# Selling Information Through Consulting

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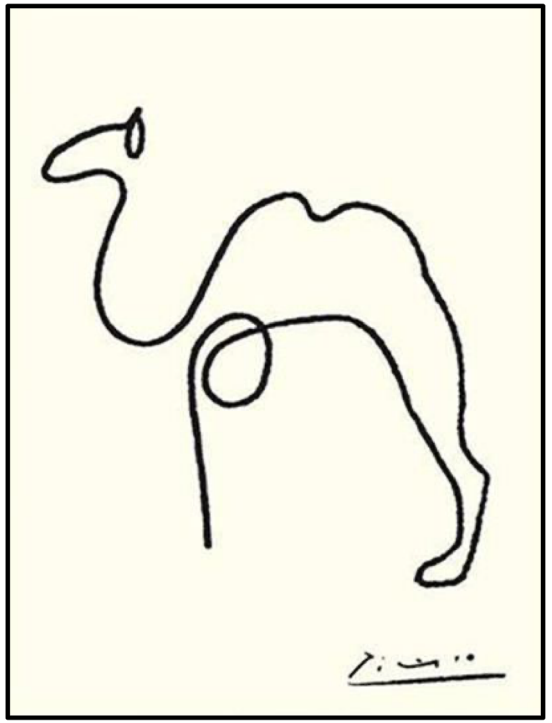
# We know a lot about selling items...



How do we sell a piece of information?

# Market of Information

Consumer Information

Credit Reports

Recommendation

acxiom

nielsen

ORACLE

EQUIFAX

experian

TransUnion

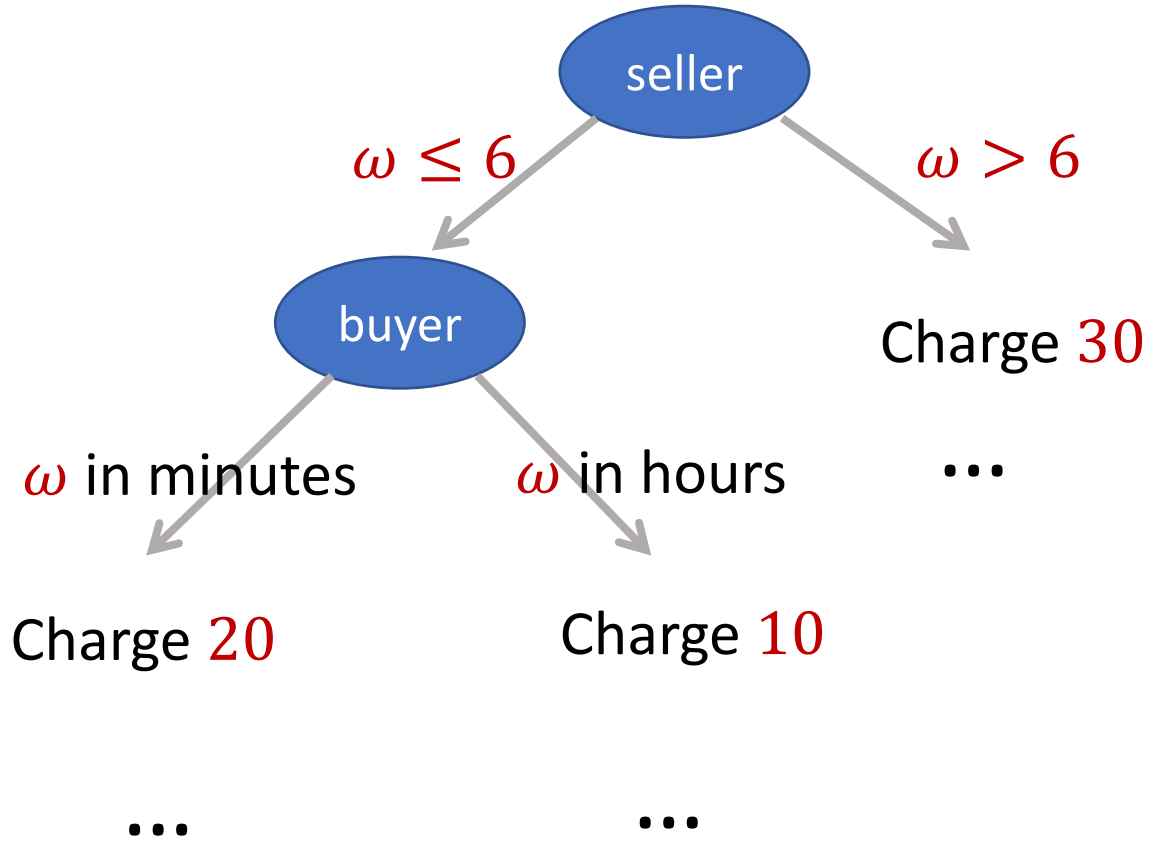
tripadvisor

Waze

# How is information different from items?

- Can be partially revealed
- e.g. FlightAware


flight delay $\omega$



# Outline

Selling information to **budget-constrained** buyers

1. **Motivation**
   - Why budget-constrained buyers?
2. **Our Results**
   - Model
   - Main theorem
   - Proof ideas

# Motivation

- Babaioff et al. [2012]: buyers with unlimited budget

- ***Budget-constrained buyers***

# Motivation

- Suppose Haifeng @Boston is going to give a talk @NYC. Safe if the flight delay $\leq 4$ hrs
- Based on historical data & weather, $0 \leq$delay $\leq 10$
- Wait/take a train instead

| loss | delay $\leq 4$ | delay $> 4$ |
|---|---|---|
| wait | 0 | $+\infty$ |
| take a train | 100 | 100 |

- No information: will take a train
- Knowing the exact delay, take a train only when delay $> 4$
- Save 4/10*100 in expectation
- Willing to pay up to 40

# Motivation

- Suppose FlightAware knows the exact delay and wants to sell it.
- 2 buyers:

| | loss | prior belief |
|---|---|---|
| buyer 1 | 400 | [0, 7.9984] |
| buyer 2 | 600 | [0, 8.0016] |

- Babaioff et al. [2012]: when delay $> 4$ w.p. $0.5$, the optimal mechanism is to
  - charge $\approx 250000$ when delay $\leq 4$,
  - pay $\approx 249600$ to the buyer when delay $> 4$.

- But the buyer's expected gain $\approx 250$.

# Our Results

***Consider budget-constrained buyers***

- Simple optimal mechanism
- Compute via a polynomial-size LP
  - Improves the previous exponential-size LP by Babaioff et al. [2012] that solves one-round optimal mechanism when there is no budget constraint

Adding budget constraint makes the problem easier

# Model: information buyer

- An unknown state of the world, random variable $\omega$
- The buyer needs to choose an action $a \in A$ that leads to utility $u(a, \omega)$
- The buyer has a private type $\theta$ that represents
  1. his belief about $\omega$: a probability distribution over $\Omega$
  2. his utility function $u_\theta(a, \omega)$
- **The buyer has a private budget $b$ that can be used to purchase additional information about $\omega$**

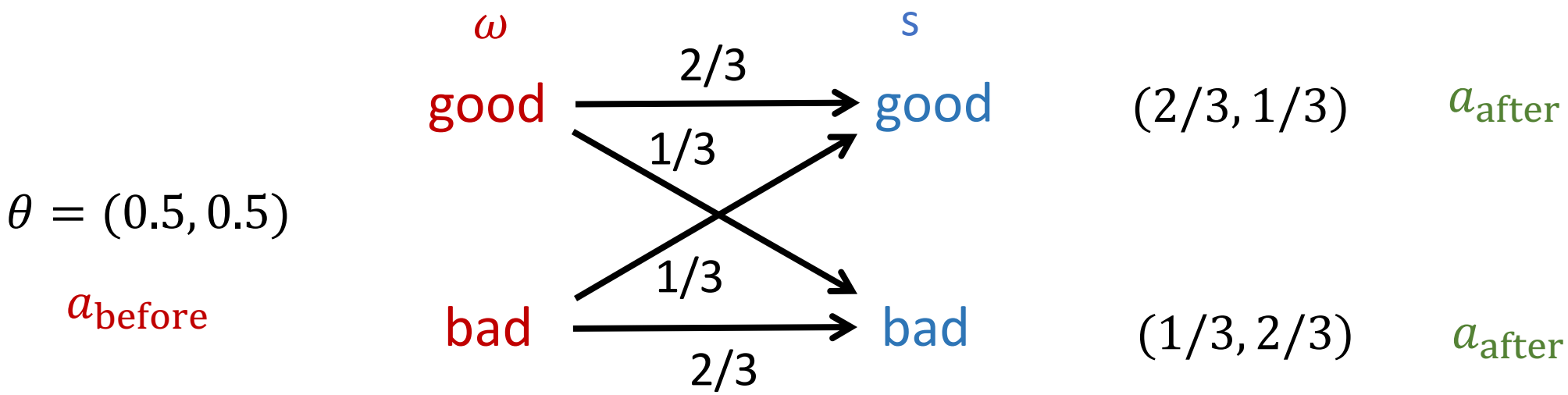
# Model: information seller

- Fully observes $\omega$, has a budget $M$
- Knows the utility function $u_\theta(a, \omega)$ and the distribution $\mu(\theta, b, \omega)$
- Goal: sell $\omega$ to maximize the expected revenue

***What are the mechanisms that can possibly be used?***

# Model: information as signals

- **Partial information**: a random variable (signal) $s$ that is correlated with the state of the world $\omega$
- Example: binary $\omega$ = good/bad, reveals $\omega$ w.p. $2/3$




- **Value of Information:** the gain from knowing $s$

$$V_\theta(s) = \mathrm{E}_\theta[u(a_{\text{after}}, \omega)] - \mathrm{E}_\theta[u(a_{\text{before}}, \omega)]$$

# Model: mechanisms

- A menu of (partial information $s$ , price $p_s$ )
- The seller can interact with the buyer in multiple rounds Babaioff et al. [2012]

- ~~Multiple-round mechanisms~~
- Recommend actions to the buyer and then charge some prices

# Our Contribution

***Consulting Mechanism:***

1. Ask the buyer to report his type $\hat{\theta}$ and deposit his budget $\hat{b}$.
2. ***For each reported $\hat{\theta}$ and $\hat{b}$,*** according to $\omega$, (randomly) decides an action to recommend and an amount of refund, the amount of which is either 0 or $\hat{b} + M$.

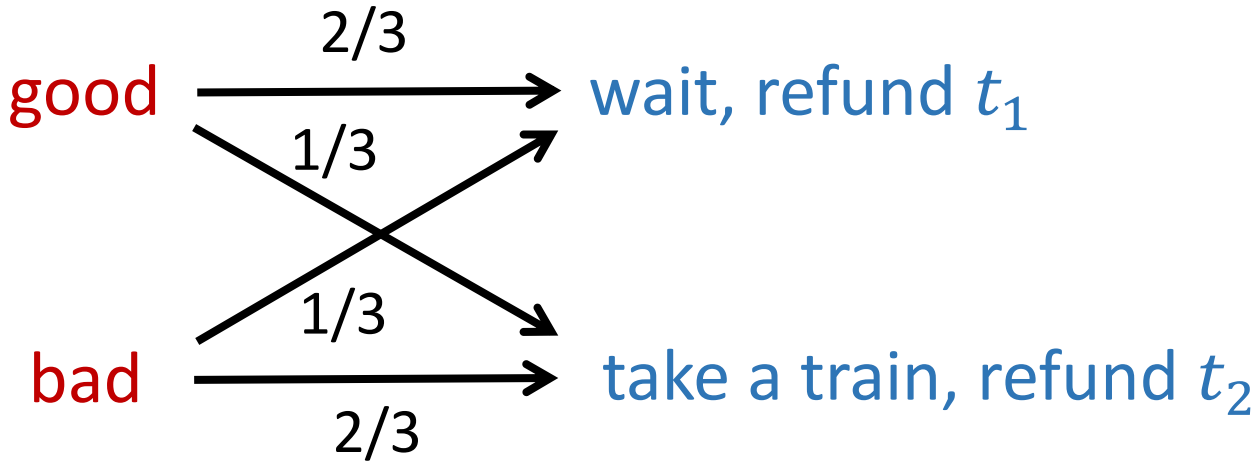
- Buyers with different reported $\hat{\theta}$ and $\hat{b}$ will get different recommendations

# Our Contribution

***Consulting Mechanism:***

1. Ask the buyer to report his type $\hat{\theta}$ and deposit his budget $\hat{b}$.
2. For each reported $\hat{\theta}$ and $\hat{b}$, ***according to $\omega$, (randomly) decides an action to recommend and an amount of refund,*** the amount of which is either 0 or $\hat{b} + M$.

- For each $\hat{\theta}$ and $\hat{b}$:



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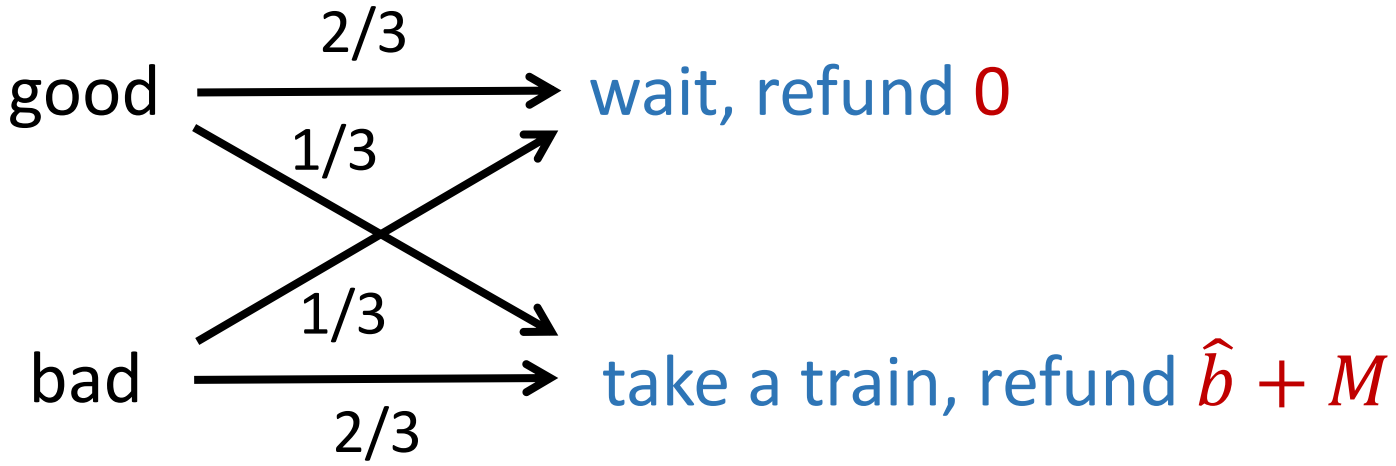
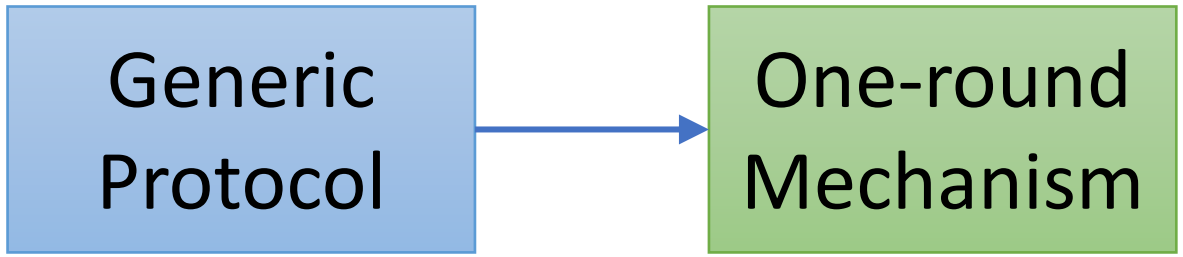


# Our Contribution

***Theorem:*** There always exists an IC and IR Consulting Mechanism

that achieves no less revenue than any (possibly multiple-round) mechanisms.

# Proof ideas




do not know how to compute

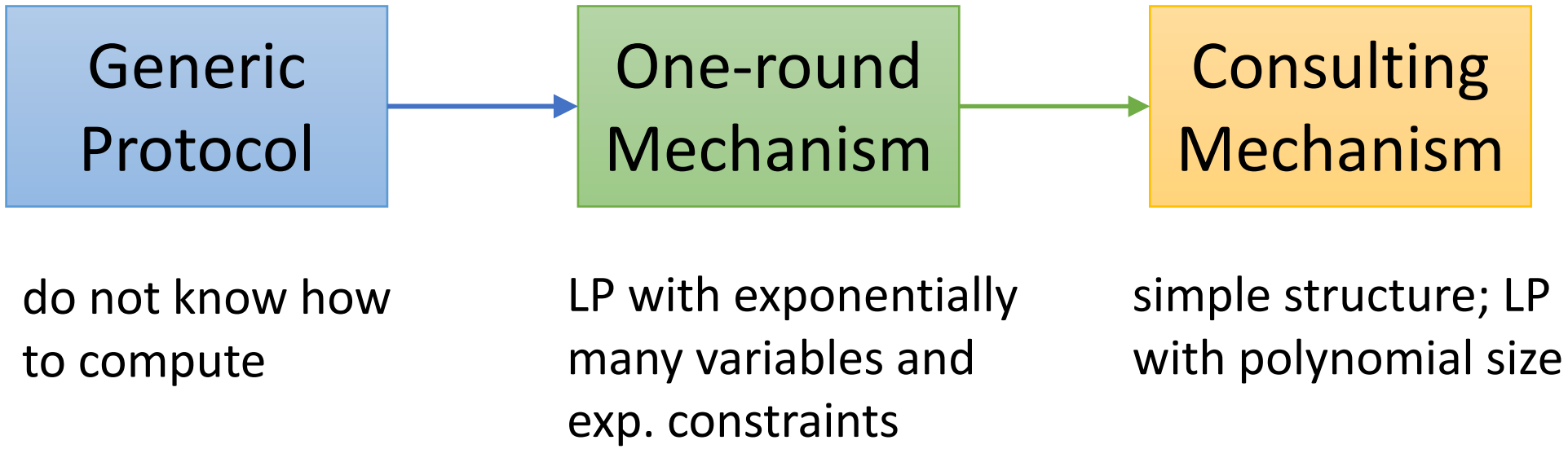
LP with exponentially many variables and exp. constraints

# One-round Mechanism

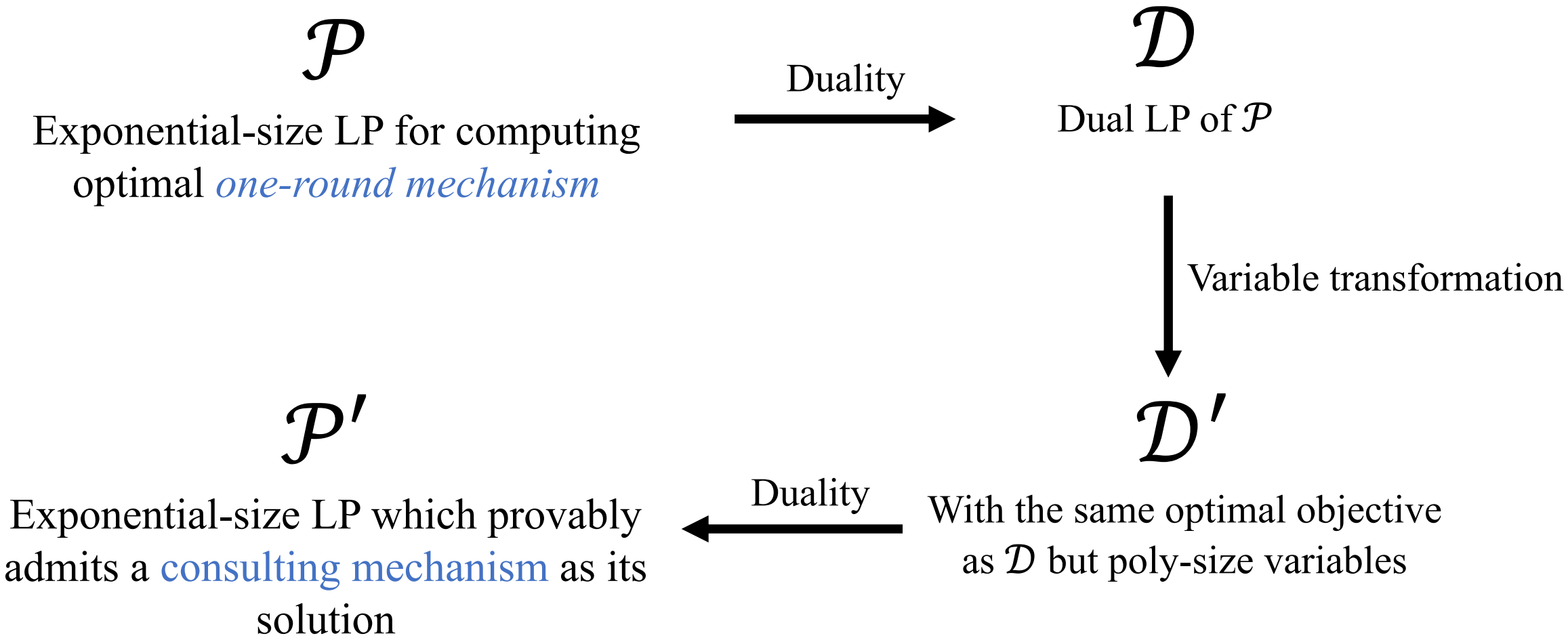
1. Ask the buyer to report his type $\hat{\theta}$ and deposit his budget $\hat{b}$.
2. According to the reported $\hat{\theta}$ and $\hat{b}$, give the buyer a piece of partial information $s$ and refund $t(s)$.

- LP with exponentially many variables and exponentially many constraints

# Proof ideas


Generic Protocol
One-round Mechanism
Consulting Mechanism
do not know how to compute
LP with exponentially many variables and exp. constraints
simple structure; LP with polynomial size

# Proof ideas


$\mathcal{P}$
Exponential-size LP for computing optimal *one-round mechanism*
Duality
$\mathcal{D}$
Dual LP of $\mathcal{P}$
Variable transformation
$\mathcal{D}'$
With the same optimal objective as $\mathcal{D}$ but poly-size variables
Duality
$\mathcal{P}'$
Exponential-size LP which provably admits a consulting mechanism as its solution

# Proof ideas

$\mathcal{P}'$

Exponential-size LP which provably admits a consulting mechanism as its solution

- Two possible payments
- Can be reduced to a consulting mechanism

# Polynomial-size LP

- Variables: for each $\theta \in \Theta$ and $b \in B$ , probability of recommending $a \in A$ and charge one of the two possible payments.
- Objective: expected revenue
- Constraints:
  - Individual Rationality
  - Incentive Compatibility

# Summary

Selling information to budget-constrained buyers

- Simple one-round optimal mechanism: consulting mechanisms
- Compute the optimal mechanism via a polynomial-size LP

# Thanks & Questions?