AM108 1:00pm

Class 18: Lorenz 63

Goals for the day:

1. Identify some properties of the Lorenz system.

Team Problems: The Lorenz system is

$$\begin{aligned}\dot{x} &= \sigma(y - x)\\ \dot{y} &= rx - y - xz\\ \dot{z} &= xy - bz\end{aligned}$$

1. Find all fixed points of the Lorenz system. These fixed points are ordered triples.
2. Let $r < 1$. Consider the function $V = \frac{1}{\sigma}x^2 + y^2 + z^2$. Show that function decreases along trajectories. Explain how that rules out closed orbits.

   As an intermediate step, after taking the time derivative, you will want to complete the square using the x terms

   This function is called a **Liapunov function** for the system. You can use this decreasing function to argue that every trajectory in the system approaches $(0, 0, 0)$, so it is a **globally stable** fixed point.
3. The characteristic equation for the eigenvalues of the Jacobian at C_+ and C_- is

   $$\lambda^3 + (\sigma + b + 1)\lambda^2 + (r + \sigma)b\lambda + 2b\sigma(r - 1) = 0.$$

   At the Hopf bifurcation, there is a pair of imaginary eigenvalues, $\lambda_+ = i\omega$ and $\lambda_- = -i\omega$. There must be a third eigenvalue, too, λ_3. By assuming all three of these are solutions, meaning that they are roots of the polynomial, find λ_3 and an implicit relationship for r_H, the value of r at the Hopf.
4. Show that the z-axis is an invariant line of the Lorenz equations, so a trajectory that starts on the axis stays on the axis.