

EARLY SCIENCE AND MEDICINE

A Journal for the Study of Science, Technology and Medicine in the Pre-modern Period

Aims & Scope

Early Science and Medicine is an international journal dedicated to the history of science, medicine and technology from the earliest times through to the end of the eighteenth century. It takes a particular interest in the text-based traditions that link antiquity with the Latin, Arabic, and Byzantine Middle Ages, the Renaissance, and the early modern period. It favors diachronic studies in a variety of forms, including commented text editions and monographic studies of historical figures and scientific questions or practices. The main language of the journal is English, although contributions in French and German are also accepted.

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ical entity but connotes the pushes, pulls, and exerted pressure of ordinary experience. The moving radius principle helped to advance the science of motion from accounts dependent on weight to explanations of the behavior of rigid bodies as such.³⁰



Structures of Argument and Concepts of Force in the *Aristotelian Mechanical Problems*

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Abstract

I argue that the main goal of the *Mechanical Problems*, a short treatise transmitted in the *Corpus Aristotelicum*, is to explain the working of technology in terms of the concepts of Aristotelian natural philosophy. The author's explanatory strategy is to reduce the thirty-five "problems" or questions that he discusses to one or more of three simple models: the circle, balance, and lever. The conceptual foundation of this reduction program is a principle concerning circular motion, viz. that a point on the circumference of a larger circle moves more quickly than one on a smaller circle, assuming that the circles turn about the same center at the same angular speed. I analyze the author's argument for this principle and his application of it throughout the text, especially to the analysis of the lever. The main conclusions are (1) that the author's justification of the circular motion principle is based on an innovative geometrical analysis of motion, not on a highly theoretical conceptualization of force; and (2) while the author is aware of a reciprocal relationship between weights and distances from the fulcrum in the case of the lever, his explanation of this fact makes no reference to the conditions for static equilibrium.

* Department of the Classics, 204 Boylston Hall, Harvard University, Cambridge, MA 02138, U.S.A. (mjschief@fas.harvard.edu). This paper owes its existence to a kind invitation from Peter McLaughlin to participate in a panel at the History of Science Society annual meeting in November 2007; I am grateful to all who participated in that event for their comments and questions. My approach to the *Mechanica* has been shaped by many discussions with colleagues in Department I of the Max Planck Institute for the History of Science in Berlin, especially Peter Damerow, Malcolm Hyman, and Jürgen Renn; I hope that they may find something of interest in the final product. I am also grateful to Bill Newman and Edith Sylla for their assistance and patience, and to Jean de Groot for sharing a draft of her paper with me. Finally it is a pleasure to dedicate the paper to John Murdoch, as a small token of appreciation for everything I have learned from him over the years and for his unstinting support.

³⁰ I am grateful to Edith Sylla for comments on an earlier version of this paper.

Keywords

mechanics, technology, nature, natural philosophy, force, power, weight, reduction, models, lever, balance, circle, circular motion, steelyard

1. Introduction

The *Mechanical Problems* or *Mechanica*, a short treatise transmitted in the *Corpus Aristotelicum*, is a work of the first importance for the history of science. It consists of an introduction emphasizing the wondrous behavior of mechanical devices and arguing that that behavior can be traced back to the properties of circular motion; this is followed by a discussion of thirty-five “problems” or questions that are principally concerned with the operation of various technological devices. Attributed to Aristotle in antiquity and the early modern period, the *Mechanica*’s rediscovery in the Renaissance spurred the composition of an extensive commentary literature.¹ There is a direct historical link between the *Mechanica* and the emergence of classical mechanics in the work of Galileo and his contemporaries.

While historians of mechanics have generally recognized the importance of the *Mechanica*, they have not always interpreted the text on its own terms. The author’s approach has typically been viewed as representative of a “dynamical” tradition, since it makes use of conceptions of motion and force, in contrast with the “statical” tradition that eschews considerations of motion and traces its origin to Archimedes’ *On the equilibrium of plane figures*. Such a division is fundamental to the most thorough and incisive recent treatment of the *Mechanica*, F. Krafft’s *Dynamische und statische Betrachtungsweise in der antiken Mechanik* (Wiesbaden, 1970). But the assumption of a division between statics and dynamics is a highly problematic starting point for analyzing the history of ancient mechanics, if for no other reason than that no ancient writer distinguishes explicitly between the two approaches. Even in the Renaissance, when such

¹ The fundamental bibliographical survey of this literature remains P.L. Rose and S. Drake, “The Pseudo-Aristotelian *Questions of Mechanics* in Renaissance Culture,” *Studies in the Renaissance*, 18 (1971), 65–104.

a distinction does emerge, writers tend to emphasize the points of agreement between the Aristotelian treatise and Archimedes’ approach.² Stimulated in part by the question of Aristotelian authorship, scholars have tended to view the argument of the *Mechanica* against the background of Aristotle’s scattered remarks on mechanics in works such as the *Physics* and *De caelo*. Thus Duhem, in his *Les origines de la statique*, explained the author’s argument for the “law of the lever” (the inverse proportionality of weights and distances on the lever arm) as a direct application of the statements of proportionality between mover, moved, distance, and time found in *Physics* VII 5.³ Contrary voices have been raised, but generally not by historians of mechanics.⁴

In this paper I will attempt to sketch out a more satisfactory interpretation of the *Mechanica*’s methodology and conceptual foundations. My starting point is the claim that the author’s principal concern is to provide a theoretical explanation of the working of technology. The *Mechanica* thus reflects a distinction between theoretical mechanics—a body of knowledge consisting of relations between concepts such as weight, force, and motion—and practitioners’ knowledge, i.e. the knowledge acquired in the design and use of technology. That the first text concerned with mechanics in the Greek tradition aims at the explanation of practitioners’ knowledge is an important reminder that technology in Greco-Roman antiquity often developed in advance of theory. A striking example of this is provided by the steelyard or balance with unequal arms, the topic of problem 20 in the *Mechanica*. The existence of the

² See e.g. the opening of Guidobaldo del Monte, *In duos Archimedis aequiponderantium libros paraphrasis* (Pesaro, 1588), esp. 4: *Quare Archimedes Aristotelem sequi videtur ...*

³ P. Duhem, *Les origines de la statique*, 2 vols. (Paris, 1905 and 1906), 1:5–6; 2:292–293.

⁴ H. Carteron offered a trenchant critique of Duhem in the introduction to his *La notion de force dans le système d’Aristote* (Paris, 1923; Eng. trans. as “Does Aristotle Have a Mechanics?” in J. Barnes et al., eds., *Articles on Aristotle*, vol. 1 (London, 1975), 161–174), though his main interest was not mechanics as such. The best recent critique of the Duhemian line is F. de Gandt, “Force et science des machines,” in J. Barnes et al., eds., *Science and Speculation: Studies in Hellenistic Theory and Practice* (Cambridge, 1982), 96–127.

steelyard is reliably attested for the late fifth century BC by a passage of Aristophanes' *Peace* (performed in 421 BC, some 100 years before the *Mechanica* was written if not more).⁵ It has often been assumed that the construction of the steelyard implies knowledge of the "law of the lever," i.e. the claim that static equilibrium requires the inverse proportionality of weights and distances from the fulcrum or balance point. But the fact that the use of the steelyard precedes by at least a century the first written attempt to explain its operation makes it seem quite likely that it was developed without knowledge of such general theoretical principles.⁶

2. Problems and Explanation

The *Mechanica* begins with some general reflections on the relationship of art or *technê* (the Greek term can refer to any kind of result-oriented expertise) and nature:

Among things that occur according to nature (*kata phusin*), we wonder at those whose cause is unknown; among things that occur unnaturally (*para phusin*), we wonder at those that come about by means of art (*technê*) for the benefit of human beings. For in many cases nature acts in a way opposed to what is useful for us. For nature always acts in the same way and simply, while what is useful changes in many ways. Whenever, then, it is necessary to do something unnatural (*para phusin*), because of the difficulty we are at a loss (*aporia*) and have need of art (*technê*). For this reason, we also call that part of art that assists in such difficulties (*aporiai*) a device (*mêchanê*). For as the poet Antiphon said, so it is: "By means of art we gain mastery (*kratoumen*) over things in which we are conquered by nature." (847a11–21)⁷

⁵ Aristophanes, *Peace* 1240–1249, with the comments of S.D. Olson, *Aristophanes' Peace: Edited with Introduction and Commentary* (Oxford, 1998), 304. For the *Mechanica* a date in the late fourth or early third centuries BC seems the most likely possibility, though certainty is impossible.

⁶ For a modern parallel see J. Renn and M. Schemmel, *Waagen und Wissen in China: Bericht einer Forschungsreise* (Preprints of the Max Planck Institute for the History of Science 136) (Berlin, 2000).

⁷ Translations from the *Mechanica* are my own, based on the text of M. Bortechia, *Aristotele MHXANIKÁ* (Studia Aristotelica 10) (Padua, 1982). Unfortunately her edition includes no figures and does not number the individual problems; I follow the

The notion that *technê* can rescue human beings from a situation of need or helplessness (*aporia*) by means of a device or strategem (*mêchanê*) has a long history in Greek thought, and is brilliantly summarized in the quotation from the poet Antiphon that crowns the passage: *technê* enables human beings to overcome their natural disadvantages. The emphasis on the "unnatural" (*para phusin*) character of what is produced by *technê* has often been taken to suggest that the author conceives of the art-nature relationship as a fundamentally antagonistic one. In fact, however, the primary contrast between art and nature here is between the complexity and utility of the former as against the simplicity and regularity of the latter. The phrase *para phusin* has a number of senses in Aristotle, including (1) "beyond nature" in the sense of outside the order of nature, i.e. supernatural; (2) "contrary to nature," i.e. opposed to a thing's natural tendencies; (3) "forced" as opposed to unconstrained; and (4) "unusual" as opposed to what is natural or normal. I have argued elsewhere that the author's conception of the art-nature relationship is best understood as the idea that art goes "beyond nature" in the sense that it brings about results that nature left to its own devices does not. Insofar as art "masters" nature, it is by the creative combination of natural principles to go beyond what nature does on its own.⁸

The introduction continues by specifying the nature of mechanical problems more precisely. They are cases in which "the lesser master the greater, and things with small inclination (*rhopê*) move great weights (*barê*)" (847a21–23). Mechanical problems occupy an intermediate position between physics and mathematics: their subject matter (*to peri ho*) is revealed by physics, while their explanation (the 'how', *to hôs*) is made clear through mathematics (847a24–28).⁹

traditional designation as found in the editions of O. Apelt (Leipzig, 1888) and W.S. Hett (*Aristotle: Minor Works* [London and Cambridge, MA, 1936]).

⁸ For the idea that art completes what nature leaves unfinished see Aristotle, *Physics* II 8, 199a8–20. I develop an interpretation of ancient mechanics along these lines in M. Schiefsky, "Art and Nature in Ancient Mechanics," in B. Bensaude-Vincent and W. Newman, eds., *The Artificial and the Natural: An Evolving Polarity* (Cambridge, MA, 2007), 67–108.

⁹ This remark recalls Aristotle's notion of subordinate sciences, according to which mathematics supplies the explanatory element in disciplines concerned with the

Next, the author mentions the lever as the paradigm example of a mechanical problem:

Among the puzzles in this class are those that concern the lever (*mochlos*). For it seems strange that a great weight (*baros*) is moved by a small force (*ischus*), and that, too, where a greater weight is involved. For the very same weight, which a man cannot move without a lever, he quickly moves by taking in addition the weight of the lever. (847a28–b15)

The key feature of mechanical problems is thus that they seem to violate the natural or normal relationship between force (*ischus*) and weight (*baros*). Normally, a large force is required to move a large weight, but the lever makes it possible to do so with a small force.

That the author's aim is to explain why this is so is made clear in the remainder of the introduction, which argues that the "original cause" (*tês aitiias tēn archēn*, 847b16) of all mechanical phenomena lies in the remarkable nature of the circle. The circle unites opposites such as concave and convex, as well as opposite motions, forward and backward (since a point on a revolving circle that sets out in one direction ends up back at its starting point); but its most remarkable feature is that

although the line from the center is one, none of the points on it moves at the same speed (*isotachōs*) as any other, but that which is farther from the fixed extremity always moves more quickly (*thattōn*). (848a15–17)

That is, as a radius (the "line from the center") sweeps out a circle, a point on the radius that is farther from the center moves quicker than one that is closer in. The final part of the introductory section (848a19–37) describes a mechanical device whose operation depends on the fact that the circle combines opposite movements. It consists of a number of circles in contact with one another like a set of interlocking gears, so that when the first one rotates it

physical world (*Metaphysics* 1078a14ff.; *Posterior Analytics* 75b14 ff., 76a23–25). I discuss these passages in Schiefsky, "Art and Nature," 90–94.

causes the second to turn in the opposite direction, and so on. The section concludes with the following important remark:

And so craftsmen, seizing on this natural property (*phusis*) of the circle, construct an instrument by concealing the principle (*archē*), so that only the wondrous character (*to thaumaston*) of the device (*mēchanēma*) is apparent, while its cause (*aition*) is unclear. (848a34–37)

Here several fundamental concepts of Aristotelian natural philosophy—nature, principle, and cause—are associated with one another and opposed to wonder: the operation of the device is wondrous only if the causal principle (*archē*) that explains it is concealed. Moreover the device is constructed not by working against the nature (*phusis*) of the circle but by making creative use of it. Mechanics, no less than any other *technē*, demands knowledge of natural principles.

The details of the author's explanatory program will concern us in the next section. But it should now be clear that his overall aim, as announced in the introduction, is to give an explanation of the working of technology in terms of the concepts of Aristotelian natural philosophy. The existence of a text like the *Mechanica* makes good sense against the background of the development of technology in the Greek world of the fifth and fourth centuries BC. I have already mentioned the steelyard, whose operation vividly demonstrates the ability of a small weight to 'overpower' a large one. But this is only one example among many. The surgical works of the Hippocratic Corpus refer to the use of machines for setting fractures and dislocations and reflect on their ability to achieve great effects.¹⁰ The development of torsion artillery at the end of the fifth century provided a dramatic example of the possibilities of enhancing human strength by mechanical contrivance, and of the relevance of technology to military and political affairs.¹¹ No less important

¹⁰ [Hippocrates] *De articulis* (On joints) 72; *De fracturis* (On fractures) 31: "For of all the instruments devised by human beings, the strongest are these three: the turning of the winch, leverage, and the use of the wedge."

¹¹ For this dating and for a general account of the development of torsion artillery see E.W. Marsden, *Greek and Roman Artillery: Historical Development* (Oxford, 1969).

than the rise of technology, however, is that of natural philosophy: for the "problems" generated by technology are only problems when viewed in light of a conception of what is natural or normal. The explicit formulation of relationships between such concepts as force and weight is thus also an important part of the background to the *Mechanica*. To invert Aristotle's famous dictum, the *Mechanica* shows that in an important sense, philosophy is the starting point of wonder.

3. The Reduction Program

The basic explanatory strategy of the treatise, announced in the introduction, is to "reduce" (*anagein*) each problem to the lever, the balance, or the circle:

Now the things that come about with the balance are reduced (*anagetai*) to the circle, those that come about with the lever to the balance, and practically everything else that comes about with mechanical movements to the lever. (848a11-14)

The notion of reduction as the analysis of a complex entity into its simpler components is familiar from a number of Aristotelian contexts. In the *Prior Analytics* Aristotle claims that all syllogisms can be reduced to the first figure (29b1-2, 40b17-22, 41b3-5). In *De generatione et corruptione* II 2 (329b15-330a29) he claims that qualities such as hard and soft can be reduced to the four primary qualities of hot, cold, wet, and dry; the idea seems to be that the derivative qualities can be understood as kinds or forms of the primary ones. Compared to these somewhat obscure examples, the reduction program in the *Mechanica* stands out for its clarity and simplicity. The reduction is carried out by identifying correspondences between the device or situation in question and the lever, balance, or circle. Each of these three conceptual schemata or models has a fixed number of components or 'slots' designated by a standard terminology, as specified in the following table:

Model	Slots
circle	center (<i>kentron</i>), radii (<i>hai ek tou kentrou</i>)
balance	beam (<i>zugon</i>), cord (<i>spartion</i>) (the beam is suspended from the cord)
lever	lever (<i>mochlos</i>), fulcrum (<i>hupomochlion</i>), weight (<i>baros</i>), mover (<i>ho kinôn</i>)

While the terminology for the slots is quite consistent, each slot can be filled by different objects depending on the particular situation. Thus the 'mover' slot in the lever model can be filled by a weight (*baros*: problem 3), a power (*dunamis*: problems 5, 6), or a person (e.g. a sailor or workman: problems 5, 29).

As an example we may consider problem 4, which asks why rowers in the middle of a boat contribute most to its movement. The situation is analyzed using the lever model (references to the slots of the model are italicized):

(a) Why do those in the middle of the ship contribute most to its movement? (b) Is it because the oar is a *lever*? For the thole-pin becomes a *fulcrum* (for this is stationary), and the *weight* is the sea, which the oar pushes away; the *mover* of the *lever* is the sailor. (c) And the *mover* of the *weight* always moves more *weight*, the more distant it is from the *fulcrum*; for in this way the radius is greater, and the thole-pin, being a *fulcrum*, is a center (850b10-16).

Slot	Identification
lever (<i>mochlos</i>)	oar
weight (<i>baros</i>)	sea
fulcrum (<i>hupomochlion</i>)	thole-pin
mover (<i>ho kinôn</i>)	sailor

The picture is that of a ship that is wider in the middle than at the bow or stern. In such a vessel, there is a greater length of oar inside the ship for a rower amidships than for one in the bow or stern; hence the greater mechanical effect achieved by the former. The order of exposition here is typical: the question (a) is followed by an identification of the slots of the relevant model (b), followed by an explanation (c) that gives the reason why the model in question accounts for the phenomenon. The questions (a) are normally

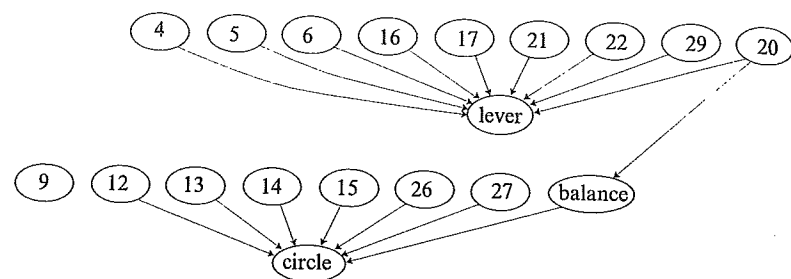


Figure 1: Reduction map

introduced by the phrase *dia ti* ('Why'), and the identifications (b) by the phrase *ē dioti* ('Is it because'). The model identification is not always so straightforward, and there is substantial variation in the structure of particular problems. But the general pattern of question, identification, and explanation is characteristic of the author's approach.

By analyzing the model identifications for each problem it is possible to build up a picture of the author's overall procedure, represented in graphical form in figure 1. The graph includes all problems for which there is an explicit model identification in the text, with the exception of problem 1 (where the balance is reduced to the circle) and problem 3 (where the lever is reduced to the balance). Of the 35 problems, 18 (including 1 and 3) meet this criterion. In the first few problems the author makes good on his promise (848a11-14) to reduce the balance to the circle (1), the lever to the balance (3), and all other mechanical phenomena to the lever (4-6). But this does not describe his procedure throughout: a significant number of problems are reduced directly to the circle. Moreover, the graph reveals that the balance plays a relatively unimportant role in the explicit model identifications. Aside from problem 3 (the lever itself) the balance is explicitly invoked only in the case of the steelyard (problem 20), which is identified with both the balance and the lever.

Several of the problems that lack explicit model identifications are nonetheless closely linked to the circle or lever models. First there are two cases of indirect reduction: problem 7, which refers back to the analysis of the rudder as a lever in problem 5, and

problem 19, which refers to the analysis of the wedge as a lever in 17. Problem 8, "why are round and circular figures easier to move," draws heavily on the analysis of the circle in problem 1. In problem 18, the author claims that the compound pulley operates in a way analogous to the lever (853a37-b2); but this seems to mean only that it produces a large effect with little input, since the slots of the lever model are not identified. Problems 2 and 10 deal with aspects of the balance but without reducing it to the circle; problems 11 and 35 concern circular movement, but again there is no explicit identification of model slots. Problems 23 and 24 explore conceptual difficulties raised by the analysis of problem 1 (the composition of motions in the former case and the differential rotation of larger and smaller circles in the latter, the famous "wheel of Aristotle"). That leaves seven problems (25, 28, and 30-34) which have no connection with the circle, balance, or lever. Most of these, however, concern either technology (25, 28) or phenomena that are somehow problematic for Aristotelian natural philosophy, such as projectile motion (32-34) or the tendency of objects in motion to continue moving (31).

Thus there is a fair degree of coherence in the selection of problems. Approximately half contribute directly to the author's project of explaining technology by reference to circle, balance, and lever. Most of the rest have some connection with the basic models; only a few stand entirely outside the explanatory program announced in the introduction. Even so, they arguably have a place in a collection of problems concerned with phenomena involving forced motion that are *prima facie* difficult to explain in the context of Aristotelian philosophy.

To sum up, the reduction program of the *Mechanica* is a remarkable specimen of analytical procedure—a movement from the complex to the simple—as contrasted with the synthetic procedure, or movement from first principles to conclusions, that is typical of many Greek mathematical texts. The distinctive feature of the author's method is the identification of the circle, balance, and lever models; the structure of these models, expressed in a consistent terminology, guides his analysis throughout. I turn now to a closer examination of the conceptual foundations of the two most important models, the circle and lever.

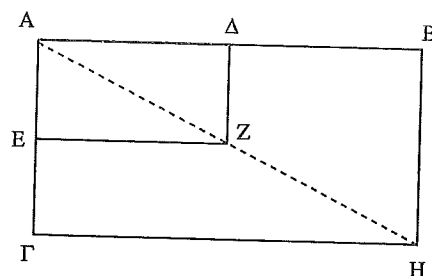


Figure 3: Rectilinear motion in the *Mechanica*, Problem 1

components is constantly changing (848b23-35). Using this method, circular motion is analyzed as consisting of one component directed to the center and a second at right angles to it. Thus in figure 2, in the time that point B moves to H along the circumference, the two components of its motion are represented by lines BK and KH: the distances covered in the vertical and horizontal directions, respectively.¹³

The second stage of the argument runs from 849a6-19. Having established that the line that traces out the circle “is carried with two motions” (*pheretai duo phoras*, 848b9-10, b36), the author states the following principle relating force and speed, which I will refer to as the “deflection principle”:

If, of two things moved from the same force (*apo tēs autēs ischuos*), one is deflected (*ekkrouto*) more and the other less, it is reasonable that the one that is deflected more will move more slowly (*braduteron*) than the one that is deflected less. And this seems to happen with the greater and the lesser of the lines from the center that trace out the circles. For because the extremity of the lesser is closer to the fixed (center) than that of the greater, the extremity of the lesser travels more slowly

nothing like this is stated in the text. The assumption of a constant ratio between the components of motion does not imply that the object's speed is constant; it implies only that both components change in the same way. It is also worth remarking that the diagram (fig. 3) is quite underdetermined by the text. Nothing in the text establishes that the figure is a rectangle, nor is anything said about the exact ratio of AΔ to AB.

¹³ On this section of the argument and on the problematic but crucial passage at 848b35-849a6, where the method of analyzing motion into its components is first applied to the circle, see the contribution of J. de Groot to this volume.

(*braduteron*), as though drawn back in the opposite direction, toward the middle. (849a6-14)

Note that there is no claim here of a quantitatively precise relationship between force and speed. The idea is simply that, if two things are impelled with the same force and one is deflected or turned aside more than the other, the one that is deflected more will move more slowly (i.e. take more time to cover the same distance).¹⁴ Evidently force or *ischus* is associated with the speed of movement: being moved “by the same force” (*ischus*) should result in the same speed, but it does not because of the greater deflection of the point closer to the center. At this point (849a14-19) the author identifies the component of circular motion directed towards the center as “unnatural” (*para phusin*) and that which is at right angles to it as “natural” (*kata phusin*); the point of this identification seems to be that the component of motion towards the center is the result of constraint.¹⁵

In the third and final part of the argument, which runs from 849a19 to 849b19, the author applies the deflection principle to the case of circular motion by arguing the point closer to the center is deflected more than the one that is farther out. He does this by arguing that the amount of “unnatural” or *para phusin* motion is smaller in the larger circle, for a given amount of “natural” motion. In figure 2, BY and XZ represent the “unnatural” components of motion of the larger and smaller circles for the same “natural” motion

¹⁴ Cf. 851a16-17: “the same magnitude moved by the same force (*tēi autēi ischui*) advances farther in air than in water.”

¹⁵ Many commentators (e.g. Krafft, *Betrachtungsweise*, 33; De Gandt, “Force,” 122) understand *kata phusin* here to refer to a heavy body's motion towards its natural place, i.e. downwards. On this interpretation the diagram of figure 2 should be rotated 90 degrees clockwise. This is supported by the fact that problem 1 concerns the motion of the arms of a balance. However, it is clear from the sequel that the author understands this analysis of circular motion to be a very general one that is applicable to circles in any orientation, such as the potter's wheel which turns in a plane parallel to the ground (pr. 8, 851b19-21; cf. 852a1-13). It therefore seems more likely that the *kata phusin/para phusin* contrast here should be understood as a contrast between free and constrained motion: an object moving in a circle would fly off in a straight line, but for the constraint imposed by the deflection (*ekkroutis*) towards the center.

($\Omega Y = \Theta Z$). But $BY < XZ$, since ΩY and ΘZ are equal, and "equal straight lines inscribed in unequal circles at right angles to the diameter cut off a smaller segment of the diameter in the greater circles" (849a35-38).¹⁶ Thus, point B is deflected less than point X; by the deflection principle, it travels more quickly. It is only at this point (849b1) that time enters the argument: in the time that X moves to Θ on the smaller circle, B moves to a point *beyond* Ω on the larger circle (since its motion is quicker).¹⁷ But how far does it go? Here the assumption that the two points lie along a single radius is crucial. Because we have two points (X and B) that are fixed on the same radius, at the end of the motion, the ratio between the "unnatural" and "natural" components must be the same for both points. And this condition is satisfied only when point B has moved to H (for $BK:KH = XZ:Z\Theta$).¹⁸

To recap: in part 1 the author argues that circular motion, like rectilinear motion, can be analyzed into two components; in part 2 he states the principle that if two things are moved by the same force, the one that is more deflected will move more slowly; and in part 3 he shows that this principle applies to the case of circular motion by demonstrating that the deflection is smaller in the larger circle. It is crucial to see, first, that this last claim depends on a comparison of motions completed in *different* times: when X has moved to Θ , B will be at H; but when B has completed as much "natural" movement as X, it will have completed less "unnatural" movement. Second, it is the assumption of proportionality, grounded in the geometry of the circle, which determines where the points end up when the motion is complete. The deflection

¹⁶ This is stated without proof. For a proof based on Euclid 3.31 and 6.8, see T.L. Heath, *Mathematics in Aristotle* (Oxford, 1949), 233.

¹⁷ The introduction of time at this point is strongly marked (848a38-849b4): "So in the amount of time (*en hosōi de chronōi*) that A Θ traverses X Θ , in so much time (*en tosoutōi chronōi*) the end of BA has traversed a greater (distance) in the greater circle than B Ω . For the natural motion is equal, while the unnatural motion is less: BY is less than ZX."

¹⁸ The importance of the proportionality requirement is strongly emphasized in this part of the argument; cf. 849b4-6: "it must be proportional (*analogon*): as the natural [motion] is to the natural, so the unnatural is to the unnatural."

principle says only that greater deflection yields a slower speed; it does not establish a precise relationship between forces and distances, and so cannot explain why there is proportionality between the natural and unnatural components when the motion is complete. What the deflection principle is supposed to explain is the surprising fact that although the same force is applied to the points closer and farther from the center, the closer point moves more slowly. But the application of the principle to circular motion is based purely on geometrical considerations. The analysis of motion presented in part 1 gives the author a way of comparing the motion of the larger and smaller circles; using this method, in part 3 he shows that the point on the smaller circle is more deflected in the way I have indicated. The upshot of the whole argument is that it is the geometry of circular motion that explains the different effects of the same force exerted at different distances from the center. It is therefore a mistake to suppose that the author is trying to explain the geometrical features of circular motion by reference to forces and their effects. In this argument the effect of a force is the *explanandum*, not the *explanans*.¹⁹

One consequence of this is that the concept of force or *ischus* that is relevant here is not a highly theoretical one, as commentators such as Duhem have claimed.²⁰ *Ischus* should be thought of

¹⁹ Another way of putting the point is that the kinematics of the situation explains the dynamics, rather than vice versa; here I am in fundamental agreement with the contribution of J. de Groot in this volume. Note that there is no reference in the *Mechanica* to a "force" from the center, except in a metaphorical sense (*hōsper antisphōmenon* 849a13).

²⁰ Following Duhem, Krafft has proposed a quite different interpretation of the claim that the points closer and farther from the center are moved "by the same force." According to Krafft (*Betrachtungsweise*, 35-36), this corresponds to a situation on a balance beam where a smaller weight is attached at a greater distance from the center and a larger weight closer in, in such a way that the ratio of the weights is the inverse of that of their distances from the center. This is based on an application of the relationship stated in *Physics* VII 5 (249b27-250a4): if a given power (*dunamis*) can move a given weight over a given distance in a given time, it will move half the weight over twice the distance in the same time. As the balance beam turns, the smaller weight covers a proportionally greater distance in the same time; hence it is moved "by the same force" as the larger weight closer in. This amounts to understanding *ischus* as the

simply as a push or effort that is applied to a mechanical device. What is surprising is that the same "push" produces a quicker movement if it is applied farther from the center. As noted above, the author associates *ischus* with speed; thus the greater speed of the point on the larger circle is responsible for the greater effect produced by a force acting farther from the center.²¹ But *ischus* itself still refers to the input force that moves the radius of the circle. Here it is relevant to note that the term *ischus* has connotations of physical strength, while the other principal term for force used in Aristotelian contexts, *dunamis*, refers primarily to a capacity or ability (it is derived from the verb *dunamai*, "to be able"). The emphasis on input force as effort is understandable given the author's concern to explain practitioners' knowledge. For what is remarkable about mechanical devices from the point of view of practical experience is the fact that the same input yields a greater effect.

In keeping with the non-technical role of *ischus* in the argument of problem 1, the author frequently substitutes *baros* or "weight" for *ischus* in applying the circular motion principle. This occurs already in problem 1, where the principle is applied to the original question about the balance:

Accordingly, the reason (*aitia*) why, from the same force (*apo tēs autēs ischuos*), the point more distant from the center travels more quickly than the nearer point, and the greater line traces out the greater circle, is clear from what has been said. And why larger balances are more accurate than smaller ones is evident from the following. For the cord becomes a center (for this is stationary), and the parts on

"effective weight" exerted by a given *baros* at a certain distance from the center (cf. De Gandt, "Force," 116-118). On this interpretation, it is clear how a small weight at a large distance from the center can balance a large weight at a short distance: for the *ischus* exerted by both is the same. Yet this is not the argument that the author actually makes, either here or in the analysis of the lever (see section 5 below). The argument of problem 1 is based not on the principle of *Physics* VII, but on the deflection principle. And the *ischus* is not the *result* of the swifter motion of the larger circle, but the cause or source of its motion.

²¹ Cf. pr. 17, 853a24-25 (the wedge "is stronger (*ischuei pleon*) because of its speed"); pr. 19, 853b18-22 (the effectiveness of the axe is due to the speed of its motion); pr. 22, 854a31-34 (the nutcracker, since its action does not involve blows, loses much of the *ischus* due to movement).

each side of the beam become the radii. So (*oun*) from the same weight (*apo tou autou barous*), the extremity of the beam must be moved more quickly, the more distant it is from the cord; and some weights are not clear to sense perception when placed on small balances, while in large balances they are clear. (849b19-28)

The author identifies the cord of the balance with the center of the circle, and the parts on each side with the radii. Then he applies the circular motion principle, but with the substitution of weight (*baros*) for force (*ischus*): from the same weight (*apo tou autou barous*), a point farther from the center will be moved more quickly. The author imagines the same weight being placed on two balances, one of which has longer arms; the magnitude of the swing produced in the larger balance is larger because of the circular motion principle. Evidently he is not concerned to distinguish being moved "by the same force" and "by the same weight." In problem 3 the greater radius is said to be moved more quickly *by an equal weight* (*hupo tou isou barous*, 850a36-7; see section 5 below). Recall that the remarkable fact about the lever mentioned in the introduction (847a28-b15) is that it makes it possible to move a large weight (*baros*) by a small force (*hupo mikras ischuos*). Since problem 3 explains how a large weight (*baros*) can be moved by a small *weight*, the implication is that a small weight is associated with a small *ischus*. Finally problem 9 suggests a similar correspondence between *ischus* and *baros*, with a direct reference back to problem 1:

Why is it that we move things that are raised and drawn by means of greater circles more easily (*rhaon*) and more quickly (*thallon*)? For instance, larger pulleys rather than smaller ones, and similarly for rollers. Is it because the longer the radius, in an equal time it is moved over more space, so that it will also do the same thing if an equal weight (*tou isou barous*) is on it, just as we also said that larger balances are more accurate than smaller ones. (852a14-20)

The remaining passages in which the circular motion principle is invoked are summarized in the following table:

Pr.	Ref.	Text
8	851b36-38	"for greater (circles) are moved, and move weights, more quickly (<i>thallon</i>) by an equal force (<i>hupo tês isês ischuos</i>)"
12	852b8-9	"the greater the radius, it is moved more quickly (<i>thallon</i>)"
13	852b15-18	"the (radii) of larger circles are moved more quickly (<i>thallon</i>), and more, from the same force (<i>apo tês autês ischuos</i>), than those of smaller circles; for by the same force (<i>hupo tês autês ischuos</i>), the extremity farther from the center changes position more quickly (<i>thallon</i>)"
14	852b27-28	"the farther it is from the center, everything is moved more easily (<i>rhaon</i>)"
15	852b33-34	"the greater radius always traces out a larger circle from an equal motion (<i>apo tês isês kinêseôs</i>)"
27	857a31-32	"the greater the (distance) from the center ... it changes position more"

The author consistently refers to "the same" or an "equal" *ischus* applied at different distances from the center; *ischus* is consistently used of input force or effort. Of the nineteen instances of the term, fourteen are expressions of agency (expressed with the prepositions *apo* or *hupo*, or the dative of agent). We may briefly consider two examples. (1) Problem 13 asks (852b11-13): "Why are larger handles more easy to move around a spindle than smaller ones, and in the same way thinner windlasses are more easily moved by the same force (*hupo tês autês ischuos*) than thicker ones?" The author applies the circle model and then goes on to state the circular motion principle at 852b15-18, as summarized in the table above. Here the point is that the same effort applied to two different devices (windlasses with longer or shorter arms) has different effects.²² (2) Problem 18 asks why it is that the use of a compound pulley enables a large weight to be raised "even if the pulling force (*hê helkousa ischuos*) is small (853a37)." The author continues (853a37-39): "Is it because the same weight is moved from a smaller force (*apo elat-tonos ischuos*) if the lever is used than from the hand?" Again we

²² Evidently the thinner windlasses have longer arms.

have a comparison between the input force exerted in two different situations, in this case with and without mechanical aid.²³

5. The Lever

I begin with a translation of the key passage of problem 3 (850a30-b6):

(a) Why do small powers (*dunameis*) move great weights (*barê*) by means of the lever, as was also said at the beginning, even though the weight of the lever is taken in addition? It is easier to move a smaller weight, and it is smaller without the lever. (b) Is it because the lever is the reason, since it is a balance (*zugon*) with the cord (*spartion*) attached below and divided into two unequal parts? For the fulcrum (*hupomochlion*) is the cord (*spartion*): for both of these remain stationary, like the center (*kentron*). (c) And since the greater radius is moved more quickly by an equal weight (*hupo tou isou barous*); and there are three things involved in the lever, the fulcrum (cord and center), and two weights, the mover (*ho kinôn*) and the moved; so (*oun*) as the weight moved is to that which moves it, the length is to the length, inversely. (d) And it will always cause movement (*kinêsei*) more easily (*rhaon*), the more distant it is from the fulcrum. (e) The reason is the one stated before, that the line that extends more from the center traces out a greater circle, so that from the same force (*apo tês autês ischuos*) the mover (*to kinoun*) that is farther from the fulcrum will change position more.

²³ If any term is a candidate for expressing the idea of "effective weight" in the *Mechanica* (cf. n. 20 above) it is *rhôpê* ("inclination"), not *ischus*. Thus at the end of problem 1 the author writes that "the magnitude of the inclination (*rhôpê*) produced by the same weight (*hupo tou autou barous*) is much larger in larger balances" (849b32-34). Here *rhôpê* refers to the visible "swing" of the balance, its deviation from equilibrium. The terms *rhôpê* and the associated verb *rhepein*, "to incline," are closely connected with the balance in Greek literature of the fifth and fourth centuries BC (cf. LSJ s.v.). In Aristotle *rhôpê* refers to the downward tendency possessed by all heavy bodies (e.g. *De caelo* 297a28). In the *Mechanica* the term refers either to a disturbance of the balance from a state of equilibrium (as in pr. 1), to something that produces such a disturbance (at 850a13-16 it is used a small weight added to one side of the balance), or to a downward tendency as in Aristotle (858a15, cf. 858a21). There is no evidence in the *Mechanica* for the idea that equilibrium is produced by equal *rhôpê* on either side of the balance, as in Archimedes. Nor is *rhôpê* consistently used of the effect produced by a weight on the balance: recall that mechanical problems are situations in which "things with a small *rhôpê* move great weights (*barê*)" (847a22-23), implying that a small *baros* has a small *rhôpê*.

The overall structure is the author's usual one of question (a), model identification (b), and explanation (c-e). Note first that the question (a) is phrased in terms of small "powers" (*dunameis*) moving large "weights" (*barê*); *dunamis* thus takes the place of *ischus* in the original statement of the lever's operation at 847a28-b15 ("at the beginning"). In (b) the lever is identified with a balance having unequal arms and supported from below.²⁴ In (c) we have a statement of the circular motion principle formulated in terms of weight (*baros*), an analysis of the lever as consisting of fulcrum and two weights (mover and moved), and a claim concerning the relationship of weights and distances: "as the weight moved is to that which moves it, the length is to the length, inversely." The "mover" slot of the lever model is here filled by a weight (*baros*). In (d) the author claims that the movement will be easier, the farther the mover is from the fulcrum. Finally, (e) cites the circular motion principle as the explanation: the same mover placed farther from the fulcrum will change position more "from the same force" (*apo tês autês ischuos*), and this is why it causes motion more easily. The substitution of *ischus* for *baros* in (c) is striking, and confirms the analysis of section 4 above.

Despite the author's claim to reduce the lever to the balance, the references to the "center" in both (b) and (c) signal the circle model, and it is in fact the circular motion principle that supplies the explanation (c, e). The notion of the equilibrium of forces on either side of the fulcrum plays no role in the argument. Nor is it clear that the statement of the relationship between weights and distances is meant to express an exact proportionality. The idea may just be that any change in the relationship of the weights is compensated by the inverse change in that of the distances: for example, if one of the weights is diminished, its distance from the fulcrum must be increased.²⁵ It is also not entirely clear whether the relationship between weights and distances is supposed to be an inference from

²⁴ Problem 2 distinguishes between the cases where the balance is suspended from above the beam and from below.

²⁵ The author uses the term *antipeponthen* (850b2), which expresses the idea of reciprocal proportion in Euclid. But the Euclidean way of putting the point would be to say that the weights "are reciprocally proportional" (*antipeponthasi*) to the distances

the circular motion principle as stated in the first part of (c), or a statement of an additional fact about the lever parallel to the remark that it involves a fulcrum and two weights. That it is a consequence of the circular motion principle is suggested by the particle *oun*, which frequently has an inferential sense (e.g. 849b24, 851a21, 851b4, 854a24, 854a29). But no argument for this is given. Insofar as there is an argument here it is to support claim (d), that "it (the mover) will always cause movement (*kinêsei*) more easily (*rhaon*), the more distant it is from the fulcrum." This is because, as is said in (e), the same mover placed farther from the fulcrum will change position (*metastêsetai*) more. So again we have the circular motion principle applied to explain the greater ease of movement produced by increasing the distance from the center. The same force, applied farther from the center, has a greater effect because of its quicker movement.

This emphasis on the effect of increasing the separation from the fulcrum is apparent in the citations of the lever principle throughout the text:

Pr.	Ref.	Text
4	850b14-15	"the mover of the weight always moves (<i>kinei</i>) more weight, the more distant it is from the fulcrum"
6	851b2-4	"the farther the fulcrum, the same power (<i>hê autê dunamis</i>) moves the same weight more easily (<i>rhaon</i>) and more quickly (<i>thattōn</i>)"
16	853a11-12	"the longer the (distance) from the fulcrum, the more it must be bent"
20	854a12-13	"the longer the length of the lever from the fulcrum, the more easily (<i>rhaon</i>) ... it causes motion (<i>kinei</i>)"
22	854b7-8	"the more the lever extends from the fulcrum, it moves (<i>kinei</i>) more easily (<i>rhaon</i>), and more, from the same force (<i>apo tês ischuos tês autês</i>)"
29	857b15-16	"the more distant (the mover) is from the weight, the more easily (<i>rhaon</i>) it causes motion (<i>kinei</i>)"

(cf. Eucl. 11.34, 12.9, 12.15), not (as here, more literally) "what the weight suffers in relation to the weight, the length suffers in relation to the length, inversely."

Each of these statements explains the consequences of increasing the distance between the mover and the fulcrum. Doing so increases the ease of movement (3d, 6, 20, 22, 29), the speed of movement (6), or the amount of movement (3e, 22; cf. 16). The mover remains the same while its distance from the center is increased. The author evidently understands the quicker or greater movement of the *weight* to follow from the quicker or greater movement of the *mover*. Only in problem 4 is there any reference to the fact that the mover can move more weight if its distance from the fulcrum is increased.²⁶ Finally we should note that the correlations between distance from the fulcrum and effect are expressed not as exact ratios but rather using vaguer locutions such as "the more ... the more" (e.g. *hosôî ... tosoutôî* in the Greek), a further indication that establishing an exact proportion between weights and distances is not the author's concern.

The author is clearly aware of some sort of reciprocal relationship between weights and distances in the lever. And the fact that he attempts to reduce the lever to the balance suggests that he recognizes the balance with unequal arms as an illustration of this relationship. But it is really *only* an illustration. The analysis of the lever is based entirely on the circular motion principle, and the main point that is established in problem 3 and applied throughout the text is that the *same* input (whether force, power, or weight) will have a greater effect, the farther it is from the fulcrum.

6. Conclusion

There is much more of interest in this short text. But I hope that the analysis I have presented, however partial, at least points the way to a clearer picture of its place in the history of mechanics. If my analysis is correct, then it is a mistake to look to the *Mechanica* for an application of the principles relating force and movement in the *Physics* or for the development of innovative concepts of force. But the author succeeds remarkably well in devising an

²⁶ However, 850b14-15 might also be translated "the mover of the weight always moves (*kineî*) the weight more, the more distant it is from the fulcrum."

explanatory framework to address his main challenge: the ability of technological devices to contravene what seemed to be the obvious notion that one needs a large force to move a large weight. The three simple models of circle, balance, and lever supply the author with the conceptual tools to analyze a wide variety of mechanical devices. The flexibility in his use of terms such as "force," "weight," and "power" is matched by standardization in the terminology for the model slots, indicating the crucial importance of the latter. Despite its importance as an illustration of the ability of a small force to balance a large weight, the balance plays a relatively unimportant role in the author's reduction program. This reflects the fact that his argument for the circular motion principle begins not from considerations of equilibrium, but rather from an innovative analysis of circular motion. It is only after Archimedes that the exact proportionality between weights and distances on the balance beam in a state of equilibrium comes to be the core of theoretical mechanical knowledge.