AM108 1:00pm

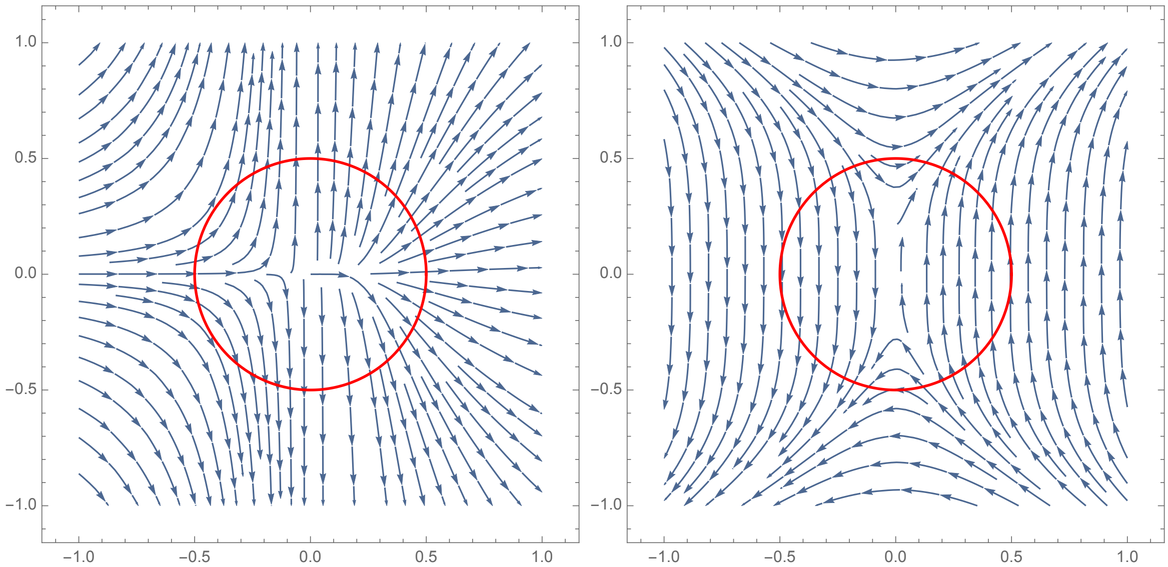
Class 11: index theory in 2D

Goals for the day:

1. Use index theory to restrict possibilities in a phase portrait.
2. Identify the index of unusual fixed points.

Team problems:

1. (6.8.8) A smooth vector field on the phase plane is known to have exactly three closed orbits. Two of the cycles, C_1 and C_2, lie inside the third cycle, C_3. However, C_1 does not lie inside C_2 or vice versa.
   (a) Sketch the arrangement of the three cycles.
   (b) Show there there must be at least one fixed point in the region bounded by C_1, C_2, and C_3. What can you say about its type?
   (c) How does this change, if at all, if C_1 now lies inside of C_2?
2. For each of the following systems, locate the fixed points and calculate the index associated with the fixed point. Explain why the index makes sense.
   (a) (6.8.2) $\dot{x} = x^2, \dot{y} = y$.
   (b) (6.8.4) $\dot{x} = y^3, \dot{y} = x$.
3. Consider a system with a saddle point at the origin and a stable spiral at $(1, 0)$, and no other fixed points. Based on index theory, where could closed trajectories potentially exist in this system (can they enclose both fixed points? one? none?)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