Multi-criteria Group Decision Making Using A Modified Fuzzy TOPSIS Procedure

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***Abstract*-In this paper we propose a modified Fuzzy Technique for Order Performance by Similarity to Ideal Solution (modified Fuzzy TOPSIS) for the Multi-criteria Decision Making (MCDM) problem when there is a group of decision makers. Regarding the value of the truth that a fuzzy number is greater than or equal to another fuzzy number, a new distance measure is proposed in this paper. This distance measure calculates the distance of each fuzzy number from both Fuzzy Positive Ideal Solution (FPIS) and Fuzzy negative Ideal Solution (FNIS). Then, the alternative which is simultaneously closer to FPIS and farther from FNIS will be selected as the best choice. To clarify our proposed procedure, a numerical example is discussed.**

I. INTRODUCTION

Decision making problem is the process of finding the best option from all of the feasible alternatives. In almost all such problems the multiplicity of criteria for judging the alternatives is pervasive. That is, for many such problems, the decision maker wants to solve a multiple criteria decision making (MCDM) problem. A MCDM problem can be concisely expressed in matrix format as:

$$\boldsymbol{D} = \begin{array}{c} \\ A_1 \\ A_2 \\ \vdots \\ A_m \end{array} \begin{array}{c} \begin{array}{cccc} C_1 & C_2 & \cdots & C_n \end{array} \\ \left[\begin{array}{cccc} x_{11} & x_{12} & \cdots & x_{1n} \\ x_{21} & x_{22} & \cdots & x_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ x_{m1} & x_{m2} & \cdots & x_{mn} \end{array}\right] \end{array}, \quad (1)$$

$$W = [w_1 \ w_2 \ \cdots \ w_n], \quad (2)$$

where $A_1, A_2, ..., A_m$ are possible alternatives among which decision makers have to choose, $C_1, C_2, ..., C_m$ are criteria with which alternative performance are measured, x_{ij} is the rating of alternative A_i with respect to criterion C_j and w_j is the weight of criterion C_j .

A survey of the MCDM methods has been presented by Hwang and Yoon [1]. Technique for Order Performance by Similarity to Ideal Solution (TOPSIS), one of the known classical MCDM methods, also was first developed by Hwang and Yoon [1]. It bases upon the concept that the chosen alternative should have the shortest distance from the Positive Ideal Solution (PIS), i.e., the solution that maximizes the benefit criteria and minimizes the cost criteria; and the farthest from the Negative Ideal Solution (NIS), i.e., the solution that maximizes the cost criteria and minimizes the benefit criteria.

In classical MCMD methods, including classical TOPSIS, the ratings and the weights of the criteria are known precisely. However, under many conditions, crisp data are inadequate to model real-life situations since human judgments including preferences are often vague and cannot estimate his preference with an exact numerical value. A more realistic approach may be to use linguistic assessments instead of numerical values, that is, to suppose that the ratings and weights of the criteria in the problem are assessed by means of linguistic variables. Lingual expressions, for example, low, medium, high, etc. are regarded as the natural representation of the judgment. These characteristics indicate the applicability of fuzzy set theory in capturing the decision makers' preference structure. Fuzzy set theory aids in measuring the ambiguity of concepts that are associated with human being's subjective judgment. Moreover, since in the group decision making, evaluation is resulted from different evaluator's view of linguistic variables, its evaluation must be conducted in an uncertain, fuzzy environment.

There are many examples of applications of fuzzy TOPSIS in literature (For instance: The evaluation of service quality [2]; Inter company comparison [3]; The applications in aggregate production planning [4], Facility location selection [5] and large scale nonlinear programming [6]). The modifications proposed in this paper can be implemented in all real world applications of Fuzzy TOPSIS.

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This study includes modifications in Fuzzy Multiple Criteria Decision-Making (MCDM) theory to strengthen the comprehensiveness and reasonableness of the decision making process using Fuzzy TOPSIS. Considering the fuzziness in the decision data and group decision making process, linguistic variables are used to assess the weights of all criteria and the ratings of each alternative with respect to each criterion. It is possible to convert the decision matrix into a fuzzy decision one and construct a weighted normalized fuzzy decision matrix once the decision makers' fuzzy ratings have been pooled. According to the concept of TOPSIS, we define the Fuzzy Positive Ideal Solution (FPIS) and the Fuzzy Negative Ideal Solution (FNIS). Then, we use a new method to calculate the distance between two triangular fuzzy ratings. Using the idea of comparison between two fuzzy numbers, we calculate the distance of each alternative from FPIS and FNIS, respectively. In other words a new distance measure for Fuzzy TOPSIS is proposed in this paper. Finally, a closeness coefficient of each alternative is used to determine the ranking order of all alternatives. The higher value of closeness coefficient indicates that an alternative is closer to FPIS and farther from FNIS simultaneously.

The remainder of this paper is organized as follows. Section II presents some necessary definitions and formulations. Section III describes our modified procedure, and in section IV of this paper, to highlight and clarify our proposed procedure, a numerical example is discussed. Finally, section V briefly concludes.

II. Definitions And Formulations

In this section we will cover some basic definitions and formulas that are used in our paper.

Definition 1. A fuzzy set *A* in a universe of discourse X is characterized by a membership function $\mu_A(x)$ which associates with each element x in X a real number in the interval [0,1]. The function value is termed the grade of membership of x in A (defined by Zadeh [7]).

Definition 2. A fuzzy set A of the universe of discourse X is convex if and only if for all x1 and x2 in X :

$$\mu_A(\lambda x_1 + (1-\lambda)x_2) \geq Min(\mu_A(x_1), \mu_A(x_2)) \tag{3}$$

where $\lambda \in [0,1]$.

Definition.3. A fuzzy set A of the universe of discourse X is called a normal fuzzy set implying that $\exists x \in X : \mu_A(x) = 1$.

Definition 4. A fuzzy number ñ is a fuzzy subset in the universe of discourse X that is both convex and normal. Fig. 1 shows a fuzzy number of the universe of discourse X which is both convex and normal.

Definition 5. The α-cut of fuzzy number ñ is defined:

$$\tilde{n}^{\alpha} = \{x_i : \mu_{\tilde{n}}(x_i) \geq \alpha, x_i \in X\} \tag{4}$$

where $\alpha \in [0,1]$.

$\tilde{n}^{\alpha}$, defined in Eq. (4), is a non-empty bounded closed interval contained in X and it can be denoted by $\tilde{n}^{\alpha} = [\tilde{n}_l^{\alpha}, \tilde{n}_u^{\alpha}]$ where $\tilde{n}_l^{\alpha}$ and $\tilde{n}_u^{\alpha}$ are the lower and upper bounds of the closed interval, respectively (defined by

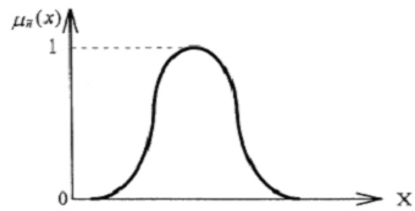

Fig. 1. A fuzzy number ñ.

Zimmermann [8]). Fig. 2 shows a fuzzy number ñ with α-cuts, where: $\tilde{n}^{\alpha_1} = [n_l^{a_1}, n_u^{a_1}], \tilde{n}^{\alpha_2} = [n_l^{a_2}, n_u^{a_2}]$.

Definition 6. A triangular fuzzy number ñ can be defined by a triplet (n_1, n_2, n_3) shown in Fig. 3. The membership function $\mu_{\tilde{n}}(x)$ is defined as:

$$\mu_{\tilde{n}}(x) = \begin{cases} \dfrac{x-n_1}{n_2-n_1}, & n_1 \leq x \leq n_2, \\ \dfrac{x-n_3}{n_2-n_3}, & n_2 \leq x \leq n_3, \\ 0 & otherwise. \end{cases} \tag{5}$$

In this paper, without loosing integrity and just to simplify the calculations, we assume the fuzzy triangular numbers to be symmetric, i.e. for every α-cut, n_2 is equal to $(n_l^{\alpha} + n_u^{\alpha})/2$ or simply in Eq. (5) assume $n_2 = (n_1 + n_3)/2$.

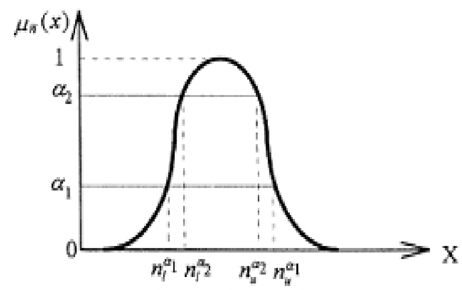

Fig. 2. Fuzzy number ñ with α-cuts.

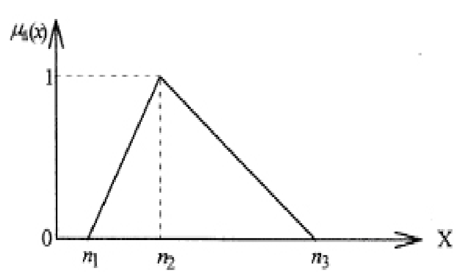

Fig. 3. A triangular fuzzy number ñ.

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Definition 7. Let $\tilde{m} = (m_1, m_2, m_3)$ and ñ= (n_1, n_2, n_3) be two triangular fuzzy numbers. If $\tilde{m} =$ ñ then $m_i = n_i, \forall i = 1,2,3$.

Definition 8. If ñ is a triangular fuzzy number and $\tilde{n}_l^{\alpha} > 0$ and $\tilde{n}_u^{\alpha} \le 1$ for $\alpha \in [0,1]$, then ñ is called a normalized positive triangular fuzzy number (see for example [9]).

Definition 9. $\tilde{D}$ is called a fuzzy matrix, if at least an entry in $\tilde{D}$ is a fuzzy number [10].

Definition 10. A linguistic variable is a variable whose values are linguistic terms (defined by Zadeh [7]). The concept of linguistic variable is very useful in dealing with situations which are too complex or too ill-defined to be reasonably described in conventional quantitative expressions [7]. For example, "weight" is a linguistic variable, its values can be very low, low, medium, high, very high, etc. These linguistic values can also be represented by fuzzy numbers.

Definition 11. Let $\tilde{m}$ =(m_1, m_2, m_3) and $\tilde{n}$ =(n_1, n_2, n_3) be two triangular fuzzy numbers, in available methods in literature of Fuzzy TOPSIS method with triangular fuzzy numbers (see for example [11]), the vertex method is usually defined to calculate the distance between fuzzy numbers as:

$$d(\tilde{m},\tilde{n}) = [\frac{1}{3}((m_1 - n_1)^2 + (m_2 - n_2)^2 + (m_3 - n_3)^2))]^{0.5}. \quad (6)$$

Here, we develop a new distance measure using the fuzzy comparison function that can be substitute with (6). Suppose fuzzycmp($\tilde{m}, \tilde{n}$) represents the fuzzy comparison function of two given discrete fuzzy numbers ($\tilde{m}$ and $\tilde{n}$). We define the fuzzycmp($\tilde{m}, \tilde{n}$) be the truth of that the fuzzy number $\tilde{m}$ be greater than or equal to the fuzzy number $\tilde{n}$. Using fuzzy logic, it can be logically defined as:

$$\text{fuzzycmp}(\tilde{m}, \tilde{n}) = \max[\min\{\mu_{\tilde{m}}(m_i), \mu_{\tilde{n}}(n_j)\}], \forall i, j : m_i \ge n_j \quad (7)$$

where m_i and n_j are the universe elements of discrete fuzzy numbers $\tilde{m}$ and $\tilde{n}$, respectively. Using (7), we define the distance of two fuzzy numbers $\tilde{m}$ and $\tilde{n}$:

$$d(\tilde{m}, \tilde{n}) = \left| \text{fuzzycmp}(\tilde{m}, \tilde{n}) - \text{fuzzycmp}(\tilde{n}, \tilde{m}) \right|. \quad (8)$$

Eq. (8) describes the idea that more closer two fuzzy numbers be, more nearer are the truths of one being greater than or equal to the other. Using (8) a new Fuzzy TOPSIS procedure for multi-criteria group decision making will be described in next session.

III. Modified Fuzzy Topsis Method

The will be described method is very suitable for solving the group decision making problem under fuzzy environment. In this paper, the importance weights of various criteria and the ratings of qualitative criteria are considered as linguistic variables. These linguistic variables can be expressed in positive triangular fuzzy numbers as Tables I and II.

A. Calculations

The importance weight of each criterion can be obtained by either directly assign or indirectly using pairwise comparisons [12]. Here, it is suggested that the decision makers use the linguistic variables (shown in Tables I and II) to evaluate the importance of the criteria and the ratings of alternatives with respect to various criteria. Assume that a decision group has K persons, then the importance of the criteria and the rating of alternatives with respect to each criterion can be calculated as:

$$\tilde{x}_{ij} = \frac{1}{K}[\tilde{x}_{ij}^1 (+) \tilde{x}_{ij}^2 (+) ... (+) \tilde{x}_{ij}^K] \quad (9)$$

$$\tilde{w}_j = \frac{1}{K}[\tilde{w}_j^1 (+) \tilde{w}_j^2 (+) .... (+) \tilde{w}_j^K] \quad (10)$$

where $\tilde{x}^k{}_{ij}$ and $\tilde{w}^k{}_j$ are the rating and the importance weight of the *k*th decision maker and (+) indicates the fuzzy arithmetic summation function.

As stated previously, a fuzzy multi-criteria group decision-making problem can be concisely expressed in matrix format as:

$$\tilde{D} = \begin{bmatrix} \tilde{x}_{11} & \tilde{x}_{12} & \cdots & \tilde{x}_{11} \\ \tilde{x}_{21} & \tilde{x}_{22} & \cdots & \tilde{x}_{2n} \\ . & . & & . \\ . & . & \cdots & . \\ . & . & & . \\ \tilde{x}_{m1} & \tilde{x}_{m2} & \cdots & \tilde{x}_{mn} \end{bmatrix}, \quad (11)$$

$$\tilde{W} = [\tilde{w}_1, \quad \tilde{w}_2, \quad ..., \quad \tilde{w}_n] \quad (12)$$

where $\tilde{x}^k{}_{ij}$ and $\tilde{w}^k{}_j$ are linguistic variables that can be shown by triangular fuzzy numbers : $\tilde{x}_{ij} = (a_{ij}, b_{ij}, c_{ij})$ and $\tilde{w}_j = (w_{j1}, w_{j2}, w_{j3})$.

To avoid the complicated normalization formula used in classical TOPSIS, in some papers (see for example [11]) the linear scale transformation is used to transform the various criteria scales into a comparable scale. Therefore, it is possible to obtain the normalized fuzzy decision matrix denoted by $\tilde{R}$:

$$\tilde{R} = [\tilde{r}_{ij}]_{m \times n} \quad (13)$$

where *B* and *C* are the set of benefit criteria and cost criteria, respectively, and:

$$\tilde{r}_{ij} = (\frac{a_{ij}}{c_j^*}, \frac{b_{ij}}{c_j^*}, \frac{c_{ij}}{c_j^*}), j \in B; \quad (14)$$

$$\tilde{r}_{ij} = (\frac{a_j^-}{c_{ij}}, \frac{a_j^-}{b_{ij}}, \frac{a_j^-}{a_{ij}}), j \in C; \quad (15)$$

$$c_j^* = \max_i \ c_{ij}, \ \ if \ \ j \in B; \quad (16)$$

$$a_j^- = \min_i \ a_{ij}, \ \ if \ \ j \in C. \quad (17)$$

The normalization method mentioned above is to preserve the property that the ranges of normalized triangular fuzzy numbers belong to [0,1]. In this paper, to avoid these

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computations and make a more easy and practical procedure, we simply define all of fuzzy numbers in this interval to omit the need of normalization method. Constructing the fuzzy numbers scalable and in [0,1], we avoid calculations (14) through (17) and therefore we have: $\tilde{r}_{ij} = \tilde{x}_{ij}$ and $\tilde{R} = \tilde{D}$.

Considering the different importance of each criterion, one can now construct the weighted normalized fuzzy decision matrix as:

$$\tilde{V} = [\tilde{v}_{ij}]_{m\times n}, i = 1,2,...,m, j = 1,2,...,n, \quad (18)$$

where $\tilde{v}_{ij} = \tilde{r}_{ij}(\cdot)\tilde{w}_j$.

According to the weighted normalized fuzzy decision matrix, we know that the elements $\tilde{v}_{ij}$ are normalized positive triangular fuzzy numbers and their ranges belong to the closed interval [0, 1]. Then, we can define the Fuzzy Positive-Ideal Solution (FPIS, A*) and Fuzzy Negative Ideal Solution (FNIS, A^-) as:

$$\begin{aligned} A^* &= (\tilde{v}_1^*, \tilde{v}_2^*, ..., \tilde{v}_n^*), \\ A^- &= (\tilde{v}_1^-, \tilde{v}_2^-, ..., \tilde{v}_n^-), \end{aligned} \quad (19)$$

where $\tilde{v}_j^*$=(1.0, 1.0, 1.0) and $\tilde{v}_j^-$=(0.0, 0.0, 0.0).

The distance of each alternative A_i (i=1,2,…,m) from A^* and A^- can be calculated as:

$$\begin{aligned} d_i^* &= \sum_{j=1}^{n} d(\tilde{v}_{ij}, \tilde{v}_j^*), \forall i = 1,2,...,m, \\ d_i^- &= \sum_{j=1}^{n} d(\tilde{v}_{ij}, \tilde{v}_j^-), \forall i = 1,2,...,m, \end{aligned} \quad (20)$$

where $d(\bullet,\bullet)$ is the new distance measurements between two fuzzy numbers as is proposed by Eq. (8). Moreover, a closeness coefficient is usually defined to determine the ranking order of all alternatives once the d_i^* and d_i^- of each alternative A_i (i=1,2,…,m) has been calculated. The closeness coefficient of each alternative is calculated as [11]:

$$CC_i = \frac{d_i^-}{d_i^* + d_i^-}, \forall i = 1,2,...,m. \quad (21)$$

Obviously, according to Eq. (21), an alternative A_i would be closer to FPIS (i.e. A^* defined in Eq. (19)) and farther from FNIS (i.e. A^- defined in Eq. (19)) as CC_i approaches 1. In other words, the closeness coefficient calculated by Eq. (21), can determine the ranking order of all alternatives and indicate the best one among a set of given feasible alternatives.

B. Implementing procedure

The discussed modified calculations for multi-criteria group decision making fuzzy TOPSIS procedure can now be implemented in following general implementing steps proposed by Chen [11]:

Step 1. Form a committee of decision-makers, then identify the evaluation criteria.

Step 2. Choose the appropriate linguistic variables for the importance weight of the criteria and the linguistic ratings for alternatives with respect to criteria.

Step 3. Aggregate the weight of criteria to get the aggregated fuzzy weight $\tilde{w}_j$ of criterion C_j, and pool the decision makers' opinions to get the aggregated fuzzy rating $\tilde{x}_{ij}$ of alternative A_i under criterion C_j.

Step 4. Construct the (normalized) fuzzy decision matrix (Eq. (13) or in our procedure Eq.(11)).

Step 5: Construct the weighted (normalized) fuzzy decision matrix (Eq.(18)).

Step 6: Determine FPIS and FNIS (Eq. (19)).

Step 7: Calculate the distance of each alternative from FPIS and FNIS, respectively using Eq. (20) with modified distance measure defined by Eq (8).

Step 8: Calculate the closeness coefficient of each alternative (Eq. (21)).

Step 9: According to the closeness coefficient, determine the ranking order of all alternatives.

IV. NUMERICAL EXAMPLE

The proposed modified Fuzzy TOPSIS procedure and required calculations have been coded using MATLAB 6.5 on a Pentium III platform running windows XP and many real world applications have been tested implementing the written procedure. Hereby, to illustrate our proposed approach of this paper we will discuss a numerical example. Assume that university ''X'' desires to hire a professor for teaching fuzzy theory course. A committee of three expert decision makers, D1, D2 and D3 has been formed to conduct the interview with three eligible candidates, namely A1, A2 and A3, and to select the most suitable candidate. Five benefit criteria are considered:

(1) Publications and researches (C1),
(2) Teaching skills (C2),
(3) Practical experiences in industries and corporations (C3),
(4) Past experiences in teaching (C4),
(5) Teaching discipline (C5).

The proposed method is applied to solve this problem and the computational procedure is summarized as follows:

Step 1. The decision makers use the linguistic weighting variables (shown in Table I) to assess the importance of the criteria (shown in Table III).

Step 2. The decision makers use the linguistic rating variables (shown in Table II) to evaluate the rating of alternatives with respect to each criterion. Final aggregated results are calculated and presented in Table IV as the linguistic fuzzy decision matrix.

Step 3. The linguistic evaluations (shown in Tables III and IV) are converted into symmetric triangular fuzzy numbers in order to construct the fuzzy decision matrix.

TABLE I
Linguistic Variables For The Importance Weight Of Each Criterion

Very low (VL)	(0.0, 0.0, 0.1)
Low (L)	(0.0; 0.1, 0.25)
Medium low (ML)	(0.15, 0.3, 0.45)
Medium (M)	(0.35, 0.5, 0.65)
Medium high (MH)	(0.55, 0.7, 0.85)
High (H)	(0.8, 0.9, 1.0)
Very high (VH)	(0.9,1.0,1.0)

TABLE II
Linguistic Variables For The Ratings

Very poor (VP)	(0.0, 0.0, 0.1)
Poor (P)	(0.0, 0.1, 0.2)
Medium poor (MP)	(0.2, 0.3, 0.5)
Fair (F)	(3.5, 0.5, 0.65)
Medium good (MG)	(0.5, 0.7, 0.9)
Good (G)	(0.65, 0.8, 9.5)
Very good (VG)	(0.9, 1.0, 1.0)

Step 4. The (normalized) fuzzy decision matrix is constructed using Eq. (13) or simply in this paper Eq. (11).
Step 5. The weighted normalized fuzzy decision matrix is constructed using Eq. (18).
Step 6. FPIS and FNIS are defined as :
$A^* = [(1, 1, 1); (1, 1, 1); (1, 1, 1); (1, 1, 1); (1, 1, 1)]$,
$A^- = [(0, 0, 0); (0, 0, 0); (0, 0, 0); (0, 0, 0); (0, 0, 0)]$.
Step 7. The distance of each candidate from FPIS and FNIS are calculated, respectively, using Eq. (20) and the new distance measure proposed by Eq. (8).
Step 8. The closeness coefficient is calculated for each candidate. The results are:

$$CC_1 = 0.56;\ CC_2 = 0.82;\ CC_3 = 0.69$$

Step 9. According to the these closeness coefficients, the ranking order of the three candidates will be A2, A3 and A1, respectively. Obviously, the best selection is candidate A2 having a greater closeness coefficient.

TABLE III
The Importance Weight Of Each Criterion Given By Decision Makers For The Numerical Example Of This Paper

Criterion \ Decision Makers	D1	D2	D3
C1	H	VH	MH
C2	VH	VH	VH
C3	VH	H	H
C4	VH	VH	VH
C5	M	MH	MH

TABLE IV
The Final Aggregated Results Obtained From Ratings Given By DecisionMakers For The Numerical Example Of This Paper

Criterion	Alternative	Linguistic Variable
C1	A1	MG
	A2	G
	A3	VG
C2	A1	G
	A2	VG
	A3	MG
C3	A1	F
	A2	VG
	A3	G
C4	A1	VG
	A2	VG
	A3	G
C5	A1	F
	A2	VG
	A3	G

V. CONCLUSION

In this paper we considered the multi-criteria decision making problem when there is a group of decision makers. While crisp data are inadequate to model the real life situations in MCDM, we modified available procedures in the TOPSIS technique when decision makers use linguistic variables. Regarding the value of the truth that a fuzzy number is greater than (or equal to) another fuzzy number, a new distance measure was proposed in this paper to calculate the distance of each fuzzy number from both Fuzzy Positive Ideal Solution (FPIS) and Fuzzy negative Ideal Solution (FNIS).

In section II of this paper, definitions was given and in section III we completely described our procedure. To clarify our proposed procedure, a numerical example was given and discussed in section IV. Using our written functions in MATLAB 6.5 we solved the numerical example and presented the computational steps of our modified proposed fuzzy TOPSIS procedure.

REFERENCES

[1] Hwang, C.L., Yoon, K., ''Multiple Attributes Decision Making Methods and Applications'', Springer, Berlin Heidelberg, 1981.

[2] Tsuar, S.H., Chang, T.Y., Yen, C.H., ''The evaluation of airline service quality by fuzzy MCDM'', Tourism Management, 2002, 23, pp. 107–115.

[3] Deng, H., Yeh, C.H., Willis, R.J., ''Inter-company comparison using modified TOPSIS with objective weights'', Computers & Operations Research, 2000, 27, pp. 963-973.

[4] Wang, R.C., Liang, T.F., ''Application of fuzzy multi-objective linear programming to aggregate production planning'', Computers & Industrial Engineering, 2004, 46, pp. 17–41.

[5] Chu, T.C., ''Facility location selection using fuzzy TOPSIS under group decisions'', International Journal of Uncertainty, Fuzziness and Knowledge-Based Systems , 2002, 10(6), pp. 687-701.

[6] Abo-Sina, M.A., Amer, A.H. ''Extensions of TOPSIS for multi-objective large-scale nonlinear programming problems'', Applied Mathematics and Computation, 2004, Article in press.

[7] L.A. Zadeh, ''Fuzzy sets'', Inform. and Control, 1965, 8, pp. 338-353.

[8] H.J. Zimmermann, ''Fuzzy Set Theory and its Applications'', 2nd edn., Kluwer Academic Publishers, Boston/Dordrecht/London, 1991.

[9] D.S. Negi, ''Fuzzy analysis and optimization'', Ph.D. Thesis, Department of Industrial Engineering, Kansas State University, 1989.

[10] Buckley, J.J., ''Fuzzy hierarchical analysis'', Fuzzy Sets and Systems, 1985, 17, pp. 233-247.

[11] Chen, C.T., ''Extensions of the TOPSIS for group decision-making under fuzzy environment'', Fuzzy Sets and Systems, 2000, 114, pp. 1-9.

[12] Hsu, H.M., Chen, C.T., ''Fuzzy hierarchical weight analysis model for multicriteria decision problem'', J. Chinese Inst. Industrial Eng., 1994, 11 (3), pp. 129-136.

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