

## Internet Appendix for “A Model of Shadow Banking”\*\*

At  $t = 0$ , a generic intermediary  $j$  solves the optimization problem:

$$\begin{aligned} \max_{(D, I_H, I_L, S_H, S_L, T_H, T_L)} \quad & [R \cdot (I_H + T_H - S_H) + p_H(S_H - T_H)] \\ & + [E_\omega(\pi_\omega) \cdot A \cdot (I_L - S_L) + E_\omega(\pi_\omega) \cdot A \cdot T_L + p_L(S_L - T_L)] \\ & + D - I_H - I_L + w_{int} - rD, \end{aligned} \quad (\text{A.1})$$

Subject to:

$$w_{int} + D - I_H - I_L - p_H(T_H - S_H) - p_L(T_L - S_L) \geq 0, \quad (\text{A.2})$$

$$R \cdot (I_H + T_H - S_H) + \pi_r \cdot A \cdot T_L - rD \geq 0, \quad (\text{A.3})$$

$$I_H - S_H \geq 0, \quad (\text{A.4})$$

$$I_L - S_L \geq 0, \quad (\text{A.5})$$

where we drop subscript  $j$  for ease of notation. Denote by  $\mu$  the multiplier attached to the resource constraint (A.2), by  $\gamma$  the multiplier attached to the riskless debt constraint (A.3), by  $\theta_H$  and  $\theta_L$  the multipliers attached to the securitization constraints (A.4) and (A.5), respectively. We also denote by  $\nu$  the multiplier attached to the aggregate constraint  $1 - \int_j I_{H,j} dj \geq 0$ , which must be considered by the intermediary when investing the last unit of H.

The first derivatives of the Lagrangian with respect to the choice variables are then equal to:

$$I_H : \quad R - 1 - \mu + \gamma R - \nu + \theta_H, \quad (\text{A.6})$$

$$T_H : \quad R - p_H - \mu p_H + \gamma R, \quad (\text{A.7})$$

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$$S_H : \quad -R + p_H + \mu p_H - \gamma R - \theta_H, \quad (\text{A.8})$$

$$D : \quad 1 - r + \mu - \gamma r, \quad (\text{A.9})$$

$$I_L : \quad E_\omega(\pi_\omega) \cdot A - 1 - \mu + \theta_L, \quad (\text{A.10})$$

$$T_L : \quad E_\omega(\pi_\omega) \cdot A - p_L - \mu p_L + \gamma \pi_r \cdot A, \quad (\text{A.11})$$

$$S_L : \quad -E_\omega(\pi_\omega) \cdot A + p_L + \mu p_L - \theta_L. \quad (\text{A.12})$$

Together with investor optimization, (A.2) to (A.12) yield the model's equilibrium. The conditions determining investors' optimal consumption-saving problem are easy. Given investors' preferences [Equation (1)], the marginal benefit for an investor  $i$  of lending  $D_i$ , purchasing  $T_{H,i}$  riskless projects and  $T_{L,i}$  pools of risky projects are respectively equal to:

$$D_i : \quad -1 + r, \quad (\text{A.13})$$

$$T_{H,i} : \quad -p_H + R, \quad (\text{A.14})$$

$$T_{L,i} : \quad -p_L + \pi_r \cdot A, \quad (\text{A.15})$$

Consider now what (A.2) to (A.15) imply for the model's equilibrium. First, note that (A.9) and (A.13) imply that in equilibrium  $r \geq 1$ , otherwise no investor is willing to lend. Thus, the only feasible lending pattern is for investors to lend resources to intermediaries who have productive projects and can therefore afford to pay  $r \geq 1$ .

**Proof of Lemma 1** Consider first how intermediaries optimally finance a riskless investment  $I_H > 0$  using borrowing and securitization. With respect to capital suppliers, investors (or lending intermediaries) prefer securitization  $T_{H,i}$  when it yields a higher return than bonds  $D_i$ , i.e. when  $R/p_H > r$ . The reverse occurs when  $R/p_H < r$ . When  $R/p_H = r$ , capital suppliers are indifferent. On the demand side, if  $R/p_H > r$ , then by (A.8) and (A.9) intermediaries prefer debt  $D_j$  to securitization  $S_{H,j}$ . If  $R/p_H < r$ , the reverse is true. This implies that in equilibrium:

$$R/p_H = r, \quad (\text{A.16})$$

namely bonds and securitization should yield the same return. From Equations (A.8) and (A.9) one can also see that when (A.16) holds, the shadow cost of securitizing riskless projects is weakly higher than that of issuing bonds because  $\theta_H \geq 0$ . We thus focus on equilibria where riskless projects are not securitized, namely  $S_H = T_H = 0$  (and  $\theta_H = 0$ ).

Next, consider the securitization of risky projects. Suppose that intermediaries engage in risky investment  $I_L > 0$  and securitize  $S_L > 0$  of it. Investors buy securitized claims if they yield them more than riskless bonds, i.e. if  $\pi_r A/p_L \geq r = R/p_H$ . By plugging this condition into Equation (A.11) and by using (A.9), one finds that if investors demand securitized risky claims  $T_{L,i}$ , then intermediaries demand an infinite amount of them, which cannot occur in equilibrium. Formally, if  $\pi_r A/p_L \geq r = R/p_H$  the benefit of increasing  $T_{L,j}$  is positive, because it is larger than that of increasing  $T_{H,j}$  (and the latter benefit must be equal to zero, for riskless projects to be securitized). But then, in equilibrium it must be that  $\pi_r A/p_L < r$  and the available securitized risky claims are traded among intermediaries, namely  $T_{L,j} = S_{L,j}$ . Equations (A.11) and (A.12) show that starting from a no securitization situation (i.e.  $\theta_L = 0$ ), purchasing securitized projects is strictly beneficial (and so  $T_{L,j} = S_{L,j} > 0$ ) if the debt constraint (A.3) is binding, namely when  $\gamma > 0$ .

**Proof of Proposition 1** Since  $R \geq E_\omega(\pi_\omega) \cdot A$ , investment in  $H$  is preferred to that in  $L$  if  $H$  is available, i.e. when  $v = 0$  in (A.6). In this case, the marginal benefit of  $I_H$  in (A.6) is larger than that of  $I_L$  in (A.10) provided:

$$R - E_\omega(\pi_\omega) \cdot A \geq \theta_L - \theta_H - \gamma R. \quad (\text{A.17})$$

Since riskless projects are not securitized ( $\theta_H = 0$ ), Equation (A.17) is satisfied if  $\gamma R \geq \theta_L$ , i.e. if the riskless project boosts leverage more than securitization. This is true if securitization does not occur (i.e.  $\theta_L = 0$ ) but also if it does. In the latter case, the fact that risky projects are only traded among intermediaries, i.e.  $T_{L,j} = S_{L,j}$ , calls for (A. 11) to be equal to minus (A.12). This requires  $\gamma \pi_r A = \theta_L$  and thus implies  $\gamma R \geq \theta_L$ . Hence, intermediaries invest in  $H$  until investment in such activity is equal to 1. Beyond that limit, intermediaries invest also in  $L$ .

Consider the equilibrium if  $w_{int} + w \leq 1$  (case a)). Here  $v = 0$ , all wealth goes to finance  $H$ , the riskless debt constraint is not binding ( $\gamma = 0$ ) since  $H$  is self-financing. By plugging  $r = 1 + \mu$  from (A.9) into (A.6), we find that  $r = R$ . Thus, equilibrium prices are ( $r = R, p_H = 1, p_L$ ) where  $\pi_r A/R \leq p_L \leq 1$  and quantities are ( $D = w, I_H = w_{int} + w, I_L = 0, S_H = S_L = T_H = T_L = 0$ ). Investors lend their wealth at  $t = 0$  by purchasing riskless bonds that promise  $R$  at  $t = 2$ . No lending or trading occurs at  $t = 1$ , because after  $\omega$  is learned investors and intermediaries have the same preferences and value all assets equally. The consumption patterns of intermediaries is  $C_0 = C_1 = 0, C_2 = R w_{int}$ , that of investors is  $C_0 = 0, C_1 = 0, C_2 = R w$ .

Consider the equilibrium when  $w_{int} + w > 1$ . Now activity  $H$  is exhausted, i.e.  $v > 0$ . There are two cases to consider, depending on whether  $E_\omega(\pi_\omega) \cdot A$  is higher or lower than one.

1) If  $E_\omega(\pi_\omega) \cdot A \leq 1$ , then intermediaries do not invest in  $L$ . To see this: by Equation (A.10),  $I_L > 0$  can only be optimal if securitization is valuable, i.e. if  $\theta_L > 0$ . For this to be the case, the resale price of the project must be higher than the investment cost, i.e.  $p_L \geq 1$  by (A.12). But no intermediary is willing to buy at  $p_L \geq 1$ , as the project yields less than 1. Thus, if  $E_\omega(\pi_\omega) \cdot A < 1$  we have  $S_L = T_L = I_L = 0$ . In this equilibrium it must be that  $r = 1$ . Indeed, if  $r > 1$  investors lend all of their wealth  $w$  and intermediaries' budget constraint becomes slack ( $\mu = 0$ ) because  $w_{int} + w > I_H = 1$ . But then equation (A.9) implies  $\gamma < 0$ , which is impossible.

Thus, in equilibrium  $r = 1$  and intermediaries' debt can take any value  $D \in (1 - w_{int}, \min(w, R))$  by the riskless debt constraint (A.3). For a given  $D$ , the consumption of intermediaries is  $C_0 = w_{int} + D - 1$ ,  $C_1 = 0$ ,  $C_2 = R - D$ , that of investors is  $C_0 = w - D$ ,  $C_1 = 0$ ,  $C_2 = D$ . Once more, there is neither trading nor lending at  $t = 1$ .

2) If  $E_\omega(\pi_\omega) \cdot A > 1$ , intermediaries wish to invest in  $L$ . There are three cases.

2.1) If  $w$  is sufficiently low, intermediaries finance  $L$  by using debt with a slack riskless debt constraint, i.e.  $\gamma = 0$ , and without securitization, so that  $\theta_L = 0$  and  $S_L = T_L = 0$ . In this case, (A.9) and (A.10) imply  $r = E_\omega(\pi_\omega) \cdot A > 1$ . As a consequence, investors lend all of their wealth and  $D = w$  and  $I_L = w + w_{int} - 1$ . The riskless debt constraint (A.3) is slack for  $R \geq E_\omega(\pi_\omega) \cdot A \cdot w$ , so this allocation is an equilibrium for  $w \in (1 - w_{int}, R/E_\omega(\pi_\omega) \cdot A]$ .

2.2) If  $w$  increases further, intermediaries start to securitize risky projects, so that  $S_L = T_L > 0$ , but not yet to the full amount of investment, i.e.  $\theta_L = 0$ . In this case because of  $\theta_L = \gamma \pi_r \cdot A$  by (A.11) and (A.12), the debt constraint holds with equality even though there is no shadow cost (i.e.  $\gamma = 0$ ), so that it is still the case that  $r = E_\omega(\pi_\omega) \cdot A > 1$  by (A.9) and (A.10). Investors lend  $w$  to intermediaries and the equilibrium level of securitization  $S_L$  is determined along the debt constraint (A.3) as follows:

$$E_\omega(\pi_\omega) \cdot A \cdot w = R + \pi_r \cdot A \cdot S_L, \quad (\text{A.18})$$

which implicitly identifies the level of securitization  $S_L$  increasing in  $w$ . This allocation constitutes an equilibrium (thus satisfying  $\theta_L = 0$ ) only if  $S_L < I_L = w + w_{int} - 1$ , which corresponds to the condition:

$$w \leq w^* \equiv \frac{R/A + \pi_r (w_{int} - 1)}{E_\omega(\pi_\omega) - \pi_r}. \quad (\text{A.19})$$

Condition (A.19) implies that this configuration is an equilibrium for  $w \in (R/E_\omega(\pi_\omega) \cdot A, w^*]$ .

2.3) If  $w$  increases beyond  $w^*$ , securitization hits the constraint  $S_L = I_L$ , i.e.  $\theta_L > 0$ . The debt constraint is binding, i.e.  $\gamma > 0$ , as in this case  $\gamma\pi_r A = \theta_L$ , and the interest rate drops to  $r < E_\omega(\pi_\omega) \cdot A$  by (A.9) and (A.10). As long as  $r \geq 1$ , investors lend  $w$  to intermediaries and the equilibrium interest rate is implicitly determined along the debt constraint (A.3) as follows:

$$r \cdot w = R + \pi_r \cdot A \cdot (w + w_{int} - 1), \quad (\text{A.20})$$

which implicitly identifies a function  $r(w)$  that monotonically decreases in  $w$  and approaches  $r = \pi_r \cdot A$  as  $w \rightarrow +\infty$ . Due to A.1, since  $\pi_r \cdot A < 1$  there is a threshold level of wealth  $w^{**}$  such that  $r(w^{**}) = 1$ . Obviously then, intermediaries cannot absorb investors' wealth beyond  $w^{**}$ .

Once more, in all equilibria 2.1) – 2.3) nothing happens to lending and trading at  $t = 1$ . It is straightforward to derive agents' consumption patterns.

**Proof of Proposition 2** The construction of the equilibrium is identical to the one discussed in the proof of proposition 1, except the now we replace  $\pi_r$  with  $\pi_d$  in the debt constraint (A.3) and in (A.15). We also replace  $E_\omega(\pi_\omega)$  with  $E_\omega^N(\pi_\omega)$  in the intermediary's objective. Compare now the extent of securitization under neglected risk and under rational expectations. Because securitization linearly increases in investor wealth, we have that  $S_L^N > S_L$  for all  $w$  provided  $w^{*N} < w^*$  because in this case risk-neglecting intermediaries max out securitization for lower values of  $w$ . Since at these values the level of investment is the same under neglected risk and RE,  $I_L = w + w_{int} - 1$ , securitization is higher in the former regime. After some algebra, one can find that  $w^{*N} < w^*$  for all  $w_{int} \leq 1$  if:

$$E_\omega^N(\pi_\omega) - \pi_d > E_\omega(\pi_\omega) - \pi_r \Leftrightarrow \varphi_g \varphi_r (\pi_g - \pi_d) > (\varphi_g + \varphi_d)^2 (\pi_d - \pi_r),$$

which is fulfilled provided the expectational error  $(\pi_d - \pi_r)$  is small. If the condition above is not met, it might be that  $w^{*N} > w^*$ . In such a case, for  $w$  on the left neighbourhood of  $R/E_\omega(\pi_\omega | t = 0) \cdot A$  securitization is higher under neglected risk (there is no securitization under RE yet). For  $w$  above  $w^{**}$ , investment and securitization are also higher under neglected risk. However for  $w$  intermediate securitization might be higher under RE.

Consider now the interest rate. When  $w^{*N} > w^*$ , by the expressions  $r^N(w) = [R + \pi_d \cdot A \cdot (w + w_{int} - 1)]/w$  and  $r(w) = [R + \pi_r \cdot A \cdot (w + w_{int} - 1)]/w$ , it is obvious that  $r^N \geq r$ . But even if  $w^{*N} < w^*$ , it is easy to see that  $r^N > r$ . Consider in fact the interest rate under neglected risk when  $w = w^*$ . Then, if  $r^N < r$  at any wealth level, it must be that  $r^N < r$  also at  $w = w^*$ . It is easy to see that  $(\pi_d - \pi_r) > 0$  and  $1 - w_{int} > 0$  imply that at  $w = w^*$  we have  $r^N > r$ . Thus, the interest rate under neglected risk is never below the interest rate under rational expectations, so that  $r^N \geq r$ .

Finally, consider leverage  $D$ . It is immediate to see that until wealth level  $w^{**}$  leverage and investment are the same under neglected risk and RE (i.e.  $D = w$ ), but that for  $w > w^{**}$  leverage and investment are strictly higher under neglected risk, confirming that  $D^N \geq D$ . Indeed, since  $\pi_r \cdot A < 1$  we have that  $w^{**} = [R - \pi_r \cdot A(1 - w_{int})]/(1 - \pi_r \cdot A)$ , which increases in  $\pi_r$  (because  $R > A$  and  $1 - w_{int} > 0$ ), implying that  $w^{**} < w^{**N}$ .

**Proof of Proposition 3** We again focus on the case where  $E_\omega(\pi_\omega | t = 0) \cdot A > 1$ . From the proof of Proposition 1 we know that for  $w \leq R/E_\omega(\pi_\omega | t = 0) \cdot A$  there is no securitization and thus fragility does not arise [i.e. we are in cases a) and b)]. For  $w \geq w^{*,N} \equiv \frac{R/A + \pi_d(w_{int} - 1)}{E_\omega(\pi_\omega | t = 0) - \pi_d}$  we

know that securitization is maximal, namely  $I_H^N = S_L^N$ . We are in case d), in which intermediaries have no spare resources at  $t = 1$ . In this case, intermediaries cannot buy back any of the debt claims from investors, and in equilibrium  $V_1 = (1-q_l)\pi_r \cdot A \cdot S_{L,j}^N$ , which is investors' reservation value. Plugging in equilibrium values, we find that in this case:

$$V_1 = (1-q_l) \cdot \pi_r \cdot A \cdot (w_{int} + \min(w, w^{*,N}) - 1)$$

where  $w^{*,N} = [R - \pi_d \cdot A \cdot (1 - w_{int})] / (1 - \pi_d \cdot A)$ .

The most interesting case arises when  $w$  lies in  $(R / E_\omega(\pi_\omega | t = 0), w^{*,N})$ . In this range, securitization is pinned down by the riskless debt constraint  $E_\omega(\pi_\omega | t = 0) \cdot A \cdot w = R + \pi_d \cdot A \cdot S_{L,j}^N$ , which implies:

$$I_{L,j}^N - S_{L,j}^N = \frac{R/A}{\pi_d} - (1 - w_{int}) - (\varphi_g + \varphi_r) \left( \frac{\pi_g}{\pi_d} - 1 \right) w,$$

which decreases in investors' wealth, attaining a maximum value in the relevant wealth interval of  $I_{L,j}^N = w_{int} + R / E^N(\pi_\omega | t = 0) - 1$  and reaching a minimum of 0 at  $w^{*,N}$ .

Since the wealth available to early intermediaries at any  $w$  is equal to  $q_l [A \cdot (I_{L,j}^N - S_{L,j}^N) - (\pi_d - \pi_r) A S_{L,j}^N]$ , the equilibrium value of risky debt  $V_1$  as a function of  $(I_{L,j}^N - S_{L,j}^N)$  can be in one of the following configurations. If  $q_l [A \cdot (I_{L,j}^N - S_{L,j}^N) - (\pi_d - \pi_r) A S_{L,j}^N] > (1-q_l) E(\pi_\omega | q_l) \cdot A \cdot S_{L,j}^N$ , case a) of Proposition 3, then the market value of risky debt is equal to intermediaries' reservation value  $(1-q_l) E(\pi_\omega | q_l) \cdot A \cdot S_{L,j}^N$ . If  $q_l [A \cdot (I_{L,j}^N - S_{L,j}^N) - (\pi_d - \pi_r) A S_{L,j}^N] < (1-q_l) \pi_r \cdot A \cdot S_{L,j}^N$ , case c) of Proposition 3, then the market value of risky debt is equal to investors' reservation

value. Otherwise, case b) of Proposition 3, the market value of risky debt is equal to early intermediaries' wealth  $q_I[A \cdot (I_{L,j}^N - S_{L,j}^N) - (\pi_d - \pi_r)A S_{L,j}^N]$ .

These conditions on  $I_{L,j}^N / S_{L,j}^N$  that partition in cases a), b), c), are implicitly conditions on  $w$  because  $I_{L,j}^N / S_{L,j}^N$  monotonically decreases in  $w$ .