

Meeting 2: Crash Course in Grounding (Pt. 2)

I. Grounding: Notation

Today I will assume that grounding is a relation between a plurality of facts (the grounds) and a single fact (the grounded).

As is standard, I use ‘plurality’ in such a way that a single fact on its own counts as a limiting case of a plurality of facts. (It is for this reason that it is better to take the grounds to be a plurality rather than a set: when $[p]$ on its own grounds $[q]$, we want to say that $[p]$, not $\{[p]\}$, is the grounds of $[q]$.)

Some notation I will be using:

f, g, h , etc. are variables ranging over individual facts

ff, gg, hh , etc. are plural variables ranging over pluralities of facts

‘ $[p]$ ’ denotes the fact that p

‘ $[p_1], \dots, [p_n]$ ’ denotes the plurality of facts consisting in $[p_1], \dots$, and $[p_n]$

‘ $[q] \leftarrow [p_1], \dots, [p_n]$ ’ is shorthand for ‘ $[q]$ is fully grounded in $[p_1], \dots, [p_n]$ (taken together)’

‘ $[p_1], \dots, [p_n] \rightarrow [q]$ ’ is shorthand for ‘ $[p_1], \dots, [p_n]$ (taken together) fully ground $[q]$ ’

‘ $[p_1], \dots, [p_n] \mapsto [q]$ ’ is shorthand for ‘ $[p_1], \dots, [p_n]$ (taken together) partially ground $[q]$ ’

(Unlike Fine, I allow that pluralities of facts, not just individual facts, can be partial grounds. My notation here deviates from Rosen’s: I put the bar on the grounds side of the arrow because the grounding relation at issue is partial on that side of things.)

‘ $q \text{ b/c } p_1, \dots, p_n$ ’ is shorthand for ‘ $[q]$ obtains fully because $[p_1], \dots, [p_n]$ all obtain’

‘ $p_1, \dots, p_n \text{ c/b } q$ ’ is shorthand for ‘ $[q]$ obtains fully because $[p_1], \dots, [p_n]$ all obtain’

(Fine uses ‘ $<$ ’ instead of ‘c/b’ for the last of these, but I find that notation confusing, since it is easy to see ‘ $<$ ’ as the head of an arrow, but when graphing grounding relations it is standard to draw arrows from the grounds to the grounded.)

When needed, I will take all of the standard notations from set theory and repurpose them for pluralities. So ‘ $f \in ff$ ’ means ‘ f is one of ff ’, ‘ $ff \subseteq gg$ ’ means ‘ ff is a subplurality of gg ’, etc.

I don’t follow Rosen in assuming that “facts are structured entities built up from worldly items—objects, relations, connectives, quantifiers, etc.—in roughly the sense in which sentences are built up from words” (“Metaphysical Dependence: Grounding and Reduction,” p. 114). Even if we allow that ‘the fact that ___’ is hyperintensional, so there can be distinct facts that necessarily co-obtain, I think we should leave open the possibility that, say, $[\phi\text{-ing is not required}] = [\text{Not-}\phi\text{-ing is permitted}]$.

I also don’t assume that every fact is expressible in the form $[p]$, for some value of ‘ p ’. For example, maybe the facts about what it’s like to be in pain are not of the form $[p]$.

II. Grounding: Some Distinctions

Some distinctions that various authors draw between types of grounding (starting with the least controversial and moving on to the more controversial):

- *full vs. partial grounding*

There is an intuitive difference between some facts *fully grounding* another fact and those facts only *being part of what grounds* that same fact (or, equivalently, between one fact obtaining *entirely because of* some facts and that fact only obtaining *in part because of* those facts).

The standard approach is to define *partial grounding* in terms of *full grounding*, like so:

$$(D) \quad \text{ff}_1 \mapsto g =_{df} (\exists \text{ff}_2: \text{ff}_1 \subseteq \text{ff}_2)(\text{ff}_2 \rightarrow g)$$

Fine provides a quick argument that *full grounding* cannot be defined in terms of *partial grounding*:

$[p \vee q]$ and $[p \& q]$ have the same partial grounds. But $[p \vee q]$ and $[p \& q]$ have different full grounds. “And so how are we to distinguish between [their] full grounds . . . if appeal is only made to their partial grounds?” (“Guide to Ground,” p. 50).

I agree with Fine’s conclusion. But it would be better to have a more airtight argument for it.

main problem: $[p \vee q]$ and $[p \& q]$ have the same partial grounds *when both obtain*; but maybe an analysis of full ground in terms of partial ground can make use of the fact that these two facts do not always co-obtain.

additional problem: Although $[p \vee q]$ and $[p \& q]$ have the same partial grounds, they are not partial grounds of the same things.

Some authors deny (D) because they claim there can be cases in which $[p]$ partially grounds $[q]$ even though there is no ff such that $[p]$ is one of ff and ff fully grounds $[q]$.

However, it is difficult to find plausible examples for which this is true. Some Williamsonians claim that $[p]$ can partially ground $[S \text{ knows that } p]$ even though there is no ff such that $[p]$ is one of ff and ff fully grounds $[S \text{ knows that } p]$, but few agree with them.

- *immediate vs. mediate grounding*

Intuitively, $[p]$ immediately grounds $[p \vee q]$; and $[p \vee q]$, $[r]$ immediately ground $[(p \vee q) \& r]$; whereas $[p]$, $[r]$ only mediate ground $[(p \vee q) \& r]$.

It is tempting to define *immediate grounding* as a lack of *mediate grounding*, maybe like so (where I focus on the case of one–one immediate grounding, to keep things simple):

$$f \text{ immediately grounds } h =_{df} \neg(\exists g)(f \text{ grounds } g \& (g \text{ grounds } h))$$

But Fine points out that there can be cases in which one fact both mediately and immediately grounds a second fact; for example, consider $[\text{The insignia is crimson}]$ and $[\text{Either something is red, or the insignia is crimson}]$.

Fine instead proposes that we take *mediate grounding* to be the transitive closure of *immediate grounding*, via the following recursive definition:

base clause: If ff immediately grounds h , then ff mediate grounds h .

recursive clause: If ff immediately grounds g ; and g , gg mediate grounds h ; then ff , gg mediate grounds h .

closure clause: There is no other way for h to be mediately grounded.

Three worries about this suggestion, in increasing order of significance:

first worry: Taking all immediate grounds to also be mediate grounds means that the prefix ‘im-’ in ‘immediate’ is a misnomer.

second worry: It is being assumed here is that a relation’s transitive closure is always an indirect version of that relation. However, this is not in general true (consider kicking and hating). So why is the transitive closure of immediate grounding itself an indirect form of that relation?

third worry: This rules out the possibility of a *densely mediated* grounding structure, in which a set of facts is such that every grounding relation that holds between them is mediated by other grounding relations holding between those facts and other intermediate facts also in the set.

- *factive vs. non-factive grounding*

Grounding as we have understood it so far is doubly factive: “ p b/c q ” entails both “ p ” and “ q .”

Fine toys around with the idea of a non-factive grounding connective—call it ‘b/c_{non-factive}’—which is such that “ p b/c_{non-factive} q ” can be true even if “ p ” or “ q ” is false (“Guide to Ground,” pp. 48–50).

I myself have serious doubts that this notion makes sense. Is there a non-factive form of knowledge which is just like knowledge except “S knows_{non-factive} that p ” doesn’t entail that p ? Is there a non-factive form of singular causation between events which is just like singular causation between events except that “ c caused_{non-factive} e ” doesn’t entail that c and e occurred?

- *strict vs. weak grounding*

The notion of ground we’ve been working with up to this point Fine calls *strict ground*, since it does not allow a truth to ground itself.

He contrasts this with a notion of *weak ground*, which subsumes the notion of strict ground but also allows a fact to ground itself.

According to Fine, cases involving weak-but-not-strict grounding can be picked out using the locution “For __ and for __ and . . . is for __.”

Fine: “We might think of strict ground as moving us down in the explanatory hierarchy. . . . Weak ground, on the other hand, may also move us sideways in the explanatory hierarchy” (ibid., p. 52)

It is natural to think of *strict ground* as the core notion here, but Fine’s inclination is to take *weak ground* to be more fundamental (mostly because doing so leads to a tidier logic and a simpler semantics).

However, this is another case where I am suspicious of a distinction Fine makes. It is extremely difficult to make sense of what Fine means by ‘weak ground’, and certainly it is not a notion on which we have an intuitive grip. (See deRosset’s “What Is Weak Ground?”)

- *grounds vs. enablers*

The following thesis is accepted by *grounding necessitarians* and denied by *grounding contingents*:

(N) If $[p_1], \dots, [p_n] \rightarrow [q]$, then $\Box((p_1 \& \dots \& p_n) \supset q)$.

Contingentists typically motivate their view by appealing to the following distinction:

grounds: Facts that make something the case.

enablers: Background conditions that “turn on” a grounding relation between the grounds and the grounded.

disablers: Background conditions that “turn off” a grounding relation between (what would otherwise be) the grounds and the grounded.

For example, Cohen reads Dancy as proposing that there are cases in which [I promised to ϕ] grounds [I ought to ϕ], with [My promise was not given under duress], [I am able to ϕ], and [There is no greater reason not to ϕ] acting as enablers without themselves being grounds.

I don’t think it’s plausible that the no-greater-reason fact is an enabler, especially if [I have most reason to ϕ] is an intermediate ground in between [I promised to ϕ] and [I ought to ϕ], as is being assumed here.

I also don’t think it’s plausible to hold that [My promise was not given under duress] is an enabler; rather, if [My promise was given under duress] obtained, it would act as a disabler. (Here I disagree with Dancy and Cohen when they claim *the absence of an enabler is a disabler*.)

In addition to enablers and disablers, there might be enablers of enablers, disablers of enablers, etc.

Cohen characterizes enablers/disabers as “[f]acts whose presence or absence is required for the first kind of fact to play their distinctive role: necessary background conditions for the former to do their grounding” (p. 78). But this is a mistaken way of characterizing enablers/disabers, for two reasons:

first problem: It is too extensional a way of characterizing them. In the promising example, the obtaining of [Every member of the set {me} is able to ϕ] is required for [I promised to ϕ] to do its grounding work but is not itself an enabler.

second problem: Sometimes either of two facts (which do not necessarily co-obtain) can serve as an enabler, but in such cases we need not view their disjunction as the true enabler.

Cohen thinks the enabler vs. ground distinction allows us to distinguish two notions of full ground: *full exclusive grounds* (i.e. the full grounds, as people usually use this term) and *full inclusive grounds* (i.e. the full grounds plus all the enablers, enabler enablers, etc., as people usually use these terms).

I don’t see that there is any case to be made for using the term ‘grounds’ for the second of these. No one who makes the analogous distinction between *causes* and *enabling conditions* thinks that there are two uses of the term ‘cause’, the second of which picks out both causes (in the first sense) and enabling conditions.

Cohen makes the extremely interesting suggestion that “the relation of being an enabling condition for something is not an asymmetric relation” (p. 79).

Her example: [Bale is a good forward soccer player] enables [Cristiano Ronaldo has characteristics C_1 , C_2 , etc.] to ground [Cristiano Ronaldo is a good forward soccer player], and [Cristiano Ronaldo is a good forward soccer player] enables [Bale has characteristics C_1^* , C_2^* , etc.] to ground [Bale is a good forward soccer player].

I worry, though, that she has conflated the grounds of [Cristiano Ronaldo is a good forward soccer player] with the causes of [Cristiano Ronaldo has played well].

III. The Pure Logic of Grounding

Fine helpfully distinguishes between

the pure logic of grounding: The study of what follow from (and what entails) grounding claims, without considering the internal structure of the facts that ground or are grounded.

the impure logic of grounding: The study of what follows from (and what entails) grounding claims, when the internal structure of the facts that ground and are grounded is taken into consideration.

Here are a few standardly assumed (but not universally accepted) principles in the pure logic of grounding (some authors formulate these as *rules of inferences* rather than *material conditionals*, and others formulate them as *schemata* rather than *universal generalizations*, but for our purposes those differences won’t matter):

- *Asymmetry (of partial grounding):* $(\forall f)(\forall g)((f \mapsto g) \supset \neg(g \mapsto f))$
- *Irreflexivity (of partial grounding):* $(\forall f)\neg(f \mapsto f)$
- *Transitivity (of full grounding):* $(\forall f)(\forall g)(\forall h)((f \rightarrow g) \& (g \rightarrow h) \supset (f \rightarrow h))$
- *Cumulative Transitivity or Cut (for full grounding):* $(\forall ff)(\forall g)(\forall gg)(\forall h)((ff \rightarrow g) \& (g, gg \rightarrow h) \supset (ff, gg \rightarrow h))$
- *Non-Monotonicity (of full grounding):* $\neg(\forall ff)(\forall g)(\forall h)((ff \rightarrow g) \supset (ff, h \rightarrow g))$

A stronger version of Non-Monotonicity does not hold, on which $(\forall ff)(\forall g)(\forall h: h \notin ff)((ff \rightarrow g) \supset \neg(ff, h \rightarrow g))$. (What would be a counterexample?)

Fine’s argument for Non-Monotonicity: if there is at least one grounding truth, so that $ff \rightarrow g$ for some ff and g , then if Non-Monotonicity were false it would follow that $ff, g \rightarrow g$. But this is a violation of Irreflexivity.

Fine argues that Cut, as formulated above, entails the following extremely controversial principle:

- *Amalgamation*: $(\forall ff_1)(\forall ff_2)(\forall g)((ff_1 \rightarrow g) \& (ff_2 \rightarrow g)) \supset (ff_1, ff_2 \rightarrow g)$

His argument (“Guide to Ground,” p. 57): suppose $ff_1 \rightarrow [p]$ and $ff_2 \rightarrow [p]$. We also have $[p]$, $[p] \rightarrow [p \& p]$. So by Cut, $ff_1, [p] \rightarrow [p \& p]$, and by Cut again, $ff_1, ff_2 \rightarrow [p \& p]$. But how can we have $ff_1, ff_2 \rightarrow [p \& p]$ without also having $ff_1, ff_2 \rightarrow [p]$?

But we need not accept this argument. The problem, in my opinion, is that Fine, Rosen, and others have formulated Cut incorrectly. Intuitively, the principle does not hold when $g \in gg$. So let’s reformulate it as:

- *Cut (correct formulation)*: $(\forall ff)(\forall g)(\forall gg)(\forall h)((ff \rightarrow g) \& (g, gg \rightarrow h)) \supset (ff, gg^{ff/g} \rightarrow h)$,

where $gg^{ff/g}$ is the result of substituting ff for g in gg (if it occurs).

The intuitions that push us toward Transitivity and Cut also push us toward these stronger variants of them:

- *Hypertransitivity*: $(\forall f)(\forall g)(\forall h)((f \rightarrow g) \& (g \rightarrow h)) \supset ([f \rightarrow g], [g \rightarrow h] \rightarrow [f \rightarrow h])$
- *Hypercut*: $(\forall ff)(\forall g)(\forall gg)(\forall h)((ff \rightarrow g) \& (g, gg \rightarrow h)) \supset ([ff \rightarrow g], [g, gg \rightarrow h] \rightarrow [ff, gg^{ff/g} \rightarrow h])$

With Hypercut in hand, it is very natural to define *mediate grounding* in terms of *grounding simpliciter*, like so:

$$ff \text{ mediately grounds } h =_{df} (ff \rightarrow h) \& (\exists g)(\exists hh, \exists gg: ff = hh, gg)([hh \rightarrow g], [g, gg \rightarrow h] \rightarrow [hh, gg^{hh/g} \rightarrow h])$$

Indeed, I would argue that this is the intuitive way of understanding mediate grounding: when f grounds h because there is a g such that f grounds g and g grounds h , that’s just what it is for f to mediate ground h .

It is much more difficult to define *immediate grounding* in terms of *grounding simpliciter*. An informal attempt:

$$ff \text{ immediately grounds } h =_{df} ff \text{ grounds } h, \text{ and it is not the case that } [ff \text{ grounds } h] \text{ is grounded only by way of } [hh \rightarrow g] \text{ and } [g, gg \rightarrow h], \text{ for some } g, hh, \text{ and } gg \text{ such that } ff = hh, gg.$$

III. The Impure Logic of Grounding

Here are a few standardly assumed (but, again, not universally accepted) principles in the impure logic of grounding:

- (\vee) If p , then $[p] \rightarrow [p \vee q]$; and if q , then $[q] \rightarrow [p \vee q]$.
- ($\&$) If p and q , then $[p], [q] \rightarrow [p \& q]$.
- (\exists) If Fa , then $[Fa] \rightarrow [(\exists x)Fx]$. (Or instead: If Fa and a exists, then $[Fa], [a \text{ exists}] \rightarrow [(\exists x)Fx]$.)
- (\forall) If Fa_1, Fa_2, \dots , and $T(a_1, a_2, \dots)$, then $[Fa_1], [Fa_2], \dots, [T(a_1, a_2, \dots)] \rightarrow [(\forall x)Fx]$,

where $[T(a_1, a_2, \dots)]$ = the totality fact = $[a_1, a_2, \dots \text{ are all of the individuals that there are}]$.

Some comments:

- (\vee) and ($\&$) illustrate the important difference between *joint grounding* and *overdetermined grounding*. When it is both the case that p and the case that q , then $[p]$ and $[q]$ together jointly ground $[p \& q]$, but each on its own fully grounds $[p \vee q]$.
- In addition to (\vee), Fine holds that if p and q , then $[p], [q] \rightarrow [p \vee q]$, but if we reject Amalgamation we need not follow him in this.
- Fine points out that if we accept (\exists), we should not also hold that $[a \text{ exists}] = [(\exists x)(x = a)]$, or else we’ll be saddled with the implausible claim that $[a = a]$ grounds $[a \text{ exists}]$ whenever a exists.

- Similarly, if we accept (\forall) , we should not also hold that $[T(a_1, a_2, \dots)] = [(\forall x)(x = a_1 \vee x = a_2 \vee \dots)]$, or else we will have two implausible results: (i) $[T(a_1, a_2, \dots)]$ partially grounds itself, and (ii) each of $[a_1 = a_1]$, $[a_2 = a_2]$, \dots partially grounds $[T(a_1, a_2, \dots)]$.
- It is fairly common these days to hold that (\exists) does not hold with full generality because of counterexamples like the following (first pointed out by Fine in “Some Puzzles of Ground”):

Plausibly, [Something is true] either grounds or is identical to [\langle Something is true \rangle is true]. But according to (\exists) , [\langle Something is true \rangle is true] grounds [Something is true]. Therefore, [Something is true] grounds [Something is true], in violation of Irreflexivity.

- In “Metaphysical Dependence: Grounding and Reduction,” Rosen only accepts (\forall) in the case of “accidental regularities” such as [Everyone in this room is shorter than twenty feet] but denies it for other sorts of universal facts, such as [Every bachelor is unmarried], which he claims to be grounded in [It lies in the nature of bachelorhood that every bachelor is unmarried].

Since then Rosen has revised his position and now accepts it in full generality (putting aside those cases that generate Fine-style paradoxes of ground). He now holds that non-accidental universal facts such as [Every bachelor is unmarried] are grounded both “from above,” via [It lies in the nature of bachelorhood that every bachelor is unmarried], and “from below,” via (\forall) (or, perhaps, via a variant of (\forall) that holds for restricted quantifiers).

- Although the above principles are plausible in many individual cases, it just not clear to me that they hold in full generality, even after we’ve dealt with Fine-style paradoxes of ground.

For example, maybe [PHIL 159 meets on Tuesdays] is entirely grounded in [PHIL 159 meets on Tuesdays and Thursdays], which in turn is entirely grounded in [The Director of Undergraduate Studies decided that PHIL 159 meets on Tuesdays and Thursdays] (together perhaps with a fact about his authority to decide the matter). By Asymmetry, this would be a counterexample to $(\&)$.

Another example: suppose you all are running a race that works as follows. You run for a really long time. At some point, the king decrees that the race is over. When he decrees this, whomever is in the lead wins. If several people are tied for the lead, they are all winners. If no one is in the lead (perhaps because all the runners have died), the king is the winner. Then we have:

[Someone won the race] \leftarrow [The king decreed that the race is over].

[Person X won the race] \leftarrow [The king decreed that the race is over], [Person X was in the lead when the king decreed this].

But it’s not clear that we need to hold, in addition, that [Someone won the race] \leftarrow [Person X won the race]. If so, then we have a counterexample to (\exists) .

The general thought behind these counterexamples: in social-ontology cases in which our decisions (or actions, or attitudes) make something true, it’s not clear that our decisions (or actions, or attitudes) always immediately ground a group of atomic facts which then make all the logically more complex facts obtain.