Extrapolative Beliefs and Exchange Rate Markets

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Abstract

This paper analyzes exchange rate dynamics and proposes a potential mechanism explaining excess return predictabilities in exchange rate markets. Using a board data sample, this paper concludes that holding currencies with higher contemporaneous interest rates earns initial excess positive bond returns. However, the sign of the excess return is a function of time. Higher contemporaneous interest rates reverse to predict negative excess returns in the medium run. In the long run, interest differentials do not predict excess returns. This paper argues that investors not only rely on fundamentals, i.e. interest differentials, but also extrapolate past exchange rates when forming expectations of future exchange rate levels. The proposed extrapolative model can potentially reconcile empirical excess return patterns and is consistent with survey evidence from investor forecasts.

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Declarations of interest: none

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1 Introduction

There are two major approaches in "valuing" currencies: the demand-based approach and the fundamental approach. Standard finance theories follow the second approach and believe that the "correct" valuation of any asset is its fundamental value.

The fundamental of assets can be decomposed into two components: the flow utility and the future valuation (the expectation of future derived utilities). For stocks, prices fundamentally relate to current dividends and future expected dividend streams. Bond prices depend on expected interests accrued. Analogously, exchange rates are fundamentally pinned down by differences between interest rates on the long and the short legs of a currency pair.

The relationship between interest rate differentials and bilateral exchange rates allows economists to model behaviors of exchange rates. Formally, let $s_{fh,t}$ be the log of the exchange rate at time t in terms of home (h) currency per foreign (f) currency, i_t and i_t^* be the respective 1-period home and foreign nominal interest rates of default-free bonds at time t, $x_{fh,t} = i_t - i_t^*$ be the time-t interest rate differential, $\mathbb{E}_t(s_{fh,t+k})$ be the logarithm of time-t expectation of k-period-ahead spot rate, and t be the time with a unit equals to 1 period.

Holding home bonds yields an interest of i_t, while holding foreign bonds exposes investors to additional exchange depreciation risk. Precisely, 1-period return of holding foreign bonds is equal to $i_t^* + \mathbb{E}_t(s_{fh,t+1}) - s_{fh,t}$. In expectation, returns earned from holding home bonds should be equalized to returns from holding foreign bonds. That is, the following should hold:

$$
\begin{aligned}
s_{fh,t} &= \mathbb{E}_t(s_{fh,t+1}) - (i_t - i_t^*) \\
&= \mathbb{E}_t(s_{fh,t+1}) - x_{fh,t} \\
&= \mathbb{E}_t(s_{fh,t+T}) - \sum_{j=0}^{T-1} \mathbb{E}_t(x_{fh,t+j}).
\end{aligned} \tag{1}
$$

The above relationship implies that interest rate differentials are fundamentals pinning down exchange levels. Yet, Meese and Rogoff (1983) and previous literature find that a random walk predicts exchange rates better than macroeconomic models (including an interest rate path, an inflation path, and etc.) in the short run. Obstfeld and Rogoff (2000) call this weak explanatory power of macroeconomic fundamentals as "the exchange-rate disconnect puzzle".

Equation (1) also links the volatility of exchange rates with the volatility of interest rate differentials. Yet, Backus et al. (1993), Bekaert (1996), and Moore and Roche (2002) document an excess exchange volatility beyond a movement in interest rate differentials. This stylized fact registers another puzzle called "the excess volatility puzzle".

Define $\rho_{fh,t+1} = s_{fh,t+1} - s_{fh,t} - x_{fh,t}$ to be the realized 1-period excess return on holding foreign over home bonds. The second equality from equation (1) implies that this expected excess return should be zero. High interest-rate currencies should depreciate against low interest-rate ones to equalize bond returns. This no-arbitrage condtion is called the uncovered interest parity (UIP).

Empirical studies unanimously find that the UIP does not hold. Bilson (1981) and Fama (1984) run the following regression:

$$s_{fh,t+1} - s_{fh,t} = a + b(i_t - i_t^*) + u_{t+1}. \tag{2}$$

If the UIP holds, regression coefficients from equation (2) should have coefficients $a = 0$ and $b = 1$. Empirically, a coefficient in front of an interest rate differential (b) is estimated to be consistently less than one and usually even lower than zero. This poses the UIP puzzle. Froot and Thaler (1990) and Engel (1996) are examples of older empirical surveys. Such patterns are robust even in contemporary studies. The "carry trade", which an investor borrows a low interest currency to lend in a high interest rate currency, is consistently profitable, as documented in Lustig and Verdelhan (2007), Brunnermeier et al. (2008), Burnside et al. (2011a), etc.[1] Frankel and Poonawala (2010) show that the UIP puzzle is less severe but still persists among emerging currencies.

International economists attempt to rationalize the UIP deviation as well as other exchange rate puzzles using two main methods. The first approach is the risk-based explanation with a key underlying idea that currencies with higher interests are riskier and require higher returns to compensate for such risk. Verdelhan (2010) uses Campbell and Cochrane (1999)'s external habit model and argues that investing in foreign currency is riskier in bad times precisely when the foreign interest rates are low relative to those of domestic. Colacito and Croce (2011), Bansal and Shaliastovich (2012), and Colacito and Croce (2013) resort to long-run risk models, while Farhi and Gabaix (2015) focus on the rare disaster risk. Gabaix and Maggiori (2015) argue that segmented markets and finan-

[1] Indeed, there are empirical evidence showing that the same phenomena are present in all asset classes. Beside currencies, Koijen et al. (2018) find strong predictability of carry across global equities, global bonds, commodities, US treasuries, credits, and options. Similarly, Asness et al. (2013) document consistent value and momentum return premia across diverse markets and asset classes including stock markets, equity index futures, government bonds, currencies, and commodities. Lettau et al. (2014) argue that the downside risk CAPM can jointly explain cross section of currencies, equity, equity index options, commodities, and sovereign bond returns.

cial frictions in form of the limited risk-bearing capacity by financial intermediaries lead to deviations from the UIP.

The second approach ignores higher-order cumulants resulting in risk and departs instead from the rational expectation assumption. De Grauwe and Dewachter (1993) and De Grauwe and Grimaldi (2006) argue that the interaction between fundamentalists and investors with a simple forecast rule can explain most of the empirical exchange-rate puzzles. Gourinchas and Tornell (2004) assume investors confuse trend changes in interest rate differentials for level changes and slowly update their beliefs, resulting in predictabilities in excess returns. Burnside et al. (2011b) use investor overconfidence to explain deviations from the UIP. Bacchetta and van Wincoop (2010) formalize Froot and Thaler (1990)'s delayed overshooting and explain how a delayed portfolio adjustment creates excess return patterns.

Recently, Engel (2016) and Valchev (2015) revisit the relationship of interest differentials and exchange rates over time horizon. Both document that the pattern of the UIP deviations is a function of a time horizon, as earlier discovered in Bacchetta and van Wincoop (2010). Higher interest rates predict positive excess returns of holding higher interest bonds initially. However, such patterns reverse in the medium run when higher interest rates predict negative excess returns. In the long run, there is no predictable excess return from interest rate differentials.

Formally, following Valchev (2015), define $\rho_{fh,t+k} = s_{fh,t+k} - s_{fh,t+k-1} - x_{fh,t+k-1}$ as the k-period-ahead realized excess return of holding foreign over home bonds. I consider the following regression when the period is set to monthly for $k = 1, 2, ..., 180$:

$$\rho_{fh,t+k} = \alpha_k + \beta_k x_{fh,t} + \epsilon_{t+k}. \tag{3}$$

Using a sample of 52 currency pairs, I confirm earlier documented patterns and note that (1) $\beta_1 < 0$, i.e. higher interest currencies do not depreciate as much as predicted by forward premiums over the next period, (2) there exists $h \geq 2$ such that $\beta_h > 0$. That is, higher interest currencies eventually earn negative excess returns with respect to the UIP benchmark at some point in the future, (3) $\lim_{k\to\infty} \beta_k = 0$, which implies that there is no excess return in the long run, and (4) $\sum_{k=1}^{\infty} \beta_k \geq 0$. Higher interest currencies have levels as strong as implied by interest differentials.

Notably, most of current risk-based and deviation-from-rationality explanations fail to reconcile with new empirical patterns. They are unable to explain the reversal in the sign of excess returns. Some exceptions include Engel (2016) and Itskhoki and Mukhin (2017), where an extra exogenous liquidity shock is introduced into the system. Valchev (2015) endogenizes this added shock by introducing an interaction between monetary and fiscal policies to create an endogenous convenience yield. Bacchetta and van Wincoop (2017) show that the delayed adjustment can in fact explain the predictability reversal puzzle, while Chernov and Creal (2018) jointly explain the pattern of the UIP deviations and the dynamic of forward rates over time by incorporating the stationary of the real exchange rate into a model of stochastic discount factor.

Previous literature decomposing the forward discount bias into the risk premium and the expectational error components finds that risk alone cannot fully capture deviations from the UIP. Prominently, Froot and Frankel (1989) use survey data to decompose the bias and reject that all bias is due to the risk premium but cannot reject that all bias is attributed to expectational errors. Bacchetta et al. (2009) argue that the excess return predictability in foreign exchanges (and in other financial markets) is related to the predictability of expectational errors.

Additionally, there is some evidence that human expectations do not follow rational expectations. A controlled experiment in Hommes et al. (2008) shows that expectations of risky assets deviate from rational expectation and seem to be driven by trend chasing behaviors. Greenwood and Shleifer (2014) document discrepancies between expected returns and return expectations and use mutual fund flows to suggest that investors act according to their expectations. Barberis et al. (2015) use survey data from Greenwood and Shleifer (2014) to parametrize the functional form of extrapolation that can fit stock market returns.

Focusing on exchange rate expectations, Frankel and Froot (1987) use different surveys of the yen/dollar exchange and conclude that expectations exhibit bandwagon effects in the short horizon. One of Ito (1990)'s findings is that investors expectations on the yen/-dollar rate violate the rational expectation hypothesis. Expanding the scope of exchange rate pairs, Chinn and Frankel (1994) find that forecasts of minor currencies exhibit smaller biases than those of major currencies. Chinn and Frankel (2002) widen the scope of survey data sources and find that forecasts are biased, and the risk premium is less variable than expected depreciations.

Above evidence argues that deviations from rational expectations deserve more attention. This paper uses a behavioral-based model in fitting the newly observed UIP pattern over time. The baseline model features investors with extrapolative beliefs. These investors are aware of fundamentally-implied exchange levels but still incorporate past exchange depreciations when forming their expectations.

Higher interest currencies have stronger-than-average exchange rates. Such elevated levels lead investors to form too optimistic beliefs of such currencies in the next period.

This extrapolation leads higher interest currencies to not depreciate as much as implied by interest differentials initially. The magnitude of this extrapolative force diminishes over time. In the medium run, mean-reverting interest differentials dominate and drive exchange levels lower. Investors then extrapolate the depreciation, causing higher interest currencies to over-depreciate during some periods in the future. Eventually, both the interest differential and the extrapolative force vanish. UIP holds in the long run.

The paper is structured as follows. Section 2 documents empirical patterns of deviations from the UIP over time. Both Engel (2016) and Valchev (2015) use US Dollar as a based currency and focus only most developed currency pairs. I expand the scope of the test by including developed and developing currencies from different regions around the world. I then test whether the choice of base currencies matters. Section 3 discusses what observed patterns say about the relationship between interest rates and exchange rates over time. Section 4 presents an extrapolative model with predictions in accordance to the observed foreign exchange dynamics. Section 5 tests some of the model assumptions and implications. In particular, survey data is used to check whether investors have extrapolative beliefs. I summarize and reiterate my findings in Section 6.

2 UIP Over Time Horizon

This section revisits stylized facts on exchange rates. The uncovered interest parity (UIP) states that high interest currencies should depreciate against low interest ones in order to equalize bond returns. Since expectation of spot rates are not tradable, the UIP needs not always hold.

Many trading strategies let investors bet on return differentials between home and foreign bonds. Investors can long and/or short country-specific bond series leaving themselves exposed to interest differentials between two currencies in a particular currency pair. Alternatively, traders can trade spot rates against forward rates to expose themselves to interest differentials. Formally, let $F_{fh,t+1}$ be a forward rate that investors agree at time t to exchange currencies at time $t+1$ in term of home per foreign currency. No arbitrage implies

$$\begin{aligned} F_{fh,t,t+1} &= S_{fh,,t} \cdot e^{i_t - i_t^*} \\ &= S_{fh,,t} \cdot e^{x_{fh,t}}. \end{aligned} \tag{4}$$

Equation (4) states that spot and forward rates can convey interest differentials. This method of constructing interest differentials has advantages over a subtraction of two interest rate series, since it is invariant to the choice of interest rate series. Since money markets are structured differently in different countries, conventional benchmarks for each country vary. Previous studies of empirical UIP patterns use eurocurrency rates, which are interests on bonds deposited in banks outside the home market, as benchmark rates. However, eurocurrency data is limited especially in emerging markets.

This paper uses two main datasources for exchange rates: Bloomberg and Datastream. Spot and forward rates in Bloomberg are 5pm New York close (21:00 GMT) levels. Datastream contains two series: the World Markets PLC/Reuters (WM/R) series and the Thomson/Reuters (T/R) series. WM/R provides 4pm London fixing (15:00 GMT) rates and has more comprehensive currency coverages. T/R is more limited in terms of currency coverages but has time series that go back further in the past history.

The paper pulls interest rate data solely from Datastream. Eurocurrency rates are used whenever available. Alternative rates such as deposit rates are used as supplements whenever eurocurrencies are unavailable.[2]

The paper constructs five sets of time-series data for exchange rates and interest rate differentials from the earliest available to 7 June 2017.

Dataset	**Exchange Rates**	**Interest Rate Differentials**
WM	Spots from WM/R	Using spots vs. forwards from WM/R
BBG	Spots from Bloomberg	Using spots vs. forwards from Bloomberg
TR	Spots from T/R	Using spots vs. forwards from T/R
i	Combine spot series from WM/R, Bloomberg, and T/R	Interest rate data from Datastream
Combine	Spots from WM/R, BBG, T/R, and i	Corresponding interest differentials from respective datasets

The remainder of this section focuses on the last dataset, that is the "Combine" method. This dataset contains the most comprehensive cross sectional sample and the longest time-series. A detailed construction can be found in Appendix A.

I consider the following set of regressions.

$$s_{fh,t+k} - s_{fh,t+k-1} = \alpha_k^1 + \gamma_k x_{fh,t} + \epsilon_{t+k}^1 \tag{5}$$

$$x_{fh,t+k-1} = \alpha_k^2 + \lambda_k x_{fh,t} + \epsilon_{t+k-1}^2 \tag{6}$$

$$s_{fh,t+k} - s_{fh,t+k-1} - x_{fh,t+k-1} = \alpha_k^3 + \beta_k x_{fh,t} + \epsilon_{t+k}^3 \tag{7}$$

By construction, $\gamma_k - \lambda_k = \beta_k$. This section discusses regression results when t is monthly and $k = 1, 2, 3, ..., 180$ in a unit of month.

[2] Appendix A provides comprehensive discussion of interest rate series from each country.

The analysis includes 52 currencies: Argentine Peso, Australian Dollar, Austrian Schilling, Belgian Franc, Brazilian Real, British Pound, Canadian Dollar, Chilean Peso, Chinese Yuan, Colombian Peso, Czech Koruna, Danish Krone, Egyptian Pound, Euro, Finnish Markka, French Franc, Deutsche Mark, Greek Drachma, Hong Kong Dollar, Hungarian Forint, Icelandic Krona, Indonesian Rupiah, Indian Rupee, Ireland Pound, Israeli New Shekel, Italian Lira, Japanese Yen, South Korean Won, Kuwaiti Dinar, Malaysian Ringgit, Mexican Peso, Netherlands Antillean Guilder, New Zealand Dollar, Norwegian Krone, Pakistani Rupee, Philippine Peso, Polish Zloty, Portuguese Escudo, Russian Ruble, Saudi Arabian Riyal, Singaporean Dollar, South African Rand, Spanish Peseta, Swedish Krona, Swiss Franc, Taiwanese Dollar, Thai Baht, Turkish Lira, United Arab Emirates Dirham, US Dollar, Uruguayan Peso, and Venezuelan Bolivar.

For countries that have since joined the European currency union, fixed conversion factors against the Euro are used to construct hypothetical exchange levels. These factors were set when the respective European legacy currency was fixed to the Euro.

I run the set of regressions (5), (6), and (7) on both country-specific time series and pooled panels. Standard errors are adjusted for heteroskedasticity, serial correlation, and cross-country correlation using Newey and West (1987) for time-series regressions and Driscoll and Kraay (1998) for panel regressions.

I assume and later verify that interest rate differentials follow an autoregressive process of order one with an autocorrelation of $\lambda \in [0, 1)$ and an independent and identically distributed innovation ϵ_t that is normally distributed with mean zero and variance σ^2. That is,

$$x_{fh,t} = \lambda x_{fh,t-1} + \epsilon_t, \quad \epsilon_t \sim \mathcal{N}(0, \sigma^2) \quad \text{and} \quad cov(\epsilon_t, \epsilon_{t-1}) = 0. \tag{8}$$

Figure 1 plots coefficients γ_k, λ_k, and β_k for different k if the UIP were to hold.

An assumption of AR(1) interest differentials means that the coefficient λ_k from the regression equation (6) is equal to λ^k. The middle plot of Figure 1 illustrates the evolution of λ_k over time.

If the UIP holds, exchange rates should move to offset differentials in interest rates and nullify excess returns in holding foreign versus home bonds. That is, γ_k must equal to λ_k so that $\beta_k \equiv 0$.

Moving away from the hypothetical world, I display patterns observed in the data. I analyze four different ways of pooling the data.

1. Pooled panel: includes all data from 52 countries.

2. Rich panel: includes data from countries whose gross domestic product (GDP) based

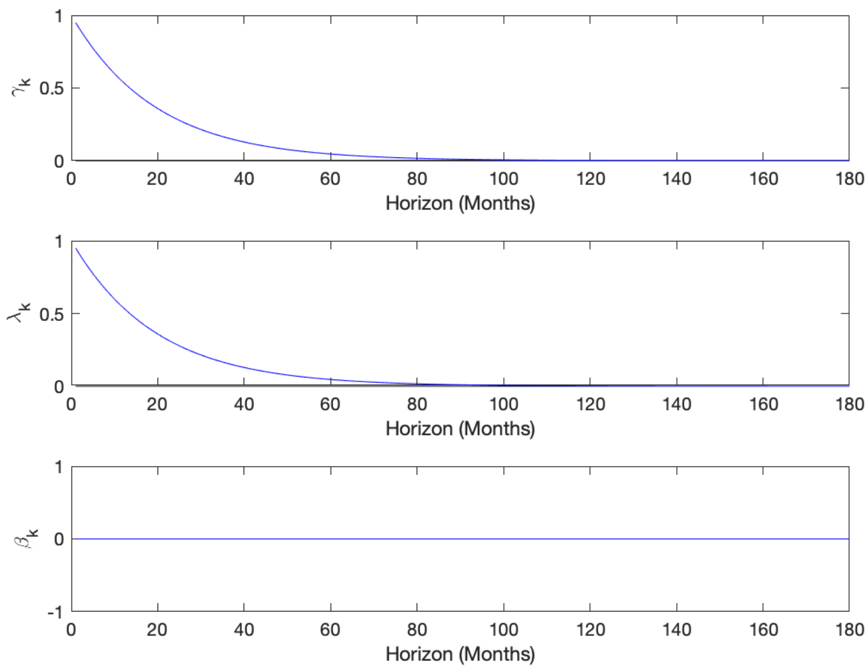


Figure 1: Coefficients under the UIP

on the purchasing power parity (PPP) per capita is not less than the median in each respective fiscal year.[3]

3. Poor panel: includes data from countries whose GDP per capita is below the median in each respective fiscal year.

4. G7: includes data from the G7 countries (Canada, France, Germany, Italy, Japan, the United Kingdom, and the United States).

2.1 Bilateral Exchange Rates

US Dollar as a Base Currency

Following existing literature, the paper first looks at estimated regression coefficients when the United States is a home country. Figure 2 plots estimated coefficients along with 95% confidence bands from panel regressions of equations (5), (6), and (7) using the "Combine" data for the G7 countries.

[3]GDP based on PPP per capita is in a unit of current international dollars. Details on the GDP-based categorization are in Appendix A.

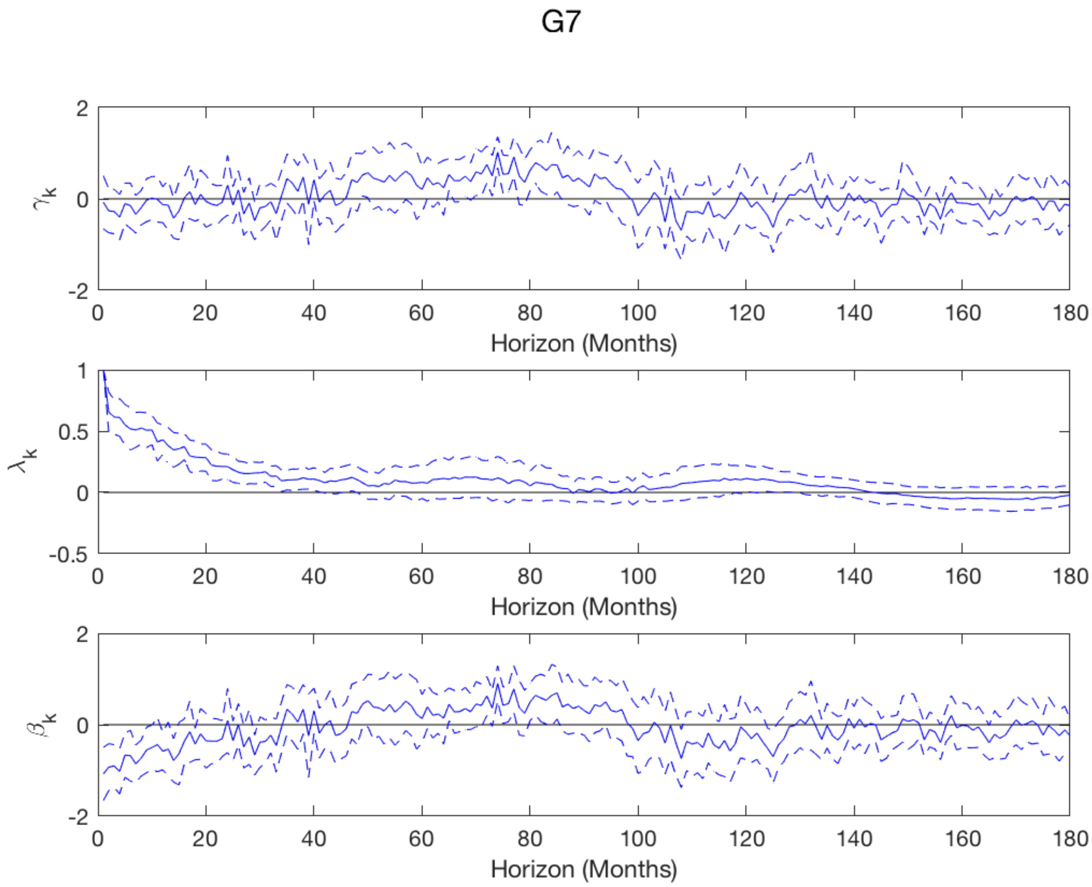


Figure 2: $k = 1$ Panel Coefficients for G7 Using Monthly Data and "Combine" Method

I make the following observations from Figure 2.

1. λ_k decays smoothly, implying that interest rate differentials roughly follow a stationary AR(1) process.

2. Exchange rates are mostly unpredictable. γ_k is almost always indistinguishable from zero except for a medium k. For a medium k, γ_k is significantly positive, implying that higher contemporaneous interest rates predict exchange depreciations some time in the future.

3. β_1 is negative, reiterating the standard UIP puzzle. Higher interest currencies fail to depreciate as much as implied by forward premiums, resulting in positive excess returns in holding higher interest bonds.

4. β_k stays positive initially. There are positive excess returns in holding bonds of higher interest currencies initially.

5. β_k turns negative for some k around 70 - 90. Higher contemporaneous interest rates predict negative excess returns of holding such bonds around 6 years after.

6. β_k reverts back to zero and stays indistinguishable from zero for all $k \geq 100$. There is no predictable excess returns from interest differentials eventually.

7. The sign of $\sum_{k=1}^{\infty} \beta_k$ appears indistinguishable from zero.

Such patterns are robust across different samples. Figure 3 plots point estimates from the same set of regression equations for four different ways of pooling the data. Additional robustness check can be found in Appendix B.

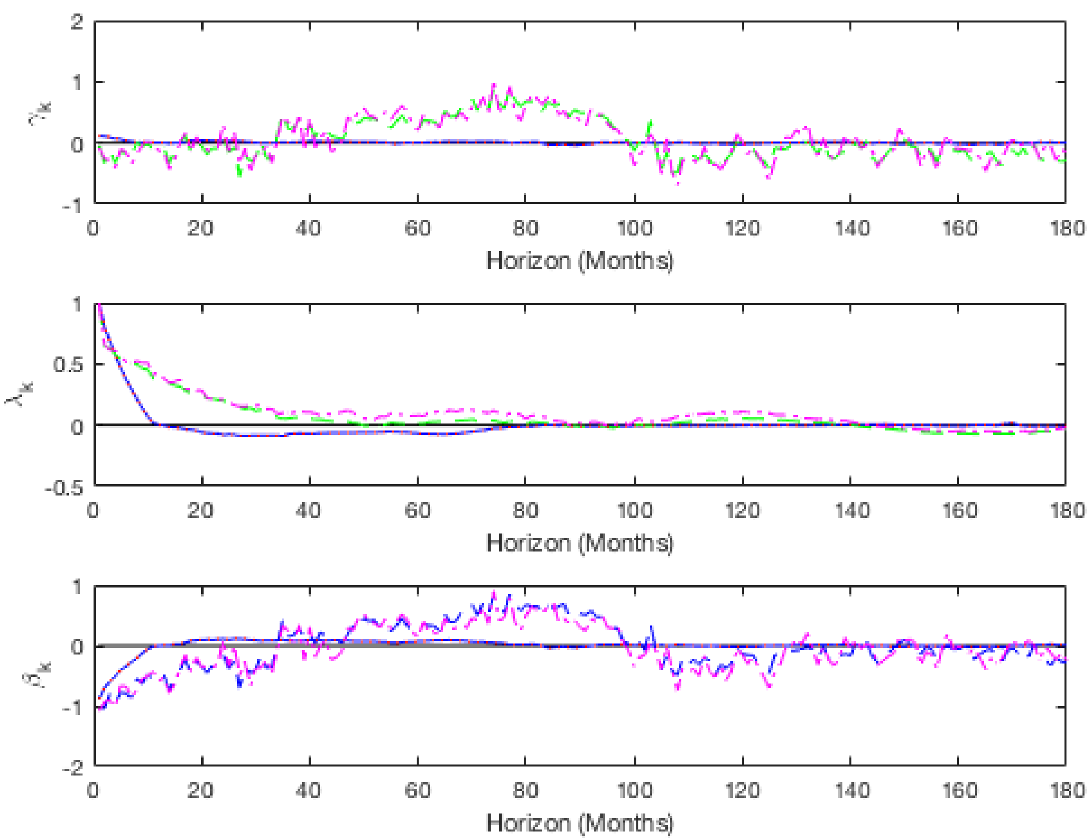


Figure 3: $k = 1$ Panel Coefficients Using Monthly Data and "Combine" Method (Pooled Panel in Solid Blue, Rich Panel in Dashed Green, Poor Panel in Dotted Red, G7 in Dash-Dot Magenta)

Figure 3 confirms the stationary AR(1) assumption of interest rate differentials and the dynamics of β (starting with a negative beta, turning positive in the medium run, and converging to zero eventually).

Alternative Home Countries

This section explores the robustness of the above empirical patterns by looking at alternative home countries. Figure 4 plots estimated coefficients along with 95% confidence bands when the Euro is used as a based currency.

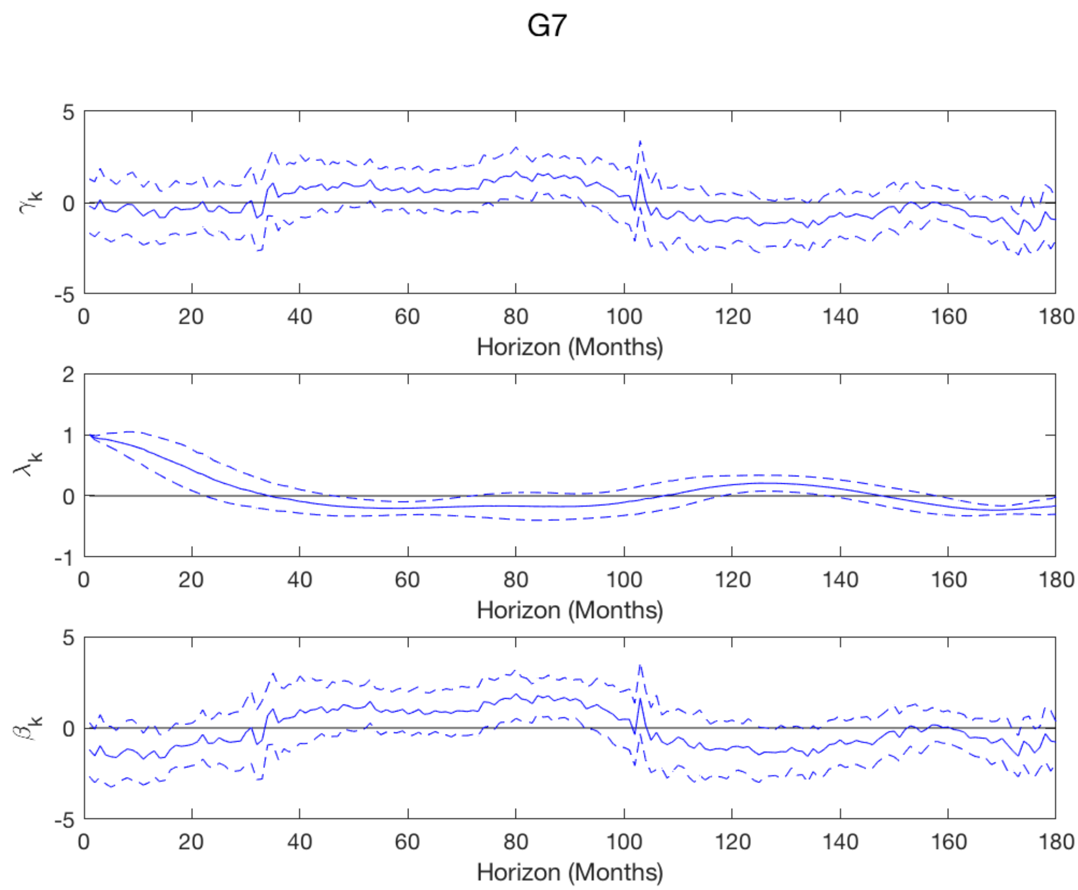


Figure 4: $k = 1$ Panel Coefficients for G7 Using Monthly Data and "Combine" Method When EUR is a Home Currency

Similar patterns unfold when EUR instead of USD is used as home currency. However, the statistical significance is compromised for the standard UIP puzzle.

2.2 Absolute Exchange Rates

This section attempts to control for the base currency effect. Let $S_{j,t}$ be the country-j absolute exchange rate at time t. I define the absolute exchange rate for country j as the relative price of non-tradable to tradable goods for country j, i.e.

$$S_{j,t} = \frac{U_{c_{it}^{NT}}}{U_{c_{it}^{T}}}, \tag{9}$$

where $U_{c_{it}^{NT}}$ is the marginal consumption utility of nontradable good, and $U_{c_{it}^{T}}$ is the marginal consumption utility of tradable good respectively.

I do not observe the marginal consumption utility directly but can explore a relationship between absolute and bilateral exchange rates. In particular, the bilateral exchange rate $S_{fh,t}$ is defined as:

$$S_{fh,t} = \frac{S_{f,t}}{S_{h,t}}. \tag{10}$$

I proxy for the absolute exchange rate using $\tilde{S}_{j,t}$, where $\tilde{s}_{j,t} = s_{jh,t} - \frac{1}{n}\sum_{k=1}^{n} s_{kh,t}$, when n is a total number of currency pairs, and the smaller cases represent the logarithm of the upper-case variables. Combining equations (9) and (10) yield

$$\tilde{s}_{j,t} = s_{j,t} - \frac{1}{n}\sum_{j=1}^{n} \ln\left(\frac{U_{c_{jt}^{NT}}}{U_{c_{jt}^{T}}}\right). \tag{11}$$

The constructed absolute exchange rate, $\tilde{s}_{j,t}$, will be a good proxy of absolute exchange rate, $s_{j,t}$, if the second term is close to zero, i.e. when the country-average of log marginal consumption utility from nontradable good is roughly the same as the country-average of log marginal consumption utility from each respective country tradable good.

Let $\tilde{x}_{j,t}$ be a proxy for the country-j absolute interest differentials defined analogously by

$$\tilde{x}_{j,t} = x_{jh,t} - \frac{1}{n}\sum_{j=1}^{n} x_{kh,t}. \tag{12}$$

Proxies for absolute exchange rates and absolute interest differentials tell us where country j's currency and interest rate stand relative to the equally-weight basket of currencies and interest rates, respectively.

This section pools all 51 bilateral exchange rates against the US Dollar and defines absolute exchange rates to be deviations from the mean. That is, $n = 51$ in my sample. Figure 5 plots estimated coefficients along with 95% confidence bands from panel regressions of equations (5), (6), and (7) using the "Combine" data for the G7 countries.

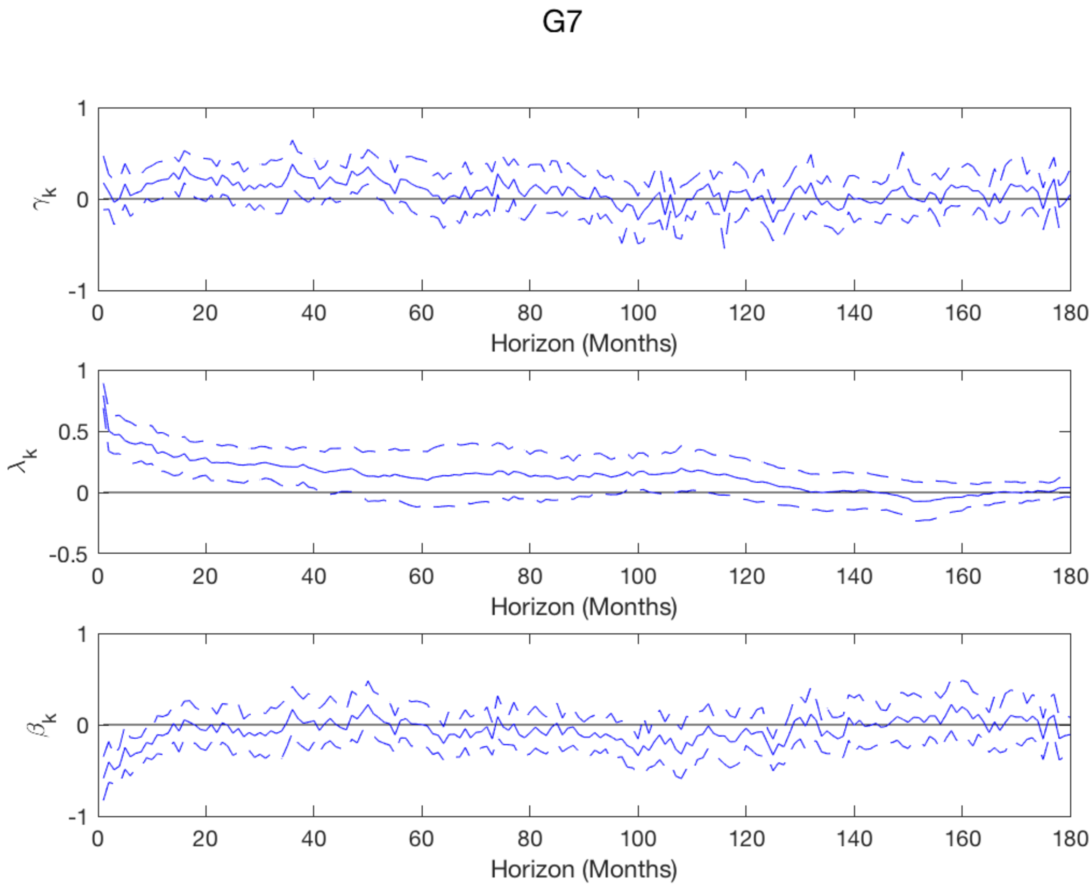


Figure 5: $k = 1$ Panel Regression for G7 Using Absolute Proxies, Monthly Data and "Combine" Method

Compared to patterns from bilateral exchange rates, absolute exchange rates lose the statistical significance of the reversal in the sign of excess returns.

3 Excess Return Predictability and Exchange Rate Dynamics

Results from Section 2 confirm recent empirical findings in Engel (2016) and Valchev (2015). The patterns are robust and are only partially affected by the choice of a base currency. From here onwards, the paper focuses on the US Dollar as a base currency case and drops a fh subscript on S, s, F, f, and x whenever it generates no possible confusion.

This section discusses implications on exchange rate dynamics from excess return predictabilities observed in the data. A significantly negative $\hat{\beta}_1$ implies that higher interest currencies do not depreciate as much as implied by forward premiums, creating positive excess returns in holding higher (versus lower) interest bonds initially. This reiterates the classical UIP puzzle.

$\hat{\beta}_k$ stays negative for awhile before turning positive for medium k around 70 - 90. This implies that higher interest currencies over-depreciate roughly six years later. In other words, higher contemporaneous interest rates forecast significantly negative returns in holding higher interest bonds in the medium run.

Eventually, $\lim_{k\to} \hat{\beta}_k = 0$. There is no predictable excess return from interest differentials in the long run.

The documented empirical patterns imply that, with respect to the UIP benchmark, higher interest currencies under-depreciate, then over-depreciate before reverting back to the implied movement pattern. Such dynamics reiterate previously known puzzles in exchange-rate economics.

The initial depreciation underpins the UIP puzzle. The failure of interest differentials in predicting exchange movements, as evident from close-to-zero γ_1, supports the exchange rate disconnect puzzle. The cycle of under- and over-depreciation of exchange rates with respect to fundamentals highlights the excess volatility puzzle.

Most importantly, the reversal in the sign of excess returns begs a quest for new models. Existing theoretical UIP literature lacks forces that drive the change in the sign of excess returns.

Risk-based models rely on the argument that higher interest currencies are riskier and thus demand higher returns to compensate for the risk. Most of this class of model contain only one risk and can only explain why $\hat{\beta}_k$ is negative.

Models with deviations from rational expectations rely on diverse explanations. Most papers feature frictions that result in the sluggishness in an exchange rate adjustment. The slow adjustment however fails to explain the reversal in signs.

Unlike Engel (2016), this paper does not find a strong evidence of $\sum_{k=1}^{\infty} \hat{\beta}_k \geq 0$. Our samples indicate that the sum seems to be indistinguishable from zero. The discussion below illustrates how the sign of the sum has an implication on the "level" of exchange rates.

For simplicity, assume that exchange rate is conditionally stationary, i.e. $\lim_{T\to\infty} s_{t+T}$ exists and is well-defined. Investors have correct beliefs regarding this long-run exchange rates, i.e. $\lim_{T\to\infty} s_{t+T} = \lim_{T\to\infty} s_{t+T}^{UIP}$.

Taking equation (3) as given and summing across k, I have

$$\begin{aligned}\sum_{k=1}^{\infty}\alpha_k + \sum_{k=1}^{\infty}\beta_k x_t + \sum_{k=1}^{\infty}\epsilon_{t+k} &= \sum_{k=1}^{\infty}\rho_k \\ &= \sum_{k=1}^{\infty}(s_{t+k} - s_{t+k-1} - x_{t+k-1}) \\ &= \lim_{T\to\infty} s_{t+T} - s_t - \sum_{k=1}^{\infty} x_{t+k-1}.\end{aligned}$$

Assuming $\sum_{k=1}^{\infty}\alpha_k = 0, \sum_{k=1}^{\infty}\beta_k \geq 0$, and $\sum_{k=1}^{\infty}\epsilon_{t+k} = 0$ implies that, for $x_t \geq 0$,

$$\begin{aligned}\lim_{T\to\infty} s_{t+T} - s_t - \sum_{k=1}^{\infty} x_{t+k-1} &= \sum_{k=1}^{\infty}\beta_k x_t \\ &\geq 0 \\ &= \sum_{k=1}^{\infty}(s_{t+k}^{UIP} - s_{t+k-1}^{UIP} - x_{t+k-1}) \\ &= \lim_{T\to\infty} s_{t+T}^{UIP} - s_t^{UIP} - \sum_{k=1}^{\infty} x_{t+k-1} \\ \Leftrightarrow s_t^{UIP} &\geq s_t.\end{aligned}$$

The above discussion illustrates that the sign of $\sum_{k=1}^{\infty}\beta_k$ indicates the strength of exchange rate, s_t, compared to the level implied by the UIP, s_t^{UIP}. Higher contemporaneous interest currencies are at least as strong (weak) as levels implied by interest differentials if the sum of excess return regression coefficients is non-negative (non-positive).

Evidence from Engel (2016) conveys that the sum is positive indicating that there is a level puzzle, i.e. higher interest currencies are too strong.

As with a reversal in the sign of excess returns, existing strands of the theoretical UIP literature cannot explain this level puzzle. If higher interest currencies are riskier, their currencies should be weaker than implied by forward premiums. On the other hand, slow adjustments mean that higher interest currencies do not appreciate enough initially.

This section argues that newly documented patterns invalidate most of existing theoretical UIP models and thus warrants a search for new models.

4 Extrapolative Model

4.1 Bubbles and Exchange Rates

Embedding the bubble framework in exchange rates can reconcile most puzzles in exchange-rate economics. I refer to bubbles as price deviations from underlying asset's intrinsic values.

Viewing exchange rates as an asset class, the exchange rate disconnect puzzle is simply a bubble phenomenon in exchange rates. Traditional bubble episodes are accompanied by excess price and return volatilities. The over- and under-valuation of exchange rates with respect to forward premiums draw close parallel to patterns of bubble's boom and bust.

The evolution of excess returns resembles the typical bubble episode. Initial positive excess returns represent an emerging phase of the bubble. These positive excess returns last for awhile, reflecting the flamboyant life of the bubble. At one point, the bubble bursts. Excess returns turn negative before adjusting slowly toward fundamentals. Exchange rate dynamics evidently point to the existence of bubbles in exchange rate markets.

There are two main types of bubbles in the finance literature: rational bubbles and behavioral bubbles. While models of rational bubbles may potentially explain the life cycle patterns of exchange rates, I focus mainly on behavioral bubbles. Under rational bubble regimes, little is known about what governs the evolution of price movements and which factors contribute toward extra volatility components. In contrast, behavioral bubbles offer more structures and often specify the origin of the bubble development.

Famous existing frameworks for behavioral bubbles include disagreement bubbles and extrapolative bubbles. Scheinkman and Xiong (2003) follow the basic insight of Harrison and Kreps (1978) and show how overconfidence among traders with heterogenous beliefs can lead to bubbles in asset prices. Barberis et al. (2018) present bubbles that are created from investors' extrapolation.

4.2 Extrapolative Beliefs

This paper acknowledges many sources of biases in beliefs but will focus on extrapolative beliefs. Evidence of extrapolative behaviors is prevalent. Earlier study of Dominguez (1986) finds a extrapolative component in exchange expectations. Case et al. (2012) document that one-year lagged house price appreciation explain almost perfectly the home buyers' expectations of future home price appreciation. Greenwood and Shleifer (2014) pull different data sources and register strong evidence of extrapolation in stock market returns. Smith et al. (1988) and Haruvy et al. (2007) recognize extrapolative expectations during well-defined experimental price bubbles.

In exchange rate economics, investors can extrapolate two main objects: interest rates and exchange rates. Investors learn information regarding short- and medium-term interest differentials from forward rates. As an information on interest rates is readily available, I assume that investors have rational expectations about interest differentials but are subjected to behavioral biases when forming their expectations on exchange rates.

This section presents the baseline model with a large home country and an infinitesimally small foreign country. A bond market equilibrium is therefore entirely determined by investors in the large home country.[4] I index a continuum of home investors by $j \in [0,1]$. Each investor has a wealth of W_t^j denominated in the home currency at time t and makes an investment decision whether to invest in home or foreign bonds. I normalize units of bonds in both countries such that their prices in the home currency are equal to 1.

Assume that the return on home (foreign) bonds are exogenously given by $i_t(i_t^*)$. Each investor holds $B_{H,t}^j(B_{F,t}^j)$ units of home (foreign) bonds respectively to maximize the next-period consumption C_{t+1}^j. The optimization problem of each home investor j is as follows:

$$\begin{aligned} &\max_{B_{H,t}^j, B_{F,t}^j} C_{t+1}^j \\ \text{subject to} \quad & W_t^j = B_{H,t}^j + B_{F,t}^j \\ \text{and} \quad & C_{t+1}^j = B_{H,t}^j \exp(i_t) + B_{F,t}^j[\exp(i_t^*)\frac{S_{t+1}}{S_t} - \exp(i_t)]. \end{aligned}$$

The solution to the above optimization problem is

$$B_{F,t}^j = \begin{cases} \infty, & \text{for } s_t < \mathbb{E}_t^j(s_{t+1}) - x_t \\ [0,\infty] & \text{for } s_t = \mathbb{E}_t^j(s_{t+1}) - x_t \\ 0, & \text{for } s_t > \mathbb{E}_t^j(s_{t+1}) - x_t \end{cases}.$$

Expectations of exchange depreciations affect individual holding of foreign bonds.

Let $\mathbb{E}_t^F(s_{t+k})$ be the time-t rationally-expected k-horizon-ahead exchange rate. Rationally-

[4]This assumption allows us to work around the famous Siegel's paradox from Siegel (1972).

expected exchange rates are pinned down by interest rate differentials as per below:

$$\mathbb{E}_t^F(s_{t+k}) = \lim_{T\to\infty} \mathbb{E}_t(s_{t+T}) - \sum_{h=0}^{\infty} \mathbb{E}_t(x_{t+k+h}). \tag{13}$$

Next, denote the time-t extrapolative k-horizon-ahead exchange rate by $\mathbb{E}_t^X(s_{t+k})$. I define,

$$\mathbb{E}_t^X(s_{t+k+1}) = \mathbb{E}_t^F(s_{t+k+1}) + \gamma(\mathbb{E}_t^X(s_{t+k-1}) - \mathbb{E}_t^X(s_{t+k-2})), \tag{14}$$

where $\gamma \in [0,\infty)$ governs the degree of a behavioral bias.

Equation (14) implies that extrapolative investors are aware of fundamentals affecting exchange rates but at the same time are subjected to some degrees of behavioral biases. This bias induces investors to incorporate past depreciations when forming exchange expectations.

The gap between an extrapolative expectation and a fundamental level is a function of a recent depreciation. Positive γ means that past depreciations result in weaker expectations. $\gamma = 0$ reflects the complete rational case.

I assume that all investors have homogenous extrapolative beliefs regarding next-period exchange rates, i.e.

$$\mathbb{E}_t^j(s_{t+1}) = \mathbb{E}_t^X(s_{t+1}) \quad \forall j \in [0,1]. \tag{15}$$

Market clearing conditions for non-zero fixed-supply home and foreign bonds require

$$s_t = \mathbb{E}_t^X(s_{t+1}) - x_t. \tag{16}$$

For simplicity, assume that interest differentials follow a stationary autoregressive process of order one with an autocorrelation coefficient of $\lambda \in [0,1)$ and an independently identically distributed innovation ϵ_t normally distributed with mean zero and variance σ^2, i.e.

$$x_t = \lambda x_{t-1} + \epsilon_t, \;\; \epsilon_t \sim \mathcal{N}(0,\sigma^2). \tag{17}$$

Investors have rational beliefs on interest differential process x_t.

4.3 Equilibrium Exchange Rate

Proposition 1 (Equilibrium Exchange Rate). *The equilibrium exchange rate s_t satisfies*

$$s_t = \lim_{T\to\infty} \mathbb{E}_t(s_{t+T}) - \frac{x_t}{1-\lambda} + \gamma(s_{t-1} - s_{t-2}). \tag{18}$$

Proposition 1 states that an equilibrium exchange rate is pinned down by a current interest differential as well as a 1-period lagged depreciation. $\gamma = 0$ recovers the fundamental exchange level. When $\gamma > 0$, a recent past exchange change affects an equilibrium exchange level. If a foreign currency recently depreciates against a home currency ($s_{t-1} > s_{t-2}$), a foreign currency will be weaker than a fundamentally implied level in an equilibrium.

I make another simplifying assumption. Following Gourinchas and Tornell (2004), I assume that the nominal exchange rate is conditionally stationary, i.e. $\lim_{T\to\infty} \mathbb{E}_t(s_{t+T})$ is well-defined and is denoted by $\bar{s}_t$. Investors have a correct belief regarding this long-run level. I acknowledge that a nominal exchange rate is often cointegrated in the data. The stationary assumption is made to aid the mathematical analysis.

Under the stationary assumption, an equilibrium exchange rate s_t is defined as below:

$$s_t = \bar{s}_t - \frac{x_t}{1-\lambda} + \gamma(s_{t-1} - s_{t-2}). \tag{19}$$

4.4 Model Implications

This section illustrates how exchange rate dynamics evolve under extrapolative beliefs. In particular, I explore how the above model explains foreign exchange market anomalies.

Proposition 2 (The Level Puzzle). *For $\gamma < \frac{1}{\lambda(1-\lambda)}$, $cov(s_t, x_t) \leq cov(s_t^{UIP}, x_t)$. This implies $\sum_{k=1}^{\infty} \beta_k \geq 0$.*

Currencies with higher contemporaneous interest rates are at least as strong as implied by interest differentials (under the UIP). The equality holds when $\gamma = 0$. That is, equilibrium exchange rates are completely pinned down by an interest differential path.

Proposition 2 can reconcile Engel (2016)'s finding that currencies with higher contemporaneous interest rates are at least as strong as implied by the UIP.

Proposition 3 (The UIP Puzzle). *The regression coefficient β_1 from the regression equation (7) is as follows:*

$$\beta_1 \begin{cases} = 0 & \text{for } \gamma = 0 \\ < 0 & \text{for } \gamma \in \left(0, \frac{1}{\lambda(1-\lambda)}\right) \end{cases}$$

Proposition 3 states that as long as investors do not extrapolative excessively, extrapolation leads exchange rates to deviate from the UIP. In particular, higher interest currencies do not depreciate enough over the following period to nullify excess returns. The UIP is recovered whenever investors are rational and do not extrapolation, i.e. when $\gamma = 0$.

Proposition 4 (Reversion in Excess Returns)**.** *For $0 < \gamma < \frac{1}{\lambda(1-\lambda)}$, there exists $h \geq 2$ such that $\beta_h > 0$. For $\gamma = 0$, $\beta_k \equiv 0$ for all k.*

Proposition 4 follows directly from combing Proposition 2 and Proposition 3. When investors do not extrapolate, i.e. $\gamma = 0$, there is no predictable excess return from interest differentials at any horizon. Investors with non-explosive extrapolative beliefs, on the other hand, experience the reversal in the sign of excess returns. Higher contemporaneous interest rates under-depreciate initially but will over-depreciate at some later period.

Proposition 5 (Long-Run Reversion to the UIP)**.** *For $\gamma \in [0, 1)$. $\lim_{k\to\infty} \beta_k = 0$.*

In any case, interest rate differentials have no predictive power of excess returns in the long run.

Proposition 6 (Excess Volatility Puzzle)**.** *For $0 < \gamma < \frac{1}{\lambda(1-\lambda)}$, $var(s_t) \geq var(s_t^{UIP})$.*

Extrapolative beliefs potentially contribute to a higher volatility of exchange rates (in excess of variations in interest differentials).

All proofs are in Appendix D. Investor beliefs affect their trading behaviors, which in turn pin down equilibrium exchange rates. When home interest rates are higher than average, home currencies are unusually strong. With extrapolative beliefs, investors form even more optimistic forecasts of next-period home levels resulting in even stronger equilibrium home currencies in the current period. Higher contemporaneous home levels increase extrapolators' expectations even more. This chain reaction results in initial positive excess returns in holding higher interest currencies.

It is not surprising that a sufficient high extrapolative coefficient may result in an explosive path of exchange rates. An initial appreciation may lead investors that extrapolate excessively to form extremely optimistic forecasts. As the recent appreciation feeds into the belief formation process, this initial appreciation may lead to everlasting appreciations.

Readers may wonder what is defined as an excessive extrapolation. Counteracting extrapolative beliefs in the above model is the depreciating force from a stationary AR(1) assumption of interest differentials. In an absence of an extrapolation, there is a natural force pulling high interest currencies back to their long-run levels. Extrapolative behaviors add another force governing exchange rate changes.

The interaction between the extrapolative force and the interest differential force is as follows. Initially, the extrapolative force counteracts the interest differential force. Investors extrapolate a recent appreciation of high interest currencies. Such action dampens the supposed depreciation, resulting in initial positive excess returns.

A non-explosive extrapolation guarantees an existence of an equilibrium exchange rate as well as an eventual reversal in the sign of excess returns. As time passes, the extrapolative force will get weaker in magnitude and becomes dominated by the interest differential force. Immediately after that point in time, the extrapolative force reinforces the interest differential force, leading to over-depreciation of high contemporaneous interest currencies. Negative excess returns are registered, as observed in the empirical data.

Eventually, both the interest differential force and the extrapolative force die off. A minimal extrapolation means an eventual reversion to the UIP.

As the extrapolative force makes exchange levels more dispersed, it naturally results in excess volatilities. In addition to the interest differential variation, there are two added components of the exchange rate variation. The first component is the exchange depreciation entering the price volatility with a magnifying factor that is equal to the square of an extrapolative coefficient (γ^2). This term always contributes to a higher volatility. The second component is the interaction between the interest differential force and the extrapolative force mentioned earlier. As discussed, these two forces sometimes cancel each other and at other times reinforce each other. As shown in Appendix D, the interaction also contributes to a higher volatility.

I illustrate the simulated exchange rate path in Figure 6. Without extrapolation, exchange rates will mirror the path of interest differentials. With extrapolative investors, exchange rates become more volatile. A momentum in investor expectations causes exchange rates to fluctuate around their fundamental levels.

4.5 Model Discussion

The proposed model is fairly tractable. The expectation formation process nests the complete fundamental case. The baseline model can generate excess returns as observed in the data. Section 5 provides an empirical evidence to the model assumptions. In particular, survey data is used to test whether investors indeed extrapolate.

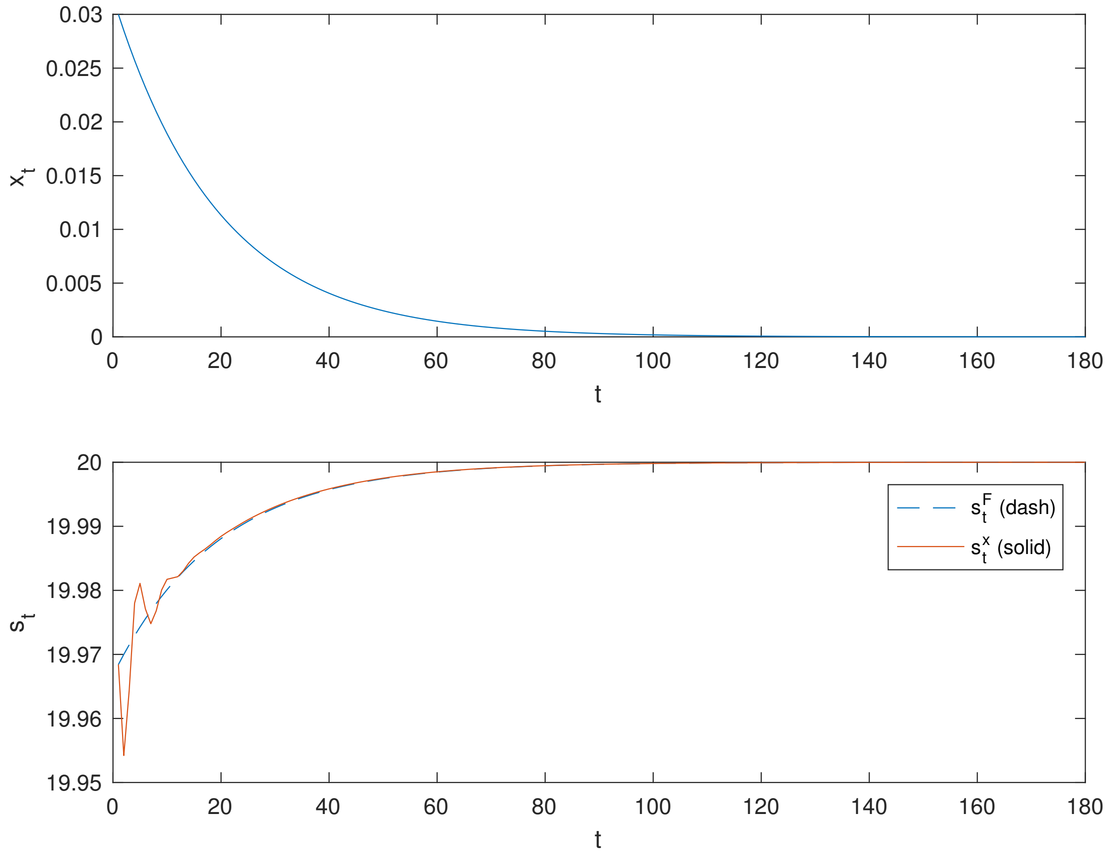


Figure 6: The upper figure displays the interest differential path under an AR(1) assumption with $\lambda = 0.95$. The lower figure displays exchange rates under the UIP (dashed) and under extrapolative beliefs with $\gamma = 0.5$ (solid)

5 Testing Model Assumptions and Implications

This section attempts to support some key assumptions made in the baseline model in Section 4. I begin by examining the AR(1) assumption of interest rate differentials and then focus on the essential question whether investors indeed extrapolate. I conclude this section by comparing my findings to an existing extrapolation literature.

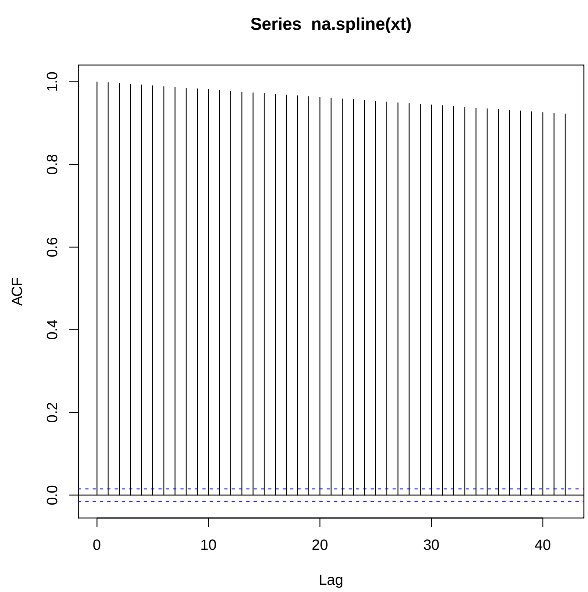
5.1 The AR(1) Assumption of Interest Rate Differentials

This section checks the validity of the AR(1) assumption of the 1-month interest rate differentials. Each country's daily time series data on the 1-month interest differentials against the United States is tested whether it follows an autoregressive process of order one. I proceed first by testing for the stationary of the process using the Dickey-Fuller test and then using the Akaike Information Criterion (AIC) to choose the order of the autoregressive model.

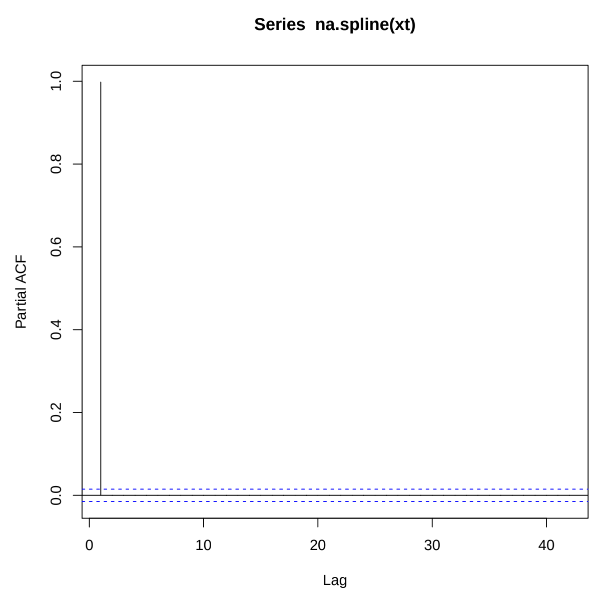
Table 4 in Appendix C indicates that a majority of countries has an augmented Dickey-Fuller p-value that is less than 0.05. I reject the null hypothesis of a unit root with a 95% confidence level for these countries. The null of a unit root can only be rejected with a 90% confidence level for Argentina and Turkey. The high p-value for Austria, Euro, and Colombia makes it impossible to reject the null of a unit root in those countries.

The last column of Table 4 in Appendix C shows that the AIC criterion picks the lag order of 1 for all countries.

Combining the p-value with an optimal order points to an evidence of a stationary AR(1) structure of interest differentials. I also plot the autocorrelation function (ACF) and the partial autocorrelation function (PACF) to confirm the AR/MA structure of interest differentials.



(a) ACF Plot



(b) PACF Plot

Figure 7: Switzerland Time Series. Monthly Data. i Method.

Figure 7 illustrates the ACF and PACF plots using the Switzerland data. The ACF plot slowly decays over time ruling out the pure MA structure as well as suggesting a relatively high autoregressive coefficient. The PACF plot spikes at one and cuts off completely thereafter, strongly supporting the AR(1) structure.

Results from other countries have exactly identical patterns (decaying ACF and cut-off-after-1 PACF). Such prominent features serve as a clear evidence for the AR(1) structure of interest rate differentials.

I perform the same analysis for the 1-, 3-, 6-, 12-, and 24-month interest rate differentials using both daily and monthly data. The results from the ADF test, the AR fitting, the ACF plot, and the PACF plot retain same patterns in all these different samples.

I conclude that the assumption of an autoregressive process of order 1 for the interest differentials is valid.

5.2 Evidence from Survey Data

This section explores an evidence of irrationality in exchange rate markets. The study of investor beliefs requires data on expectations since individual beliefs are rarely elicited. I obtain consensus forecasts from the Forecasts Unlimited Inc. (FX4casts.com). Appendix A describes this dataset in more details. In short, FX4casts.com gathers survey consensus from large financial institutions. The data contains monthly historical spots as well as 1-, 3-, 6-, 12-, and 24-month-ahead spot forecasts of 32 currencies along with their confidence intervals.

Patterns of excess returns displayed in Section 2 are robust to the choice of a period step, as shown in Appendix C. This section provides an empirical evidence from 3-month forecasts instead of 1-month forecasts, as the 3-month data started on August 1986 while the 1-month data only started on July 2008. I complement spots and forecasts with interest rate differentials data from the "i" method.

Survey-Expected Excess Returns

Analogous to the analysis performed in section 2, this subsection examines deviations from the UIP when expected depreciations are used instead of realized depreciations. In particular, I analyze the following regressions.

$$\rho_{t+h} = s_{t+h} - s_t - x_t = \kappa_1 + \eta_1 x_t + \xi_{1,t+h} \tag{20}$$

$$\mathbb{E}_t^e(\rho_{t+h}) = \mathbb{E}_t^e(s_{t+h}) - s_t - x_t = \kappa_2 + \eta_2 x_t + \xi_{2,t} \tag{21}$$

The realized excess return from holding foreign bonds from time t for h periods is denoted by $\rho_{t+h} = s_{t+h} - s_t - x_t$. Investor's expected excess return of holding foreign instead of home bonds is denoted by $\mathbb{E}_t^e(\rho_{t+h}) = \mathbb{E}_t^e(s_{t+h}) - s_t - x_t$.

Regression equation (20) is the standard UIP regression, while regression equation (21) tests whether the UIP holds when investor forecasts are used instead of realized rates.

Estimated $\hat{\eta}_2$ is less negative than estimated $\hat{\eta}_1$ in Table 1. This implies that deviations from the UIP are less severe in the survey data. Investors are aware that higher interest currencies should depreciate over the next period and form their forecasts to reflect weaker exchange levels than next-period realized rates. Significantly positive $\hat{\eta}_2$ indicates

Table 1: Excess Returns when the Period Step is 3 months ($h = 3$)

	All		G7		Rich		Poor	
	ρ_{t+h}	$\mathbb{E}_t^e(\rho_{t+h})$	ρ_{t+h}	$\mathbb{E}_t^e(\rho_{t+h})$	ρ_{t+h}	$\mathbb{E}_t^e(\rho_{t+h})$	ρ_{t+h}	$\mathbb{E}_t^e(\rho_{t+h})$
x_t	-1.070***	-0.452*	-1.392	-0.963***	-1.299	-0.579*	-1.010**	-0.419*
	(-3.65)	(-2.55)	(-1.88)	(-3.45)	(-1.83)	(-2.23)	(-3.18)	(-2.04)
constant	-0.0004	-0.0012	0.0015	-0.0024	0.0028	-0.0001	-0.0056	-0.0028
	(-0.10)	(-0.61)	(0.49)	(-1.87)	(0.78)	(-0.11)	(-0.81)	(-0.65)
N	4311	4311	927	927	2784	2784	1527	1527
Adj. R-Square	0.0173	0.0057	0.0139	0.0530	0.0112	0.0195	0.0231	0.0043

Notes: Monthly Data: August 1986 - June 2017. "i" Method. t statistics using Driscoll-Kraay standard error in parentheses. * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$.

that there are still positive excess returns in holding higher-interest currencies in their expectations.

Survey Exchange Rates

The proposed model argues that investors extrapolate by incorporating past depreciations when forming forecasts of future exchange rates. Consider the following regressions:

$$\mathbb{E}_t^e(s_{t+h}) = \kappa_3 + \eta_3 x_t + \xi_{3,t} \tag{22}$$

$$\mathbb{E}_t^e(s_{t+h}) = \kappa_4 + \eta_4 x_t + \gamma_4(s_{t-h} - s_{t-2h}) + \xi_{4,t}. \tag{23}$$

Equation (22) regresses exchange forecasts on interest rate differentials, while equation (23) tests whether past depreciations have any additional predictive power in addition to interest differentials.[5]

$\hat{\eta}_3$ and $\hat{\eta}_4$ from regression equations (22) and (23) are significantly negative in both the G7 and the rich-country samples. Currencies of these countries are generally stronger when their interest rates are higher. The poor-country sample on the other hand has insignificantly positive estimates of η_3 and η_4. In the lower-than-median GDP per capita countries, higher interest rates may correlate with other characteristics associated with weaker currencies. For example, higher interest rates among poor countries may signal

[5]The Frisch-Waugh-Lovell theorem states that the estimated coefficient $\hat{\gamma}_4$ from the regression equation (23) is the same as the estimate from the following regression:

$$\mathbb{E}_t^e(s_{t+h}) - \widehat{\mathbb{E}_t^e(s_{t+h})} = \kappa_4 + \gamma_4(s_{t-h} - s_{t-2h}) + \xi_{4,t},$$

where $\widehat{\mathbb{E}_t^e(s_{t+h})}$ is the predicted forecast levels from interest differentials. That is, $\widehat{\mathbb{E}_t^e(s_{t+h})} = \hat{\kappa}_3 + \hat{\eta}_3 x_t$, where $\hat{\kappa}_3$ and $\hat{\eta}_3$ are regression coefficients from equation (22).

Table 2: Survey Exchange Rates when the Period Step is 3 months ($h = 3$)

	All		G7		Rich		Poor	
	$\mathbb{E}_t^e(s_{t+h})$		$\mathbb{E}_t^e(s_{t+h})$		$\mathbb{E}_t^e(s_{t+h})$		$\mathbb{E}_t^e(s_{t+h})$	
x_t	-0.208	-0.251	-7.558**	-6.793**	-6.595*	-5.813*	1.852	1.482
	(-0.13)	(-0.16)	(-3.14)	(-2.86)	(-2.18)	(-1.99)	(0.78)	(0.66)
$s_{t-h} - s_{t-2h}$		0.543***		0.443***		0.450***		0.554**
		(3.40)		(3.98)		(3.35)		(2.77)
constant	-2.365***	-2.362***	-1.237***	-1.236***	-1.639***	-1.638***	-3.670***	-3.668***
	(-103.66)	(-106.37)	(-85.03)	(-86.29)	(-103.35)	(-104.45)	(-72.11)	(-73.91)
N	4272	4272	920	920	2763	2763	1509	1509
Adj. R-Square	0.0000	0.0251	0.0544	0.0850	0.0348	0.0613	0.0037	0.0233

Notes: Monthly Data: August 1986 - June 2017. "i" Method. t statistics using Driscoll-Kraay standard error in parentheses. * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$.

the inflation problem leading investors to form less optimistic forecasts of such currencies. This signaling channel is absent or less prominent in more developed countries with better reputations on the inflation management.

Notably, Table 2 displays significantly positive $\hat{\gamma}_4$ in all samples. Controlling for interest rate differentials, a 1% past depreciation leads to between 0.44% to 0.55% decrease in forecast levels. I view this as evidence for extrapolative beliefs among investors.

Equilibrium Exchange Rates

As investor beliefs affect their trading behaviors, exchange forecasts should have an impact on contemporaneous equilibrium exchange rates. This section explores whether extrapolative beliefs leave some traces on realized exchange rates.

Replacing survey exchange rates with equilibrium exchange rates yields analogs of regression equations (22) and (23) as below:

$$s_t = \kappa_5 + \eta_5 x_t + \xi_{5,t} \tag{24}$$

$$s_t = \kappa_6 + \eta_6 x_t + \gamma_6(s_{t-h} - s_{t-2h}) + \xi_{6,t}. \tag{25}$$

Table 3 shows that estimated $\hat{\eta}_5$ and $\hat{\eta}_6$ from regression equations (24) and (25) have similar patterns when realized exchange rates are used instead of survey levels as a regressand. Among the G7 countries and the rich countries, a 1% increase in the foreign against home interest rates leads to around 7% - 7.5% stronger foreign currencies. Again, estimates of η_5 and η_6 have opposite signs (positive instead of negative) and are no longer significant in the poor-country sample.

Table 3: Equilibrium Exchange Rates when the Period Step is 3 months ($h = 3$)

	All		G7		Rich		Poor	
	s_t		s_t		s_t		s_t	
x_t	-0.643	-0.686	-7.534**	-6.823**	-7.005*	-6.278*	1.408	1.035
	(-0.44)	(-0.49)	(-3.18)	(-2.91)	(-2.31)	(-2.14)	(0.64)	(0.51)
$s_{t-h} - s_{t-2h}$		0.531***		0.412***		0.418**		0.560**
		(3.40)		(3.60)		(3.04)		(2.88)
constant	-2.363***	-2.360***	-1.234***	-1.234***	-1.639***	-1.638***	-3.664***	-3.663***
	(-110.04)	(-113.11)	(-85.08)	(-86.06)	(-101.19)	(-102.02)	(-78.34)	(-80.56)
N	4272	4272	920	920	2763	2763	1509	1509
Adj. R-Square	0.0005	0.0288	0.0531	0.0790	0.0377	0.0596	0.0028	0.0290

Notes: Monthly Data: August 1986 - June 2017. "i" Method. t statistics using Driscoll-Kraay standard error in parentheses. * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$.

Significantly positive estimates of γ_6 from table 3 indicate that the 1-period lag depreciation has an additional predictive power beyond interest differentials on equilibrium exchange rates. Specifically, a 1% recent depreciation leads to around 0.41% - 0.55% weaker exchange levels. Past depreciations have roughly the same effect on equilibrium exchange rates as on survey expected levels.

Discussion

Above evidence supports the model assumption of extrapolative beliefs among investors. Investors appear to take into account not only fundamentals (interest rate differentials) but also past exchange changes when forming forecasts. In particular, investors extrapolate the 1-period lagged depreciation. As investor beliefs affect trading trading behaviors, past depreciations lead to lower equilibrium exchange rates.

Results on extrapolations are robust to different specifications as discussed in Appendix C.

5.3 Relations to Existing Extrapolation Literature

Using survey data to fit regression equations (23) and (25) derive an extrapolative coefficient γ around 0.41% - 0.55% (when the period step is 3 months). This subsection compares the proposed model with previous extrapolative literature.

Greenwood and Shleifer (2014) document discrepancies between expected returns and return expectations and uses mutual fund flows to conclude that investors act according to their expectations. Empirical tests in this paper point to the same conclusion that investor expectations affect equilibrium exchange rates. Extrapolative coefficients γ are roughly the same in both the expectation formation regression (23) and the equilibrium exchange regression (25).

Barberis et al. (2015) use survey data from Greenwood and Shleifer (2014) to parametrize the functional form of extrapolation to fit stock market returns. Their underlying mechanism is similar to the proposed model. When past price changes are positive, extrapolators expect stock markets to perform well in the future pushing the current price even higher. Their model features heterogenous agents with rational investors trading with extrapolators. Their parametrization results in 50% of each group. The proposed baseline model needs only one type of investors since the model has a built-in depreciating force from the an AR(1) assumption of interest differentials. Future research may extend the baseline model to include heterogenous agents, but the key underlying idea will apply.

Jin and Sui (2017) model different functional forms of extrapolation and uses survey expectations from Greenwood and Shleifer (2014) to get the parametrized weight between fundamental and behavioral beliefs around 0.5 (0 indicates complete rational, while 1 indicates fully extrapolative).

As there are different ways of modeling extrapolation, it is hard to reconcile extrapolative coefficients across different models. Previous discussion centers around the sign and not the magnitude of extrapolation. Therefore, there exists no consensus extrapolative coefficient readily available.

As pointed out by previous studies, equilibrium exists only when the extrapolative component is not too high. Otherwise, optimistic future prices will push the current price higher and so on. The infinite feedback loop makes equilibrium vanishes.

The baseline model includes only the 1-period lagged depreciation. In this sense, extrapolators quickly forget all but most recent changes. There are ongoing debates on whether this is realistic. Greenwood and Shleifer (2014) argue that investor expectations depend mostly on recent returns, while Malmendier and Nagel (2011) and Malmendier and Nagel (2015) suggest that distant past events might also play a role.

The framing of survey questions as well as the forecast horizon seem to affect how far back investors look into the past. Investors look back only for a couple months when forming short-term forecasts but incorporate almost their entire experiences when forming long-term forecasts. It is possible to extend my current model to include more lags at the cost of computational complexity.

6 Conclusion

The paper revisits the relationship between interest rates and exchange rates. As documented in Engel (2016) and Valchev (2015), deviations from the UIP vary with time horizon. The paper decomposes excess returns in holding higher versus lower interest bonds into two main components: exchange depreciations and interest rate differentials. Using a large scope of currencies, the paper finds that exchange rate changes are mostly unpredictable by interest rate differentials. While the interest rate differentials appear to follow an autoregressive process of order 1, exchange rates behave much more like a random walk. The failure of exchange depreciation to offset interest differentials results in excess return predictabilities.

The paper confirms recent findings that there are positive excess turns in holding higher interest bonds initially. Such excess returns reverse to negative at some periods in the future. In the long run, the UIP appears to hold. Such patterns are robust when expanding the currency scope to cover both developed and developing currencies across different continents. The patterns persist regardless of whether the US Dollar is used as a base currency.

Observed empirical patterns especially the reversal in the sign of excess returns invalidate many of existing theoretical UIP models. The paper proposes a simple behavioral model based on extrapolation that is consistent with observed patterns of excess returns.

Higher interest rates are associated with stronger-than-average currencies. Extrapolative investors then form optimistic views of next-period levels resulting in even stronger contemporaneous exchange rates. Momentum in investor beliefs leads to initial persistent positive excess returns in holding higher interest bonds.

As interest differentials follow a stationary autoregressive process of order 1, there exists a built-in depreciating force that pulls exchange rates back to their long-run levels. The interaction between the depreciating force and the extrapolative force results in the eventual reversal in the sign of excess returns. Both forces lose magnitude with time leading exchange rates to revert back to the UIP level in the long run.

I use survey data to show that investors indeed extrapolate exchange rates. The proposed extrapolative model is consistent with patterns of excess returns and evidence from survey data.

A Data Appendix

A.1 Daily vs. Monthly Data

"Daily" data pulls information from every trading day, while "Monthly" data picks only the last trading day of each month to construct the month-end data.

A.2 Exchange Rates Data and Interest Rates Data

WM

World Markets PLC/Reuters (WM/R) provides daily 4pm London fixing (15:00 GMT) spot and forward rates. I combine bilateral exchange rates with US Dollar (USD) as a base currency with those with British Pound (GBP) as a base currency. Most GBP series are longer except for the Euro.

Bloomberg

Bloomberg provides daily 5pm New York Close (21:00 GMT) spot and forward rates for a majority of currencies in the study. The data ranges from 1 December 1983 to 7 June 2017. The FXTF function on Bloomberg terminal reveals a list of currencies (AUD, EUR, IEP, NZD, and GBP) with special forward-points convention. Pakistani Rupee only has data of onshore forward points.

TR

Thomson Reuters (T/R) provides daily 5pm New York Close (21:00 GMT) spot and forward rates. Again, I complement the USD series with the GBP ones.

For above datasets, I calculate implied interest rate differentials x_t using the following formula:

$$x_t = log(\frac{S_t}{F_t}).$$

i

This method pulls daily data of annual Eurocurrency rates provided by Intercapital from Datastream. The data covers the period from 2 Jan 1970 to 7 June 2017. The mnemonics for the Eurocurrency rates are ECxxxyy, where xxx is the country code and yy represents the horizon (for example, 1M for 1 month). As the eurocurrency rates are often missing or incomplete for non-OECD countries, the paper uses the following alternatives in the empirical studies.

1-month VIBOR and Real 1-month implied rates are used for Austria and Brazil respectively, as these rates are roughly in line with forward-implied rates. The paper uses

TR Chinese Yuan 1-month deposit for China, as the TR deposit rate is quite compatible to the discontinued Eurocurrency rates. Finland Euro-Markka 1-month ICAP/TR rate is used for Finland. For Greece, I combine the ECGRD1M with earlier observations from the Greek deposit rate. From the year 2002 onwards, interest rates for Greece follow the common Euro 1-month rate. The TR deposit rate is combined with earlier observations from The Taiwan deposit rate for Taiwan. For Thailand, I complement the ECTHB1M with later observations from the TR deposit rate.

All interest rates are annually adjusted and are in percentage. The paper calculates interest rate differentials using the following formula:

$$\begin{aligned} x_t &= i_t - i_t^* \\ &= (1 + \frac{i^{\text{raw}}}{100})^{\frac{1}{12}} - (1 + \frac{i^{*\text{raw}}}{100})^{\frac{1}{12}}. \end{aligned}$$

Combine

The paper ranks the data quality in the following order from the most reliable to the least reliable: WM, BBG, TR, and i. WM is ranked first because it appears to be the most accurate and the most recent. BBG used by a majority of active currency investors is augmented to the WM series whenever the WM data is missing. TR with more sparse data is then used. I rank the spot and forward pairs above the "i" method as both come from the same source. The "i" method combines the spot series from the previous 3 methods and calculates interest rate differentials from interest rate series from Datastream. I note that there are slight discrepancies of spot rates and interest differentials among each dataset due to different recording times. These differences appear to be minimal.

Even though WM is expansive in term of the currency coverage, its forward data only starts in the early 90s. Bloomberg data is as extensive as WN with an addition of Uruguay forward data. Data from TR is sparse in term of coverage but goes back earlier in time. The interest differentials from the "i" method cover all countries of interests and run the furthest back.

A.3 GDP-based Categorization

The paper uses the time series of the GDP per capita, current prices (purchasing power parity in the unit of international dollars per capita) provided by the International Monetary Fund (http://www.imf.org/external/datamapper/PPPPC@WEO/ARG/AUS/AUT/BEL/BRA/CAN/CHL/CHN/CZE/DNK/EGY/EU).

This data has an annual frequency dating back to 1980. I use the 1980 level to proxy for levels prior t0 1980. There is no available data for the Euro area, so the paper uses the "whole European union" series as a proxy.

Another popular measure of categorization is MSCI market classification of countries into developed and emerging countries. The paper does not explore this method.

A.4 Survey Data from the Forecasts Unlimited Inc.

Background of the Forecasts Unlimited Inc. (FX4casts.com)

The Currency Forecasters' Digest was started in August 1984. It was sold to the Financial Times in September 1994 and was renamed to the Financial Times Currency Forecaster. The company was repurchased and renamed Biz4casts.com in January 1999. It has been renamed FX4casts.com since December 2002. Throughout the change in the company's ownership, the production staff remained the same with an addition of Marsha Kameron in January 1988. This ensures the consistency of the data collected.

Contributors of Consensus Forecasts

The current contributors of the forecast data are Allied Irish Bank, ANZ Bank, Bank of America/Merrill Lynch, Bank of New York Mellon, Barclays Capital, Bayerische Landesbank, BNP Paribas, Canadian Imperial Bank of Commerce, Credit-Agricole, Citigroup, Commerzbank, Credit Suisse - First Boston, Danske Bank, Deka, Deutsche Bank, DnBNOR, Economist Intelligence Unit, Goldman Sachs, Handel Banken, HSBC, IHS Global Insight, ING Bank, Intesa Sanpaolo, JP Morgan Chase, Julius Bear, Lloyds TSB, Macquarie Capital Securities, Moody's Economy.com, Morgan Stanley, National Australia Bank, Nomura, Nordea, PNC Financial, Rabobank, Royal Bank of Canada, Royal Bank of Scotland, Scotia Bank, SEB, Societe Generale, Standard Chartered, SunTrust, Swedbank, Tokyo-Mitsubishi UFJ, Toronto Dominion, UBS Warburg, UniCreditHVB, Vontobel, and Westpac.

The list has changed over the 30-year period to reflect mergers among banks and financial institutions but always contains major intermediaries in the exchange rate markets.

Data

Data contains monthly spot rates and consensus 3-, 6-, and 12-month forecasts for 32 currencies. The series start on August 1986 for 10 currencies: British Pound, Danish Krone, Euro (with Deutsche Mark prior to January 1999), Norwegian Krone, Swedish Krona, Swiss Franc, Australian Dollar, New Zealand Dollar, Japanese Yen, and Canadian Dollar. The series start on October 2001 for the remaining 22 countries: Czech Koruna, Hungarian Forint, Polish Zloty, Russian Rouble, Turkish Lira, Chinese Renminbi, Hong Kong Dollar, Indian Rupee, Indonesian Rupiah, New Zealand Dollar, Philippine Peso, Singapore Dollar, South Korean Won, Taiwan Dollar, Thai Baht, Argentine Peso, Brazilian Real, Chilean

Peso, Colombian Peso, Mexican Peso, Venezuelan Bolivar, and South African Rand.

The 95% confidence intervals for all 32 currencies are available starting October 2001. The 1- and 24-month forecasts data for all 32 currencies become available starting July 2008.

The data only contains consensus forecasts for the Euro with no data for each individual European currency.

I note that this dataset from FX4casts.com has been used previously in academic research. Bacchetta et al. (2009) and Gourinchas and Tornell (2004) are examples of previous articles using the consensus forecasts from this source.

B Empirical Patterns Appendix

The main paper focuses on results for the G7 sample using monthly data from the "Combine" method when the period step is 1 month. This section provides some robustness check by looking at different ways of pooling the data.

B.1 Different Samples

Figure 8, Figure 9, and Figure 10 plot estimated coefficients along with 95% confidence bands from panel regressions of equations (5), (6), and (7) using the "Combine" data for the higher-than-median GDP per capita sample, the lower-than-median GDP per capita sample, and the entire sample respectively.

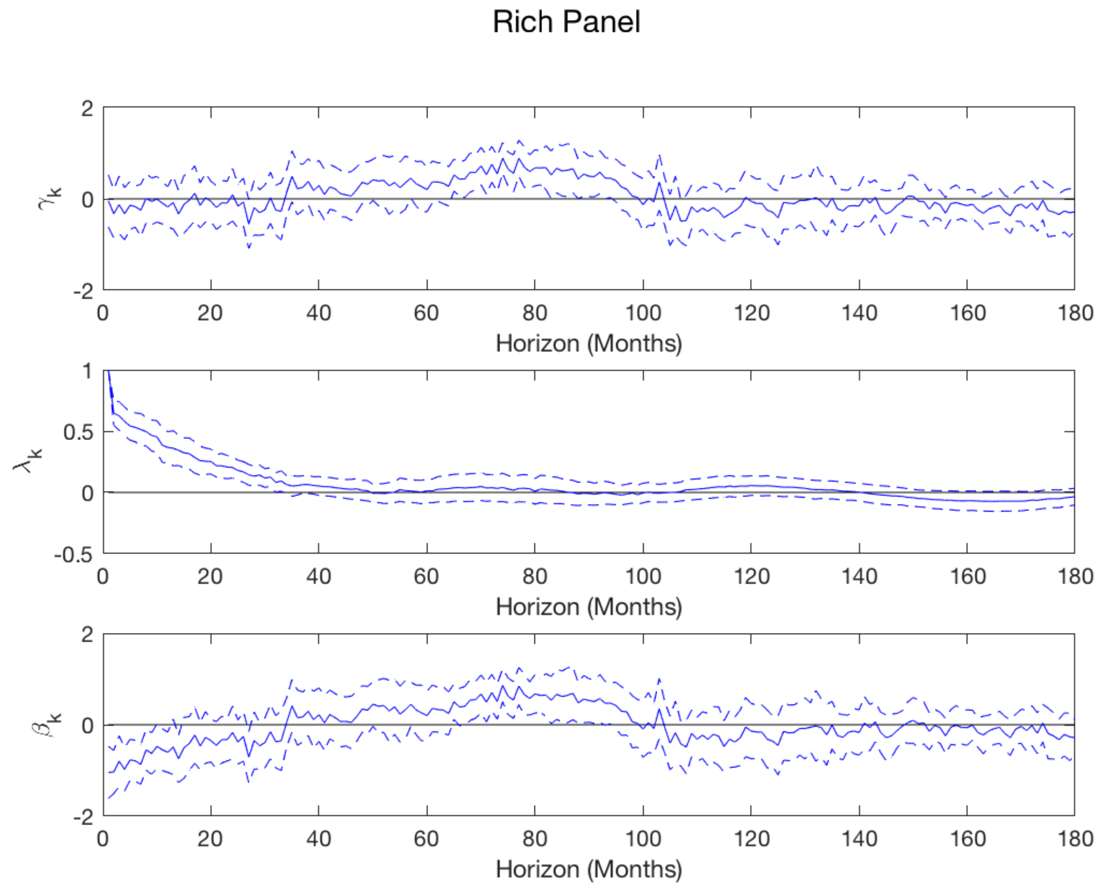


Figure 8: $k = 1$ Panel Coefficients for High GDP per Capita Using Monthly Data and "Combine" Method

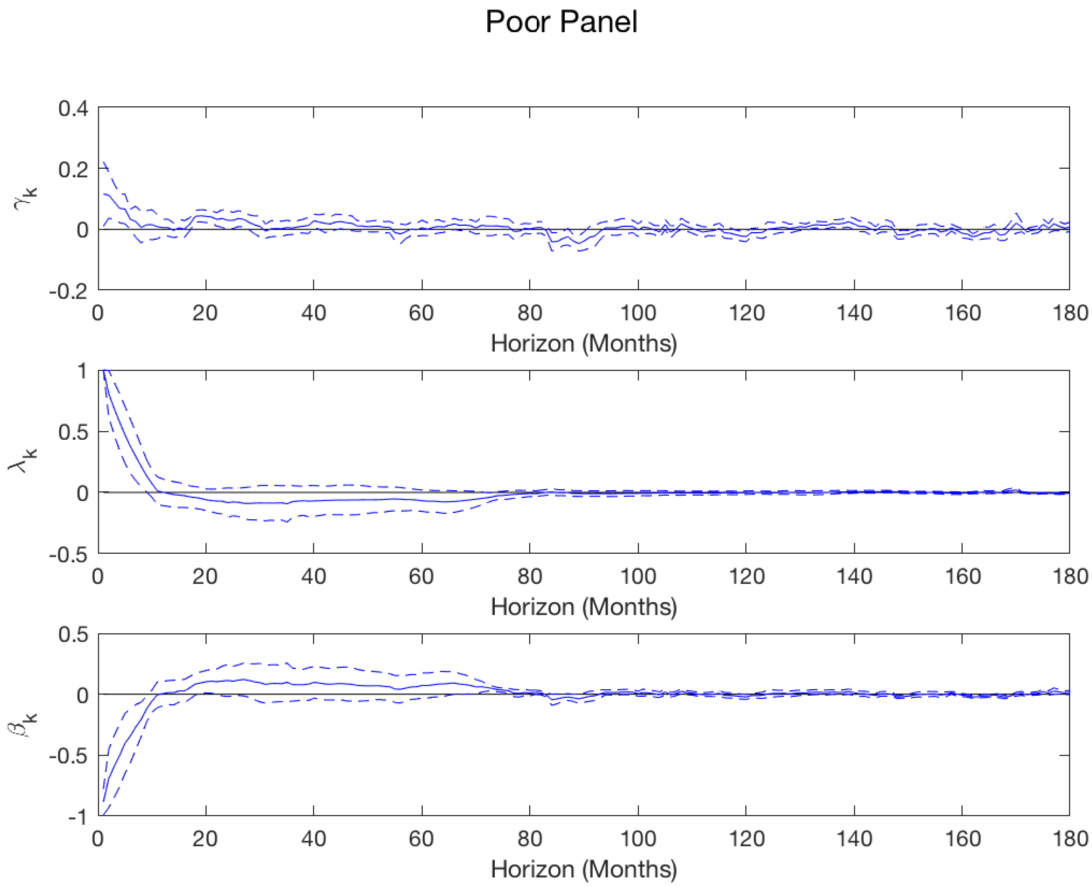


Figure 9: $k = 1$ Panel Coefficients for Low GDP per Capita Using Monthly Data and "Combine" Method

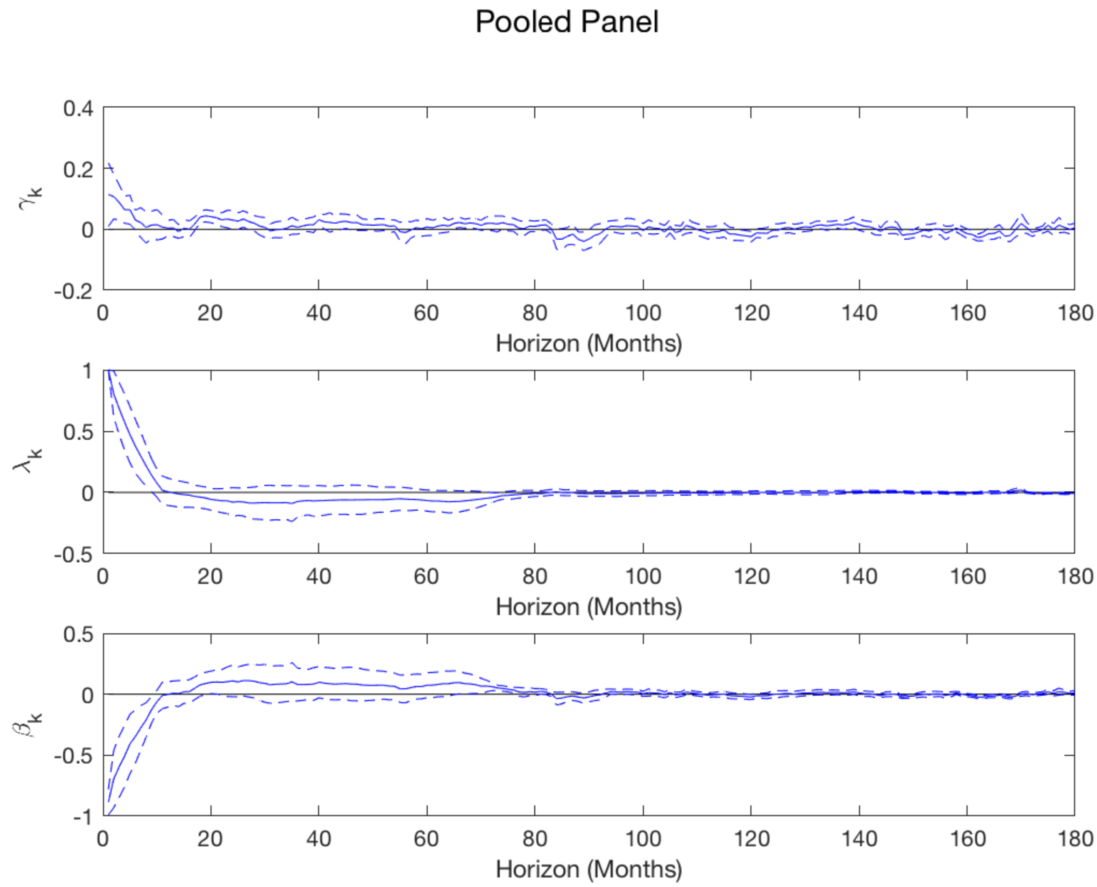


Figure 10: $k = 1$ Panel Coefficients for All 51 Currencies Using Monthly Data and "Combine" Method

Key patterns hold across different samples.

B.2 Data Frequency

Figure 11 plots estimated coefficients along with 95% confidence bands from panel regressions of equations (5), (6), and (7) using the "Combine" data for the G7 sample when USD is a base currency.

Instead of month-end data, daily data is now used. For a comparison, the numbers of observations per country are 11,049 and 508 for the daily and the monthly data respectively.

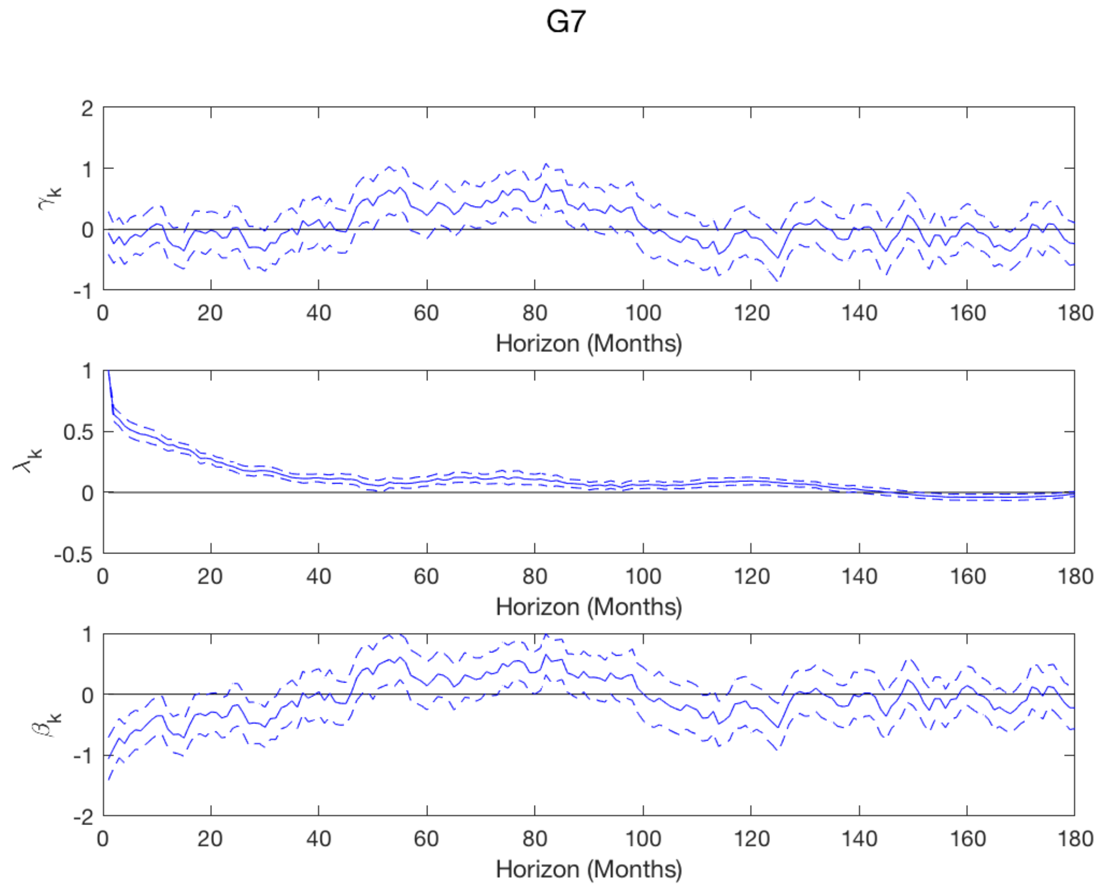


Figure 11: $k = 1$ Panel Coefficients for G7 Using Daily Data and "Combine" Method

Empirical patterns observed in the paper are robust when daily data is used instead of the month-end data.

B.3 Alternative Datasets

The main paper displays results from the "Combine" method. This section illustrates empirical patterns from alternative datasets.

WM

Figure 13, Figure 14 Figure 15, and Figure 16 plot estimated coefficients along with 95% confidence bands from panel regressions of equations (5), (6), and (7) for the G7 countries using the "WM", the "BBG", the "TR", and the "i" data respectively.

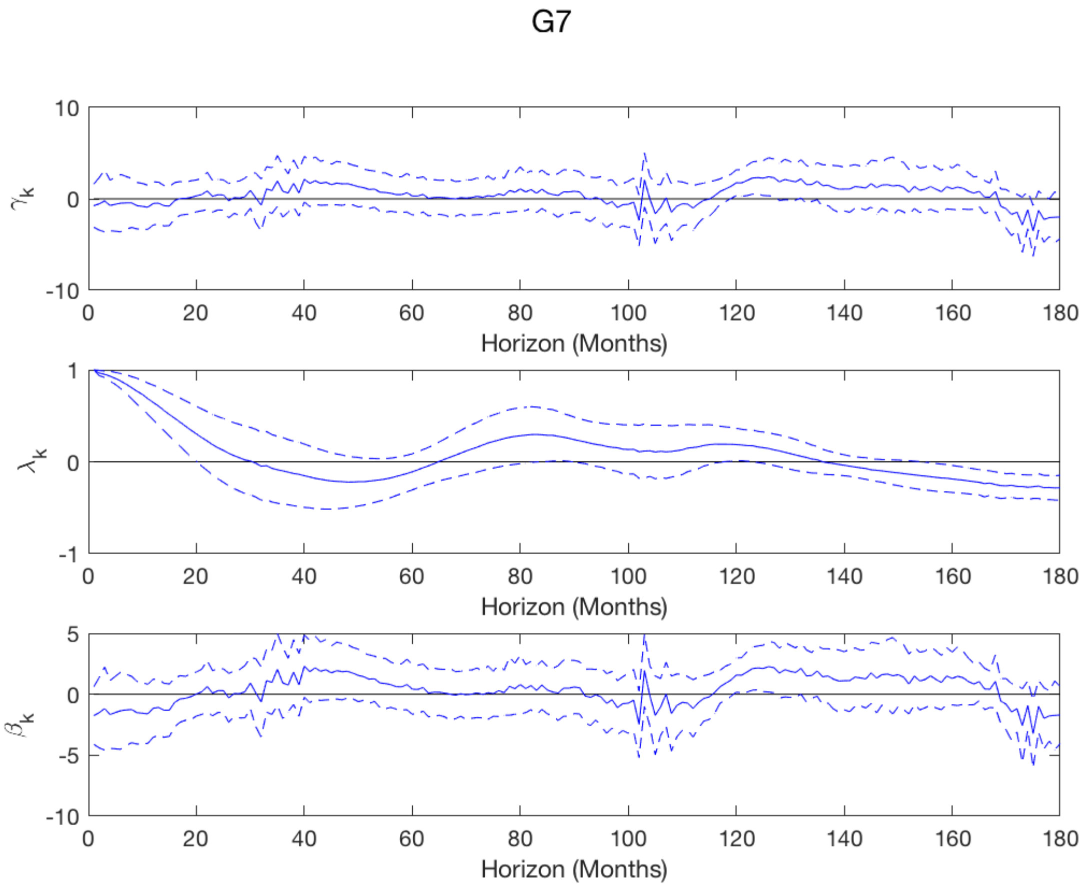


Figure 12: Coefficient

Figure 13: $k = 1$ Panel Coefficients for G7 Using Monthly Data and "WM" Method

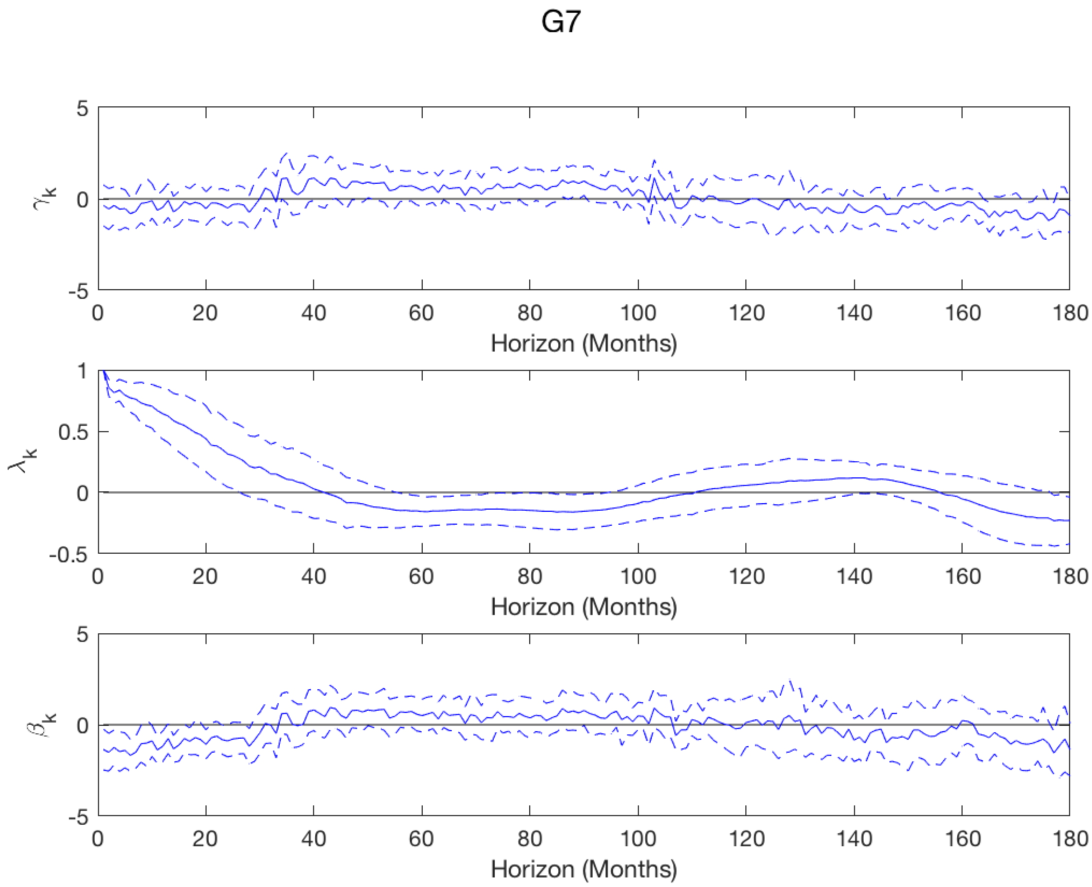


Figure 14: $k = 1$ Panel Coefficients for G7 Using Monthly Data and "BBG" Method

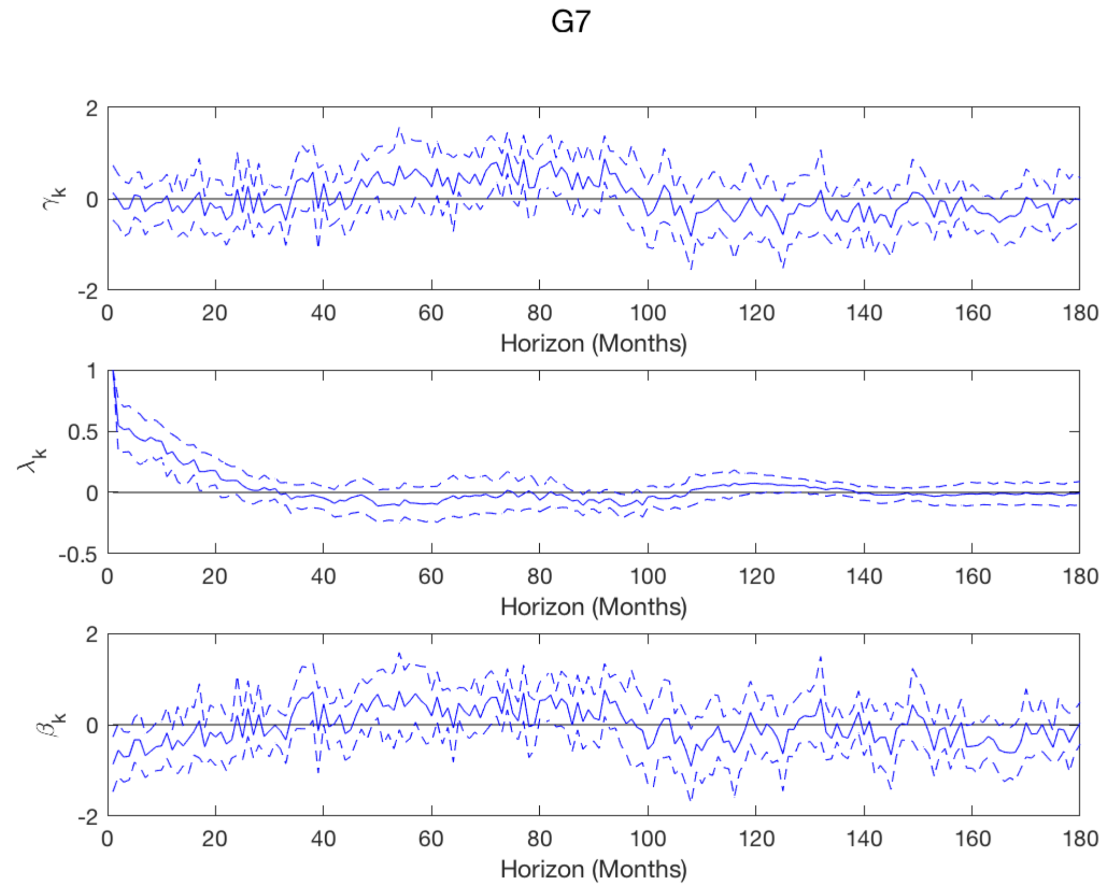


Figure 15: $k = 1$ Panel Coefficients for G7 Using Monthly Data and "TR" Method

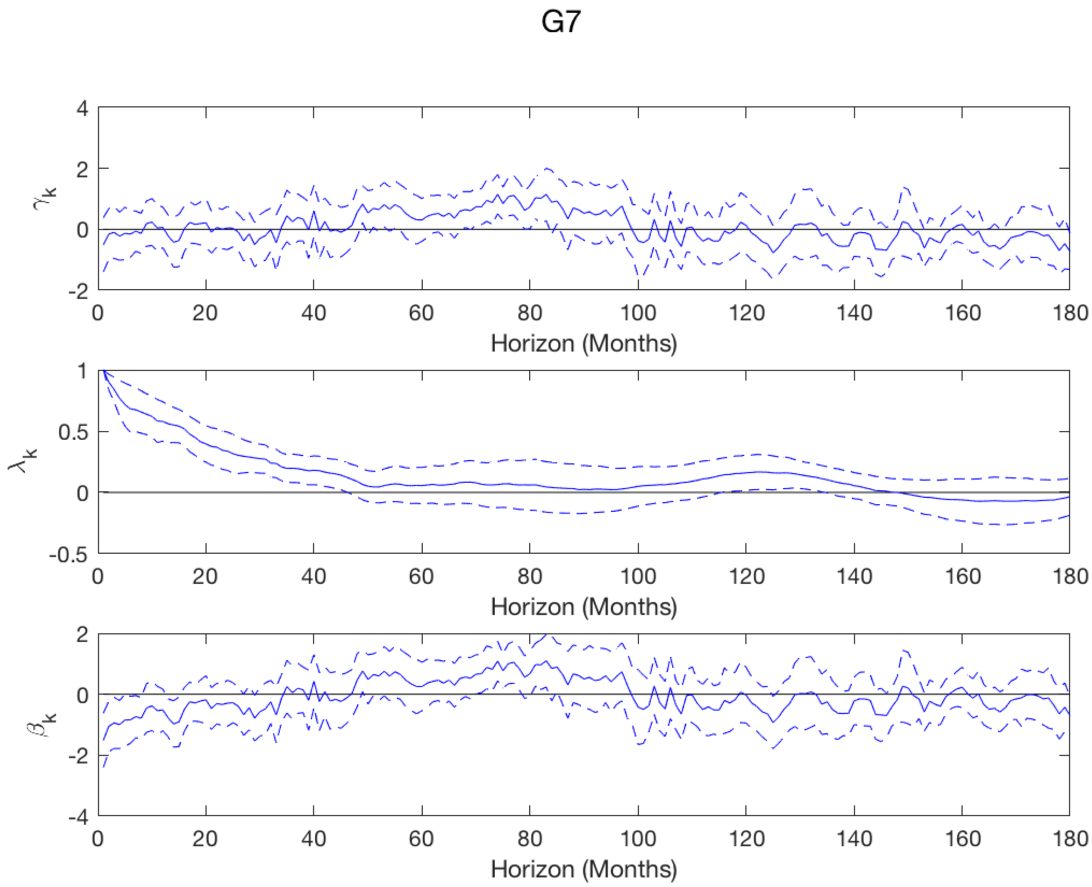


Figure 16: $k = 1$ Panel Coefficients for G7 Using Monthly Data and "i" Method

This section checks the robustness of empirical patterns emphasized in the main paper by looking at alternative datasets for exchange rates and interest rate differentials. Point estimates from alternative datasets show same patterns. The statistical significance is lost in all but the "i" method. This is potentially due to the shorter interest differentials samples.

B.4 Varying Period Step

The main paper displays empirical patterns when the period step is fixed at 1 month. This section checks whether such patterns are robust to different period steps. In particular, Figure 17, Figure 18, and Figure 19 plot estimated coefficients along with 95% confidence bands from panel regression of equations (5), (6), and (7) for the G7 countries when the period step is 3, 6, and 12 months respectively.

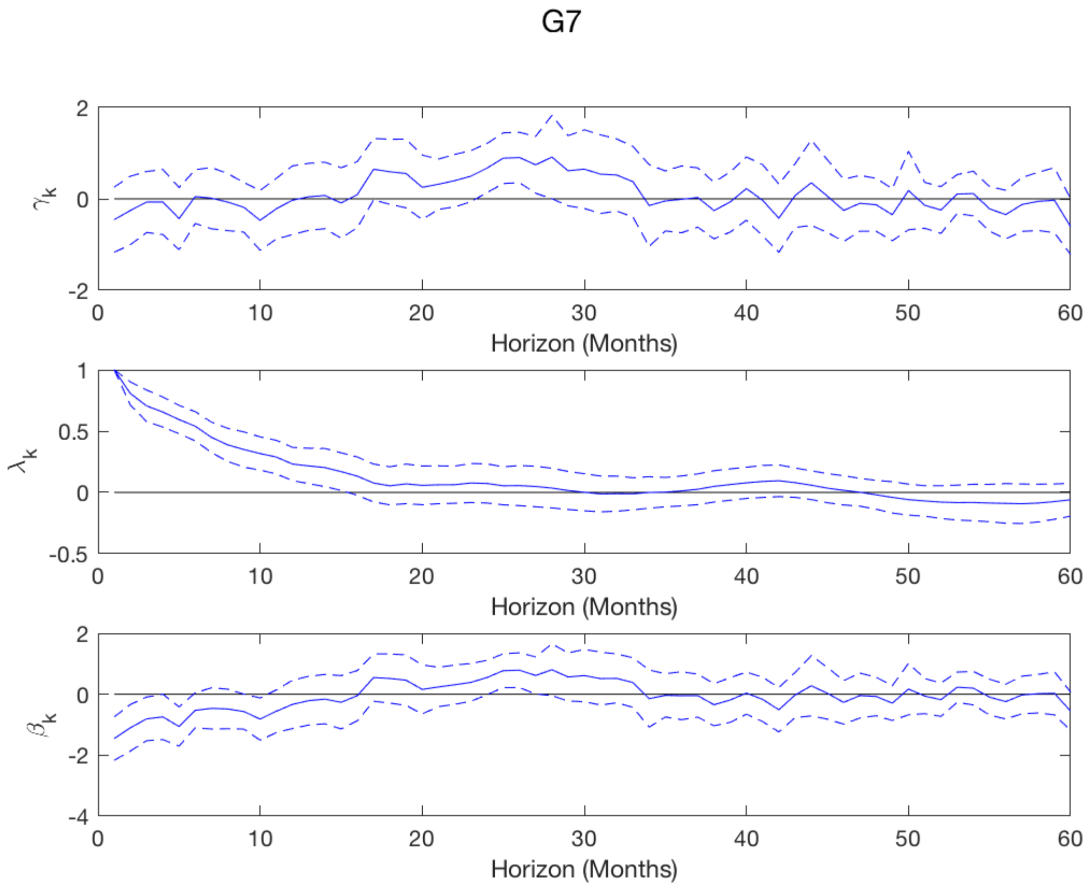


Figure 17: $k = 3$ Panel Coefficients for G7 Using Monthly Data and "Combine" Method

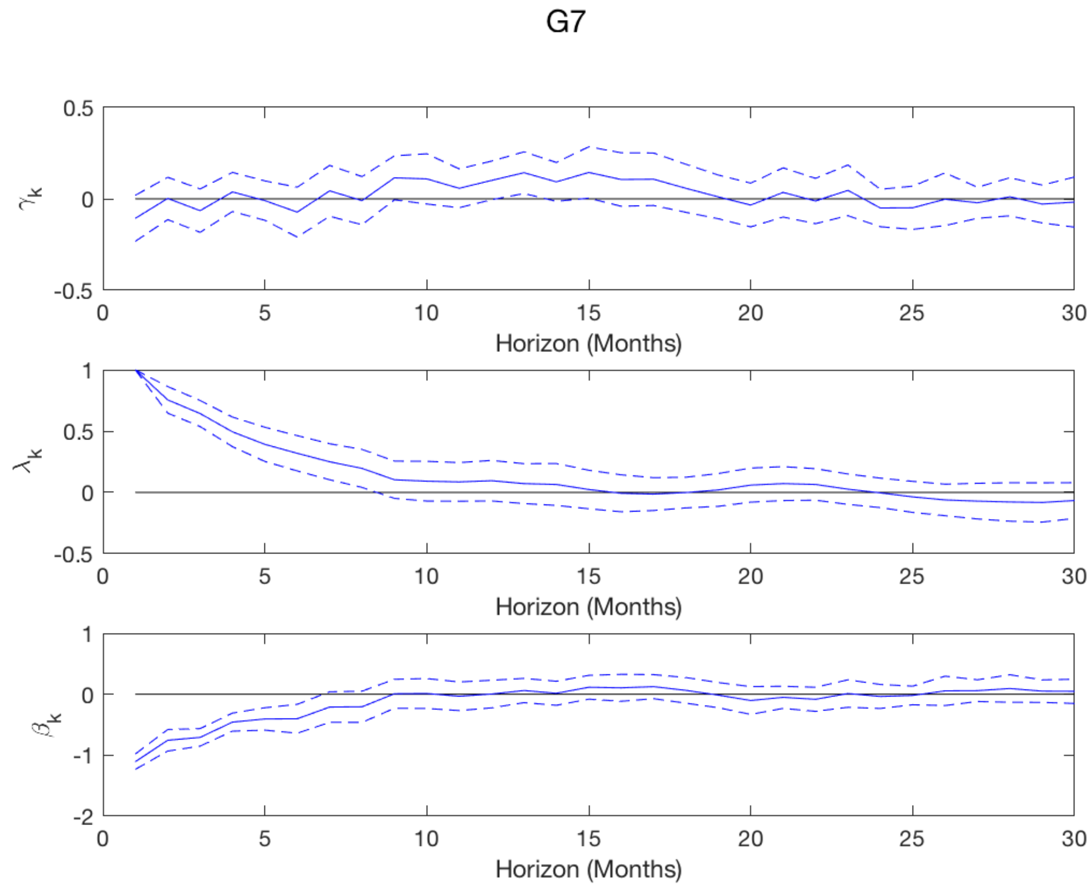


Figure 18: $k = 6$ Panel Coefficients for G7 Using Monthly Data and "Combine" Method

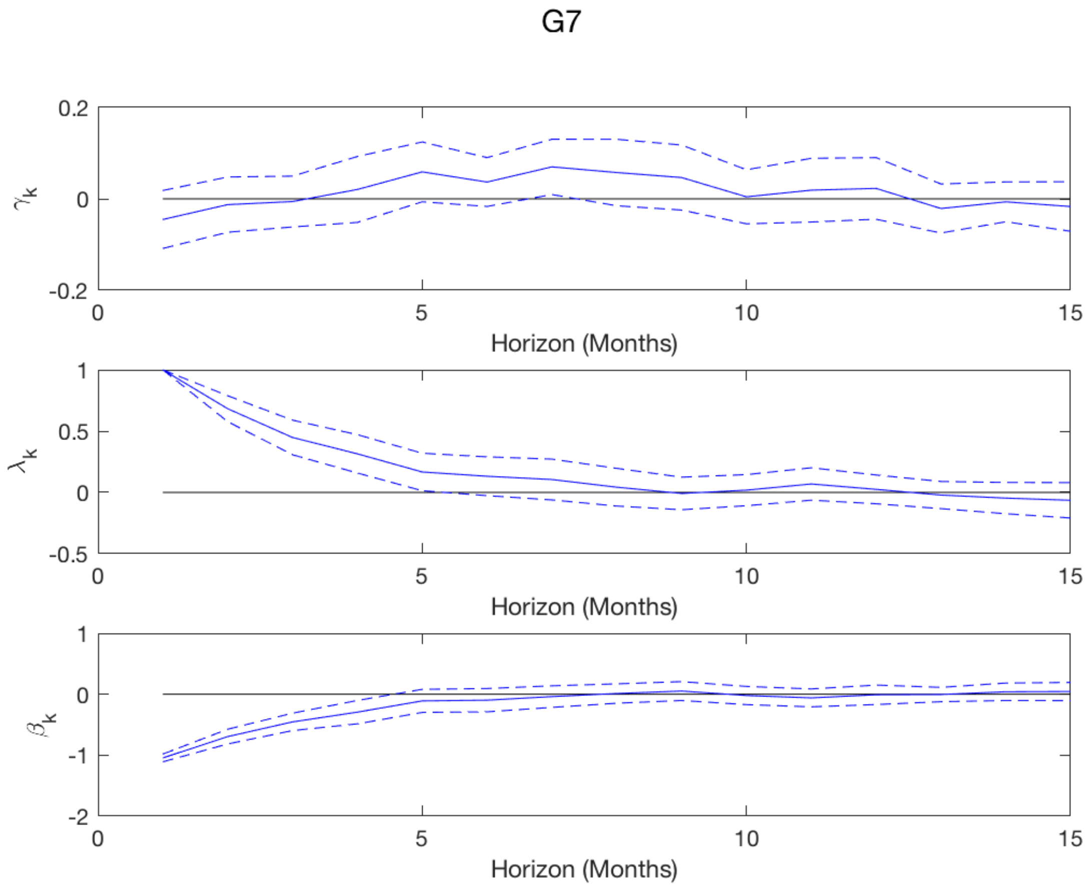


Figure 19: $k = 12$ Panel Coefficients for G7 Using Monthly Data and "Combine" Method

Patterns of excess returns are robust to varying period steps. As expected, plots become smoother as the period step gets longer. The reversal in the sign of β remains but loses some statistical significance when $k = 6$ and 12.

C Empirical Evidence Appendix

C.1 The AR(1) Assumption of Interest Rate Differentials

Table 4: The Unit Root Test and Optimal Order of the Autoregressive Model for Each Country's 1-month Interest Rate against the US Rate

Currency Code	Country	P-Value	Order
ARS	Argentina	0.0672	1
AUD	Australia	0.01	1

ATS	Austria	0.99	1
BEF	Belgium	0.01	1
BRL	Brazil	0.01	1
CAD	Canada	0.01	1
CLP	Chile	0.01	1
CNY	China	0.0120	1
CZK	Czech	0.01	1
DKK	Denmark	0.01	1
EGP	Egypt	-	-
EUR	Euro	0.1529	1
FIM	Finland	0.0272	1
FRF	France	0.01	1
DEM	Germany	0.01	1
GRD	Greece	0.0464	1
HKD	Hong Kong	0.01	1
HUF	Hungry	0.01	1
ISK	Iceland	0.01	1
IDR	Indonesia	0.01	1
INR	India	0.01	1
IEP	Ireland	-	-
ILS	Israel	0.0110	1
ITL	Italy	0.01	1
JPY	Japan	0.01	1
KRW	Korea	0.01	1
KWD	Kuwait	0.01	1
MYR	Malaysia	0.01	1
MXN	Mexico	-	-
NLG	Netherlands	0.01	1
NZD	New Zealand	0.0132	1
NOK	Norway	0.01	1
PKR	Pakistan	0.01	1
PHP	Philippines	0.01	1
PLN	Poland	0.01	1
PTE	Portugal	0.0299	1
RUB	Russia	0.01	1

SAR	Saudi Arabia	0.01	1
SGD	Singapore	0.0124	1
ZAR	South Africa	0.01	1
ESP	Spain	0.01	1
SEK	South Korea	0.0423	1
CHF	Switzerland	0.01	1
TWD	Taiwan	0.01	1
THB	Thailand	0.0252	1
TRY	Turkey	0.0674	1
AED	UAE	0.0338	1
GBP	UK	0.01	1
UYU	Uruguay	-	-
COP	Colombia	0.2714	1
VEF	Venezuela	0.01	1

Notes: Results from the 1-month interest rate differentials daily time-series data from the "i" method. The missing data is filled using the polynomial interpolation (spline interpolation). The column "P-Value" shows the p-value of the Augmented Dickey-Fuller test for the null of a unit root against an alternative hypothesis of a stationary process. The column "Order" displays the order of the fitted autoregressive model chosen by minimizing the AIC.

The "i" method provides no available interest rate data for 4 countries: Egypt, Ireland, Mexico, and Uruguay. Most of the remaining countries have the Augmented Dickey-Fuller test's p-value that is less than 0.05.

C.2 Implied Interest Rate Differentials

Section 5 in the paper displays the results using the 3-month interest rate differentials data from the "i" method. This section replicates the analysis using the constructed interest rate differentials data from the "Combine" method.

Survey-Expected Excess Returns

Table 5 replicates Table 1 in the main paper.

Survey-expected excess returns point to less severe UIP deviations in the G7 and the rich-country samples but more severe deviations in the poor-country sample.

Survey Exchange Rates and Equilibrium Exchange Rates

Table 5: Excess Returns when the Period Step is 3 months ($h = 3$)

	All		G7		Rich		Poor	
	ρ_{t+h}	$\mathbb{E}_t^e(\rho_{t+h})$	ρ_{t+h}	$\mathbb{E}_t^e(\rho_{t+h})$	ρ_{t+h}	$\mathbb{E}_t^e(\rho_{t+h})$	ρ_{t+h}	$\mathbb{E}_t^e(\rho_{t+h})$
x_t	-0.573***	-0.984***	-1.832***	-1.277***	-1.197*	-0.615***	-0.555***	-0.995***
	(-6.59)	(-11.03)	(-3.73)	(-6.31)	(-2.36)	(-3.79)	(-6.59)	(-11.26)
constant	0.00222	-0.00409*	0.00123	-0.00121	0.00203	0.0000439	0.00143	-0.0100**
	(0.58)	(-2.23)	(0.44)	(-1.06)	(0.58)	(0.04)	(0.31)	(-2.77)
N	5075	5075	989	989	3120	3120	1955	1955
Adj. R-Square	0.0490	0.243	0.0273	0.0832	0.0136	0.0294	0.0853	0.266

Notes: Monthly Data: August 1986 - June 2017. "Combine" Method. t statistics using Driscoll-Kraay standard error in parentheses. * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$.

The analysis in this section is crucial in establishing extrapolative beliefs among investors. Controlling for fundamentals, I test whether past depreciations have any additional effect on survey forecasts and equilibrium exchange rates.

Table 6 and Table 7 replicate Table 2 and Table 3, respectively.

Table 6: Survey Exchange Rate when the Period Step is 3 months ($h = 3$)

	All		G7		Rich		Poor	
	$\mathbb{E}_t^e(s_{t+h})$		$\mathbb{E}_t^e(s_{t+h})$		$\mathbb{E}_t^e(s_{t+h})$		$\mathbb{E}_t^e(s_{t+h})$	
x_t	0.626	0.333	-8.998***	-8.425***	-3.441*	-3.066	0.948	0.613
	(0.92)	(0.45)	(-5.30)	(-5.16)	(-2.02)	(-1.92)	(1.20)	(0.73)
$s_{t-h} - s_{t-2h}$		0.580***		0.484***		0.547***		0.564*
		(3.54)		(4.34)		(4.07)		(2.39)
constant	-2.343***	-2.342***	-1.100***	-1.099***	-1.680***	-1.679***	-3.412***	-3.411***
	(-110.33)	(-112.46)	(-83.94)	(-85.40)	(-110.50)	(-113.21)	(-88.82)	(-90.12)
N	5022	5022	982	982	3095	3095	1927	1927
Adj. R-Square	0.0020	0.0292	0.0821	0.112	0.0144	0.0544	0.0060	0.0248

Notes: Monthly Data: August 1986 - June 2017. "Combine" Method. t statistics using Driscoll-Kraay standard error in parentheses. * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$.

Using the interest rate data from the "Combine" method yields similar results as in the main paper. Investors expect currencies to generally be stronger when their interest rates are higher except in the poor-country sample when the signaling channel confounds the results.

Coefficients in front of past depreciations are significantly positive. Past depreciations

Table 7: Equilibrium Exchange Rate when the Period Step is 3 months ($h = 3$)

	All		G7		Rich		Poor	
	s_t		s_t		s_t		s_t	
x_t	0.431	0.145	-8.665***	-8.135***	-3.806*	-3.453*	0.764	0.429
	(0.65)	(0.20)	(-5.23)	(-5.10)	(-2.21)	(-2.12)	(0.98)	(0.51)
$s_{t-h} - s_{t-2h}$		0.565***		0.446***		0.514***		0.564*
		(3.52)		(3.92)		(3.78)		(2.48)
constant	-2.341***	-2.339***	-1.099***	-1.098***	-1.680***	-1.679***	-3.405***	-3.404***
	(-116.37)	(-118.39)	(-83.76)	(-84.90)	(-108.17)	(-110.38)	(-95.96)	(-97.31)
N	5022	5022	982	982	3095	3095	1927	1927
Adj. R-Square	0.0011	0.0308	0.0771	0.103	0.0173	0.0517	0.0048	0.0282

Notes: Monthly Data: August 1986 - June 2017. "Combine" Method. t statistics using Driscoll-Kraay standard error in parentheses. * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$.

lead investors to have more pessimistic forecasts with estimated effects in the same ballpark as in the main paper. That is, controlling for interest rate differentials, a 1% past depreciation leads to between 0.48% to 0.58% lower level forecasts.

Effects of interest rate differentials and past depreciations on equilibrium exchange rates are similar to those on survey rates. Estimated coefficients from Table 6 and Table 7 have similar magnitudes. Controlling for interest rate differentials, a 1% past depreciation leads to between 0.45% to 0.57% lower current-period equilibrium exchange rates.

The above analysis confirms that results in Section 5 are robust to different sources of interest rate data. Using implied interest differentials supports that currency investors hold extrapolative beliefs.

C.3 Varying Period Step

This section explores whether survey data yields consistent evidence across different period steps.

Section 2 illustrates results when the period step is set to 1 month, while Section 5 switches to use the period step of 3 months due to the limited data availability for the 1-month forecasts.[6]

Excess Returns

[6] FX4casts.com starts collecting the 1-month forecasts only in July 2008, roughly 22 years after the earliest observations for the 3-, 6-, and 12-month forecasts.

Table 8, Table 9, and Table 10 compare realized excess returns with survey-expected excess returns when the time steps are 1, 6, and 12 months, respectively.

Table 8: Excess Returns when the Period Step is 1 month ($h = 1$)

	All		G7		Rich		Poor	
	ρ_{t+h}	$\mathbb{E}_t^e(\rho_{t+h})$	ρ_{t+h}	$\mathbb{E}_t^e(\rho_{t+h})$	ρ_{t+h}	$\mathbb{E}_t^e(\rho_{t+h})$	ρ_{t+h}	$\mathbb{E}_t^e(\rho_{t+h})$
x_t	-0.0965	-0.974***	12.63	-3.508**	1.559	-2.741***	-0.244	-0.817***
	(-0.10)	(-3.88)	(2.01)	(-3.02)	(0.56)	(-3.67)	(-0.25)	(-4.09)
constant	-0.00212	-0.00353**	-0.00192	-0.00193**	-0.00122	-0.00234***	-0.00298	-0.00501**
	(-0.71)	(-3.29)	(-0.64)	(-2.94)	(-0.41)	(-5.79)	(-0.65)	(-2.70)
N	2141	2141	300	300	1125	1125	1016	1016
Adj. R-Square	0.0000	0.0089	0.0356	0.0226	0.0009	0.0492	0.0001	0.0066

Notes: Monthly Data: July 2008 - June 2017. "i" Method. t statistics using Driscoll-Kraay standard error in parentheses. * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$.

Table 9: Excess Returns when the Period Step is 6 months ($h = 6$)

	All		G7		Rich		Poor	
	ρ_{t+h}	$\mathbb{E}_t^e(\rho_{t+h})$	ρ_{t+h}	$\mathbb{E}_t^e(\rho_{t+h})$	ρ_{t+h}	$\mathbb{E}_t^e(\rho_{t+h})$	ρ_{t+h}	$\mathbb{E}_t^e(\rho_{t+h})$
x_t	-1.330***	-0.482***	-1.458*	-0.356	-1.372	-0.132	-1.311***	-0.565***
	(-5.64)	(-3.61)	(-2.27)	(-1.53)	(-1.86)	(-0.53)	(-6.29)	(-3.63)
constant	-0.00324	0.00357	0.00271	-0.00507*	0.00419	0.00188	-0.0161	0.00503
	(-0.43)	(1.42)	(0.48)	(-2.13)	(0.65)	(0.74)	(-1.41)	(1.31)
N	4080	4080	916	916	2618	2618	1462	1462
Adj. R-Square	0.0585	0.0627	0.0289	0.0098	0.0232	0.0013	0.0923	0.207

Notes: Monthly Data: August 1986 - June 2017. "i" Method. t statistics using Driscoll-Kraay standard error in parentheses. * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$.

The 1-month sample with a small number of observations has insignificant estimated coefficients using realized excess returns. That is, the standard UIP puzzle is absent. Survey-expected returns, on the other hand, suggest that holding higher interest bonds yields significantly positive excess returns over holding lower interest bonds. Coefficients in front of interest differentials are significantly negative in all samples.

The predictability of excess returns in both the 6- and 12-month samples shares the same patterns as in the 3-month sample. The standard UIP puzzle is recovered with more comprehensive data. There is no survey-expected excess returns in the G7 and the rich-country samples. In the poor-country samples, investors expect positive excess returns in

Table 10: Excess Returns when the Period Step is 12 months ($h = 12$)

	All		G7		Rich		Poor	
	ρ_{t+h}	$\mathbb{E}_t^e(\rho_{t+h})$	ρ_{t+h}	$\mathbb{E}_t^e(\rho_{t+h})$	ρ_{t+h}	$\mathbb{E}_t^e(\rho_{t+h})$	ρ_{t+h}	$\mathbb{E}_t^e(\rho_{t+h})$
x_t	-1.282***	-0.622***	-1.597***	-0.142	-1.404**	-0.00844	-1.260***	-0.727***
	(-8.83)	(-8.68)	(-3.34)	(-0.95)	(-2.63)	(-0.05)	(-9.72)	(-12.27)
constant	-0.00769	0.00679	0.00342	-0.00729*	0.00594	0.00601	-0.0318	0.00492
	(-0.61)	(1.83)	(0.38)	(-2.20)	(0.58)	(1.50)	(-1.67)	(1.22)
N	4054	4054	901	901	2618	2618	1436	1436
Adj. R-Square	0.137	0.203	0.0698	0.0033	0.0505	0.0000	0.217	0.468

Notes: Monthly Data: August 1986 - June 2017. "i" Method. t statistics using Driscoll-Kraay standard error in parentheses. * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$.

holding higher interest currencies. The magnitude of excess returns is lower in the survey expectation than in the realized data. Investors are aware that high interest currencies should depreciate over the next period, and deviations from the UIP are mitigated in the survey data.

Survey Exchange Rates

Again, this section attempts to understand how investors form forecasts. I test whether investors incorporate past depreciations in their expectations.

Table 11, Table 12, and Table 13 examine potential auxiliary effects of past depreciations on expected levels when the period steps are 1, 6, and 12 months, respectively.

Table 11: Survey Exchange Rate when the Period Step is 1 month ($h = 1$)

	All		G7		Rich		Poor	
	$\mathbb{E}_t^e(s_{t+h})$		$\mathbb{E}_t^e(s_{t+h})$		$\mathbb{E}_t^e(s_{t+h})$		$\mathbb{E}_t^e(s_{t+h})$	
x_t	35.08***	34.47***	-133.2***	-134.1***	-82.72***	-82.06***	45.88***	45.12***
	(5.55)	(5.41)	(-4.08)	(-4.20)	(-4.48)	(-4.48)	(6.38)	(6.19)
$s_{t-h} - s_{t-2h}$		0.489*		0.509*		0.311		0.487
		(2.09)		(2.54)		(1.86)		(1.53)
constant	-2.527***	-2.527***	-1.019***	-1.018***	-1.769***	-1.769***	-3.372***	-3.374***
	(-95.20)	(-96.10)	(-67.28)	(-68.74)	(-122.89)	(-121.85)	(-96.66)	(-97.48)
N	2170	2170	304	304	1140	1140	1030	1030
Adj. R-Square	0.0813	0.0888	0.319	0.337	0.269	0.278	0.148	0.153

Notes: Monthly Data: July 2008 - June 2017. "i" Method. t statistics using Driscoll-Kraay standard error in parentheses. * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$.

Table 12: Survey Exchange Rate when the Period Step is 6 months ($h = 6$)

	All		G7		Rich		Poor	
	$\mathbb{E}_t^e(s_{t+h})$		$\mathbb{E}_t^e(s_{t+h})$		$\mathbb{E}_t^e(s_{t+h})$		$\mathbb{E}_t^e(s_{t+h})$	
x_t	-0.217	-0.0724	-3.290**	-2.822*	-3.254*	-2.613	0.811	0.712
	(-0.35)	(-0.12)	(-2.61)	(-2.30)	(-2.17)	(-1.81)	(1.24)	(1.19)
$s_{t-h} - s_{t-2h}$		0.504**		0.354**		0.393**		0.558
		(2.76)		(3.27)		(2.87)		(1.97)
constant	-2.376***	-2.374***	-1.229***	-1.229***	-1.504***	-1.504***	-3.926***	-3.923***
	(-125.86)	(-132.06)	(-84.21)	(-86.92)	(-101.29)	(-103.52)	(-108.38)	(-112.13)
N	4053	4053	911	911	2606	2606	1447	1447
Adj. R-Square	0.0003	0.0631	0.0426	0.0838	0.0359	0.0790	0.0051	0.0720

Notes: Monthly Data: August 1986 - June 2017. "i" Method. t statistics using Driscoll-Kraay standard error in parentheses. * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$.

Table 13: Survey Exchange Rate when the Period Step is 12 months ($h = 12$)

	All		G7		Rich		Poor	
	$\mathbb{E}_t^e(s_{t+h})$		$\mathbb{E}_t^e(s_{t+h})$		$\mathbb{E}_t^e(s_{t+h})$		$\mathbb{E}_t^e(s_{t+h})$	
x_t	0.0962	0.197	-1.207	-1.019	-1.216	-0.959	0.902	0.805*
	(0.23)	(0.57)	(-1.89)	(-1.73)	(-1.79)	(-1.49)	(1.85)	(2.17)
$s_{t-h} - s_{t-2h}$		0.584***		0.416***		0.371***		0.800***
		(4.37)		(6.09)		(3.68)		(3.80)
constant	-2.314***	-2.310***	-1.223***	-1.222***	-1.454***	-1.455***	-3.901***	-3.891***
	(-125.54)	(-160.90)	(-82.15)	(-90.22)	(-104.54)	(-112.77)	(-111.37)	(-158.22)
N	3941	3941	887	887	2574	2574	1367	1367
Adj. R-Square	0.0002	0.165	0.0243	0.139	0.0225	0.104	0.0146	0.267

Notes: Monthly Data: August 1986 - June 2017. "i" Method. t statistics using Driscoll-Kraay standard error in parentheses. * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$.

Investors incorporate fundamentals into the exchange calculation in the expected way for the G7 and the rich-country samples. The estimated coefficients are most negative in the 1-month samples and increase monotonically to close to zero as the period step lengthens.

The reduction in absolute magnitudes with the length of period step arises naturally. An AR(1) structure implies that the 3-month autoregressive coefficient is roughly the 1-month coefficient to the power of 3. For stationary processes, the 1-month coefficient is less than 1. Any positive integer power of a number less than 1 is declining with the size of the power.

Coefficients in front of interest differentials have the opposite sign in the poor-country sample. They are all positive, but only that of the 1-month is significantly so. This suggests that only shorter rates are used as a signal for the inflation management problem.

Significantly positive estimated coefficients in front of past depreciations suggest that investors indeed extrapolate.

Equilibrium Exchange Rates

Table 14, Table 15, and Table 16 display the predictive power of interest differentials and past depreciations on realized exchange rates when the period steps are 1, 6, and 12 months accordingly.

Table 14: Equilibrium Exchange Rates when the Period Step is 1 month ($h = 1$)

	All		G7		Rich		Poor	
	s_t		s_t		s_t		s_t	
x_t	12.98*	12.91*	-21.43**	-20.91**	-19.03*	-18.38*	21.55*	21.22*
	(2.00)	(2.02)	(-3.05)	(-2.99)	(-2.32)	(-2.28)	(2.25)	(2.24)
$s_{t-h} - s_{t-2h}$		0.627**		0.550**		0.468*		0.638*
		(3.20)		(3.22)		(2.51)		(2.40)
constant	-2.279***	-2.279***	-1.235***	-1.235***	-1.621***	-1.621***	-3.422***	-3.422***
	(-105.54)	(-107.12)	(-85.54)	(-85.92)	(-101.48)	(-102.00)	(-72.71)	(-73.68)
N	4467	4467	928	928	2862	2862	1605	1605
Adj. R-Square	0.0227	0.0322	0.0473	0.0588	0.0334	0.0415	0.0715	0.0788

Notes: Monthly Data: July 2008 - June 2017. "i" Method. t statistics using Driscoll-Kraay standard error in parentheses. * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$.

Results on equilibrium exchange rates share almost exactly same patterns as results on survey forecasts. The estimated coefficients in front of interest differentials are significantly negative in all the G7 and the rich-country samples. The estimates' magnitude declines with the length of the period step. As before, the estimated coefficients in front of interest differentials are positive in all the poor-country samples. These results point to the perceived inflation risk among developing countries.

Across all samples, past depreciations affect equilibrium exchange rates even after controlling for interest differentials. Investors extrapolate in a way that past depreciations weaken realized exchange rates.

Table 15: Equilibrium Exchange Rate when the Period Step is 6 months ($h = 6$)

	All		G7		Rich		Poor	
	s_t		s_t		s_t		s_t	
x_t	-0.570	-0.435	-3.858**	-3.403**	-4.077**	-3.499*	0.644	0.552
	(-0.94)	(-0.76)	(-3.23)	(-2.92)	(-2.71)	(-2.41)	(1.04)	(0.97)
$s_{t-h} - s_{t-2h}$		0.470**		0.344**		0.354*		0.519
		(2.69)		(3.23)		(2.49)		(1.95)
constant	-2.377***	-2.375***	-1.223***	-1.224***	-1.505***	-1.505***	-3.924***	-3.922***
	(-130.52)	(-136.11)	(-83.79)	(-85.63)	(-95.37)	(-96.49)	(-114.22)	(-117.87)
N	4053	4053	911	911	2606	2606	1447	1447
Adj. R-Square	0.0018	0.0558	0.0561	0.0934	0.0511	0.0829	0.0034	0.0651

Notes: Monthly Data: August 1986 - June 2017. "i" Method. t statistics using Driscoll-Kraay standard error in parentheses. * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$.

Table 16: Equilibrium Exchange Rate when the Period Step is 12 months ($h = 12$)

	All		G7		Rich		Poor	
	s_t		s_t		s_t		s_t	
x_t	-0.401	-0.303	-2.006***	-1.797**	-2.147**	-1.874**	0.704	0.615
	(-1.02)	(-0.90)	(-3.36)	(-3.31)	(-3.14)	(-2.87)	(1.59)	(1.81)
$s_{t-h} - s_{t-2h}$		0.572***		0.462***		0.395***		0.739***
		(4.62)		(6.38)		(3.53)		(3.77)
constant	-2.320***	-2.316***	-1.214***	-1.213***	-1.457***	-1.458***	-3.901***	-3.892***
	(-132.25)	(-161.74)	(-80.52)	(-87.17)	(-95.83)	(-100.52)	(-121.55)	(-167.73)
N	3941	3941	887	887	2574	2574	1367	1367
Adj. R-Square	0.0026	0.158	0.0611	0.190	0.0597	0.138	0.0098	0.248

Notes: Monthly Data: August 1986 - June 2017. "i" Method. t statistics using Driscoll-Kraay standard error in parentheses. * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$.

D Mathematical Proofs

D.1 Proof of Proposition 1

Proof. $\forall k \geq 0$,

$$\begin{aligned}
s_t &= \mathbb{E}_t^X(s_{t+1}) - x_t \\
&= \mathbb{E}_t^F(s_{t+1}) + \gamma(s_{t-1} - s_{t-2}) - x_t \\
&= \lim_{T\to\infty} \mathbb{E}_t(s_{t+T}) - \sum_{h=0}^{\infty} \mathbb{E}_t(x_{t+1+h}) + \gamma(s_{t-1} - s_{t-2}) - x_t \\
&= \lim_{T\to\infty} \mathbb{E}_t(s_{t+T}) - \frac{x_t}{1-\lambda} + \gamma(s_{t-1} - s_{t-2})
\end{aligned}$$

The first and second equalities follow from the market clearing condition (16) and the relationship between extrapolative and fundamental beliefs (14) respectively. The third equality is a direct result of the definition of fundamental exchange rates from the equation (13). The last equality uses the AR(1) assumption of interest differentials. $\square$

D.2 Proof of Proposition 2

Proof. Using the time-series lag operator L to rewrite the equilibrium exchange rate equation (18) yields the first equality as per below.

$$\begin{aligned}
s_t &= \lim_{T\to\infty} \mathbb{E}_t(s_{t+T}) - \frac{x_t}{1-\lambda} + \gamma(Ls_t - L^2 s_t) \\
(1 - \gamma(L - L^2))s_t &= \bar{s}_t - \frac{x_t}{1-\lambda} \\
s_t &= \frac{1}{1 - \gamma(L - L^2)}(\lim_{T\to\infty} \mathbb{E}_t(s_{t+T}) - \frac{x_t}{1-\lambda}) \\
&= (1 + \gamma(L - L^2) + \gamma^2(L - L^2)^2 + ...+)(\lim_{T\to\infty} \mathbb{E}_t(s_{t+T}) - \frac{x_t}{1-\lambda}) \\
cov(s_t, x_t) &= cov((1 + \gamma(L - L^2) + \gamma^2(L - L^2)^2 + ...+)(\lim_{T\to\infty} \mathbb{E}_t(s_{t+T}) - \frac{x_t}{1-\lambda}), x_t)
\end{aligned}$$

The stationary assumption of exchange rates implies that $cov(\lim_{T\to\infty} \mathbb{E}_t(s_{t+T}), x_t) = 0$

resulting in:

$$
\begin{aligned}
cov(s_t, x_t) &= cov((1+\gamma(L-L^2)+\gamma^2(L-L^2)^2+...+)(-\frac{x_t}{1-\lambda}), x_t) \\
&= -\frac{cov(x_t+\gamma(x_{t-1}-x_{t-2})+\gamma^2((x_{t-2}-x_{t-3})-(x_{t-3}-x_{t-4}))+..., x_t)}{1-\lambda} \\
&= -[\frac{1+\gamma\lambda(1-\lambda)+\gamma^2\lambda^2(1-\lambda)^2+...}{1-\lambda}]var(x_t) \\
&= -[\frac{1}{1-\lambda}\cdot\frac{1}{1-\gamma\lambda(1-\lambda)}]var(x_t) \\
&\leq -[\frac{1}{1-\lambda}]var(x_t) \\
&= cov(-\frac{x_t}{1-\lambda}, x_t) \\
&= cov(\lim_{T\to\infty}\mathbb{E}_t(s_{t+T})-\frac{x_t}{1-\lambda}, x_t) \\
&= cov(s_t^{\text{UIP}}, x_t).
\end{aligned}
$$

The above infinite geometric series has a finite sum only if $\gamma\lambda(1-\lambda) < 1$. The inequality holds with equality whenever $\gamma = 0$, i.e. when investors have no behavioral bias.

Below shows the relationship between the covariance inequality and the exchange level compared to the UIP-implied level.

$$
\begin{aligned}
cov(s_t, x_t) &\leq cov(s_t^{\text{UIP}}, x_t) \\
cov(\mathbb{E}_t(s_{t+T})-s_t-\sum_{k=1}^{\infty}(x_{t+k-1}), x_t) &\geq cov(\mathbb{E}_t(s_{t+T})-s_t^{\text{UIP}}-\sum_{k=1}^{\infty}(x_{t+k-1}), x_t) \\
\frac{cov(\mathbb{E}_t(s_{t+T})-s_t-\sum_{k=1}^{\infty}(x_{t+k-1}), x_t)}{var(x_t)} &\geq \frac{cov(\mathbb{E}_t(s_{t+T})-s_t^{\text{UIP}}-\sum_{k=1}^{\infty}(x_{t+k-1}), x_t)}{var(x_t)} \\
\frac{cov(\sum_{k=1}^{\infty}\mathbb{E}_t(s_{t+k}-s_{t+k-1}-x_{t+k-1}), x_t)}{var(x_t)} &\geq \frac{cov(\sum_{k=1}^{\infty}\mathbb{E}_t((s_{t+k}^{\text{UIP}}-s_{t+k-1}^{\text{UIP}}-x_{t+k-1}), x_t)}{var(x_t)} \\
\frac{\sum_{k=1}^{\infty}(cov(\mathbb{E}_t(s_{t+k}-s_{t+k-1}-x_{t+k-1}), x_t))}{var(x_t)} &\geq \frac{\sum_{k=1}^{\infty}(cov(\mathbb{E}_t(s_{t+k}^{\text{UIP}}-s_{t+k-1}^{\text{UIP}}-x_{t+k-1}), x_t))}{var(x_t)} \\
\sum_{k=1}^{\infty}\frac{cov(\mathbb{E}_t(\rho_k), x_t)}{var(x_t)} &\geq 0 \\
\sum_{k=1}^{\infty}\beta_k &\geq 0
\end{aligned}
$$

The covariance inequality indicates the sign of the sum $\sum_{k=1}^{\infty}\beta_k$, which in turn relates the equilibrium exchange rate (s_t) with the level implied by the UIP (s_t^{UIP}).

□

D.3 Proof of Proposition 3

Proof. From Proposition 2, $\frac{cov(s_t,x_t)}{var(x_t)} = -\frac{1}{1-\lambda} \cdot \frac{1}{1-\gamma\lambda(1-\lambda)}$. Similarly, calculate $\frac{cov(\mathbb{E}_t(s_{t+1}),x_t)}{var(x_t)}$.

$$
\begin{aligned}
s_{t+1} &= \mathbb{E}^X_{t+1}(s_{t+1}) \\
&= \mathbb{E}^F_{t+1}(s_{t+1}) + \gamma(s_t - s_{t-1}) \\
&= \bar{s}_{t+1} - \frac{x_{t+1}}{1-\lambda} + \gamma(s_t - s_{t-1}) \\
(1-\gamma(L-L^2))s_{t+1} &= \bar{s}_{t+1} - \frac{x_{t+1}}{1-\lambda} \\
s_{t+1} &= \frac{1}{1-\gamma(L-L^2)}(\bar{s}_{t+1} - \frac{x_{t+1}}{1-\lambda}) \\
&= (1+\gamma(L-L^2)+\gamma^2(L-L^2)^2+...)(\bar{s}_{t+1} - \frac{x_{t+1}}{1-\lambda}) \\
cov(\mathbb{E}_t(s_{t+1}),x_t) &= -\frac{cov(\mathbb{E}_t[(1+\gamma(L-L^2)+\gamma^2(L-L^2)^2+...)x_{t+1}],x_t)}{1-\lambda} \\
&= -\frac{cov(\mathbb{E}_t[x_{t+1}+\gamma(x_t-x_{t-1})+\gamma^2((x_{t-1}-x_{t-2})-(x_{t-2}-x_{t-3}))+...],x_t)}{1-\lambda} \\
&= [-\frac{\lambda+\gamma(1-\lambda)+\gamma^2\lambda(1-\lambda)^2+...}{1-\lambda}]var(x_t) \\
\frac{cov(\mathbb{E}_t(s_{t+1}),x_t)}{var(x_t)} &= -\frac{1}{1-\lambda}(\lambda + \frac{\gamma(1-\lambda)}{1-\gamma\lambda(1-\lambda)}) \\
\frac{cov(\mathbb{E}_t(s_{t+1})-s_t-x_t,x_t)}{var(x_t)} &= -\frac{1}{1-\lambda}(\lambda + \frac{\gamma(1-\lambda)}{1-\gamma\lambda(1-\lambda)}) + \frac{1}{1-\lambda}\cdot\frac{1}{1-\gamma\lambda(1-\lambda)} - 1 \\
\frac{cov(\mathbb{E}_t(\rho_1),x_t)}{var(x_t)} &= -\frac{\gamma(1-\lambda)}{1-\gamma\lambda(1-\lambda)}
\end{aligned}
$$

When $0 < \gamma < \frac{1}{\lambda(1-\lambda)}$, $\beta_1 = \frac{cov(\mathbb{E}_t(\rho_1),x_t)}{var(x_t)} < 0$. If $\gamma = 0$, $\beta_1 = 0$, and UIP holds.

□

D.4 Proof of Proposition 4

Proof. For $0 < \gamma < \frac{1}{\lambda(1-\lambda)}$, $\sum_{k=1}^{\infty} \beta_k \geq 0$ and $\beta_1 < 0$, therefore there exists $h \geq 2$ such that $\beta_h > 0$.

For $\gamma = 0$, $s_t = \bar{s}_t - \frac{x_t}{1-\lambda}$, therefore for all $k \geq 1$

$$\begin{aligned}
\beta_k &= \frac{cov(\mathbb{E}_t(s_{t+k} - s_{t+k-1} - x_{t+k-1}), x_t)}{var(x_t)} \\
&= \frac{cov(\bar{s}_{t+k} - \bar{s}_{t+k-1} + \frac{\lambda^{t+k-1}x_t - \lambda^{t+k}x_t}{1-\lambda} - \lambda^{t+k-1}x_t, x_t)}{var(x_t)} \\
&= 0
\end{aligned}$$

□

D.5 Proof of Proposition 5

Proof. I first calculate $\frac{cov(s_{t-1}, x_t)}{var(x_t)}$.

$$\begin{aligned}
\frac{cov(s_{t-1}, x_t)}{var(x_t)} &= \frac{cov((1 + \gamma(L - L^2) + \gamma^2(L - L^2)^2 + ...)(\bar{s}_{t-1} - \frac{x_{t-1}}{1-\lambda}), x_t)}{var(x_t)} \\
&= -\frac{\lambda + \lambda\gamma\lambda(1-\lambda) + \lambda\gamma^2(\lambda(1-\lambda))^2 + ...}{1-\lambda} \\
&= -\frac{\lambda}{(1-\lambda)(1-\gamma\lambda(1-\lambda))}
\end{aligned}$$

Define $A_k = \frac{cov(\mathbb{E}_t(s_{t+k}-s_{t+k-1}),x_t)}{var(x_t)}$ and $B_k = \frac{cov(\mathbb{E}_t(s_{t+k}-s_{t+k-1}-x_{t+k-1}),x_t)}{var(x_t)}$.

$$\begin{aligned}
A_0 &= \frac{cov(s_t - s_{t-1}, x_t)}{var(x_t)} \\
&= \frac{cov(s_t, x_t) - cov(s_{t-1}, x_t)}{var(x_t)} \\
&= -\frac{1}{1-\gamma\lambda(1-\lambda)} \\
B_0 &= A_0 - \frac{cov(x_{t-1}, x_t)}{var(x_t)} \\
&= A_0 - \lambda \\
&= \frac{-1-\lambda+\gamma\lambda^2-\gamma\lambda^3}{1-\gamma\lambda(1-\lambda)} \\
A_1 &= \frac{cov(\mathbb{E}_t(s_{t+1}) - s_t, x_t)}{var(x_t)} \\
&= \frac{cov(\mathbb{E}_t(s_{t+1}), x_t) - cov(s_t, x_t)}{var(x_t)} \\
&= \frac{1-\gamma+\gamma\lambda^2}{1-\gamma\lambda(1-\lambda)} \\
B_1 &= A_1 - \frac{cov(x_t, x_t)}{var(x_t)} \\
&= A_1 - 1 \\
&= \frac{-\gamma(1-\lambda)}{1-\gamma\lambda(1-\lambda)}
\end{aligned}$$

For $k \geq 2$,

$$
\begin{aligned}
s_{t+k} &= \bar{s}_{t+k} - \frac{x_{t+k}}{1-\lambda} + \gamma(s_{t+k-1} - s_{t+k-2}) \\
s_{t+k-1} &= \bar{s}_{t+k-1} - \frac{x_{t+k-1}}{1-\lambda} + \gamma(s_{t+k-2} - s_{t+k-3}) \\
\mathbb{E}_t(s_{t+k} - s_{t+k-1}) &= \bar{s}_{t+k} - \bar{s}_{t+k-1} + \frac{\mathbb{E}_t(x_{t+k-1} - x_{t+k})}{1-\lambda} + \gamma[(\mathbb{E}_t(s_{t+k-1} - s_{t+k-2})) - (\mathbb{E}_t(s_{t+k-2} - \\
&= \bar{s}_{t+k} - \bar{s}_{t+k-1} + \frac{\lambda^{k-1}x_t - \lambda^k x_t}{1-\lambda} + \gamma[(\mathbb{E}_t(s_{t+k-1} - s_{t+k-2})) - (\mathbb{E}_t(s_{t+k-2} - s_{t+} \\
cov(\mathbb{E}_t(s_{t+k} - s_{t+k-1}), x_t) &= \lambda^{k-1} var(x_t) + \gamma(cov(\mathbb{E}_t(s_{t+k-1} - s_{t+k-2}), x_t) - cov(\mathbb{E}_t(s_{t+k-2} - s_{t+k-3}), x_t) \\
A_k &= \lambda^{k-1} + \gamma(A_{k-1} - A_{k-2}) \\
B_k &= A_k - \frac{cov(\mathbb{E}_t(x_{t+k-1}), x_t)}{var(x_t)} \\
&= A_k - \lambda^{k-1} \\
&= \gamma(A_{k-1} - A_{k-2})
\end{aligned}
$$

Lemma 7. $\gamma \in [0,1)$ *is a sufficient condition for* $\lim_{k\to\infty} A_k = 0$

Proof From the recurrence relation $A_k = \gamma(A_{k-1} - A_{k-2}) + \lambda^{k-1}$, I solve for the close-form solution of A_k using characteristic polynomials.

$$
\begin{aligned}
A_k &= \gamma(A_{k-1} - A_{k-2}) + \lambda^{k-1} \qquad (26) \\
A_{k+1} &= \gamma(A_k - A_{k-1}) + \lambda^k \qquad (27) \\
(27) - \lambda\ (26) : A_{k+1} - \lambda A_k &= \gamma(A_k - A_{k-1}) - \gamma\lambda(A_{k-1} - A_{k-2}) \\
x^3 - \lambda x^2 &= \gamma x^2 - \gamma x - \gamma\lambda x + \gamma\lambda \\
x^3 - \lambda x^2 - \gamma x^2 + \gamma x + \gamma\lambda x - \gamma\lambda &= 0 \\
(x-\lambda)(x^2 - \gamma x + \gamma) &= 0 \\
x &= \lambda, \frac{\gamma \pm \sqrt{\gamma^2 - 4\gamma}}{2}
\end{aligned}
$$

Write A_k in term of 3 roots with $x_1 = \lambda$, $x_2 = \frac{\gamma+\sqrt{\gamma^2-4\gamma}}{2}$, and $x_3 = \frac{\gamma-\sqrt{\gamma^2-4\gamma}}{2}$, i.e. $A_k = a{x_1}^k + b{x_2}^k + c{x_3}^k$ for some constants a, b and c.

It is sufficient to show that $\gamma \in [0,1)$ implies $\parallel x_i \parallel < 1$ for all $i = 1, 2$, and 3 because $\lim_{k\to\infty} A_k = a \lim_{k\to\infty} {x_1}^k + b \lim_{k\to\infty} {x_2}^k + c \lim_{k\to\infty} {x_3}^k = 0$. Recall $\lambda \in [0,1)$, so $\parallel x_1 \parallel <$

1. For $\gamma \in [0,1)$, $\gamma^2 - 4\gamma \leq 0$. Therefore, $\sqrt{\gamma^2 - 4\gamma} = \sqrt{4\gamma - \gamma^2} i$.

$$\frac{\gamma + \sqrt{\gamma^2 - 4\gamma}}{2} = \frac{\gamma + \sqrt{\gamma(4-\gamma)} i}{2}$$
$$\frac{\gamma - \sqrt{\gamma^2 - 4\gamma}}{2} = \frac{\gamma - \sqrt{\gamma(4-\gamma)} i}{2}$$

Therefore $\| \frac{\gamma+\sqrt{\gamma^2-4\gamma}}{2} \| = \| \frac{\gamma-\sqrt{\gamma^2-4\gamma}}{2} \| = \sqrt{\frac{\gamma^2+\gamma(4-\gamma)}{4}} = \sqrt{\frac{4\gamma}{4}} < 1$. That is, $\gamma \in [0,1)$ is a sufficient condition for $\lim_{k\to\infty} A_k = 0$.

So, for $\gamma \in [0,1)$, $\lim_{k\to\infty} \beta_k = \lim_{k\to\infty} B_k = \lim_{k\to\infty}(A_k - \lambda^{k-1}) = \lim_{k\to\infty} A_k = 0$.

□

D.6 Proof of Proposition 6

Proof. First, recall from proposition 5 that $\frac{cov(s_{t-1},x_t)}{var(x_t)} = -\frac{\lambda}{(1-\lambda)(1-\gamma\lambda(1-\lambda))}$. Similarly, we can calculate $\frac{cov(s_{t-2},x_t)}{var(x_t)}$.

$$\begin{aligned}
\frac{cov(s_{t-2},x_t)}{var(x_t)} &= \frac{cov((1+\gamma(L-L^2)+\gamma^2(L-L^2)^2+...)(\bar{s}_{t-2}-\frac{x_{t-2}}{1-\lambda}),x_t)}{var(x_t)} \\
&= -\frac{\lambda^2+\lambda^2\gamma\lambda(1-\lambda)+\lambda^2\gamma^2(\lambda(1-\lambda))^2+...}{1-\lambda} \\
&= -\frac{\lambda^2}{(1-\lambda)(1-\gamma\lambda(1-\lambda))}
\end{aligned}$$

Now, we have

$$\begin{aligned}
var(s_t) &= var(\bar{s}_t - \frac{x_t}{1-\lambda} + \gamma(s_{t-1}-s_{t-2})) \\
&= \frac{var(x_t)}{(1-\lambda)^2} + \gamma^2 var(s_{t-1}-s_{t-2}) - \frac{\gamma}{1-\lambda} cov(s_{t-1}-s_{t-2},x_t) \\
&= \frac{var(x_t)}{(1-\lambda)^2} + \gamma^2 var(s_{t-1}-s_{t-2}) - \frac{\gamma}{1-\lambda}[-\frac{\lambda}{(1-\lambda)(1-\gamma\lambda(1-\lambda))} + \frac{\lambda^2}{(1-\lambda)(1-\gamma\lambda(1-\lambda))}]v \\
&= \frac{var(x_t)}{(1-\lambda)^2} + \gamma^2 var(s_{t-1}-s_{t-2}) + \frac{\gamma\lambda}{(1-\lambda)(1-\gamma\lambda(1-\lambda))} var(x_t).
\end{aligned}$$

Whenever $0 < \gamma < \frac{1}{\lambda(1-\lambda)}$, $var(s_t) > \frac{var(x_t)}{1-\lambda^2} = var(s_t^{\mathrm{UIP}})$.

□

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