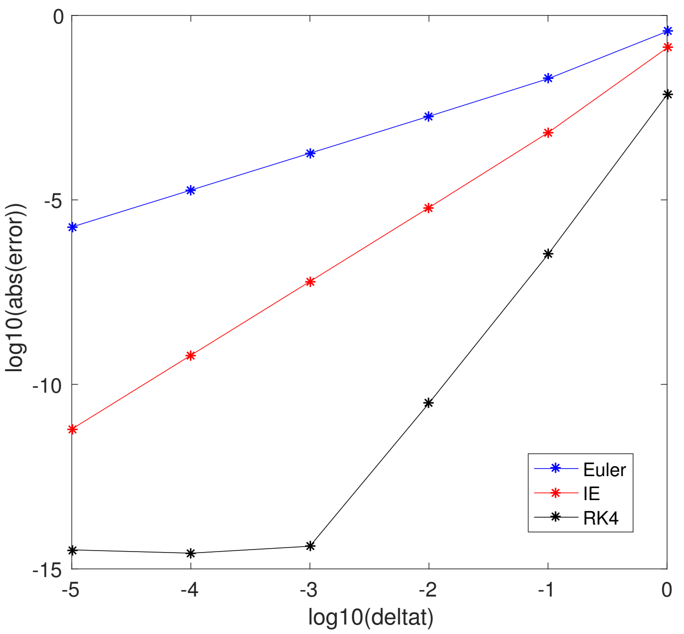
AM108 1:00pm Class 06

Goals for the day

1. Compare the integration error for Euler, Improved Euler and RK4 as a function of stepsize.
2. Interpret a time series from a system with bistability where a parameter is varying in time.
3. (time permitting) Make a concept map of the main ideas we have encountered in 1D systems.

Questions:

1. (2.8.3, 2.8.4, 2.8.5) On HW01 you looked at the run time of the Euler method and how it changed with stepsize. I modified our Matlab code to include improved Euler (IE) and Runge-Kutta 4 (RK4) as well. The plot of $\log_{10} E$ vs $\log_{10} \Delta t$ is shown below for the three methods.



   (a) On the HW, you estimated the slope of the $\log_{10} E$ vs $\log_{10} \Delta t$ points and most of you found a value of around 1. Estimate the slopes of the IE and the RK4 lines, as well. For RK4, leave out the -4 and -5 points when estimating the slope.

   (b) The slopes are for lines $\log_{10} E = m \log_{10} \Delta t + b$, so $E = c\Delta t^m$. For each method, of Δt is reduced by a factor of 10, by what factor does the error go down?

   (c) Given that each of these methods runs in about the same amount of time, which method should you choose to approximate solutions to differential equations?

   (d) For the RK4 line, how might you explain its shape for small Δt?

   Bifurcations on the real line:

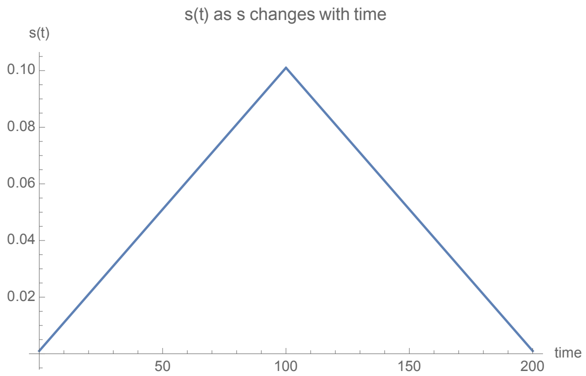
2. On the HW you are working with the nondimensional system

$$\dot{x} = s - rx + \frac{x^2}{1 + x^2}.$$

   This system arises from a model of pattern formation, where a pigment gene is being switched on by the presence of the substance s.

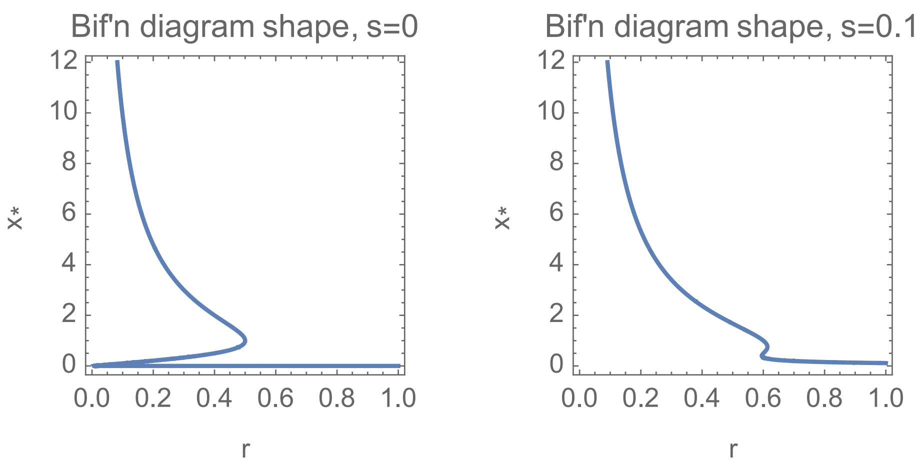
   (a) (This relates to part c of the HW Q) We will have s vary with time, first rising steadily and then falling steadily:

```
g0 = 0; dg = 0.001; changetime = 100;
g[t_] = dg +
   Piecewise[{{dg t,
      t < changetime}, {dg*changetime - dg (t - changetime),
      t > changetime}}];
Plot[g[t], {t, 0, changetime*2}, AxesLabel -> {"time", "s(t)"},
 PlotLabel -> "s(t) as s changes with time"]
```



Here are two contour plots showing the shapes of the bifurcation diagrams at two values of s.

```
f[x_] = s - r x + x^2/(1 + x^2);
p0 = ContourPlot[(f[x] /. s -> 0) == 0, {r, 0, 1}, {x, -0.1, 12},
   PlotLabel -> "Bif'n diagram shape, s=0", FrameLabel -> {"r", "x*"},
    MaxRecursion -> 5];
p1 = ContourPlot[(f[x] /. s -> 0.1) == 0, {r, 0, 1}, {x, -0.1, 12},
   PlotLabel -> "Bif'n diagram shape, s=0.1",
   FrameLabel -> {"r", "x*"}, MaxRecursion -> 5];
GraphicsGrid[{{p0, p1}}]
```



In part c of the HW, you are told that initially $x(0) = 0$, and then s (the activating signal for a genetic switch that produces pigment when it is on) is slowly increased from zero. What happens to $x(t)$? What happens if s goes back to zero?

For this class activity, consider the specific case where $r = 0.4$ and $s(t)$ evolves as given above. Start by finding the stability of the $x = 0$ fixed point that exists when $s = 0$, then reason through how the value of $x(t)$ will change as $s(t)$ changes. Sketch an approximate time series for $x(t)$.

You can modify the contour plot in Mathematica if other bifurcation diagram shapes would be helpful to you.

(b) We will use the build-in integrator in Mathematica to find $x(t)$ given $\dot{x} = s(t) - rx + \frac{x^2}{1+x^2}$, with

$s(t)$ as above. This built-in integrator is called `NDSolve`. Run the portion of the code with the `NDSolve` command to generate a numerical approximation of the time series for $x(t)$.

Compare this to the time series you sketched above.

(c) Next plot the timeseries $x(t)$ and the fixed points of the system as a function of $s(t)$. This code is also already set up for you. Why plot these two things together?

(d) Finally, plot the shape of the bifurcation diagram for $r = 0.4$ in the sx-plane along with the parameterized curve $(s(t), x(t))$ in the sx-plane that comes from the time series. How did $s(t)$ act as a switch for the system? What value of x would you consider "off" and what value would you consider "on"?

3. (Time-permitting) Grab sticky-notes from the front of the room, writing different topics and concepts from the class on sticky notes. Use your whiteboard to organize and connect the sticky-notes to show the way different ideas and definitions fit together.