AM147 11:30am

Names: ________________________________

Goals for the day:

1. How does a second order differential equation relate to a 2D system?
2. What is the relationship between eigenvalues/eigenvectors and analytic solutions of a 2D linear system?
3. Given the eigenvalues and eigenvectors of a matrix associated with a 2D linear dynamical system, construct the *phase portrait.*
4. Use a phase portrait to identify all of the possible long term behaviors of a particular 2D linear system.

Team problems:

1. Let
$$\ddot{x} + \dot{x} - 2x = 0.$$
   (a) Consider a possible solution $x(t) = Ae^{rt}$. Using the method of substitution, determine r so that this is a solution.
   (b) You should have found two possible values of r. Do those values correspond to solutions showing exponential growth or exponential decay?
   (c) The general solution of the differential equation is written as an arbitrary linear combination of $e^{r_1 t}$ and $e^{r_2 t}$. Write down the general solution.

2. In this class, we will work with systems of first order equations because the bifurcation analysis methods we are developing apply to first order equations. To convert from a second order (or higher) equation to a first order system, we create dummy variables.
   (a) Our dummy variable is $y = \dot{x}$. Find $\dot{y}$ using the equation above and rewrite the second order equation as a first order system
$$\dot{x} =$$
$$\dot{y} =$$
   (b) Using your solution to the second order equation, along with the definition $y = \dot{x}$, write a general solution to the matrix equation of the form
$$\begin{bmatrix} x(t) \\ y(t) \end{bmatrix} =$$
   This solution will turn out to be of the form $c_1 e^{r_1 t}\mathbf{v}_1 + c_2 e^{r_2 t}\mathbf{v}_2$ where $\mathbf{v}_1$ is an eigenvector corresponding to eigenvalue r_1. Show that $\mathbf{v}_1$ and $\mathbf{v_2}$ are eigenvectors of the matrix in the matrix equation and have the expected eigenvalues.
   (c) On the xy plane, sketch the solutions corresponding to the eigenvectors (these are straight lines in the plane). Denote the direction of time along the solution trajectories using an arrow.
   (d) Use continuity and local vector field information to fill in the rest of the phase portrait.
   (e) What type of fixed point in the fixed point at $(0, 0)$?
   (f) Consider the third order equation $\dddot{x} + \ddot{x} + x^2 = 0$. Using dummy variables $y = \dot{x}$ and $z = \dot{y}$, rewrite this equation as a first order system
$$\dot{x} =$$
$$\dot{y} =$$
$$\dot{z} =$$

3. Use trace and determinant to match the following systems to one of the phase portraits below. If necessary, use eigenvectors as well.

$$\begin{array}{llllllll} \dot{x} = & x & \qquad \dot{x} = & -x - y & \qquad \dot{x} = & x & \qquad \dot{x} = & x + y \\ \dot{y} = & x - y & \qquad \dot{y} = & x - 2y & \qquad \dot{y} = & -y & \qquad \dot{y} = & -2x + y \end{array}$$