Online Computational Appendix to "Optimal Taxation and Human Capital Policies over the Life Cycle"*

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1 Numerical Algorithm

1.1 Assumptions

The stochastic ability θ_t follows a geometric random walk:

$$\theta_t = \theta_{t-1}\varepsilon_t$$

where ε_t is distributed lognormal with mean 1 and variane σ_ε^2

The wage is assumed to be a CES function, of the form:

$$w_t\left(\theta_t, s_t\right) = \left(c_s \times s_t^{1-\rho} + c_\theta \times \theta_t^{1-\rho}\right)^{\frac{1}{1-\rho}}$$

with $c_s, c_\theta > 0$

The cost function is isoelastic:

$$\phi\left(\frac{y_t}{w_t\left(\theta_t, s_t\right)}\right) = \frac{\kappa}{\alpha}\left(\frac{y_t}{w_t\left(\theta_t, s_t\right)}\right)^{\alpha}$$

The utility of consumption is log:

$$u\left(c_t\right) = \log\left(c_t\right)$$

The cost function of education is linear with an adjustment factor scaled by the adjustment coefficient $c_a > 0$

$$C_t\left(s_{t-1}, e_t\right) = e_t + c_a\left(\frac{e_t}{s_{t-1}}\right)^{\eta} s_{t-1}$$

*I thank Ivan Werning for great discussions about and very generous advice on the numerical analysis.

1.2 Normalizing the problem

Variables and functions:

To reduce the number of state variables, define the normalized variables $\tilde{s}_{t-1} = s_{t-1}/\theta_{t-1}$, $\tilde{e}_t = e_t/\theta_{t-1}$, $\tilde{y}_t = y_t/\theta_{t-1}$, $\tilde{c}_t = c_t/\theta_{t-1}$.

The following relations then hold: $\tilde{s}_t = (\tilde{s}_{t-1} + \tilde{e}_t)/\varepsilon_t$

$\tilde{w}_t = w_t(\varepsilon_t, \tilde{s}_t\varepsilon_t) = w_t(\varepsilon_t, \tilde{s}_{t-1} + \tilde{e}_t) = w_t(\theta_t, s_t)/\theta_{t-1} = w_t/\theta_{t-1}$

$\phi\left(\frac{\tilde{y}_t}{\tilde{w}_t}\right) = \phi\left(\frac{y_t}{w_t}\right)$

$\tilde{C}_t = C_t(\tilde{s}_{t-1}, \tilde{e}_t) = C_t(s_{t-1}, e_t)/\theta_{t-1}$

Densities:

The densities of θ_t and ε_t are related in the following ways:

$$f(\theta_t|\theta_{t-1}) = \frac{1}{\theta_t\sigma_\varepsilon\sqrt{2\pi}} \exp\left\{-\frac{(\log\theta_t - \log\theta_{t-1} - \mu)^2}{2\sigma_\varepsilon^2}\right\}$$

hence: since $\log\varepsilon_t = \log\theta_t - \log\theta_{t-1}$

$$g_\varepsilon(\varepsilon_t) = \frac{1}{\varepsilon_t\sigma_\varepsilon\sqrt{2\pi}} \exp\left\{-\frac{(\log\varepsilon_t - \mu)^2}{2\sigma_\varepsilon^2}\right\}$$

so that:

$$f(\theta_t|\theta_{t-1}) = g_e(\varepsilon_t)/\theta_{t-1}$$

The derivatives of the densities are:

$$\frac{\partial f(\theta_t|\theta_{t-1})}{\partial\theta_{t-1}} = f(\theta_t|\theta_{t-1}) = \frac{1}{\theta_t\sigma_\varepsilon\sqrt{2\pi}} \frac{(\log\theta_t - \log\theta_{t-1} - \mu)}{\sigma_\varepsilon^2} \frac{1}{\theta_{t-1}} \exp\left\{-\frac{(\log\theta_t - \log\theta_{t-1} - \mu)^2}{2\sigma_\varepsilon^2}\right\}$$

$$\frac{\partial g_\varepsilon(\varepsilon_t)}{\partial\varepsilon_t} = \frac{-1}{\varepsilon_t\sigma_\varepsilon\sqrt{2\pi}} \frac{(\log\varepsilon_t - \mu)}{2\sigma_\varepsilon^2} \exp\left\{-\frac{(\log\varepsilon_t - \mu)^2}{2\sigma_\varepsilon^2}\right\}$$

so that

$$\tilde{g}_\varepsilon(\varepsilon_t) := g_\varepsilon(\varepsilon_t) + \varepsilon_t\frac{\partial g_\varepsilon(\varepsilon_t)}{\partial\varepsilon_t} = \theta_{t-1}^2\frac{\partial f(\theta_t|\theta_{t-1})}{\partial\theta_{t-1}} = \theta_{t-1}^2 g(\theta_t|\theta_{t-1})$$

In particular, note that

$$f(\theta_t|\theta_{t-1})\,d\theta_t = g_\varepsilon(\theta_t)\,d\varepsilon_t$$

and

$$g(\theta_t|\theta_{t-1})\,d\theta_t = \frac{\tilde{g}_\varepsilon(\varepsilon_t)}{\theta_{t-1}}d\varepsilon_t$$

Continuation Utilities:

Define $\beta_t^{\text{factor}} = 1 + \beta + ... + \beta^{T-t} = \frac{1-\beta^{T-t+1}}{1-\beta}$ and the following normalized variables:

$$\tilde{v}_t \equiv E\left(\sum_{s=t+1}^{T} \beta^{s-t-1}(\log(c_s/\theta_t) - \phi(y_s/w_s)\right) = v_t - \beta_{t+1}^{\text{factor}} \log(\theta_t)$$

$$\begin{aligned} \tilde{\omega}_t\left(\theta^t\right) &\equiv u\left(\tilde{c}_t\right) - \phi\left(\tilde{y}_t/\tilde{w}_t\right) + \beta\left(\sum_{s=t+1}^{T} \beta^{s-t-1}(\log\left(c_s/\theta_{t-1}\right) - \phi\left(\frac{y_s/\theta_{t-1}}{w_s/\theta_{t-1}}\right)\right) \\ &= u\left(\tilde{c}_t\right) - \phi\left(\tilde{y}_t/\tilde{w}_t\right) + \beta\tilde{v}_t + \beta \times \beta_{t+1}^{\text{factor}} \log\varepsilon_t \end{aligned}$$

$$\tilde{\Delta}_t \equiv \Delta_t/\theta_{t-1}$$

Rewriting the constraints in terms of the normalized variables:

Starting from the promise-keeping constraints, we can write:

$$\begin{aligned} \int \omega_t\left(\theta_t\right) f\left(\theta_t|\theta_{t-1}\right) d\theta_t &= v_{t-1} \\ &\Leftrightarrow \int \tilde{\omega}_t\left(\varepsilon_t\right) g_\varepsilon\left(\varepsilon_t\right) d\varepsilon_t = \tilde{v}_{t-1} \end{aligned}$$

The marginal utility promise-keeping constraint can be rewritten as:

$$\begin{aligned} \int \omega_t\left(\theta_t\right) g\left(\theta_t|\theta_{t-1}\right) d\theta_t &= \Delta_{t-1} \\ &\Leftrightarrow \int \tilde{\omega}_t\left(\varepsilon_t\right) \tilde{g}_\varepsilon\left(\varepsilon_t\right) d\varepsilon_t = \Delta_{t-1}\theta_{t-1} = \tilde{\Delta}_{t-1} \end{aligned}$$

$$\frac{\partial\tilde{\omega}\left(\varepsilon_t\right)}{\partial\varepsilon_t} = \frac{\tilde{y}_t}{\tilde{w}_t^2}\frac{d\tilde{w}_t}{d\varepsilon_t}\phi'\left(\frac{\tilde{y}_t}{\tilde{w}_t}\right) + \beta\frac{\tilde{\Delta}_t}{\varepsilon_t} \tag{1}$$

The full normalized program:

Let $\tilde{K}_t \equiv K_t/\theta_{t-1}$.The full program is then:

$$\tilde{K}_t\left(\tilde{v}_{t-1}, \tilde{\Delta}_{t-1}, \tilde{s}_{t-1}\right) = \min \int \left(\tilde{c}_t\left(\varepsilon_t\right) - \tilde{y}_t\left(\varepsilon_t\right) + \tilde{C}_t\left(\tilde{s}_{t-1}, \tilde{e}_t\right) + \frac{1}{R}\varepsilon_t\tilde{K}_{t+1}\right) g_\varepsilon\left(\varepsilon_t\right) d\varepsilon_t$$

subject to (with constraint multipliers in brackets on each corresponding line):

$$\begin{aligned} &\frac{\partial\tilde{\omega}\left(\varepsilon_t\right)}{\partial\varepsilon_t} = \frac{\tilde{y}_t}{\tilde{w}_t^2}\frac{d\tilde{w}_t}{d\varepsilon_t}\phi'\left(\frac{\tilde{y}_t}{\tilde{w}_t}\right) + \beta\frac{\tilde{\Delta}_t}{\varepsilon_t} \quad [\mu\left(\varepsilon_t\right)] \\ &\int \tilde{\omega}_t\left(\varepsilon_t\right) g_\varepsilon\left(\varepsilon_t\right) d\varepsilon_t = \tilde{v}_{t-1} \quad [\lambda_{t-1}] \\ &\int \tilde{\omega}_t\left(\varepsilon_t\right) \tilde{g}_\varepsilon\left(\varepsilon_t\right) d\varepsilon_t = \tilde{\Delta}_{t-1} \quad [\gamma_{t-1}] \\ &\tilde{s}_t\varepsilon_t = \tilde{s}_{t-1} + \tilde{e}_t \end{aligned}$$

with the definition

$$\tilde{g}_\varepsilon\left(\varepsilon_t\right) \equiv g_\varepsilon\left(\varepsilon_t\right) + \varepsilon_t\frac{\partial g_\varepsilon\left(\varepsilon_t\right)}{\partial\varepsilon_t} \tag{2}$$

1.3 First Order Conditions of the normalized problem

The first order conditions in the normalized problem, with respect to $\tilde{s}_t$ and $\tilde{y}_t$ are: (note that in this case, we substitute the last constraint $\tilde{s}_t \varepsilon_t = \tilde{s}_{t-1} + \tilde{e}_t$ directly to eliminate $\tilde{e}_t$.

$$[\tilde{s}_t] : \left(-\frac{\partial \tilde{C}_t}{\partial \tilde{e}_t} + \frac{\tilde{y}_t}{\tilde{w}_t} \frac{\partial \tilde{w}_t}{\partial \tilde{s}_t} + \frac{1}{R} \frac{\partial \tilde{K}_{t+1}}{\partial \tilde{s}_t} \right) - \frac{\mu(\varepsilon_t)}{g_\varepsilon(\varepsilon_t)} \frac{\tilde{y}_t}{\tilde{w}_t} \phi' \left(\frac{\tilde{y}_t}{\tilde{w}_t} \right) \frac{1}{\tilde{w}_t^2} \frac{\partial \tilde{w}_t}{\partial \varepsilon_t} \frac{\partial \tilde{w}_t}{\partial \tilde{s}_t} [\rho - 1] = 0 \tag{3}$$

with the envelope condition:

$$\frac{\partial \tilde{K}_{t+1}}{\partial \tilde{s}_t} = \int \left(\frac{\partial \tilde{C}_{t+1}}{\partial \tilde{e}_{t+1}} - \frac{\partial \tilde{C}_{t+1}}{\partial \tilde{s}_t} \right) g_\varepsilon(\varepsilon_{t+1}) \, d\varepsilon_{t+1}$$

$$[\tilde{y}_t] : \left[1 - \frac{\phi' \left(\frac{\tilde{y}_t}{\tilde{w}_t} \right)}{u'(\tilde{c}_t) \tilde{w}_t} \right] = \frac{\mu(\varepsilon_t)}{g_\varepsilon(\varepsilon_t)} \frac{\partial \tilde{w}_t}{\partial \varepsilon_t} \frac{1}{\tilde{w}_t^2} \phi' \left(\frac{\tilde{y}_t}{\tilde{w}_t} \right) \left[1 + \frac{\phi'' \left(\frac{\tilde{y}_t}{\tilde{w}_t} \right) \frac{\tilde{y}_t}{\tilde{w}_t}}{\phi' \left(\frac{\tilde{y}_t}{\tilde{w}_t} \right)} \right] \tag{4}$$

The first-order conditions with respect to $\tilde{v}_t$, $\tilde{\Delta}_t$ are:

$$[\tilde{v}_t] : \frac{1}{u'(\tilde{c}_t(\varepsilon_t))} = \frac{1}{R\beta} \varepsilon_t \lambda_t(\varepsilon_t) \tag{5}$$

$$\left[\tilde{\Delta}_t \right] : -\beta R \frac{1}{\varepsilon_t^2} \frac{\mu(\varepsilon_t)}{g_\varepsilon(\varepsilon_t)} = \gamma_t(\varepsilon_t) \tag{6}$$

While the law of motion of the co-state $\dot{\mu}(\varepsilon_t)$ is given by:

$$\dot{\mu}(\varepsilon_t) = \left(-\frac{1}{u_t'(\tilde{c}_t)} g_\varepsilon(\varepsilon_t) + \lambda_{t-1} g_\varepsilon(\varepsilon_t) - \gamma_{t-1} \left(g_\varepsilon + \varepsilon_t \frac{\partial g_\varepsilon(\varepsilon_t)}{\partial \varepsilon_t} \right) \right) \tag{7}$$

using the FOCs for $\tilde{v}_t$ and $\tilde{\Delta}_t$, we can rewrite the FOC for $\tilde{y}_t$ as:

$$[\tilde{y}_t] : \left[1 - \frac{1}{R\beta} \varepsilon_t \lambda_t(\varepsilon_t) \phi' \left(\frac{\tilde{y}_t}{\tilde{w}_t} \right) \right] = \frac{\mu(\varepsilon_t)}{g_\varepsilon(\varepsilon_t)} \frac{\partial \tilde{w}_t}{\partial \varepsilon_t} \frac{1}{\tilde{w}_t^2} \phi' \left(\frac{\tilde{y}_t}{\tilde{w}_t} \right) \left[1 + \frac{\phi'' \left(\frac{\tilde{y}_t}{\tilde{w}_t} \right) \frac{\tilde{y}_t}{\tilde{w}_t}}{\phi' \left(\frac{\tilde{y}_t}{\tilde{w}_t} \right)} \right] \tag{8}$$

1.4 Solution algorithm

The solution works backwards from period $t = T$, with the initializations $v_T(\varepsilon) = 0$, $\Delta_T(\varepsilon) = 0$, $\omega_T(\varepsilon) = u_T(c) - \phi(y/w) \; \forall \varepsilon$.

The state space is modified relative to the theory part to be $(\theta_-, \lambda_-, \gamma_-, s_-)$ instead of $(\theta_-, v_-, \Delta_-, s_-)$. With the normalization above, the state space has a generic element:

$$\sigma_{t-1} \equiv (\lambda_{t-1}, \gamma_{t-1}, s_{t-1})$$

Step 1:

Start with a guess for the continuation utility of the lowest type in a given period: $\omega_{0t} \equiv \omega_t(\underline{\varepsilon})$. For each such guess, and for all states, solve for the following choice variables in this order:

Sove for y_t as a function of (λ_t, e_t) using the first-order conditions (4) and (5).

Solve for e_t as a function of $(\lambda_t, \varepsilon_t, \mu_t; \omega_{0t}, \sigma_{t-1})$ using foc (3).

Solve for λ_t as a function of $(\varepsilon_t, \mu_t; \omega_{0t}, \sigma_{t-1})$ from (5), replacing $\tilde{c}_t$ as function of $\tilde{\omega}_t$ and $\tilde{v}_t$ and using the solutions $y_t(\lambda_t, (\lambda_t, \varepsilon_t, \mu_t; \omega_{0t}, \sigma_{t-1}))$ and $e_t(\lambda_t, \varepsilon_t, \mu_t; \omega_{0t}, \sigma_{t-1})$ just computed.

Solve for $\gamma_t(\varepsilon_t, \mu_t; \omega_{0t}, \sigma_{t-1})$.

Use the law of motion for $\mu(\varepsilon)$ in (7) to solve the differential equation for $\mu(\varepsilon)$, using (5) to replace for $\frac{1}{u_t'(\tilde{c}_t)}$ as a function of $(\varepsilon_t, \mu_t; \omega_{0t}, \sigma_{t-1})$.

Step 2:

Check the boundary condition at $\mu(\bar{\theta})$.

With the normalization performed, the boundary condition is

$$\mu(\bar{\theta}) = -\tilde{\gamma}_{t-1}\bar{\varepsilon}_t g_\varepsilon(\bar{\varepsilon}_t)$$

Repeat step 1 if the boundary condition is not met to a satisfactory tolerance level.

Iterate on the initial guess ω_{0t} until the boundary condition is met.

Step 3:

Once condition is met, work in the exact reverse order as listed in Step 1, to compute the choice variables at their equilibrium values.

Step 4:

Having obtained the choice variables $(e_t, y_t, \lambda_t, \gamma_t)$:

$\tilde{\omega}_t(\sigma_{t-1}, \varepsilon_t)$ is obtained from $\tilde{\omega}_t(\sigma_{t-1}, \varepsilon_t) = \int \frac{\partial}{\partial\varepsilon}\tilde{\omega}_t(\sigma_{t-1}, \varepsilon_t)\, d\varepsilon$ where $\frac{\partial}{\partial\varepsilon}\tilde{\omega}_t$ is as given in (1), using the subsequent period's policy function $\tilde{\Delta}_t(\sigma_t)$ interpolated at the computed solutions for $\sigma_t = (\lambda_t, \gamma_t, s_t = s_{t-1} + e_t)$.

To compute $\tilde{v}_{t-1}(\sigma_{t-1})$, and $\tilde{\Delta}_{t-1}(\sigma_{t-1})$, use their definitions:

$$\begin{aligned}
\tilde{v}_{t-1}(\sigma_{t-1}) &= \int \tilde{\omega}_t(\sigma_{t-1}, \varepsilon_t)\, g_e(\varepsilon_t)\, d\varepsilon_t \\
\tilde{\Delta}_{t-1}(\sigma_{t-1}) &= \int \tilde{\omega}_t(\sigma_{t-1}, \varepsilon_t)\, \tilde{g}_\varepsilon(\varepsilon_t)\, d\varepsilon_t
\end{aligned}$$

with $\tilde{g}_\varepsilon(\varepsilon_t)$ as defined in (2).

2 Calibration

2.1 Endogenously calibrated parameters

The exogenously calibrated parameters were explained in the main text. The two endogenous targets are explained in more detail here.

- **Ratio of the net present value of education over the net present value of income**: The US Department of Education (see Chang Wei, 2010) reports that among those enrolled full-time in college, around 67% are in 4-year programs, and 33% in 2-year programs. The OECD (OECD, 2013) estimates the total resource cost of a year of tertiary study (including research activities) to be \$29,900. Mean GDP per capita over the same period is \$47,000. Using $R = 1.053$, this yields a ratio of the net present value of education expenses to the net present value of lifetime income of around 13%. To allow for later-in-life investments, which are not taken into account in this statistic, it is assumed that college costs represent 2/3 of all lifetime investments in human capital, yielding a target ratio of the net present value of lifetime human capital expenses over the net present value of lifetime income of 19%.

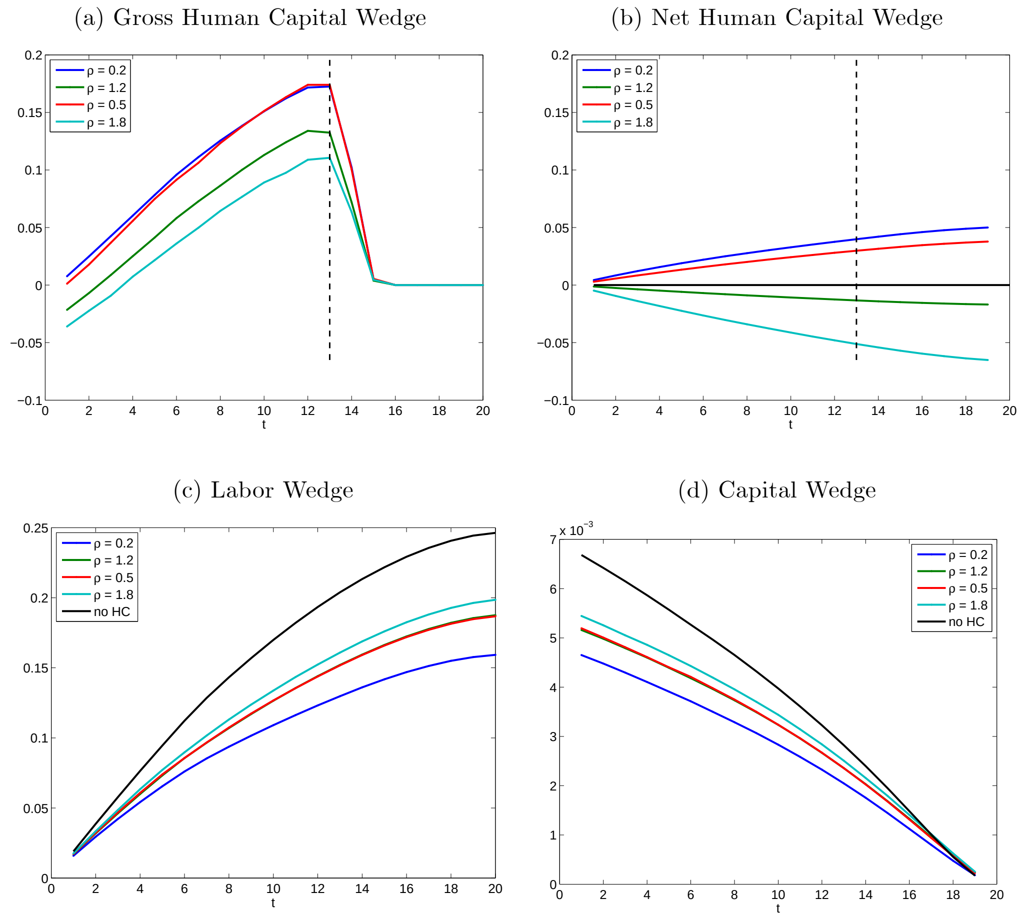
- **Wage Premium:** Autor *et al.* (hereafter AKK, 1998) report that around 38.6% of people have some type of college degree. Hence, in the baseline economy, at any given taxes and exogenous parameters, the top 38.6% in the population ranked by lifetime present value of education spending are assumed to represent the real-life college-goers. Symmetrically, the bottom 38.6% in terms of lifetime human capital spending are assumed to have no college. The middle 22.8% are omitted to emphasize the delineation between college-goers and others, and to account for the fact that some people might have some other type of education, or completed partial requirements toward a degree. Indeed, because of the continuous investments in the model, there is no sharp distinction between "college" and "no-college", and in particular, no notion of "a degree." The average wage of the "college-goers," for the periods after which human capital investments have been completed relative to the average wage of the "no college" agents, for the same years is matched to the wage premium for college estimated in the literature, as reported in the text.

2.2 Alternative calibrations

2.2.1 The effects of the Hicksian complementarity coefficient

Figures (1) illustrates the effects of changing the value of the Hicksian coefficient of complementarity $\rho_{\theta s}$. Two additional values, one below and one above 1 are explored. As mentioned in the main text, moving away from a multiplicatively separable wage, i.e., setting $\rho_{\theta s}$ further away from 1 in either direction, increases the net wedge in absolute value. When human capital has more positive redistributive or insurance value (i.e., $\rho_{\theta s}$ falls), the net subsidy is larger and grows faster over time. The opposite is true when the redistributive and insurance effects of human capital decrease ($\rho_{\theta s}$ rises). The gross wedge is only weakly monotone in $\rho_{\theta s}$; recall that the true incentive effect is in the net wedge t_{st}. The net wedge remains small overall. Panels (c) and (d) show that both the labor wedge and the

Figure 1: The effects of the complementarity coefficient on optimal wedges



(a) and (b): Dashed line marks time after which there are mostly no longer investments in human capital. Moving away from a multiplicatively separable wage ($\rho_{\theta s} = 1$) increases the net wedge in absolute value. The gross wedge is weakly monotone in $\rho_{\theta s}$.
(c) and (d): "No HC" denotes the case without human capital. Note that the lines for $\rho_{\theta s} = 1.2$ and $\rho_{\theta s} = 0.5$ overlap almost perfectly. Both the labor wedge and the capital wedge are smallest when human capital has the strongest redistributive and insurance effects ($\rho_{\theta s}$ small). The conclusion from the main text that the labor and capital wedges are smaller in the presence of human capital remains true for different values of $\rho_{\theta s}$.

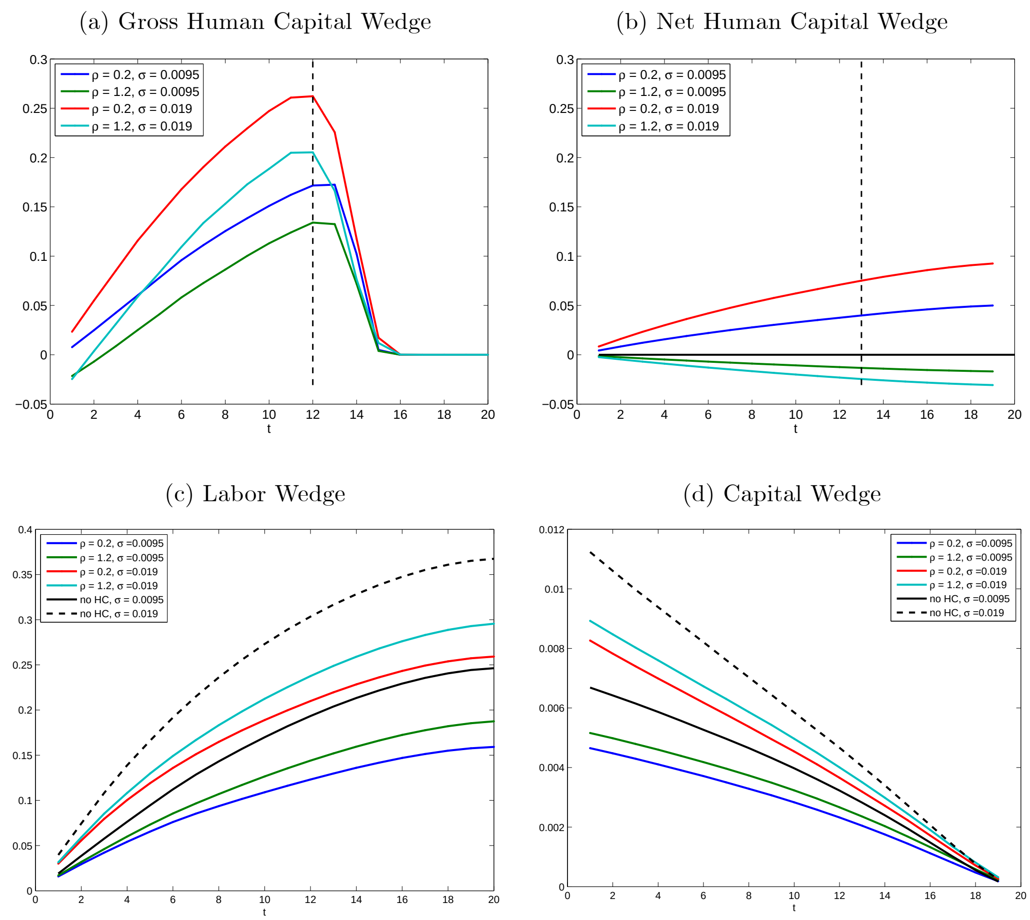
capital wedge are smallest when human capital has the strongest redistributive and insurance effects. The relation is not strictly monotone however: note how the lines or $\rho_{\theta s} = 0.5$ and $\rho_{\theta s} = 1.2$ overlap almost perfectly. The conclusion from the main text that the labor and capital wedges are smaller in the presence of human capital remains true for different values of $\rho_{\theta s}$.

2.2.2 The effects of higher volatility

Figures (2) highlight the effects of doubling the volatility of ability to 0.019. This higher volatility is close to the 0.0161 found by Storesletten *et al.* (2004) for the years 1980-1996. All other parameters are as in the main text.

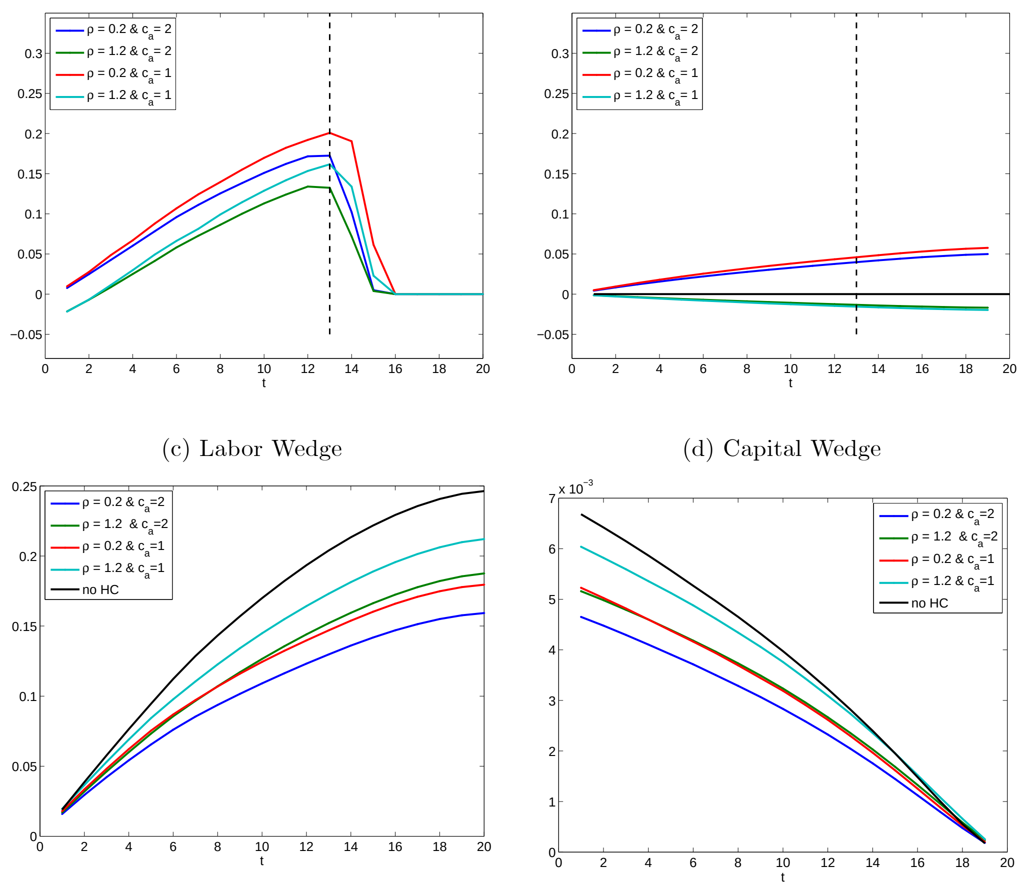
More risk increases the value of insurance over life. When human capital has a positive insurance

Figure 2: The effects of higher volatility on optimal wedges



Dashed line marks time after which there are mostly no longer investments in human capital. A higher volatility increases the optimal gross and net wedges and moves the optimum further away from full deductibility. “No HC” denotes the case without human capital. A higher volatility of ability increases the optimal labor and capital wedges. The optimal labor and capital wedges remain lower in the presence of human capital conditional on the volatility of ability.

Figure 3: The effects of the adjustment cost



(a) Gross Human Capital Wedge

(b) Net Human Capital Wedge

(c) Labor Wedge

(d) Capital Wedge

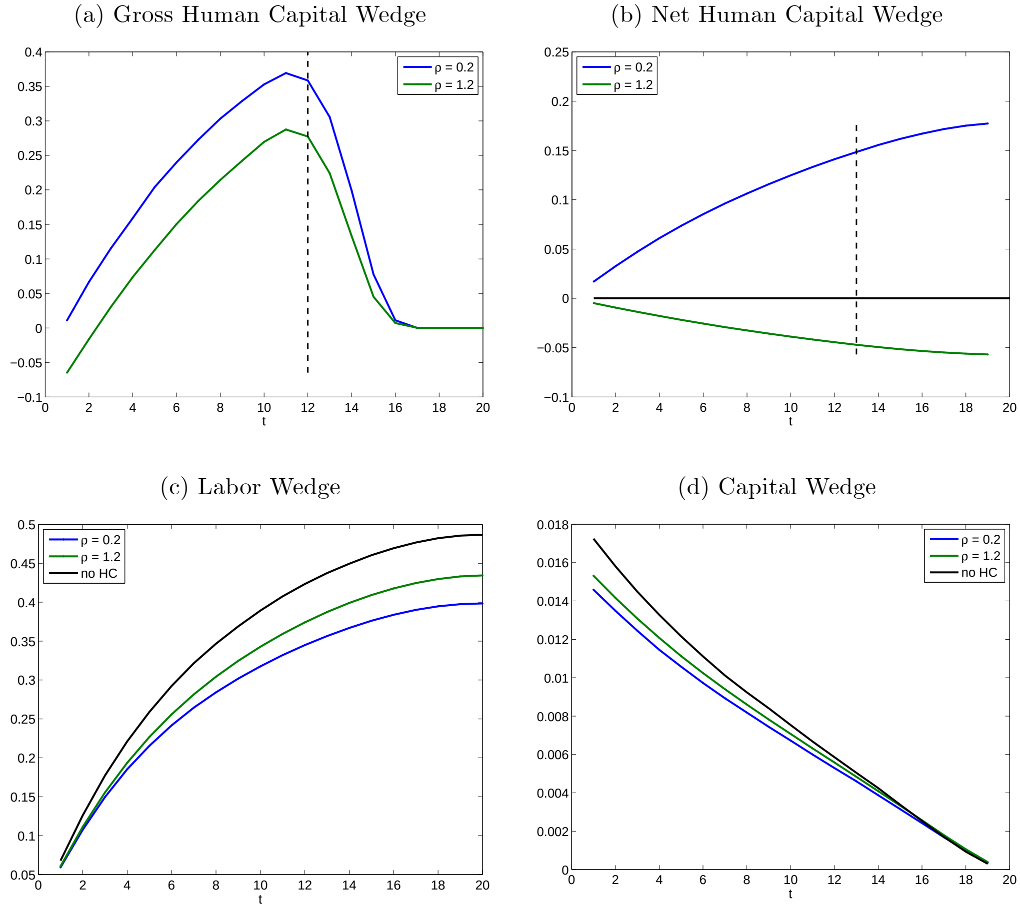
Dividing the adjustment cost parameter by 2, while adjusting the linear cost parameter c_l so as to keep meeting the target ratio of NPV of lifetime HC expenses over the NPV of lifetime income, increases the wedges only very slightly.

effect ($\rho_{\theta s} \leq 1$) it is optimal to subsidize it more than when volatility is lower. Inversely, when human capital has a negative insurance value, a higher volatility makes it optimal to tax it more on net. Note however that the labor tax also increases when volatility is higher because insurance becomes more valuable. Accordingly, part of the higher gross wedge merely compensates for the disincentive effect of labor taxes on human capital. The net wedges are still well below 10% throughout life. The labor and capital wedges remain lower when there is human capital than in the corresponding model without human capital, conditional on the volatility of ability.

2.2.3 The effects of the adjustment cost

Figure (3) shows that dividing the adjustment cost by 2, while adjusting the linear cost parameter c_l and the HC scale c_s so as to keep matching the two targets (the ratio of the NPV of lifetime HC expenses to the NPV of lifetime income and the wage premium), only slightly increases the wedges. Hence, the cost structure for HC does not make much difference for the optimal policies.

Figure 4: Wedges with very high volatility and very low adjustment cost



Dashed line marks time after which there are mostly no longer investments in human capital. A very high volatility combined with a lot adjustment cost increases the gross wedges and net wedges considerably. "No HC" denotes the case without human capital. The labor and capital wedges are higher when volatility is very high and there is a low adjustment cost to human capital.

2.2.4 A scenario with high net wedges

To obtain high net wedges, a calibration is needed which has a very large volatility of 0.038, more than double the high volatility in Storesletten *et al.* (2004). The rest of the parameters are as in the main text, except that the adjustment cost parameter is $c_a = 1$ linear cost parameter c_l is set to 0.25 to meet the target ratio of expenses over lifetime income. This scenario remains consistent with the target moments described above, except that the volatility is considerably higher than most estimates in the literature.