

1. Consider the system

$$\begin{aligned}\dot{x} &= -\mu y + xy \\ \dot{y} &= \mu x + \frac{1}{2}(x^2 - y^2).\end{aligned}$$

- (a) Find and classify fixed points of this system and find a conserved quantity if appropriate.
 - (b) Show that the line $x = \mu$ is an invariant of this system. The line $y = \frac{1}{\sqrt{3}}(x + 2\mu)$ is also an invariant of the system. You're welcome to show that but it is a bit harder, so you're not required to.
 - (c) Using the invariant lines, local fixed point information and the vector field, sketch a phase portrait for the system.
 - (d) Sketch a phase portrait with the help of a numerical tool (include the name of the tool and any code or instructions needed to reproduce your plot).
 - (e) Find equations corresponding to any heteroclinic or homoclinic orbits in the system.
2. Do problem 6.5.6 on an epidemic model. Read problem 3.7.6 for the background.
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