete-time Markov decision process problem by using uniformization (see Lippman (1975)). This uniformized version has a transition rate of $\gamma \equiv \lambda_1 + \lambda_2 + \mu_1 + \mu_2 < \infty$ applied to all states. The state of the system is described by a vector $x = (x_1, x_2)$, where $x_1 \geqslant 0$ and $x_2 \geqslant 0$ represent the serviceable and recoverable inventory levels, respectively. We denote the state space by Γ.

We define the value iteration operator T on any real-valued function f (defined on Γ) as

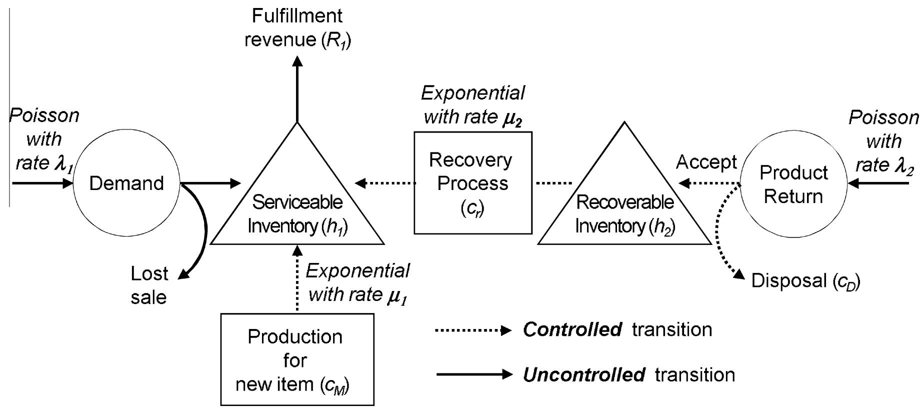

Fig. 1. The hybrid manufacturing–remanufacturing system under consideration.

$$Tf(x) = \frac{1}{\gamma}[-x_1h_1 - x_2h_2 + \lambda_1\{(R_1 + f(x-e_1))1(x_1 > 0) + (x)1(x_1 = 0)\} + \mu_1 \max\{-c_M + f(x+e_1), f(x)\} + \lambda_2 \max\{f(x+e_2), f(x) - c_D\} + \mu_2[\max\{-c_r + f(x+e_1-e_2), \times (x)\}1(x_2 > 0) + f(x)1(x_2 = 0)]], \quad (1)$$

where $1(a) = 1$ if a is true, and $1(a) = 0$ otherwise. In (1), $-x_1h_1/\gamma$ and $-x_2h_2/\gamma$ are the average holding cost of keeping x_1 units in serviceable inventory and x_2 units in recoverable inventory, respectively, until the next uniformized transition. The terms multiplied by λ_1 in the above represent sales revenue as well as the transition generated with a demand arrival. The terms multiplied by μ_1 correspond to the cost and transitions associated with a production completion, and the term multiplied by μ_2 is the cost and the transition associated with a recovery completion in addition to a self-loop when $x_2 = 0$. The terms multiplied by λ_2 represent the cost and transitions associated with a return arrival.

Let $h(x)$ be the relative value function of being in state x, and g be the optimal average profit per unit of uniformized time. Then, the average-profit optimality equation can be written as

$$g + h(x) = Th(x). \quad (2)$$

3. Structure of the joint optimal production, remanufacturing, and disposal policy

To characterize the structure of the optimal policy, we will follow Porteus (1982). The key to this approach is to identify a set of structured value function properties and to show that they are preserved under the value iteration operator. To this end, define the operators D_i on any real-valued function f (defined on Γ) as

$$D_if(x) \equiv f(x+e_i) - f(x), \quad i = 1, 2.$$

Quantities $D_1f(x)$ and $D_2f(x)$ denote the marginal values of having one more unit in serviceable inventory and recoverable inventory, respectively. Let $\mathcal{F}$ be the set of all real-valued functions defined on state space Γ such that if $f \in \mathcal{F}$, then

$$D_if(x) \geqslant D_if(x+e_j), \quad i \neq j, \ i = 1, 2, \ j = 1, 2, \quad (3)$$

$$D_if(x+e_j) \geqslant D_if(x+e_i), \quad i \neq j, \ i = 1, 2, \ j = 1, 2, \quad (4)$$

$$D_if(x) \geqslant D_if(x+e_i), \quad i = 1, 2, \quad (5)$$

$$D_1f(x) \leqslant R_1. \quad (6)$$

We say f is *submodular* if it satisfies (3). Submodularity of f implies that the marginal benefit of holding one more unit of one inventory decreases as the other inventory increases. We say f is *diagonal subordinate* if it satisfies condition (4). Condition (4) implies that the benefit of having one more unit of one inventory decreases faster in that inventory than in the other inventory. We say f is *concave* in its coordinates if it meets condition (5). If f is *concave*, the incremental benefit with one more unit of one inventory is decreasing in the level of that inventory. Note that if a function is both submodular and diagonal subordinate then it is also concave. Condition (6) states that the marginal profit of having one more unit of serviceable inventory is at most equal to the per unit revenue obtained from satisfying demand.

The following lemma provides a key step in characterizing the jointly optimal policy by stating that properties (3)–(6) are preserved under the functional operator T. The proof of this and all subsequent results are included in Appendix A.

Lemma 1 (*Preservation*). *If $f \in \mathcal{F}$, then $Tf \in \mathcal{F}$.*

The following lemma shows that the optimal policy, regardless of the size of recoverable inventory, does not prescribe to produce an item when the size of serviceable inventory is beyond a threshold level. Below this threshold, the unit sales revenue is greater than the average cost of holding x_1 units of serviceable inventory during a uniformized period of length γ^{-1}.

Lemma 2 (*Production Threshold*). *Let $N_1 := \min\{x_1 \geqslant 1: x_1h_1/\gamma > R_1\}$. If $f \in \mathcal{F}$ satisfies*

$$D_1f(x) < c_M, \quad x_1 \geqslant N_1 \quad (7)$$

and

$$D_1f(x) < R_1 - x_1h_1/\gamma, \quad x_1 < N_1, \quad (8)$$

then so does Tf.

The following lemma confirms the intuitive notion that the optimal policy will dispose of a returned item, regardless of the size of serviceable inventory, whenever the size of recoverable inventory reaches or exceeds a threshold. Below this threshold the benefit expected from accepting a returned item is greater than the recovery and the holding cost incurred.

Lemma 3 (*Disposal Threshold*). *Let $N_2 := \min\{x_2 \geqslant 1: x_2h_2/\gamma > R_1 - c_r + c_D\}$. If $f \in \mathcal{F}$ satisfies*

$$D_2f(x) < -c_D, \quad x_2 \geqslant N_2 \quad (9)$$

and

$$D_2f(x) < R_1 - c_r - x_2h_2/\gamma, \quad x_2 < N_2, \quad (10)$$

then so does Tf.

Using Lemmas 1–3, we can now characterize the jointly optimal production, remanufacturing, and disposal policy as follows:

Theorem 1 (*Jointly Optimal Policy*).

(a) *There exists an optimal average profit, g, and an optimal relative value function, h, satisfying (2) such that $h \in \mathcal{F}$.*
(b) *The jointly optimal policy can be characterized by three switching curves*

$$P(x_2) := \max\{x_1 : h(x) \leqslant h(x+e_1) - c_M\}, \tag{11}$$
$$R(x_2) := \max\{x_1 : h(x+e_2) \leqslant h(x+e_1) - c_r\}, \tag{12}$$
$$D(x_1) := \min\{x_2 : h(x+e_2) \leqslant h(x) - c_D\} \tag{13}$$

such that in state $x \in \Gamma$, it is optimal (i) to produce an item if $x_1 \leqslant P(x_2)$, (ii) to remanufacture a returned item if $x_1 \leqslant R(x_2)$, and (iii) to dispose of a returned item (when a return occurs) if $x_2 \geqslant D(x_1)$.

Proposition 1 (*Monotonicity of the Optimal Policy*). *Optimal switching curves $P(x_2)$, $R(x_2)$, and $D(x_1)$ are decreasing in x_2, increasing in x_2, and decreasing in x_1, respectively.*

Theorem 1 and Proposition 1 together state that the jointly optimal policy can be defined by three monotone stitching curves: $P(x_2)$, $R(x_2)$, and $D(x_1)$. In particular, Proposition 1 states that it is optimal to produce less of serviceable (recoverable) inventory as recoverable (serviceable) inventory increases. Since a returned item can be converted into a serviceable product, the two types of inventories serve as complements, and hence, if more units are stored in one inventory, it is optimal to store less of the other. $R(x_2)$ being increasing in x_2 implies that it is more profitable to convert returned items into serviceable ones as recoverable inventory increases.

Fig. 2 illustrates the jointly optimal production, remanufacturing, and disposal policy for a numerical example with $R = 100$, $h_1 = 1$, $h_2 = 0.7$, $c_r = 5$, $c_M = 10$, $c_D = 5$, $\lambda_1 = 0.6$, $\mu_1 = 0.5$, $\lambda_2 = 0.2$, and $\mu_2 = 1$. In Fig. 2, three monotone switching curves $P(x_2)$, $R(x_2)$, and $D(x_1)$ separate the state space into seven regions. As an example, in state $x = (1,5)$, it is optimal to produce an item and to remanufacturer a returned item. Moreover, at this state, if a product return occurs, it should be accepted for recovery.

In the following two theorems, we identify conditions under which idling and disposal actions are always optimal when the system is empty. The first condition results in the case where the marginal value of having a unit of serviceable inventory is less than the unit manufacturing cost.

Theorem 2 (*Empty System: Idling*). *If $\lambda_1 R_1 < h_1 + \lambda_1 c_M$, it is always optimal to not produce an item whenever both serviceable and recoverable inventory levels are zero.*

The second result gives a condition under which the marginal benefit of having a unit of recoverable inventory becomes always smaller than $-c_D$ when the system is empty. Hence, under this condition, it becomes optimal to dispose of a returned item based on the switching curve (13).

Theorem 3 (*Empty System: Disposal*). *If $R_1 + c_D - c_r < h_2/\mu_2$, it is always optimal to dispose of a returned item (if occurs) whenever both serviceable inventory and recoverable inventory levels are zero.*

In Theorem 3, the condition $R_1 + c_D - c_r < h_2/\mu_2$ implies that the costs incurred for the recovery activity is larger than the benefits expected from recovery. To see this, suppose that a returned item is accepted for recovery. Then, the unit disposal cost, c_D, is saved and the revenue R_1 can be gained after the returned item is recovered. However, because of the recovery lead-time, it costs $h_2/\mu_2 + c_r$ to convert the returned item into a serviceable product.

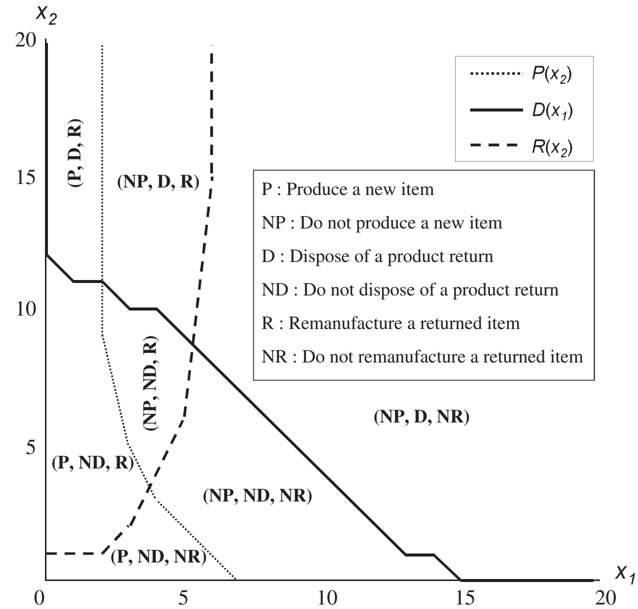

Fig. 2. Graphical representation of optimal switching curves $P(x_2)$, $R(x_2)$, and $D(x_1)$.

4. Impact of the remanufacturing and disposal options

In this section, by comparing scenarios with and without the remanufacturing and disposal options, we examine the benefit of such options for firms under different operating conditions. Our numerical examples and results are summarized in Tables 2 and 3. In Table 2, we examine the impact of the remanufacturing and disposal options on the optimal average profit by varying the values of λ_1, μ_1, λ_2, and μ_2. In Table 3, we examine the impact of the remanufacturing and disposal options on the optimal average profit by varying the values of cost related parameters R_1, h_1, h_2, c_r, c_M, and c_D. We denote by $g^{R,D}$ the optimal average profit of the model with both remanufacturing and disposal options, $g^{NR,D}$ the optimal average profit of the model without remanufacturing option but with disposal option, $g^{R,ND}$ the optimal average profit of the model with remanufacturing option but without disposal option, and $g^{NR,ND}$ the optimal average profit of the model without remanufacturing and disposal options. In Tables 2 and 3, VR (%), VD (%), and VRD (%) are obtained from $(g^{R,ND} - g^{NR,ND})/g^{NR,ND} \times 100$, $(g^{NR,D} - g^{NR,ND})/g^{NR,ND} \times 100$, $(g^{R,D} - g^{NR,ND})/g^{NR,ND} \times 100$, respectively. Therefore, they represent the percentage improvement in the average profit due to the remanufacturing option, disposal option, and both remanufacturing and disposal options, respectively.

Tables 2 and 3 provide several insights into the value of remanufacturing and disposal options. From both tables it can be observed that the average of VR (%) is smaller than the average of VD (%), that is, the remanufacturing option is on average less valuable than the disposal option. In addition, percentage improvement in the average profit due to the remanufacturing option when the disposal option is given ($g^{R,D}/g^{NR,D}$) is smaller on average than that due to the disposal option when the remanufacturing option is given ($g^{R,D}/g^{R,ND}$). These observations suggest that it can be more valuable to control the inventory of returned items through the disposal option than the remanufacturing option.

From Table 2, we also observe that the disposal option is more valuable for cases with a relatively lower demand rate, higher production rate, higher return rate, and lower remanufacturing rate. When λ_1 is low or μ_1 is high, it is likely that the optimal policy will recover less returned items to avoid a large serviceable inventory. Hence, more product returns will be disposed of, which makes the

Table 2
Impact of remanufacturing and disposal options on the optimal average profit as a function of demand and capacity parameters.

No.	R_1	h_1	h_2	c_r	c_M	c_D	λ_1	μ_1	λ_2	μ_2	$g^{NR,ND}$	$g^{R,ND}$	VR (%)	$g^{NR,D}$	VD (%)	$g^{R,D}$	VRD (%)
1	50	2	1	6	10	3	0.25	0.5	0.2	0.9	0.78	3.02	287.2	5.89	655.1	5.98	666.7
2							0.30				5.68	6.61	16.4	7.37	29.8	7.63	34.3
3							0.35				8.12	8.78	8.1	9.06	11.6	9.26	14.0
4							0.40				10.19	10.61	4.1	10.59	3.9	10.79	5.9
5							0.45				11.84	12.18	2.9	12.03	1.6	12.26	3.5
6	50	2	1	6	10	3	0.4	0.5	0.2	0.9	10.19	10.61	4.1	10.59	3.9	10.79	5.9
7								0.6			10.38	10.81	4.1	10.85	4.5	11.02	6.2
8								0.7			10.52	10.95	4.1	11.04	4.9	11.20	6.5
9								0.8			10.62	11.06	4.1	11.19	5.4	11.35	6.9
10								0.9			10.70	11.15	4.2	11.30	5.6	11.46	7.1
11	50	2	1	6	10	3	0.4	0.5	0.1	0.9	10.22	10.40	1.8	10.24	0.2	10.40	1.8
12									0.15		10.34	10.59	2.4	10.44	1.0	10.64	2.9
13									0.2		10.19	10.61	4.1	10.59	3.9	10.79	5.9
14									0.25		9.50	10.24	7.8	10.67	12.3	10.87	14.4
15									0.3		7.70	9.03	17.3	10.71	39.1	10.90	41.6
16	50	2	1	6	10	3	0.4	0.5	0.2	0.7	9.70	10.42	7.4	10.20	5.2	10.63	9.6
17										0.8	9.98	10.52	5.4	10.41	4.3	10.72	7.4
18										0.9	10.19	10.61	4.1	10.59	3.9	10.79	5.9
19										1	10.35	10.67	3.1	10.73	3.7	10.85	4.8
20										1.1	10.48	10.73	2.4	10.84	3.4	10.91	4.1
21	50	2	1	3	10	6	0.25	0.5	0.2	0.9	1.44	3.62	151.4	6.05	320.1	6.25	334.0
22							0.30				6.35	7.21	13.5	7.71	21.4	7.90	24.4
23							0.35				8.79	9.38	6.7	9.43	7.3	9.67	10.0
24							0.40				10.85	11.21	3.3	11.1	2.3	11.29	4.1
25							0.45				12.51	12.78	2.2	12.6	0.7	12.81	2.4
Average (%)													22.9		46.2		49.2

Table 3
Impact of remanufacturing and disposal options on the optimal average profit as a function of cost parameters.

No.	R_1	h_1	h_2	c_r	c_M	c_D	λ_1	μ_1	λ_2	μ_2	$g^{NR,ND}$	$g^{R,ND}$	VR (%)	$g^{NR,D}$	VD (%)	$g^{R,D}$	VRD (%)
26	100	2	1.5	4	10	2	0.4	0.4	0.25	0.8	27.46	27.9	1.6	28.38	3.4	28.62	4.2
27	125										36.76	37.19	1.2	37.66	2.4	37.91	3.1
28	150										46.09	46.59	1.1	47.08	2.1	47.33	2.7
29	175										55.66	56.15	0.9	56.52	1.5	56.82	2.1
30	200										65.23	65.72	0.8	66.03	1.2	66.34	1.7
31	100	3	1.5	4	10	2	0.4	0.4	0.25	0.8	24.46	25.48	4.2	26.09	6.7	26.48	8.3
32		3.5									23.17	24.49	5.7	24.96	7.7	25.53	10.2
33		4									21.89	23.58	7.7	24.10	10.1	24.67	12.7
34		4.5									20.61	22.74	10.3	23.30	13.1	23.96	16.3
35		5									19.34	21.92	13.3	22.50	16.3	23.26	20.3
36	100	2	0.5	4	10	2	0.4	0.4	0.25	0.8	27.92	29.53	5.8	28.72	2.9	29.72	6.4
37			0.7								27.83	29.05	4.4	28.65	2.9	29.39	5.6
38			0.9								27.74	28.71	3.5	28.58	3.0	29.1	4.9
39			1.1								27.64	28.4	2.7	28.51	3.1	28.9	4.6
40			1.3								27.55	28.13	2.1	28.44	3.2	28.74	4.3
41	100	2	1.5	6	10	2	0.4	0.4	0.25	0.8	26.84	27.4	2.1	27.90	3.9	28.19	5.0
42				7							26.53	27.15	2.3	27.67	4.3	27.99	5.5
43				8							26.21	26.9	2.6	27.46	4.8	27.81	6.1
44				9							25.90	26.65	2.9	27.28	5.3	27.64	6.7
45				10							25.59	26.4	3.2	27.11	5.9	27.48	7.4
46	100	2	1.5	4	4	2	0.4	0.4	0.25	0.8	28.19	28.63	1.6	29.54	4.8	29.7	5.4
47					7						27.83	28.27	1.6	28.9	3.8	29.09	4.5
48					10						27.46	27.9	1.6	28.38	3.4	28.62	4.2
49					13						27.1	27.54	1.6	27.92	3.0	28.17	3.9
50					16						26.74	27.18	1.6	27.48	2.8	27.73	3.7
51	100	2	1.5	4	10	3	0.4	0.4	0.25	0.8	27.46	27.9	1.6	28.34	3.2	28.59	4.1
52						4					27.46	27.9	1.6	28.31	3.1	28.56	4.0
53						5					27.46	27.9	1.6	28.27	2.9	28.52	3.9
54						6					27.46	27.9	1.6	28.24	2.8	28.49	3.8
55						7					27.46	27.9	1.6	28.21	2.7	28.45	3.6
Average (%)													3.1		4.6		6.0

role of disposal option significant. When μ_2 is low or λ_2 is high, the system can face a situation in which an excessively large remanufacturable inventory is accumulated. Therefore, for these cases, maintaining the appropriate level of remanufacturable inventory via a disposal option is important. Table 2 also shows that the remanufacturing option is more valuable for cases with the same conditions as those of the disposal option except for the production rate. Interestingly, the value of remanufacturing option is to some extent sensitive to changes in the production rate. However, comparing the results of Examples 1–5 and 21–25, the pattern of percentage improvement in the average profit due to the disposal and remanufacturing options seems to be robust to changes in c_r and c_D.

Moreover, fixing such rates, Table 3 shows the effect of cost parameters on the value of disposal and remanufacturing options. From this table we observe that both remanufacturing and disposal options have a greater percentage improvement in the average profit for cases with a lower revenue, higher holding cost of serviceable inventory, or higher remanufacturing cost. When R_1 is small, h_1 is large, or c_r is large, a smaller fraction of the demands can be fulfilled through remanufacturing. Therefore, more product returns should be disposed of, which makes the role of remanufacturing and disposal options significant. We observe that the remanufacturing option is more valuable with a lower holding cost of remanufacturable inventory while the disposal option is more valuable with a higher one. When h_2 is large, it is expected that the optimal policy is willing to decrease the remanufacturable inventory (thus, restricting the role of remanufacturing option) and dispose of more product returns (thus, enhancing the role of disposal option). Table 3 also shows that the disposal option is more valuable for cases with lower manufacturing and disposal costs. When c_M is small, product recovery will become less valuable, which makes it more significant to control the product return flow using a disposal option. When c_D is small, there is a risk that less remanufacturable inventory can be piled up because disposing of product returns is cheap. Therefore, holding the appropriate level of remanufacturable inventory through a more conservative remanufacturing policy becomes important. Another interesting observation from our results is that the value of remanufacturing option is insensitive to changes in c_M and c_D.

5. Heuristic policies

Since the optimal policy described in Section 3 is rather complex to implement in practice, we develop two types of simpler and more implementable heuristics. We then perform a computational experiment to compare the performance of the proposed heuristics with the optimal policy via a set of test examples.

The first heuristic uses two linear switching curves for jointly determining the production and disposal decisions. Under this heuristic, remanufacturing is pushed as long as items are available in the recoverable inventory. In this context, we term the first heuristic a PUSH control strategy. This heuristic can be described as follows using two integer parameters I_P and I_D:

- A new item is produced if $x_1 + x_2 \leqslant I_P$.
- Whenever $x_2 \geqslant 1$, the remanufacturing process is activated.
- An incoming product return is disposed of if $x_1 + x_2 \geqslant I_D$.

Appropriate values of two integer parameters I_P and I_D are found using a two-dimensional search.

The second heuristic uses one threshold and two linear switching curves for jointly determining the production, remanufacturing, and disposal decisions, which is motivated by the results of Theorem 1. Since remanufacturing of a returned item is postponed until it is needed, the second heuristic is termed a PULL control

Table 4
Comparison of the optimal and heuristic policies.

No.	R_1	h_1	h_2	c_r	c_M	c_D	λ_1	μ_1	λ_2	μ_2	g^{Opt}	First heuristic (Push)				Second heuristic (Pull)				
												g^H	I_P	I_D	%	g^H	I_P	I_R	I_D	%
1	120	1	0.5	2	20	3	0.2	0.5	0.2	1	19.33	19.07	2	4	1.35	19.28	1	2	5	0.26
2							0.3				29.32	28.95	3	6	1.26	29.28	2	2	8	0.14
3							0.4				38.62	38.29	4	9	0.85	38.51	3	3	12	0.28
4							0.5				47.5	47.00	6	13	1.05	47.4	5	3	12	0.21
5	80	3	1.5	5	10	5	0.2	0.8	0.2	1	8.52	8.48	1	2	0.47	8.48	0	1	2	0.47
6							0.3				14.42	14.12	2	3	2.08	14.38	1	1	3	0.28
7							0.4				20.53	20.38	2	3	0.73	20.43	1	1	3	0.49
8							0.5				26.21	25.92	3	4	1.11	26.11	2	2	5	0.38
9	100	1	0.75	5	20	1	0.2	0.5	0.2	1	14.91	14.84	2	4	0.47	14.9	1	2	4	0.07
10							0.3				22.67	22.57	3	6	0.44	22.65	2	2	6	0.09
11							0.4				30.04	29.94	4	8	0.33	30.02	3	3	9	0.07
12							0.5				36.94	36.85	5	11	0.24	36.91	4	3	11	0.08
13	120	3	1	2	10	7	0.2	0.5	0.2	0.3	14.82	14.13	2	2	4.66	14.37	1	2	2	3.04
14							0.3				24.24	23.05	3	3	4.91	23.41	2	4	3	3.42
15							0.4				33.23	31.82	4	4	4.24	32.31	3	4	4	2.77
16							0.5				41.84	40.31	5	5	3.66	40.97	4	3	5	2.08
17	80	1	0.2	2	10	1	0.2	0.5	0.2	0.3	11.96	11.12	3	3	7.02	11.66	2	2	4	2.51
18							0.3				18.46	17.32	4	4	6.18	17.96	3	3	5	2.71
19							0.4				24.7	23.43	5	5	5.14	24.08	4	4	6	2.51
20							0.5				30.65	29.29	6	6	4.44	29.96	5	4	6	2.25
21	100	3	1.5	5	20	5	0.2	0.8	0.2	0.3	10.68	9.40	1	0	11.99	9.97	1	2	3	6.65
22							0.3				17.51	16.21	2	1	7.42	16.94	1	2	3	3.26
23							0.4				24.47	22.39	3	2	8.50	23.52	2	3	4	3.88
24							0.5				31.14	29.08	3	2	6.62	30.08	2	3	4	3.40
Average of Opt Gap (%)															3.55					1.72

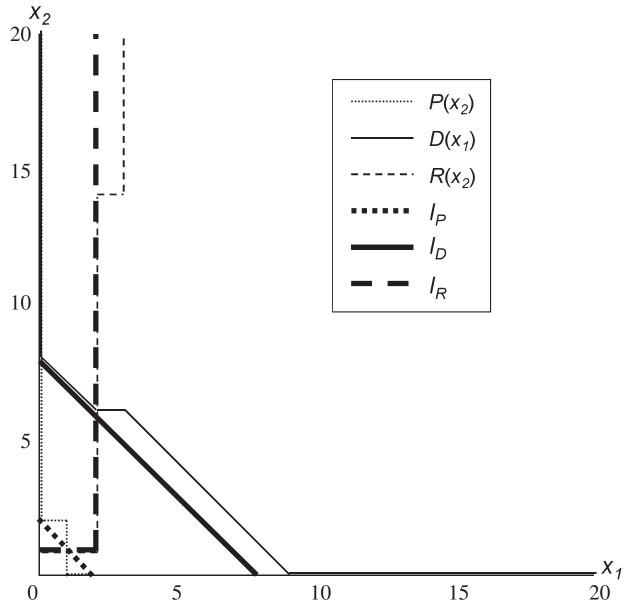

Fig. 3. Structural comparison of the optimal and the PULL heuristic policy.

strategy. This policy can be described using three integer parameters I_P, I_R, and I_D:

- A new item is produced if $x_1 + x_2 \leqslant I_P$.
- When $x_2 \geqslant 1$, the remanufacturing process is activated if $x_1 \leqslant I_R$.
- An incoming product return is disposed of if $x_1 + x_2 \geqslant I_D$.

Appropriate values of three integer parameters I_P, I_R, and I_D are found using a three-dimensional search. Since Lemmas 2 and 3 establish the upper bounds on x_1 and x_2, N_1 and N_2, respectively, it is guaranteed that the search will be terminated within a finite number of steps.

Table 4 compares the performance of the optimal and the two heuristic policies for 24 numerical examples. In this table, g^{Opt} is the average profit under the optimal policy and g^H is the average profit under the heuristic policies. Test results in Table 4 show that these simple heuristics perform very well. The average optimality gap for the first and second heuristic 3.55% and 1.72%, respectively. As is expected, the second heuristic outperforms the first one, as it provides a better approximation for the optimal remanufacturing threshold. However, it should be noted that although the second heuristic has econimical advantages over the first heuristic, implementing the second heuristic is harder, as it requires optimizing an additional parameter, I_R. Furthermore, in practice, the first heuristic may be preferable, since serviceable inventory and remanufacturable inventory can be controlled independently. From Table 4, we find that the optimality gap of the second heuristic is almost negligible when $\mu_2 = 1$ and $\lambda_2 = 0.2$ but it is relatively large when $\mu_2 = 0.3$ and $\lambda_2 = 0.2$. In fact, our extensive tests reveal that the proposed heuristic performs relatively better when λ_2/μ_2 is small. Table 4 also shows that I_P, I_R, and I_D are all weakly increasing in λ_1.

Fig. 3 illustrates the structural difference and similarity of the optimal policy and the second heuristic policy using Example 2 of Table 4. As shown in Fig. 3, remanufacturing control thresholds in the optimal and the second heuristic policies are exactly the same when $x_2 \leqslant 14$. The difference is only when $x_2 > 14$. In this case, the optimal policy remanufactures a returned item as long as $x_1 \leqslant 3$ while the heuristic policy remanufactures a returned item only when $x_1 \leqslant 2$. Fig. 3 also shows that disposal controls of the optimal and heuristic policies are the same when $x_1 \leqslant 2$ and they differ by one, that is, $D(x_1) = I_D + 1$, when $x_1 > 2$. In addition, we observe from the figure that production controls in the optimal and the second heuristic policies are the same when $x_1 \geqslant 2$. The difference is when $x_1 \leqslant 1$. When $x_1 = 1$, the optimal policy produces a new item if $x_2 \leqslant 2$ while the heuristic policy produces a new item if $x_2 \leqslant 1$. When $x_1 = 0$, the optimal policy produces a new item regardless of x_2 while the heuristic policy produces a new item only if $x_2 \leqslant 2$. These structural similarities of production, remanufacturing, and disposal controls between the optimal and heuristic policies explains why our heuristic works very well for this example, and the test suite suggests that our heuristic in general provides a very good approximation of the optimal policy.

6. Conclusions

We generated insights into the coordination of production, remanufacturing, and disposal decisions for a product recovery system where serviceable inventory can be replenished through either product recovery and remanufacturing or new manufacturing. We showed that, under an optimal policy, production, remanufacturing, and disposal decisions are jointly controlled according to three monotone switching curves. We identified the conditions which guarantee that idling and disposal actions are always optimal when the system is empty. We also showed that there exists a serviceable inventory limit above which production cannot be optimal and a recoverable inventory limit above which disposal is always optimal.

To examine the impact of remanufacturing and disposal options on the performance of the system, we implemented performance comparison between models with and without these options. We found that the the value of having these options together is higher than the sum of their individual values which declares a synergy between remanufacturing and disposing of returned items. Our results also show that the value of the disposal option is higher than that of remanufacturing on average. This highlights the importance of allowing for disposing of returned items in manufacturing–remanufacturing systems. We also generated several insights into the operational conditions under which the disposal and the remanufacturing options are more valuable.

Since the structure of the optimal policy is rather complex, we developed two simple threshold heuristics to effectively coordinate disposal, remanufacturing, and production activities. In one of the proposed heuristics, the production and disposal decisions are based on both serviceable and recoverable inventories according to threshold rules that depend on the sum of serviceable and remanufacturable inventories, and the remanufacturing decision is only based on serviceable inventory. Extensive numerical tests showed that this proposed heuristic is particularly very effective. On average, this proposed heuristic shows a 1.72% optimality gap and provides an effective tool to jointly make production, remanufacturing, and disposal decisions.

The properties and insights provided in this paper can be very useful for understanding and addressing more realistic settings with arbitrary probability distributions (i.e., other than exponential), since it is not typically tractable to establish the structure of the optimal policy when processes follow arbitrary distributions. Another direction for future research is to consider situations where the product return intensity can be controlled by the compensation given to the customer to induce the return of a used product. Such future research can further advance the practice and provide more incentives for firms to incorporate sustainability in their operations.

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Appendix A

Denote by $p_{(\cdot)}(=0,1)$ the optimal production control in state $(\cdot)$ where $p_{(\cdot)}=1$ and $p_{(\cdot)}=0$ represent *Produce* and *Do not produce* actions, respectively. Denote by $d_{(\cdot)}(=0,1)$ the optimal disposal control in state $(\cdot)$ where $d_{(\cdot)}=1$ and $d_{(\cdot)}=0$ represent *Dispose of*, and *Do not dispose of* actions, respectively. Denote by $r_{(\cdot)}(=0,1)$ the optimal remanufacturing control in state $(\cdot)$ where $r_{(\cdot)}=1$ and $r_{(\cdot)}=0$ represent *Remanufacture*, and *Do not remanufacture* actions, respectively. We present the following properties of $p_{(\cdot)}$, $d_{(\cdot)}$, and $r_{(\cdot)}$:

$$p_{(x)} \geqslant p_{(x+e_1)} (P1); \quad p_{(x+e_1)} \leqslant p_{(x+e_2)} (P2); \quad p_{(x)} \geqslant p_{(x+e_2)} (P3);$$
$$d_{(x)} \leqslant d_{(x+e_1)} (D1); \quad d_{(x+e_1)} \leqslant d_{(x+e_2)} (D2); \quad d_{(x)} \leqslant d_{(x+e_2)} (D3);$$
$$r_{(x)} \geqslant r_{(x+e_1)} (R1); \quad r_{(x+e_1)} \leqslant r_{(x+e_2)} (R2); \quad r_{(x)} \leqslant r_{(x+e_2)} (R3).$$

Properties P1 and D3 follows from concavity of f, properties P3 and D1 from submodularity of f, and properties of P2 and D2 from diagonal subordinance of f. Properties R1–R3 follow from diagonal subordinance of f.

Define $T_1f(x) = (R_1 + f(x-e_1))1(x_1>0) + f(x)1(x_1=0)$, $T_2f(x) = \max\{f(x+e_1) - c_M, f(x)\}$, $T_3f(x) = \max\{f(x+e_2), f(x) - c_D\}$, and $T_4f(x) = \max\{-c_r + f(x+e_1-e_2), f(x)\}\ 1(x_2>0) + f(x)1(x_2=0)$. Then, $Tf(x) = \frac{1}{\gamma}[-x_1h_1 - x_2h_2 + \lambda_1 T_1f(x) + \mu_1 T_2f(x) + \lambda_2 T_3 f(x) + \mu_2 T_4 f(x)]$.

Proof of Lemma 1

(i) Since $D_1Tf(x) - D_1Tf(x+e_2) = D_2Tf(x) - D_2Tf(x+e_1)$, we show that $D_1Tf(x) - D_1Tf(x+e_2) \geqslant 0$. Let $\Delta^k = D_1T_kf(x) - D_1T_kf(x+e_2)$.

1. If $x_1>0$, $\Delta^1 \geqslant 0$ by (3) since $f \in F$. If $x_1 = 0$, $\Delta^1 = 0$.
2. By P1–P3, $(p_{(x+e_1)}, p_{(x)}, p_{(x+e_1+e_2)}, p_{(x+e_2)})$ are as follows: For $(0,0,0,0)$ and $(1,1,1,1)$, $\Delta^2 \geqslant 0$ by (3). For $(0,1,0,1)$, $(1,1,0,1)$, and $(0,1,0,0)$, $\Delta^2 \geqslant f(x+e_1) - (f(x+e_1) - c_M) - (f(x+e_1+e_2) - (f(x+e_1+e_2) - c_M)) = 0$.
3. By D1–D3, $(d_{(x+e_1)}, d_{(x)}, d_{(x+e_1+e_2)}, d_{(x+e_2)})$ are as follows: For $(0,0,0,0)$ and $(1,1,1,1)$, $\Delta^4 \geqslant 0$ by (3). For $(0,0,1,1)$, $(1,0,1,1)$, and $(0,0,1,0)$, $\Delta^3 \geqslant f(x+e_1+e_2) - f(x+e_2)$ - $^3 \geqslant f(x+e_1+e_2) - f(x+e_2) - [(f(x+e_1+e_2) - c_D)$ - $e_2) - f(x+e_2) - [(f(x+e_1+e_2) - c_D) - (f(x+e_2) - c_D)] = 0$.
4. By R1–R3, $(r_{(x+e_1)}, r_{(x)}, r_{(x+e_1+e_2)}, r_{(x+e_2)})$ are as follows: For $(0,0,0,0)$ and $(0,0,0,1)$, $\Delta^4 \geqslant D_1f(x) - D_1f(x+e_2) \geqslant 0$ by (3). For $(1,1,1,1)$ and $(0,1,1,1)$, $\Delta^4 \geqslant D_1f(x+e_1-e_2) - D_1$- $^4 \geqslant D_1f(x+e_1-e_2) - D_1f(x+e_1) \geqslant 0$ by (3). For $(0,1,0,1)$, $\Delta^4 = f(x+e_1) - f(x+e_1-e_2) - (fx+\ e_1+e_2) - f(x+e_1) = D_2$- $_1+e_2) - f(x+e_1) = D_2f(x+e_1-e_2) - D_2f(x+e_1) \geqslant 0$ by (5). For $(0,0,1,1)$, $\Delta^4 = D_1f(x) - D_1f(x+e_1) \geqslant 0$ by (5).
   Therefore, $D_1Tf(x) - D_1Tf(x+e_2) = 1/\gamma[\lambda_1\Delta^1 + \mu_1\Delta^2 + \lambda_2\Delta^3 + \mu_2\Delta^4] \geqslant 0$.

(ii) Suppose that $i=1$ and $j=2$. Let $\Delta^k = D_1T_kf(x+e_2) - D_1T_kf(x+e_1)$.

1. If $x_1>0$, $\Delta^1 \geqslant 0$ by (4). If $x_1=0$, $\Delta^1 = R - D_1f(x) \geqslant 0$ by (6).
2. By P1–P3, $(p_{(x+e_1+e_2)}, p_{(x+e_2)}, p_{(x+2e_1)}, p_{(x+e_1)})$ are as follows: For $(0,0,0,0)$ and $(1,1,1,1)$, $\Delta^2 \geqslant 0$ by (4). For $(0,1,0,1)$, $(1,1,0,1)$, and $(0,1,0,0)$, $\Delta^2 \geqslant f(x+e_1+e_2) - (f(x+e_1+e_2) - c_M) - (f(x+2e_1) - (f(x+2e_1) - c_M)) = 0$.
3. By D1–D3, $(d_{(x+e_1+e_2)}, d_{(x+e_2)}, d_{(x+2e_1)}, d_{(x+e_1)})$ are as follows: For $(0,0,0,0)$ and $(1,1,1,1)$, $\Delta^3 \geqslant 0$ by (4). For $(1,0,1,0)$, $\Delta^3 = -c_D + f(x+e_1+e_2) - f(x+2e_2) - (-c_D + f(x+2e_1) - f(x+\ e_1+e_2)) \geqslant f(x+e_2) - f(x+e_1) - (f(x+2e_2) - f(x+e_1+e_2))$ (by (4)) $\geqslant 0$ by (4). $(1,1,1,0)$ and $(1,0,0,0)$ can be shown using the results of $(1,1,1,1)$ and $(0,0,0,0)$, respectively.
4. By R1–R3, $(d_{(x+e_1+e_2)}, d_{(x+e_2)}, d_{(x+2e_1)}, d_{(x+e_1)})$ are as follows: For $(0,0,0,0)$, $\Delta^4 = D_1f(x+e_2) - D_1f(x+e_1) \geqslant 0$ by (4). For $(1,1,1,1)$, $\Delta^4 = D_1f(x+e_1) - D_1f(x+2e_1-e_2) \geqslant 0$ by (4). For $(1,1,0,0)$, $\Delta^4 = D_1f(x+e_1) - D_1f(x+e_1) = 0$. For $(0,1,0,0)$ and $(1,1,0,1)$, $\Delta^4 \geqslant D_1f(x+e_1) - D_1f(x+e_1) = 0$.

Now suppose that $i=2$ and $j=1$. Let $\Delta^k = D_2T_kf(x+e_1) - D_2T_kf(x+e_2)$.

1. If $x_1>0$, $\Delta^1 \geqslant 0$ by (4). If $x_1=0$, $\Delta^1 = D_2f(x) - D_2f(x+e_2) \geqslant 0$ by (5).
2. By P1–P3, $(p_{(x+e_1+e_2)}, p_{(x+e_1)}, p_{(x+2e_2)}, p_{(x+e_2)})$ are as follows: For $(0,0,0,0)$ and $(1,1,1,1)$, $\Delta^2 \geqslant 0$ by (4). For $(0,0,1,1)$, $\Delta^2 = D_2f(x+e_1) - D_2f(x+e_1+e_2) \geqslant 0$ (by (5)). For $(0,1,0,1)$, Δ^2- $^2 = f(x+e_1+e_2) - (f(x+2e_1) - c_M) - [f(x+2e_2) - (f(x+e_1+$- $e_2) - c_M)] \geqslant f(x+e_1) - f(x+2e_1) - (f(x+e_2)) - f(x+e_1+e_2)$ (by (4)) $\geqslant 0$ by (4). $(0,1,1,1)$ and $(0,0,0,1)$ can be shown using the result of $(1,1,1,1)$ and $(0,0,0,0)$, respectively.
3. By D1–D3, $(d_{(x+e_1+e_2)}, d_{(x+e_1)}, d_{(x+2e_2)}, d_{(x+e_2)})$ are as follows: For $(0,0,0,0)$ and $(1,1,1,1)$, $\Delta^3 \geqslant 0$ by (4). For $(1,0,1,0)$, $(1,0,1,1)$, and $(0,0,1,0)$, $\Delta^3 \geqslant f(x+e_1+e_2) - c_D - f(x+e_1+e_2) - [f(x+2e_2) - c_D - f(x+2e_2)] = 0$.
4. By R1–R3, $(r_{(x+e_1+e_2)}, r_{(x+e_1)}, r_{(x+2e_2)}, r_{(x+e_2)})$ are as follows: For $(0,0,0,0)$ and $(1,1,1,1)$, $\Delta^4 \geqslant 0$ by (4). For $(0,0,1,1)$, $\Delta^4 = D_2$- $f(x+e_1) - D_2f(x+e_1) = 0$. For $(0,1,0,0)$ and $(1,1,0,1)$, Δ^4- $^4 \geqslant D_2f(x+e_1) - D_2f(x+e_1) = 0$.
   Therefore, $D_if(x+e_j) - D_if(x+e_i) = 1/\gamma[\lambda_1\Delta^1 + \mu_1\Delta^2 + \lambda_2\Delta^3 + \mu_2\Delta^4] \geqslant 0$.

(iii) $D_iTf(x) \geqslant D_iTf(x+e_j)$ (by (3)) $\geqslant D_iTf(x+e_i)$ by (4).

(iv) 1. $D_1T_1f(x) = R_1 1\{x_1=0\} + D_1f(x-e_1)1\{x_1>0\} \leqslant R_1$ by (6). Consider cases in $(p_{(x+e_1)}, p_{(x)})$. $p_{(x)} \geqslant p_{(x+e_1)}$ by P1. For $(1,1)$, $D_1T_2f(x) = D_1f(x+e_1)$ and, for $(0,0)$ and $(0,1)$, $D_1T_2f(x) \leqslant D_1 f(x) \leqslant R_1$. Hence, $D_1T_2f(x) \leqslant R_1$. Consider cases in $(d_{(x+e_1)}, d_{(x)})$. $d_{(x)} \leqslant d_{(x+e_1)}$ by D1. For $(0,0)$, $D_1 T_3f(x) = D_1f(x+e_2)$, for $(1,1)$ and $(1,0)$, $D_1T_3f(x) \leqslant D_1 f(x)$. Hence, $D_1T_3f(x) \leqslant R_1$. Consider cases in $(r_{(x+e_1)}, r_{(x)})$. $r_{(x)} \geqslant r_{(x+e_1)}$ by R1. For $(1,1)$, $D_1T_4 f(x) = D_1f(x+e_1-e_2)$. For $(0,0)$ and $(0,1)$, $D_1\ T_4f(x) \leqslant D_1f(x)$. Hence, $D_1T_4f(x) \leqslant R_1$. Therefore, $D_1Tf(x) = 1/\gamma[-h_1 + \lambda_1D_1T_1 f(x) + \mu_1D_1\ T_2f(x) + \lambda_2D_1T_3f(x) + \mu_2D_1T_4f(x)] \leqslant R_1$.

Proof of Lemma 2. We show that (7) and (8) are preserved under T. Let $\Delta^k = D_1T_kf(x)$.

(i)

1. If $x_1 > N_1$, $\Delta^1 = D_1f(x_1-1, x_2) < c_M$ by (7). If $x_1 = N_1$, $\Delta^1 = D_1$- $f(N_1-1, x_2) <\ R_1 - (N_1-1)h_1/\gamma$ by (8).
2. $p_{(x)} = 0$ by (7) and thus $p_{(x+e_1)} = 0$ by (5). Hence, $\Delta^2 = D_1$- $f(x) < c_M$ by (7).
3. Consider cases in $(d_{(x+e_1)}, d_{(x)})$. Case $(0,1)$ is excluded by D1. For $(0,0)$ and $(1,0)$, $\Delta^3 \leqslant D_1f(x)$. For $(1,1)$, $\Delta^3 = D_1f(x+e_2)$. Hence $\Delta^3 < c_M$ by (7).
4. Consider cases in $(r_{(x+e_1)}, r_{(x)})$. Case $(1,0)$ is excluded by R2. For $(0,0)$ and $(0,1)$, $\Delta^4 \leqslant D_1f(x)$. For $(1,1)$, $\Delta^4 = D_1f(x+e_1$- $-e_2) \leqslant D_1f(x)$ (by (4)). Hence, $\Delta^4 < c_M$ by (7).
   Let $\Delta = D_1Tf(x)$. When $x_1 = N_1$, $\Delta < 1/\gamma[-h_1 + \lambda_1(R_1 - (N_1 - 1)h_1/\gamma) + (\mu_1+\mu_2+\lambda_2)c_M] = 1\ \ /\gamma[-h_1 + \lambda_1/\gamma h_1 + \lambda_1(R_1 - N_1 h_1/\gamma) + (\mu_1+\mu_2+\lambda_2)c_M] < 1/\gamma(\mu_1+\mu_2+\lambda_2)c_M$ (by $\lambda_1/\gamma < 1$, $R_1 < N_1h_1/\gamma) < c_M$. When $x_1 > N_1$, $\Delta = 1/\gamma[-h_1 + \gamma c_M] < c_M$.

(ii)

1. When $x_2 > 0$, $\Delta^1 = D_1f(x_1-1, x_2) < R_1 - (x_1-1)h_1/\gamma$ by (8). Otherwise, $\Delta^1 = R_1$.
2. Consider cases in $(p_{(x+e_1)}, p_{(x)})$. $(1,0)$ is excluded by P3. For $(1,1)$, $\Delta^2 = D_1f(x+e_1) \leqslant D_1f(x)$ (by (5)). For $(0,0)$ and $(0,1)$, $\Delta^2 \leqslant D_1f(x)$. Hence $\Delta^2 < R_1 - x_1h_1/\gamma$ by (8).

3. Consider cases in $(d_{(x+e_1)}, d_{(x)})$. (0,1) is excluded by D1. For (1,1) and (1,0), $\Delta^3 \leqslant D_1 f(x)$. For (0,0), $\Delta^3 = D_1 f(x + e_2)$. Hence, $\Delta^3 < R_1 - x_1 h_1/\gamma$ by (8).
4. Consider cases in $(r_{(x+e_1)}, r_{(x)})$. Case (1,0) is excluded by R2. For (0,0) and (0,1), $\Delta^4 \leqslant D_1 f(x)$. For (1,1), $\Delta^4 = D_1 f(x + e_1 - e_2) \leqslant D_1 f(x)$ (by (4)). Hence, $\Delta^4 < R_1 - x_1 h_1/\gamma$ by (8).
 $\Delta := D_1 Tf(x)$: When $x_1 > 0, \Delta < 1/\gamma[-h_1 + \lambda_1(R_1 - (x_1 - 1)h_1/\gamma) + (\mu_1 + \mu_2 + \lambda_2)(R_1 - x_1 h_1/\gamma)] = 1/\gamma[-h_1 + \lambda_1 h_1/\gamma + \gamma(R_1 - x_1 h_1/\gamma)] < R_1 - x_1 h_1/\gamma$. When $x_1 = 0$, $\Delta < 1/\gamma[-h_1 + \lambda_1 R_1 + (\mu_1 + \mu_2 + \lambda_2)R_1] < R_1$.

Proof of Lemma 3. We show that (9) and (10) are preserved under T. Let $\Delta^k = D_2 T_k f(x)$.

(i)
1. Since demand does not affect the size of x_2, $\Delta^1 < -c_D$ by (9).
2. Consider cases in $(p_{(x+e_2)}, p_{(x)})$. (1,0) is excluded by P1. For (1,1), $\Delta^2 = D_2 f(x + e_1)$. For (0,0) and (0,1), $\Delta^2 \leqslant D_2 f(x)$. Hence, $\Delta^2 < -c_D$ by (9).
3. $d_{(x)} = 1$ by (9) and thus $d_{(x+e_2)} = 1$ by (5). Hence, $\Delta^3 = D_2 f(x) < -c_D$ by (9).
4. Consider cases in $(r_{(x+e_2)}, r_{(x)})$. Case (1,0) is excluded by R2. For (0,0), $\Delta^4 < -c_D$ by (9). For (1,1) and (1,0), $\Delta^4 < -c_D$ by (9) when $x_2 > N_2$, and $\Delta^4 < R_1 - c_r - (N_2 - 1)h_2/\gamma$ by (10) when $x_2 = N_2$.
 Let $\Delta = D_2 Tf(x)$. When $x_2 = N_2$, $\Delta < 1/\gamma[-h_2 + (\lambda_1 + \mu_1 + \lambda_2)(-c_D) + \mu_2(R_1 - c_r - (N_2 - 1)h_2/\gamma)] = 1/\gamma[-h_2 + \mu_2/\gamma h_2 + (\lambda_1 + \mu_1 + \lambda_2)(-c_D) + \mu_2(R_1 - c_r - N_2 h_2/\gamma)] < 1/\gamma[-h_2 + \mu_2/\gamma h_2 + \gamma(-c_D)]$ (using definition of N_2) $< -c_D$. When $x_2 > N_2$, $\Delta < 1/\gamma[-h_2 + \gamma(-c_D)] < -c_D$.

(ii)
1. Since demand does not affect the size of x_2, $\Delta^1 < R_1 - c_r - x_2 h_2/\gamma$ by (10).
2. Consider cases in $(p_{(x+e_2)}, p_{(x)})$. (1,0) is excluded by P1. For (1,1), $\Delta^2 = D_2 f(x + e_1)$. For (0,0) and (0,1), $\Delta^2 \leqslant D_2 f(x)$. $\Delta^2 < R_1 - c_r - x_2 h_2/\gamma$ by (10).
3. Consider cases in $(d_{(x+e_2)}, d_{(x)})$. (0,1) is excluded by D3. For (1,1) and (1,0), $\Delta^4 \leqslant D_2 f(x)$. For (0,0), $\Delta^3 = D_2 f(x + e_2) \leqslant D_2 f(x)$ by (5). Hence, $\Delta^3 < R_1 - c_r - x_2 h_2/\gamma$ by (10).
4. Consider cases in $(r_{(x+e_2)}, r_{(x)})$. Case (0,1) is excluded by R2. For (0,0), $\Delta^4 < R_1 - c_r - x_2 h_2/\gamma$ by (10). For (1,1) and (1,0), $\Delta^4 < R_1 - c_r - (x_2 - 1)h_2/\gamma$ by (10).
 Let $\Delta = D_2 Tf(x)$. When $x_2 > 0$, $\Delta < 1/\gamma[-h_2 + (\lambda_1 + \mu_1 + \lambda_2)(R_1 - c_r - x_2 h_2/\gamma) + \mu_2(R_1 - c_r - (x_2 - 1)h_2/\gamma)] = 1/\gamma[-h_2 + \mu_2/\gamma h_2 + \gamma(R_1 - c_r - x_2 h_2/\gamma)] < R_1 - c_r - x_2 h_2/\gamma$. When $x_2 = 0$, $\Delta < 1/\gamma[-h_2 + \gamma(R_1 - c_r)] < R_1 - c_r$.

Proof of Theorem 1

(i) From Lemmas 2 and 3the original problem with infinite state space can be converted into one with finite state space. Since the model has a finite action space and is unichain and aperiodic, the result follows from Theorem 8.4.5 of Puterman (2005).
(ii) The first part for optimal production control is due to P1, the second par for optimal remanufacturing control is due to R1, and the third part for optimal disposal control is due to D3.

Proof of Proposition 1. Decreasing of $P(x_2)$ in x_2 is due to P3, increasing of $R(x_2)$ in x_2 is due to R3, and decreasing of $D(x_1)$ in x_1 is due to D1.

Proof of Theorem 2. It is sufficient to show that if $\lambda_1 R < h_1 + \lambda_1 c_M$,

$$D_1 f(x) < c_M, \quad x_1 = x_2 = 0 \tag{14}$$

is preserved under operator T.

1. $D_1 T_1 f(x) = R_1 + f(x) - f(x) = R_1$.
2. By (14), $p_{(x)} = 0$. Hence, $p_{(x+e_1)} = 0$ by P1 and $D_1 T_2 f(x) = D_1 f(x) < c_M$ by (14).
3. Consider cases in $(d_{(x+e_1)}, d_{(x)})$. For (1,1) and (1,0), $D_1 T_3 f(x) \leqslant D_1 f(x) < c_M$ by (14). For (0,0), $D_1 T_3 f(x) = D_1 f(x + e_2) \leqslant D_1 f(x) < c_M$ by (14). Case (0,1) is excluded by P1.
4. Since $x_2 = 0$, $r_{(x+e_1)} = r_{(x)} = 0$ and thus $D_1 T_4 f(x) = D_1 f(x) < c_M$.
 Hence, $D_1 Tf(x) < 1/\gamma[-h_1 + \lambda_1 R_1 + (\mu_1 + \lambda_2 + \mu_2) c_M] = 1/\gamma[-h_1 + \lambda_1 R_1 + (\gamma - \lambda_1)c_M] < c_M$ (by assumption).

Proof of Theorem 3. It is sufficient to show that if $R_1 + c_D - c_r < h_2/\mu_2$,

$$D_2 f(x) < -c_D, \quad x_1 = x_2 = 0 \tag{15}$$

is preserved under operator T.

1. $D_2 T_1 f(x) = D_2 f(x) < -c_D$.
2. Consider cases in $(p_{(x+e_1)}, p_{(x)})$. Case (1,0) is excluded by P1. For (0,0) and (0,1), $D_2 T_2 f(x) \leqslant D_2 f(x)$. For (1,1), $D_2 T_2 f(x) = D_2 f(x + e_1) \leqslant D_2 f(x)$ by (3). Hence, $D_2 T_2 f(x) < -c_D$ by (15).
3. Since $D_2 f(x) < -c_D$, $f(x + 2e_2) < f(x + e_1) - c_D$ by D3, $D_2 T_3 f(x) = D_2 f(x) < -c_D$.
4. Since $x_2 = 0$, $r(x) = 0$. When $r(x + e_2) = 1$, $D_2 T_4 f(x) = f(x + e_1) - c_r - f(x) \leqslant R_1 - c_r$ by (6). When $r(x + e_2) = 0$, $D_2 T_4 f(x) = D_2 f(x) < -c_D$ (by (15)) $< R_1$.
 Hence, $D_2 Tf(x) \leqslant 1/\gamma[-h_2 + (\lambda_1 + \mu_1 + \lambda_2)(-c_D) + \mu_2(R_1 - c_r)] < 1/\gamma[-h_2 + (\gamma - \mu_2)(-c_D) + \mu_2(-c_D + h_2/\mu_2)]$ (by assumption) $= -c_D$.

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