Phil. 250z: Metaphysical Grounding
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Fine on Grounding (Pt. 2)

I. Fine on the Varieties of Ground

We saw last meeting that Fine prefers thinking of grounding as a *sentential operator*.

> However, for consistency's sake, and because I find it makes avoiding use/mention slippages easier, in this handout I will continue to follow Rosen in taking grounding to be a *relation* between *facts*. I will also continue to follow Rosen's conventions for how to notate grounding. (Translating everything here into Fine's conventions is easy enough.)

We saw last meeting that Fine distinguishes between three different "strengths" of grounding: *natural*, *normative*, and *metaphysical*.

Fine also distinguishes between a variety of different "flavors" of grounding:

1. *full vs. partial ground*

   Fine defines *partial ground* in terms of *full ground* in the same way that Rosen does:

   $[p] \leftarrowtail [q] =_{df}$ for some set of facts Γ, $[p] \leftarrow \Gamma$ and $[q] \in \Gamma$

   Fine's argument that *full ground* cannot be defined in terms of *partial ground*:

   > [*p* ∨ *q*] and [*p* & *q*] have the same partial grounds. But [*p* ∨ *q*] and [*p* & *q*] have different full grounds. "And so how are we to distinguish between [their] full grounds ... if appeal is only made to their partial grounds?" ("Guide to Ground," §5).

   I agree with the conclusion here. But it would be better to have a more airtight argument for it. Two minor problems:

   i. [*p* ∨ *q*] and [*p* & *q*] have the same partial grounds *when both obtain*; but maybe an analysis of full ground in terms of partial ground can make use of the fact that these two facts do not always co-obtain.

   ii. Although [*p* ∨ *q*] and [*p* & *q*] have the same partial grounds, they are not partial grounds of the same things.

2. *mediate vs. immediate ground*

   Intuitively, [*p*], [*q*] immediately grounds [*p* & *q*], and [*p* & *q*], [*r*] immediately grounds [(*p* & *q*) & *r*], whereas [*p*], [*q*], [*r*] only mediately grounds [(*p* & *q*) & *r*)].

   Fine defines *mediate ground* in terms of *immediate ground* via the following recursive definition:

   base clause: If [*p*] is immediately grounded in Γ, then [*p*] is mediately grounded in Γ.

   recursive clause: If [*p*] is mediately grounded in [*q*], Γ_1 and [*q*] is immediately grounded in Γ_1, then [*p*] is mediately grounded in Γ_1, Γ_2.

   closure clause: There is no other way for [*p*] to be mediately grounded.

   Two comments:

   i. Taking immediate grounds to also count as mediate grounds means that the prefix "im-" in "immediate" is a misnomer.

   ii. This rules out the possibility of *dense* grounding structures, in which we have some set of facts such that every grounding relation that holds between them is mediated by a set of other grounding relations holding between the facts in question and certain intermediate facts.

It is tempting to try to define *immediate ground* in terms of *mediate ground* as follows (here I ignore grounding's plurality on the right, to keep things simple):

> [*p*] is immediately grounded in [*q*] $=_{df}$ there does not exist a fact [*r*] such that [*p*] is grounded in [*r*], and [*r*] is grounded in [*q*].

Fine's counterexample: Intuitively, [$p \vee (p \vee p)$] is immediately grounded in [*p*], and yet [$p \vee (p \vee p)$] is grounded in [$p \vee p$], which is grounded in [*p*].

Thus, according to Fine, the best we can say is that an immediate ground "is one that need not be seen to be mediated" ("Guide to Ground," §5). However, this should not be taken to be a definition.

> (However, the Elimination Rules for logically complex truths that Fine proposes in §8 of "Guide to Ground" come very close to providing a definition of immediate ground.)

3. *strict vs. weak ground*

The sense of ground we've been working with up to this point Fine calls *strict ground*, since it does not allow a truth to ground itself.

He contrasts this with a notion of *weak ground*, which subsumes the notion of strict ground but also allows a truth of ground itself.

> According to Fine, this latter notion can be picked out using the locution "For it to be the case that *p* is for it to be the case that *q*."

Fine: "We might think of strict ground as moving us down in the explanatory hierarchy. ... Weak ground, on the other hand, may also move us sideways in the explanatory hierarchy" (GtG, §5).

According to Fine, *strict ground* can be defined in terms of *weak ground* as follows:

> [*p*] is strictly grounded in Γ $=_{df}$ [*p*] is weakly grounded in Γ, and [*p*] (on its own or with some other facts) does not weakly grounded any member [*q*] of Γ.

According to Fine, *weak ground* can be defined in terms of *strict ground* as follows:

> [*p*] is weakly grounded in Γ $=_{df}$ (necessarily?) for every fact [*q*] and set of facts Δ, if [*q*] is strictly grounded in [*p*], Δ, then [*q*] is strictly grounded in Γ, Δ.

It is natural to think of *strict grounding* as the core notion here, but Fine's inclination is to think of *weak grounding* as more fundamental (mostly because it leads to a tidier logic and a simpler semantics).

After introducing the full vs. partial and strict vs. weak distinctions, Fine makes some über-subtle distinctions between different types of partial/strict grounds. These won't matter for our purposes.

II. Fine on Amalgamation

During his exposition of the pure logic of ground, Fine argues that it is possible to derive the following:

(Amalgamation) If [*p*] $\leftarrow \Gamma_1$ and [*p*] $\leftarrow \Gamma_2$, then [*p*] $\leftarrow \Gamma_1, \Gamma_2$.

His derivation of this crucially relies on

(Cut) If [*p*] $\leftarrow$ [q_1], [q_2]; [q_1] $\leftarrow \Gamma_1$; and [q_2] $\leftarrow \Gamma_2$; then [*p*] $\leftarrow \Gamma_1, \Gamma_2$.

His argument: suppose [*p*] $\leftarrow \Gamma_1$ and [*p*] $\leftarrow \Gamma_2$. Then [p & p] $\leftarrow$ [*p*], [*p*], so, by Cut, we have [p & p] $\leftarrow \Gamma_1, \Gamma_2$. But then if Γ_1, Γ_2 is a strict full ground for [p & p], how can Γ_1, Γ_2 fail to be a full strict ground for [*p*]? ("Guide to Ground," §5; see also "The Pure Logic of Ground," p. 7.)

However, anyone who finds Amalgamation implausible is likely to deny that Cut holds when [q_1] = [q_2].

III. Fine's Paradoxes of Ground

Fine's universal grounding paradox for facts:

Assumptions:[1]

(Universal Existence)	Everything exists.
(Factual Existence)	If *p*, then [*p*] exists.
(Factual Grounding)	If [*p*] exists, then [[*p*] exists] ← [*p*].
(Universal Grounding)	If (∀*x*)*φ*(*x*) and if *a* exists, then [(∀*x*)*φ*(*x*)] ⇤ [*φ*(*a*)].

Statement of paradox:

Everything exists, by Universal Existence. So, by Factual Existence, [Everything exists] exists. Thus, by Universal Grounding, [Everything exists] ⇤ [[Everything exists] exists]. But, by Factual Grounding, [[Everything exists] exists] ← [Everything exists]. So, by transitivity, [Everything exists] ⇤ [Everything exists]. Which contradicts irreflexivity.

Fine's universal grounding paradox for propositions:

Assumptions:

(Universal Middle)	Every proposition is either true or not true.
(Propositional Existence)	The proposition <*p*> exists.
(Propositional Grounding)[2]	If *p*, then [<*p*> is true] ← [*p*].
(Disjunctive Grounding)	If *p*, then [*p* ∨ *q*] ← [*p*].
(Universal Grounding)	If (∀*x*)*φ*(*x*) and if *a* exists, then [(∀*x*)*φ*(*x*)] ⇤ [*φ*(*a*)].

Statement of paradox:

Every proposition is either true or not true, by Universal Middle. By Propositional Existence, <Every proposition is either true or not true> exists. Call this proposition <p_0>. By Propositional Grounding, [<p_0> is true] ← [p_0]. But, by Disjunctive Grounding, [<p_0> is true or not true] ← [<p_0> is true]. So, by transitivity, [<p_0> is true or not true] ← [p_0]. By Universal Grounding, [p_0] ⇤ [<p_0> is true or not true]. So, by transitivity again, [p_0] ⇤ [p_0]. Which contradicts irreflexivity.

Fine's particular grounding paradox for facts:

Assumptions:

(Particular Existence)	Something exists.
(Factual Existence)	If *p*, then [*p*] exists.
(Factual Grounding)	If [*p*] exists, then [[*p*] exists] ← [*p*].
(Existential Grounding)	If *φ*(*a*) and if *a* exists, then [(∃*x*)*φ*(*x*)] ← [*φ*(*a*)], [*a* exists].

Statement of paradox:

Something exists, by Particular Existence. So, by Factual Existence, [Something exists] exists. Thus, by Existential Grounding, [Something exists] ← [[Something exists] exists]. But, by Factual Grounding, [[Something exists] exists] ← [Something exists]. So, by transitivity, [Something exists] ← [Something exists]. Which contradicts irreflexivity.

(Fine also constructs an analogous *particular grounding paradox for propositions*.)

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1 Although Fine states his assumptions as rules of inference, I will continue to follow Rosen in stating them as true conditionals, since I'll be urging a response to these paradoxes which doesn't turn on the distinction between entailment and conditionals.

2 This combines two assumptions which Fine keeps separate.

Fine thinks there are four viable ways of responding to these paradoxes:

- *predicativism*: Deny Factual Existence and Propositional Existence. [Let us set this position aside.]
- *extreme position #1*: Deny Universal Existence, Universal Middle, and Particular Existence.
- *extreme position #2*: Deny Universal Grounding and Existential Grounding.
- *compromise position*: Deny Universal Existence, Universal Middle, and Existential Grounding.

According to Fine's compromiser, Existential Grounding is overly strong, and all we have is:

(Weak Existential Grounding) If $(\exists x)\varphi(x)$, then there exists an a such that $[(\exists x)\varphi(x)] \leftarrow [\varphi(a)]$, [$a$ exists].

Why isn't rejecting Factual Grounding and Propositional Grounding an option?

According to some deflationists/minimalists, these two principles are false because [[*p*] exists] = [*p*] and [<*p*> is true] = [*p*]. However, a similar paradox can be generated using these identities.

According to a different style of deflationist, [<*p*> is true] and [*p*] are distinct facts, but they are immediately grounded in the same things. But, again, this is enough to generate a similar paradox.

I think it is clear that *extreme position #2* is the best way to respond to these paradoxes.

- *claim #1*: Denying Universal Existence, Universal Middle, or Particular Existence is not a way of avoiding these sorts of paradoxes, unless one goes all the way to predicativism.

  Fine thinks that it's important for his puzzle that Universal Existence, Universal Middle, and Particular Existence are (or at least seem to be) logical truths.

  But all we need to generate a paradox of this sort is a *true* universal or existential generalization whose quantifier ranges over a fact or proposition *with that universal or existential generalization as its content*.

  So the following are enough to generate the paradox, if we don't go in for predicativism: [Every true proposition is true] and [Something is true].

- *claim #2*: We have good reason to reject Universal Grounding and Existential Grounding.

  One sort of counterexample to Existential Grounding (from a previous handout):

  Intuitively, [Something is true] is not grounded in [<Something is true> is true].

  Another sort of counterexample (a variant of an example from a previous handout):

  Suppose you are all running a race that works as follows. You run for a really long time. At some point, the king decrees that the race is over. When he decrees this, whomever is in the lead wins. (Let us stipulate that the race is set up in such a way that it is impossible for any two people to be tied for the lead at a given time.) Then we have:

  [Someone won the race] ← [The king decreed that the race is over]

  [Person X won the race] ← [The king decreed that the race is over], [Person X was in the lead when the king decreed this]

  (This sort of example gives us a general method of generating counterexamples to Fine's claims about the impure logic of grounding: when my authority makes something the case, what I make the case can have any logical form whatsoever.)

  Several meetings ago we considered Rosen's argument against Universal Grounding. Fine basically accepts this argument on p. 109 of "Some Puzzles of Ground."

But even if Universal Grounding and Existential Grounding are false in general, on particular occasions their consequents will be true. Can we generate a version of Fine's paradox on any of these occasions? I can't think of any cases in which we can. But that isn't a good argument for this sort of thing being impossible.