

# Optimal Taxation and Human Capital Policies over the Life Cycle

Stefanie Stantcheva

MIT

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## Human Capital and Taxes: A Two Way Interaction

- Interplay between HC policies and taxes.
- HC policies affect the income distribution - a key input for taxes.
- Taxes affect return and risk from HC investments.
- Calls for joint analysis of optimal taxation and HC policies.
- Optimal Taxation (Mirrlees) literature typically assumes exogenous ability
  - ▶ Mirrlees 1971, Saez 2001...

## Questions addressed in this paper

- How should the tax system take into account HC acquisition?
  - ▶ Should HC expenses be tax deductible?
  - ▶ What is the right tax treatment of cost of time?
  - ▶ What if HC unobservable to the govt?
- What parameters are important for HC policies, e.g., subsidies?
  - ▶ What is the lifetime evolution of optimal HC policies?
- What policy instruments implement the optimum?
  - ▶ How close can simpler policies come?

## A Model to capture main features of HC acquisition

- Dynamic lifecycle model of labor supply and HC acquisition.
- Investment in HC through **time** (training) or **monetary expenses**.
- Heterogeneous and uncertain returns  
→ Wage depends on **endogenous HC** and exogenous **stochastic ability**.
- Government faces **asymmetric info** about ability, evolution of ability, and labor effort.
  - ▶ 2 cases: HC observable or unobservable to govt. (College vs. OJT?)
  - ▶ Dynamic mechanism design with incentive compatibility constraints.

## Preview of Findings

- Characterize constrained efficient allocations over life using “wedges.”
  - ▶ Implementations proposed: Income Contingent Loans and “Deferred Deductibility” Scheme.
- Highlight important parameters for optimal policies:
  - ▶ Crucial how complementary HC is to ability and risk.
  - ▶ For training time: additional interaction with labor supply.
- Numerical analysis:
  - ▶ Full *dynamic risk-adjusted* deductibility close to optimal.
  - ▶ Simple age-dependent linear policies achieve bulk of welfare gain.

## Related literature

**Human Capital:** Becker (1964), Ben-Porath (1967), Heckman (1976), Card (1995), Heckman and Cunha (2006), Goldin and Katz (2008), Lochner and Monge-Naranjo (2011).

**Optimal Taxation:** Mirrlees (1971), Saez (2001), Kocherlakota (2005), Albanesi and Sleet (2006), Golosov, Tsyvinski, Werning (2006), Scheuer (2013), Farhi and Werning (2013).

**Taxation and HC:** Bovenberg and Jacobs (2005), Kapicka and Neira (2013), Findeisen and Sachs (2013), Krueger and Ludwig (2013).

**Dynamic Mechanism Design and First Order Approach:** Spear and Srivastava (1987), Fernandes and Phelan (2000), Doepke and Townsend (2006), Pavan *et al.* (2013), **Farhi and Werning (2013)**.

**Contribution:** Lifecycle investment, money & time, heterogeneous & uncertain HC returns, unobservable HC stock, wage function.

# Outline

- 1 Model and Solution Approach
- 2 Human Capital Expenses
- 3 Training Time
- 4 Unobservable Human Capital
- 5 Implementation

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# Model: Risky investments in Human Capital

- **Wage:**  $w_t = w_t(\theta_t, s_t, z_t)$
- **Ability  $\theta$ :**  
stochastic, Markov  $f^t(\theta_t|\theta_{t-1})$ , private info, privately uninsurable.

- Two ways of acquiring HC:

1. **Expenses**  $e_t$  at cost  $M_t(e_t)$ . Stock of HC expenses  $s_t$  :

$$s_t = s_{t-1} + e_t$$

2. **Training time**  $i_t$  at disutility cost  $\phi_t(l_t, i_t)$ . Accomplished training  $z_t$  :

$$z_t = z_{t-1} + i_t$$

- Cost composition of College versus OJT?

- Income:  $y_t = w_t l_t$

## Hicksian complementarity

- **Hicksian coefficients of complementarity:**

$$\rho_{\theta s} = \frac{w_{\theta s} w}{w_s w_\theta} \quad \rho_{\theta z} = \frac{w_{\theta z} w}{w_z w_\theta}$$

- $\rho_{\theta s} \geq 0$  : Marginal wage gain from HC  $\uparrow$  in ability.
- $\rho_{\theta s} \geq 1$  : Elasticity of wage to HC  $\uparrow$  in ability.
- If separable  $w = \theta + h(s, z) \Rightarrow \rho_{\theta s} = \rho_{\theta z} = 0$
- If multiplicative  $w = \theta h(s, z) \Rightarrow \rho_{\theta s} = \rho_{\theta z} = 1$
- If CES  $w = [\alpha_1 \theta^{1-\rho_t} + \alpha_2 s^{1-\rho_t} + \alpha_3 z^{1-\rho_t}]^{\frac{1}{1-\rho_t}} \Rightarrow \rho_{\theta s} = \rho_{\theta z} = \rho_t$

## Model: Preferences over Lifetime Allocations

- $T$  periods of work,  $T_r$  periods of retirement.
- Per period utility:  $u_t(c_t) - \phi_t(l_t, i_t)$ .
- History  $\theta^t = \{\theta_1, \dots, \theta_t\} \in \Theta^t$ , probability  $P(\theta^t) = f^t(\theta_{t+1}|\theta_t) \dots f(\theta_1)$ .
- Allocation:  $\{c(\theta^t), y(\theta^t), s(\theta^t), z(\theta^t)\}_{\theta^t}$ .
- Expected lifetime utility from allocation:

$$U(\{c(\theta^t), y(\theta^t), s(\theta^t), z(\theta^t)\}) \\ = \sum_{t=1}^{T+T_r} \int \beta^{t-1} \left[ u_t(c(\theta^t)) - \phi_t\left(\frac{y(\theta^t)}{w_t(\theta^t)}, z(\theta^t) - z(\theta^{t-1})\right) \right] P(\theta^t) d\theta^t$$

$$w_t(\theta^t) \equiv w_t(\theta_t, s_t(\theta^t), z_t(\theta^t))$$

# Government's/Planner's Goals: Insurance and Redistribution

- Govt's/Planner's goal: max expected social welfare given Pareto weights.
  - ▶ **Insurance** against earnings risk.
  - ▶ **Redistribution** across intrinsic ability heterogeneity (persistent).
  - ▶ **Incentives** for efficient work and HC investment.
- Asymmetric information about:

ability and its evolution      labor supply

↓                                      ↓

$$w_t(\theta_t, s_t, z_t) \times l_t = y_t$$

↑      ↑

2 cases: observable and unobservable HC.

→ **“direct revelation mechanism”** with incentive compatibility.

## Government's/Planner's Program: Dual Formulation

- Min expected resource cost s.t. utility targets and incentive compatibility  
→ **constrained efficiency.**

$$\min_{\{c, y, s, z\}} \sum_{t=1}^T \frac{1}{R^{t-1}} \int (c(\theta^t) - y(\theta^t) + M_t (s(\theta^t) - s(\theta^{t-1}))) P(\theta^t) d\theta^t$$

$$\text{s.t.: } U(\{c, y, s, z\}) \geq \underline{U}$$

$\{c, y, s, z\}$  is incentive compatible.

- If initial heterogeneity and non-utilitarian welfare function set any Pareto weights through  $\underline{U} = (\underline{U}(\theta_1))_{\Theta}$ .

## Incentive Compatibility Defined

- Reporting strategy:  $r = \{r_t(\theta^t)\}_{t=1}^T$ , with history  $r^t \equiv \{r_1, \dots, r_t\}$ .
- **Continuation utility** under reporting strategy  $r$ :

$$\omega^r(\theta^t) = u_t(c(r^t(\theta^t))) - \phi_t \left( \frac{y(r^t(\theta^t))}{w_t(\theta_t, s(r^t(\theta^t))), z(r^t(\theta^t))}, i(r^t(\theta^t)) \right) + \beta \int \omega^r(\theta^{t+1}) f^{t+1}(\theta_{t+1}|\theta_t) d\theta_{t+1}$$

- Under truth-telling:  $\omega(\theta^t)$  with  $r_t(\theta^t) = \theta_t$  for all  $\theta^t$ .
- **Incentive Compatibility**

$$\omega(\theta^t) \geq \omega^r(\theta^t) \quad \forall r, \forall \theta^t$$

# Solving the Government's Program: Method

- ① Solving the direct revelation mechanism:
  - ▶ Step 1: Relax program using first order approach (FOA).
  - ▶ Step 2: Formulate relaxed program recursively.
- ② Characterize optimal allocations using “wedges” or implicit taxes.
- ③ Decentralize or “implement” optimum using policy instruments.

## Step 1. Relaxing the Program: First-Order Approach

- Consider deviating strategy  $\sigma^r$  with report  $r$ :

$$\omega(\theta^t) = \max_r (u_t(c(\theta^{t-1}, r))) - \phi_t \left( \frac{y(\theta^{t-1}, r)}{w_t(\theta_t, s(\theta^{t-1}, r), z(\theta^{t-1}, r))} \right) + \beta \int \omega^{\sigma^r}(\theta^{t-1}, r, \theta_{t+1}) f^{t+1}(\theta_{t+1}|\theta_t) d\theta_{t+1}$$

- Replace by necessary **Envelope Condition**:

$$\frac{\partial \omega(\theta^t)}{\partial \theta_t} = \frac{\partial \phi_t}{\partial l_t} \frac{\partial w_t}{\partial \theta_t} \frac{l_t}{w_t} + \beta \int \omega(\theta^{t+1}) \frac{\partial f^{t+1}(\theta_{t+1}|\theta_t)}{\partial \theta_t} d\theta_{t+1}$$

- Sufficiency?**

- Conditions on allocations (Pavan *et al.* 2013).
- Ex-post verification (Werning, 2007, Farhi and Werning, 2013).



## Step 2. Recursive Formulation: Define States

- Definition of continuation utility:

$$\omega(\theta^t) = u_t(c(\theta^t)) - \phi_t \left( \frac{y(\theta^t)}{w_t(\theta^t)}, i(\theta^t) \right) + \beta \int \omega(\theta^{t+1}) f^{t+1}(\theta_{t+1}|\theta_t) d\theta_{t+1}$$

- Envelope Condition:

$$\frac{\partial \omega(\theta^t)}{\partial \theta_t} = \frac{\partial \phi_t}{\partial l_t} \frac{\partial w_t}{\partial \theta_t} \frac{l(\theta^t)}{w(\theta^t)} + \beta \int \omega(\theta^{t+1}) \frac{\partial f^{t+1}(\theta_{t+1}|\theta_t)}{\partial \theta_t} d\theta_{t+1}$$

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$$v_t(\theta^t) = \int \omega(\theta^{t+1}) f^{t+1}(\theta_{t+1}|\theta_t) d\theta_{t+1}$$

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$$v_t(\theta^t) = \int \omega(\theta^{t+1}) f^{t+1}(\theta_{t+1} | \theta_t) d\theta_{t+1}$$

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$$v_t(\theta^t) = \int \omega(\theta^{t+1}) f^{t+1}(\theta_{t+1} | \theta_t) d\theta_{t+1}$$

$$\Delta_t(\theta^t) = \int \omega(\theta^{t+1}) \frac{\partial f^{t+1}(\theta_{t+1} | \theta_t)}{\partial \theta_t} d\theta_{t+1}$$

## Step 2. Recursive Formulation: Rewrite Recursively

- Definition of continuation utility, using  $\theta_-, \theta, \theta'$ .

$$\omega(\theta) = u_t(c(\theta)) - \phi_t \left( \frac{y(\theta)}{w_t(\theta)}, z(\theta) - z_- \right) + \beta v(\theta)$$

- Envelope Condition:

$$\frac{\partial \omega(\theta)}{\partial \theta} = \frac{\partial \phi_t}{\partial l} \frac{\partial w_t(\theta)}{\partial \theta} \frac{l(\theta)}{w_t(\theta)} + \beta \Delta(\theta)$$

$$v(\theta) = \int \omega(\theta') f^{t+1}(\theta'|\theta) d\theta'$$

$$\Delta(\theta) = \int \omega(\theta') \frac{\partial f^{t+1}(\theta'|\theta)}{\partial \theta} d\theta'$$



## Recursive Formulation of Relaxed Program

$$K(v, \Delta, \theta_-, s_-, z_-, t) = \min \int (c(\theta) + M_t(s(\theta) - s_-) - w_t(\theta, s(\theta), z(\theta)) l(\theta) + \frac{1}{R} K(v(\theta), \Delta(\theta), \theta, s(\theta), z(\theta), t+1)) f^t(\theta|\theta_-) d\theta$$

$$\omega(\theta) = u_t(c(\theta)) - \phi_t(l(\theta), z(\theta) - z_-) + \beta v(\theta)$$

$$\dot{\omega}(\theta) = \frac{w_{\theta,t}}{w_t} l(\theta) \phi_{l,t}(\theta, z(\theta) - z_-) + \beta \Delta(\theta)$$

$$v = \int \omega(\theta) f^t(\theta|\theta_-) d\theta$$

$$\Delta = \int \omega(\theta) \frac{\partial f^t(\theta|\theta_-)}{\partial \theta_-} d\theta$$

over  $(c(\theta), l(\theta), s(\theta), z(\theta), \omega(\theta), v(\theta), \Delta(\theta))$

## Recursive Formulation of Relaxed Program

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$$K(v, \Delta, \theta_-, s_-, z_-, t) = \min \int (c(\theta) + M_t(s(\theta) - s_-) - w_t(\theta, s(\theta), z(\theta)) l(\theta) + \frac{1}{R} K(v(\theta), \Delta(\theta), \theta, s(\theta), z(\theta), t+1)) f^t(\theta|\theta_-) d\theta$$

$$\omega(\theta) = u_t(c(\theta)) - \phi_t(l(\theta), z(\theta) - z_-) + \beta v(\theta)$$

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## Recursive Formulation of Relaxed Program

$$K(v, \Delta, \theta_-, s_-, z_-, t) = \min \int (c(\theta) + M_t(s(\theta) - s_-) - w_t(\theta, s(\theta), z(\theta)) l(\theta) + \frac{1}{R} K(v(\theta), \Delta(\theta), \theta, s(\theta), z(\theta), t+1)) f^t(\theta|\theta_-) d\theta$$

$$\omega(\theta) = u_t(c(\theta)) - \phi_t(l(\theta), z(\theta) - z_-) + \beta v(\theta)$$

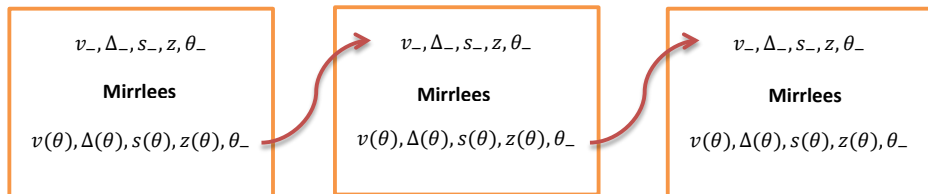
$$\dot{\omega}(\theta) = \frac{w_{\theta,t}}{w_t} l(\theta) \phi_{l,t}(\theta, z(\theta) - z_-) + \beta \Delta(\theta)$$

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over  $(c(\theta), l(\theta), s(\theta), z(\theta), \omega(\theta), v(\theta), \Delta(\theta))$

# Method summary: Repeated Mirrlees Nested in Dynamic Programming + Endogenous Human Capital Formation



# Outline

- 1 Model and Solution Approach
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## Implicit taxes and subsidies: Definition

Implicit marginal labor income tax:

$$\tau_{L_t} \equiv 1 - \underbrace{\frac{\phi_{l,t}(l_t, i_t)}{w_t u'_t(c_t)}}_{\text{MRS/MRT } l_t \text{ and } c_t}$$

Implicit marginal savings tax:

$$\tau_{k_t} \equiv 1 - \underbrace{\frac{1}{\beta R} \frac{u'_t(c_t)}{E_t(u'_{t+1}(c_{t+1}))}}_{\text{(MRS } c_t \text{ and } c_{t+1}) / \text{Return on savings}}$$

Implicit marginal HC subsidy:

$$\tau_{S_t} \equiv \underbrace{M'_t(e_t) - \beta E_t \left( \frac{u'_{t+1}(c_{t+1})}{u'_t(c_t)} M'_{t+1}(e_{t+1}) \right)}_{\text{Dynamic (risk-adjusted) Cost}} - \underbrace{(1 - \tau_{L_t}) w_{s,t} l_t}_{\text{Marginal Benefit}}$$

Implicit marginal bonus for training time:

$$\tau_{Z_t} \equiv \underbrace{\frac{\phi_{i,t}(l_t, i_t)}{u'_t(c_t)} - \beta E_t \left( \frac{u'_{t+1} \phi_{i,t+1}(l_{t+1}, i_{t+1})}{u'_t u'_{t+1}(c_{t+1})} \right)}_{\text{Dynamic (risk-adjusted, monetary) cost}} - \underbrace{(1 - \tau_{L_t}) w_{z,t} l_t}_{\text{Marginal Benefit}}$$

# A Net Human Capital Subsidy to Capture Real Incentives

## Definition (Net Wedge)

$$t_{st} \equiv \frac{\tau_{St} - \tau_{Lt} M'_t{}^d + P_t}{(M'_t{}^d - \tau_{St})(1 - \tau_{Lt})}$$

$M'_t{}^d \equiv M'_t - \beta E_t \left( \frac{u'_{t+1}}{u'_t} M'_{t+1} \right)$ : risk adjusted dynamic cost

$P_t \equiv \frac{1}{R} \frac{\tau_k}{1 - \tau_k} (1 - \tau_{Lt}) E \left( \beta \frac{u'_{t+1}}{u'_t} M'_{t+1} \right)$ : risk adjusted savings distortion

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$t_{st} = 0 \rightarrow$  **Full dynamic risk-adjusted deductibility of expenses**  
**Neutrality of tax system wrt HC.**

**Static model:**  $\tau_{St}^* = M'_t \tau_{Lt}^*$ , standard deductibility (BJ, 2005).

**Dynamic model + uncertainty:**  $\tau_{St}^* = M'_t{}^d \tau_{Lt}^* - P_t^*$

i) dynamic cost, ii) risk adjustment, iii) savings wedge.

$t_{st} > 0 \rightarrow$  positive net subsidy beyond deductibility.



## Optimal Net Subsidy: the Formula

$$t_{st}^* (\theta^t) = \frac{(\kappa (\theta^t) + \eta (\theta^t)) u'_t (c (\theta^t))}{f^t (\theta_t | \theta_{t-1})} \frac{\epsilon_{w\theta,t}}{\theta_t} (1 - \rho_{\theta s,t})$$

$\epsilon_{w\theta,t}$ : wage elasticity wrt ability.

# 1. Insurance Motive

$$t_{st}^* (\theta^t) = \frac{(\kappa (\theta^t) + \eta (\theta^t)) u'_t (c (\theta^t))}{f^t (\theta_t | \theta_{t-1})} \frac{\varepsilon_{w\theta,t}}{\theta_t} (1 - \rho_{\theta_s,t})$$



$$\kappa (\theta^t) \equiv \int_{\theta_t}^{\bar{\theta}} \left( \frac{1}{u'_t (c (\theta^{t-1}, \theta_s))} - E_{t-1} \left( \frac{1}{u'_t (c (\theta^t))} \right) \right) f (\theta_s | \theta_{t-1}) d\theta_s$$

## Insurance Motive

$\kappa (\theta^t)$  captures dispersion in marginal utilities

$\kappa (\theta^t) = 0$  if quasilinear utility or no uncertainty (fully persistent types).

## 2. Persistence and the Redistributive Motive

$$t_{st}^*(\theta^t) = \frac{(\kappa(\theta^t) + \eta(\theta^t)) u'_t(c(\theta^t)) \varepsilon_{w\theta,t}}{f^t(\theta_t|\theta_{t-1}) \theta_t} (1 - \rho_{\theta_s,t})$$



$$\eta(\theta^t) \equiv t_{st-1}^*(\theta^{t-1}) \left[ \frac{R\beta}{u'_{t-1}} \frac{1}{(1 - \rho_{\theta_s,t-1})} \frac{\theta_{t-1}}{\varepsilon_{w\theta,t-1}} \int_{\theta_t}^{\theta} \frac{\partial f(\theta_s|\theta_{t-1})}{\partial \theta_{t-1}} d\theta_s \right]$$

### Persistence and the Redistributive Motive

Persistence of ability  $\rightarrow$  persistence of policy.

$\eta(\theta^t) = 0$  with iid shocks.

Redistributive motive against initial heterogeneity remains active if persistence.

### 3. Complementarity Between HC and Ability

$$t_{st}^* (\theta^t) = \frac{(\kappa (\theta^t) + \eta (\theta^t)) u_t' (c (\theta^t)) \varepsilon_{w\theta,t}}{f^t (\theta_t | \theta_{t-1}) \theta_t} (1 - \rho_{\theta_s,t})$$

$$t_{st}^* (\theta^t) \geq 0 \Leftrightarrow \rho_{\theta_s,t} \leq 1$$

#### Labor Supply Effect:

subsidy increases wage

→ ↑ labor

→ ↑ resources.

+

#### Inequality Effect:

if  $\rho_{\theta_s} \geq 0$

HC benefits more able agents more

→ ↑ *pre-tax* inequality and risk.

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$\rho_{\theta_s} \leq 1 \Rightarrow$  subsidy ↓ *post-tax* inequality

$\Rightarrow$  has **positive redistributive and insurance effects**.

$\rho_{\theta_s} = 1 \Rightarrow t_{st}^* (\theta^t) = 0$

Benchmark case in literature  $w_t = \theta_t s_t$

Full dynamic risk-adjusted deductibility  $\approx$  Atkinson-Stiglitz result.

## Evolution of the net subsidy over time

- If  $\log(\theta_t) = \rho \log(\theta_{t-1}) + \psi_t$ , with  $f^\psi(\psi|\theta^{t-1})$  and  $E(\psi|\theta^{t-1}) = 0$ .

$$\begin{aligned} E_{t-1} \left( t_{st} \frac{\varepsilon_{w\theta,t-1}}{\varepsilon_{w\theta,t}} \frac{(1 - \rho_{\theta_s,t-1})}{(1 - \rho_{\theta_s,t})} \left( \frac{1}{R\beta} \frac{u'_{t-1}}{u'_t} \right) \right) \\ = (1 - \rho_{\theta_s,t-1}) \varepsilon_{w\theta,t-1} \text{Cov} \left( \frac{1}{R\beta} \frac{u'_{t-1}}{u'_t}, \log(\theta_t) \right) + \rho t_{st-1} \end{aligned}$$

- If HC has **positive insurance value** ( $\rho_{\theta_s} \leq 1$ ): positive drift.
  - ▶ Fading drift term  $\rightarrow$  “**Subsidy smoothing.**”
- Persistence of shocks  $\rightarrow$  persistence of policy  $t_{st}$ .

# Empirical Estimates of the Hicksian Complementarity

- **Labor/Human Capital Literature:**

Heckman, Cunha et al., 2006, Ashenfelter and Rouse, 1998, ...

- ▶ Early Childhood investments level playing field  $\Rightarrow \rho_{\theta s} \leq 1$ .
- ▶ Evidence suggests  $\rho_{\theta s}$  changes over life.
- ▶ College benefits already able people  $\Rightarrow \rho_{\theta s} \geq 0$  and  $\rho_{\theta s} \geq 1$  possible.

- **Structural Macro Literature:**

Huggett, Ventura, Yaron, 2010, Heathcote, Perri, Violante, 2010:

- ▶ Ability as the residual, assume log separability  $\Leftrightarrow \rho_{\theta s} = 1$ .

- **OJT?** Investments later in life? Scarce empirical evidence.

# Numerical Analysis: Setup

	Functional Form	Values	Source
<b>Wage</b>	$w_t = \left( \theta_t^{1-\rho} + c_s s_t^{1-\rho} \right)^{\frac{1}{1-\rho}}$	$\rho = \{0.2, 1.2\}$ $c_s = \{0.09, 0.1\}$	CHLM (2006) Match wage premium (AKK, 1998)
<b>Utility</b>	$\log(c_t) - \frac{1}{\gamma} \left( \frac{y_t}{w_t} \right)^\gamma$	$\gamma = 3$	Chetty (2012)
<b>Stochastic process</b>	$\log \theta_t = \log \theta_{t-1} + \psi_t$ $\psi_t \sim N \left( -\frac{1}{2} \sigma_\psi^2, \sigma_\psi^2 \right)$	$\sigma_\psi^2 = 0.0095$	HSV (2005)
<b>Cost</b>	$M_t(e_t) = c_l e_t + 2 \left( \frac{e_t}{s_{t-1}} \right)^2$	$c_l = 0.5$	Match expenses OECD (2013) US DoE (2010)

table 1

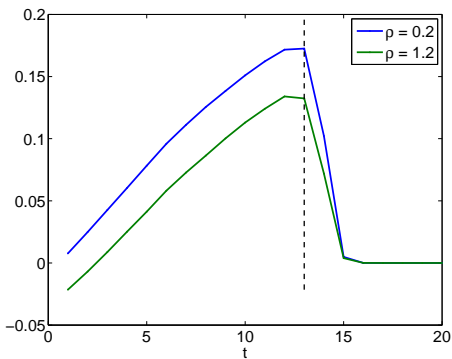
- $T = 20$ ,  $T_r = 10$ ,  $\beta = 0.95$ ,  $R = 1/\beta$ .
- Select zero net cost allocation, utilitarian planner.

▶ Match

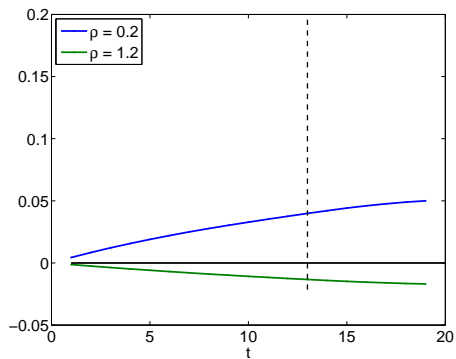
▶ Baseline

▶ Vol

# Optimal Gross and Net Human Capital Wedges



(a) Gross Wedge  $\tau_{St}$



(b) Net Wedge  $t_{st}$

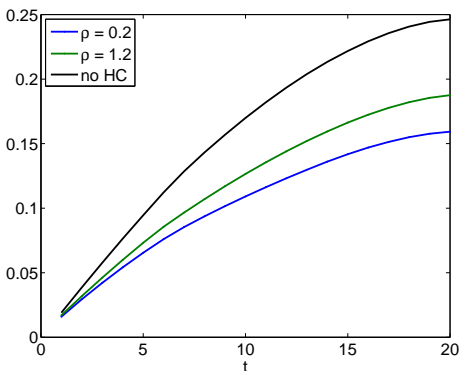
If  $\rho_{\theta_s} < 1$ ,  $\tau_s$  higher and grows faster;  $t_s > 0$  and growing.  
But: Full dynamic risk adjusted deductibility close to optimal.

► Effects of  $\rho_{\theta_s}$

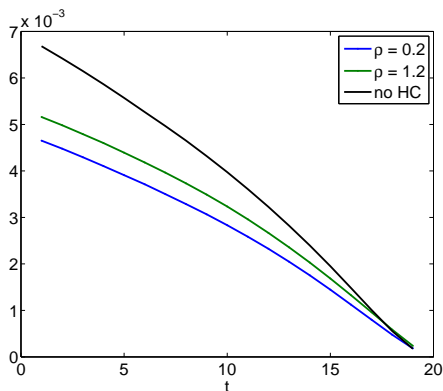
► Effects of volatility



# Labor and Capital Wedges with Human Capital



(a) Labor Wedge over time

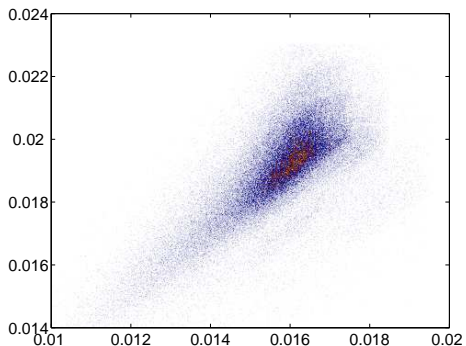


(b) Capital Wedge over time

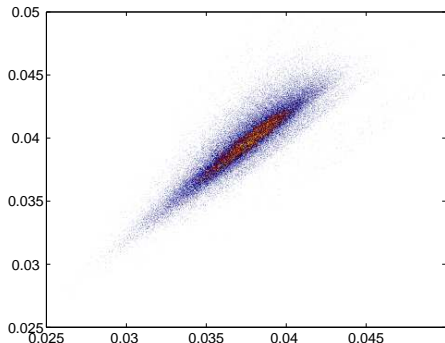
Labor and capital wedges are smaller in the presence of HC.  
Standard Inverse Euler Equation holds.

# Subsidy Smoothing over Life

$t_{st}$  against  $t_{st-1}$



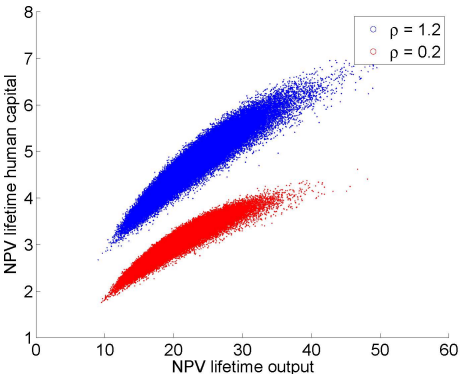
(a) at  $t = 5$



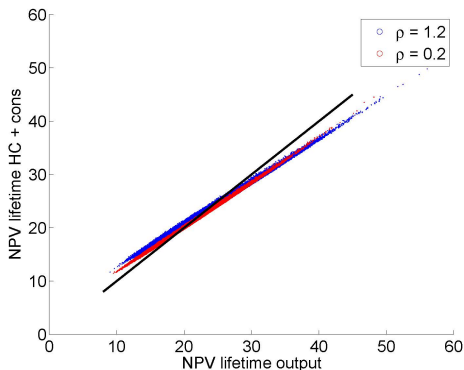
(b) at  $t = 13$

$t_s$  becomes more correlated over time as age increases because the variance of consumption growth vanishes.

# Insurance and HC Over the Life Cycle



(a) HC against lifetime income



(b) Consumption + HC against lifetime income

# Outline

- 1 Model and Solution Approach
- 2 Human Capital Expenses
- 3 Training Time**
- 4 Unobservable Human Capital
- 5 Implementation

## The net bonus on training time

Measures real incentive on training, beyond purely compensating for income and savings tax distortions.

### Definition

$$t_{zt} \equiv \frac{\tau_{Zt} - \tau_{Lt} (\phi_{zt} / u'_t)^d + P_{Zt}}{(1 - \tau_{Lt}) \left( (\phi_{zt} / u'_t)^d - \tau_{Zt} \right)}$$

$(\phi_{zt} / u'_t)^d \equiv \frac{\phi_{z,t}}{u'_t} - \beta E_t \left( \frac{u'_{t+1}}{u'_t} \frac{\phi_{z,t+1}}{u'_{t+1}} \right)$  dynamic risk adjusted cost

$P_{Zt} \equiv \frac{1}{R} \frac{\tau_k}{1 - \tau_k} E \left( \beta \frac{u'_{t+1}}{u'_t} \frac{\phi'_{z,t+1}}{u'_{t+1}} \right)$ , risk adjusted savings distortion.

## Optimal Net Bonus: Special Case of Separable Disutility

- Subsidize training on net iff positive redistributive and insurance values:

$$t_{zt}^* (\theta^t) \geq 0 \Leftrightarrow \rho_{\theta z,t} \leq 1$$

- Inverse elasticity rules for implicit taxes:

$$t_{zt}^* = \frac{\tau_{Lt}^*}{1 - \tau_{Lt}^*} \frac{\varepsilon_t^c}{1 + \varepsilon_t^u} (1 - \rho_{\theta z,t}), \quad t_{st}^* = \frac{\tau_{Lt}^*}{1 - \tau_{Lt}^*} \frac{\varepsilon_t^c}{1 + \varepsilon_t^u} (1 - \rho_{\theta s,t})$$

- Bonus and subsidy set **proportionally to their redistributive effects**:

$$\frac{t_{zt}^*}{t_{st}^*} = \frac{(1 - \rho_{\theta z,t})}{(1 - \rho_{\theta s,t})}$$

- If CES wage, identical tax treatment of expenses and time:  $t_{st}^* = t_{zt}^*$ .

## Optimal Net Bonus on Training Time

At the optimum, the net bonus is given by:

$$t_{zt}^* (\theta^t) = \frac{\tau_{Lt}^* (\theta^t)}{1 - \tau_{Lt}^* (\theta^t)} \frac{\varepsilon_t^c}{1 + \varepsilon_t^u} \left( (1 - \rho_{\theta z, t}) - \frac{\varepsilon_{\phi_{Iz, t}}}{\varepsilon_{wz, t}} \right) + \frac{1}{R} E_t \left( \frac{\tau_{Lt+1}^* (\theta^{t+1})}{1 - \tau_{Lt}^* (\theta^t)} \frac{\varepsilon_{t+1}^c}{1 + \varepsilon_{t+1}^u} \frac{w_{z, t+1}}{w_{z, t}} \frac{l_{t+1}}{l_t} \frac{\varepsilon_{\phi_{Iz, t+1}}}{\varepsilon_{wz, t+1}} \right)$$

$$t_{zt}^* (\theta^t) \geq 0 \iff \underbrace{(1 - \rho_{\theta z, t})}_{\text{Insurance effect}} \geq \underbrace{\varepsilon_{\phi_{Iz, t}} / \varepsilon_{wz, t}}_{\text{Interaction w/ Labor}} \geq 0$$

- Same “Labor Supply” and “Inequality Effect” as for HC expenses.
- In addition: Direct interaction with labor supply.  
 “Learning-and-Doing” versus “Learning-or-Doing?”

# Outline

- 1 Model and Solution Approach
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- 5 Implementation



## Unobservable HC: Modified Program

- With unobservable expenses: Euler Equation for HC ( $\tau_{St} \equiv 0$ ).

$$u'_t(c_t) M'_t(e_t) = \frac{w_{s,t}}{w_t} l_t \phi'(l_t) + \beta E_t(u'_{t+1}(c_{t+1}) M'_{t+1}(e_{t+1}))$$

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$$\Delta^s(\theta^t) \equiv -E_t(u'_{t+1}(c_{t+1}) M'_{t+1}(e_{t+1}))$$

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$$\Delta^s \equiv -E_{t-1}(u'_t(c_t) M'_t(e_t))$$

- With unobservable training: Euler Equation for training ( $\tau_{Z_t} \equiv 0$ ).

$$\phi_{z,t} = w_{z,t} l_t \frac{1}{w_t} \phi_{l,t} + \beta E_t(\phi_{z,t+1})$$

## Unobservable HC: Modified Program

- With unobservable expenses: Euler Equation for HC ( $\tau_{S_t} \equiv 0$ ).

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- With unobservable training: Euler Equation for training ( $\tau_{Zt} \equiv 0$ ).

$$\phi_{z,t} = w_{z,t} l_t \frac{1}{w_t} \phi_{l,t} - \beta \Delta_t^z(\theta)$$



## Unobservable HC: Modified Program

- With unobservable expenses: Euler Equation for HC ( $\tau_{S_t} \equiv 0$ ).

$$u'_t(c_t) M'_t(e_t) = \frac{w_{s,t}}{w_t} l_t \phi'(l_t) - \beta \Delta^s(\theta^t)$$

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$$\Delta_t^z \equiv -E_t(\phi_{z,t+1})$$

## Unobservable HC: Modified Program

- With unobservable expenses: Euler Equation for HC ( $\tau_{St} \equiv 0$ ).

$$u'_t(c_t) M'_t(e_t) = \frac{w_{s,t}}{w_t} l_t \phi'(l_t) - \beta \Delta^s(\theta^t)$$

$$\Delta^s \equiv -E_{t-1}(u'_t(c_t) M'_t(e_t))$$

- With unobservable training: Euler Equation for training ( $\tau_{Zt} \equiv 0$ ).

$$\phi_{z,t} = w_{z,t} l_t \frac{1}{w_t} \phi_{l,t} - \beta \Delta_t^z(\theta)$$

$$\Delta^z \equiv -E_{t-1}(\phi_{z,t})$$

## Unobservable HC expenses: Results

- $\tau_L$  and  $\tau_K$  indirectly provide incentives for HC accumulation.  
Lower  $\tau_L$  and higher  $\tau_K$  mimic  $t_s > 0$ .
- $\tau_L$ : Lower if  $\rho_{\theta_s}$  lower.
- $\tau_K$ : “Modified Inverse Euler” holds (hybrid model).

$$\frac{\beta R(1 - \tilde{\gamma}_t^E(\theta^t) M'_t u''_t)}{u'_t} = \int_{\underline{\theta}}^{\bar{\theta}} \frac{(1 - \tilde{\gamma}_{t+1}^E(\theta^{t+1}) M'_{t+1} u''_{t+1})}{u'_{t+1}} f^{t+1}(\theta_{t+1}|\theta_t)$$

$\tilde{\gamma}_t^E(\theta^t)$ : multiplier on agent's Euler for HC.

## Unobservable Training, Observable HC Expenses: Results

$$\underbrace{\frac{\tau_{Lt}}{1 - \tau_{Lt}} - \frac{\tau_{Lt}^*}{1 - \tau_{Lt}^*}}_{\text{Adjustment of labor tax}} = \underbrace{\frac{1}{(1 - \rho_{zs,t})}}_{\text{Relative efficiency}} \underbrace{\frac{1 + \varepsilon_t^u}{\varepsilon_t^c}}_{\text{Adjustment of subsidy}} \underbrace{(t_{st} - t_{st}^*)}_{\text{Adjustment of subsidy}}$$

- Inverse elasticity rule for available instruments.
- Sharpest instrument adjusts more to compensate for “missing bonus.”
- Subsidy for HC expenses changed iff  $\rho_{zs,t} \neq 1$ .

# Outline

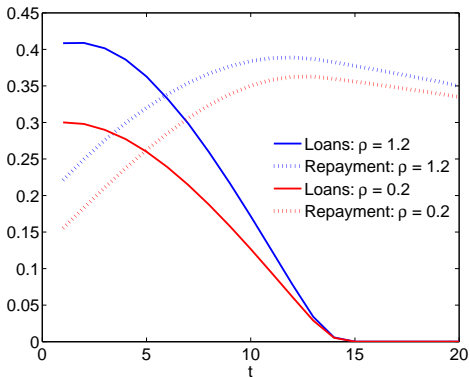
- 1 Model and Solution Approach
- 2 Human Capital Expenses
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## Implementation: Idea

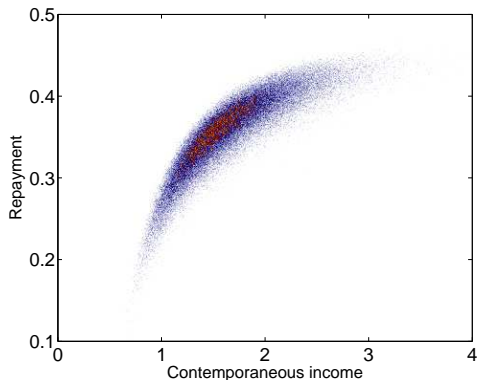
- From direct revelation mechanism to policy instruments.
- “Taxation principle”  $\rightarrow T_t(y^t, s^t)$  implements optimum.
- Indeterminacy of instruments in theory:  
Administrative constraints or political preferences in practice?
- Propose ICLs and Deferred Deductibility Scheme.

# Income Contingent Loans (ICLs)

- **Loan** covers HC cost:  $L_t(e_t) = M_t(e_t)$ .
- **Repayment** based on history of loans and earnings:  $R_t(L^{t-1}, y^{t-1}, e_t, y_t)$ .



(a) Loans and repayment as % of income



(b) Insurance role and history contingency of repayments  
▶ Agent's program

## Simplified versions of ICLs exist

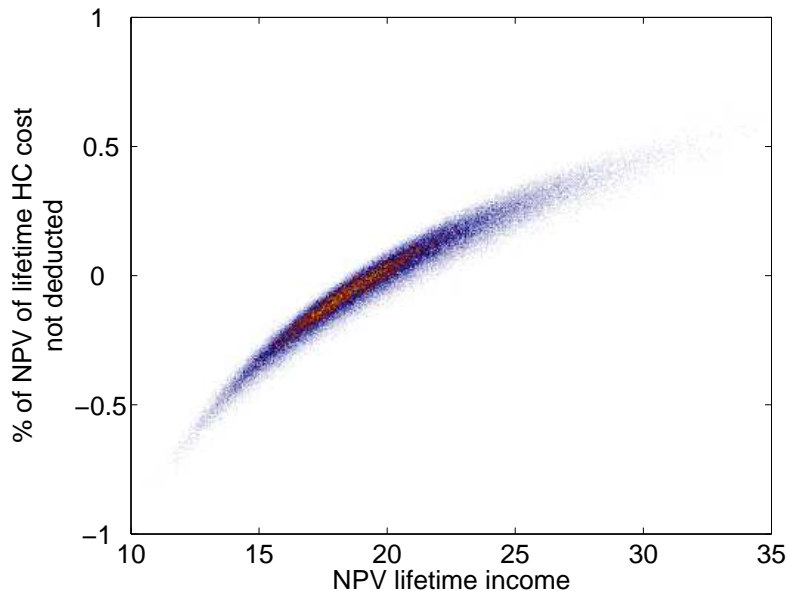
- Proposed by J. Tobin and M. Friedman.
- Tried in Sweden, Australia, NZ, UK, US, Chile, SA, Thailand.
  - ▶ US: ICR for public sector (1994), IBR since 2007 (CRRRA).
  - ▶ Australia: HECS - automatic, collected by tax authority.
- Main differences of scheme proposed here:
  - ▶ Not only for College
  - ▶ Longer history-dependence
  - ▶ Focus on both downside and upside.
- “Yale Plan” debacle (1970s): need tax power of govt (adverse selection).



# Deferred Deductibility Scheme

- Part of expense made at  $t$  deducted at  $t + j$ .
  - ▶ Nonlinear cost: deduct at MC effective at  $t + j$ , not historic MC. ▶ Nonlinear
  - ▶ Linear cost:  $(1 - \beta)\%$  of cost. ▶ Linear
  - ▶ Plus “no arbitrage” for physical capital taxation.
- Not sufficient to make HC expenses **contemporaneously deductible**:
  - ▶ Changing nonlinear tax rates
  - ▶ Savings tax
  - ▶ Risk adjustment (varying  $u'$ ).

## Out-of-pocket HC costs at different income levels



# What welfare gain can simpler policies achieve?

Policy studied:

Set linear  $\tau_{Lt}, \tau_{St}, \tau_{Kt}$  to cross-sectional average (across all histories  $\theta^t$  at age  $t$ ).

Table 2: Welfare Gains

	$\rho_{\theta_s} = 0.2$		$\rho_{\theta_s} = 1.2$	
Volatility	Medium	High	Medium	High
Welfare gain from second best	0.85%	1.60%	0.98%	1.76%
Welfare gain from linear				
age-dependent policies	0.79%	1.53%	0.94%	1.74%
as % of second best	93%	95.6%	95.5%	98.5%

# Conclusion

- Applications: entrepreneurial taxation, bequest taxation, health care?
- Open empirical questions:
  - i) Estimate  $\rho_{\theta_S}$  (also later in life investments).
  - ii) How strongly does HC respond to taxes (weaker: to net returns)?

## Bottomline

- Crucial consideration: complementarity of HC to ability and risk + direct interaction with labor time.
- Numerically:
  - i) Full dynamic risk-adjusted deductibility close to optimal.
  - ii) Simpler age-dependent linear policies perform very well.

# Appendix

# Verification Procedure

Policy Functions:

$$\begin{aligned} & c^P (v_{t-1}, \Delta_{t-1}, s_{t-1}, z_{t-1}, \theta_{t-1}, \theta_t, t), y^P (v_{t-1}, \Delta_{t-1}, s_{t-1}, z_{t-1}, \theta_{t-1}, \theta_t, t), \\ & s^P (v_{t-1}, \Delta_{t-1}, s_{t-1}, z_{t-1}, \theta_{t-1}, \theta_t, t), z^P (v_{t-1}, \Delta_{t-1}, s_{t-1}, z_{t-1}, \theta_{t-1}, \theta_t, t), \\ & \omega^P (v_{t-1}, \Delta_{t-1}, s_{t-1}, z_{t-1}, \theta_{t-1}, \theta_t, t), \Delta^P (v_{t-1}, \Delta_{t-1}, s_{t-1}, z_{t-1}, \theta_{t-1}, \theta_t, t), \\ & v^P (v_{t-1}, \Delta_{t-1}, s_{t-1}, z_{t-1}, \theta_{t-1}, \theta_t, t) \end{aligned}$$

Check for all states  $v_{t-1}, \Delta_{t-1}, s_{t-1}, z_{t-1}$ , report  $r_{t-1}, t$ , and  $\theta_t$  that:

$$\begin{aligned} \theta_t \in \operatorname{argmax}_r & u_t (c^P (v_{t-1}, \Delta_{t-1}, s_{t-1}, z_{t-1}, r_{t-1}, r, t)) \\ & - \phi_t (y^P (v_{t-1}, \Delta_{t-1}, s_{t-1}, z_{t-1}, \theta_{t-1}, \theta_t, t)) \\ & + \beta \int (\omega^P (v^P, \Delta^P, s^P, z^P, r, \theta_{t+1}, t + 1)) \end{aligned}$$

with  $\omega^P, v^P, s^P, z^P$  as defined above evaluated at  $\theta_t = r$ . [▶ Back to Main](#)

## First Period Heterogeneity

With a non-utilitarian objective, if  $\theta_1$  is interpreted as heterogeneity:

$$\kappa(\theta_1) = \int_{\theta_1}^{\bar{\theta}} \frac{1}{u_1'(c_1(\theta_s))} (1 - \lambda_0(\theta_s) u_1'(c_1(\theta_s))) f(\theta_s)$$

$$\eta(\theta_1) = 0$$

where  $\lambda_0(\theta_s)$  is the multiplier (scaled by  $f(\theta_s)$ ) on type  $\theta_s$  target utility.

With linear utility:  $1 = \int_{\underline{\theta}}^{\bar{\theta}} \lambda_0(\theta_s) f(\theta_s)$ .

[▶ Back to Main](#)

## Sufficient Statistics for $\rho_{\theta_S}$ ?

$$t_S \equiv \frac{\tau_S - \tau_L}{(1 - \tau_S)(1 - \tau_L)} = \left( \underbrace{\frac{\varepsilon'_{LS}}{\left( \varepsilon'_{LT} \frac{\varepsilon_{YEE}}{y} \right)}}_{\text{Labor supply effect}} - \underbrace{\frac{(1 - \bar{e})}{(1 - \bar{y})}}_{\rho_{\theta_S}} \right) \text{ terms}$$

$$\bar{y} \equiv \frac{E(u'_{ti}(c_{ti})y_{ti})}{E(u'_{ti}(c_{ti}))y_t}, \quad \bar{e} \equiv \frac{E(u'_{ti}(c_{ti})e_{ti})}{E(u'_{ti}(c_{ti}))e_t}$$

$\rho_{\theta_S}$  captured by relative redistributive effect of HC versus income.

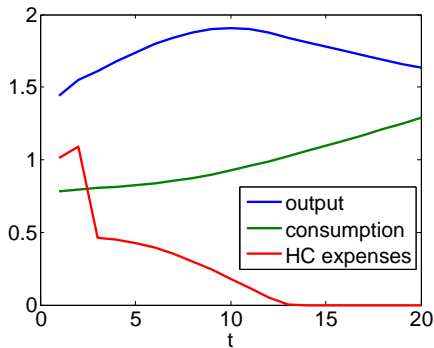
Is education less or more concentrated than income among high consumption (high ability?) people? Note that relative concentration matters.



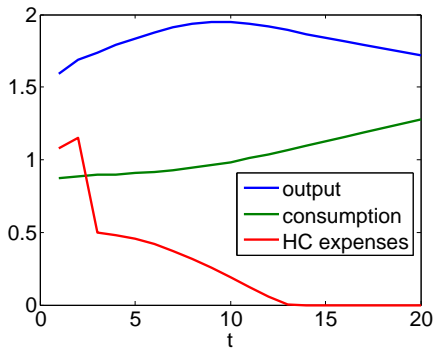
## Endogenously targeted moments

- Calibrate wage and cost function parameters ( $c_s$  and  $c_l$ ) in “**baseline**” economy:
  - ▶ Free saving and borrowing
  - ▶ Linear  $\tau_S = 35\%$  for first 2 periods;  $\tau_L = 13\%$ ;  $\tau_K = 25\%$ .
- **Wage Premium**: The top 42.7% in the population of baseline economy, ranked by educational expenses, are “college-goers” (Autor *et al*, 1998).
  - ▶ Their average wage relative to bottom 38.6% must match college wage premium.
  - ▶ Estimates: 1.58 (Murphy and Welch, 1992), 1.66 to 1.73 (Autor *et al*, 1998), 1.80 (Heathcote *et al.*, 2010). Target: 1.7.
- **NPV education expenses/NPV lifetime income**:
  - ▶ For College is 13%:  $\approx$  \$30,000 resource cost per year (OECD, 2013) for 4 yr college (67%) or 2 yr college (33%) (NCES, 2010). Mean income \$47,000.
  - ▶ Add allowance for later-in-life investments  $\rightarrow 19\%$ .

# Baseline allocations

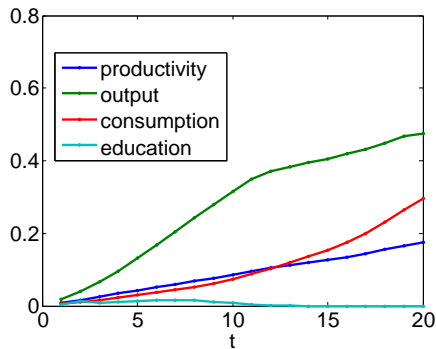


(a)  $\rho_{\theta_s} = 0.2$

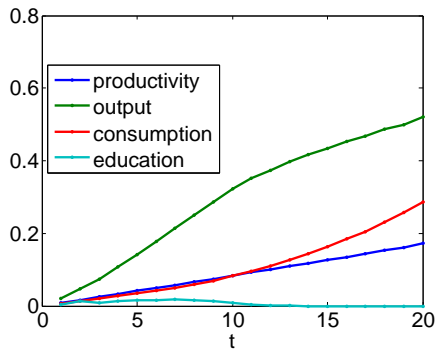


(b)  $\rho_{\theta_s} = 1.2$

# Baseline volatilities



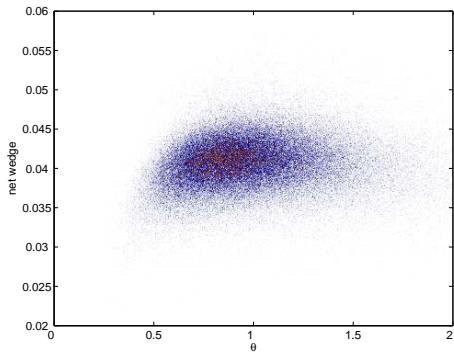
(a)  $\rho_{\theta_s} = 0.2$



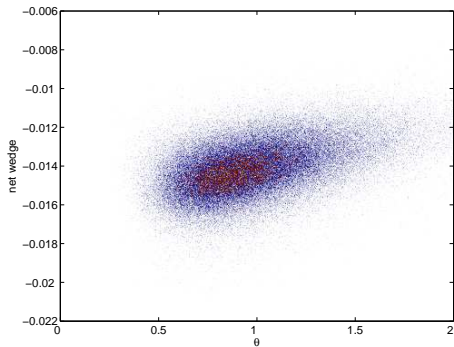
(b)  $\rho_{\theta_s} = 1.2$

# Subsidy Regressivity

Net wedge  $t_{st}$  against  $\theta_t$



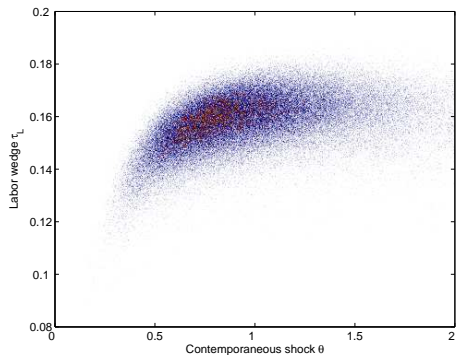
(a)  $\rho_{\theta_s} = 0.2$



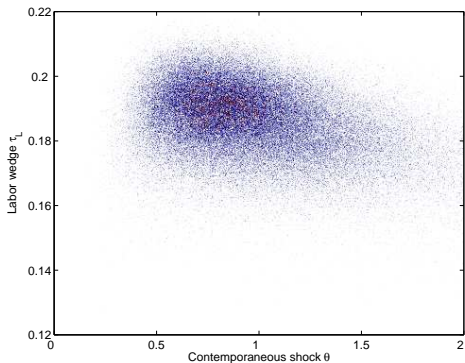
(b)  $\rho_{\theta_s} = 1.2$

# Tax Progressivity

Labor wedge  $\tau_{Lt}$  against  $\theta_t$



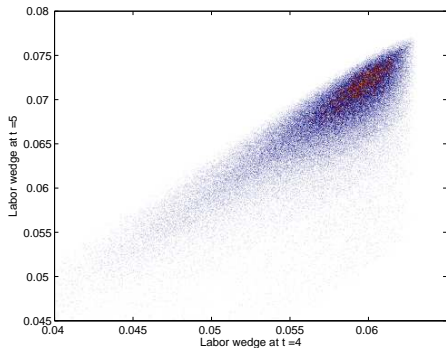
(a)  $\rho_{\theta_S} = 0.2$



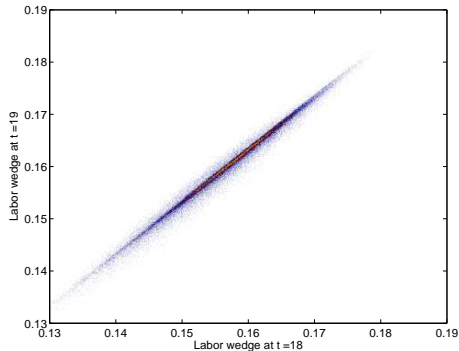
(b)  $\rho_{\theta_S} = 1.2$

# Tax Smoothing

$\tau_{Lt}$  against  $\tau_{Lt-1}$

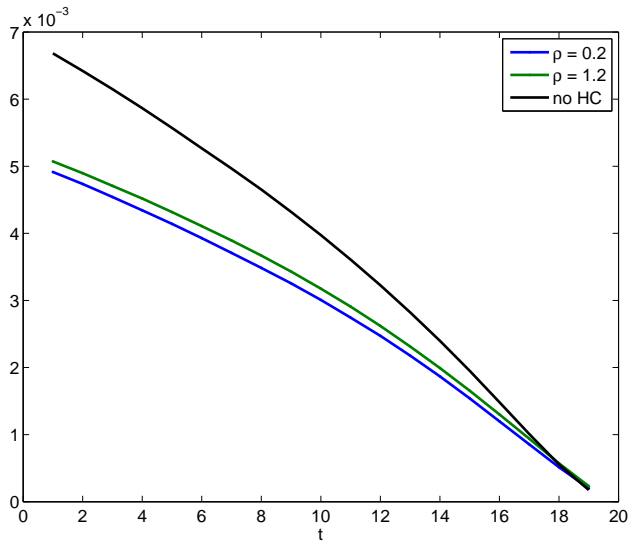


(a) at  $t = 5$

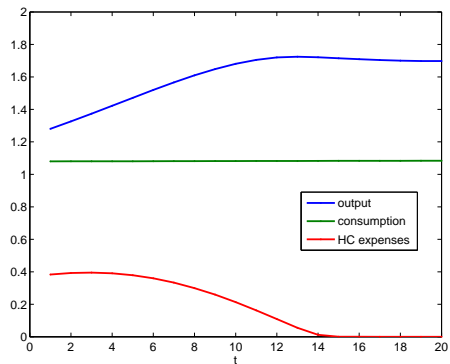


(b) at  $t = 19$

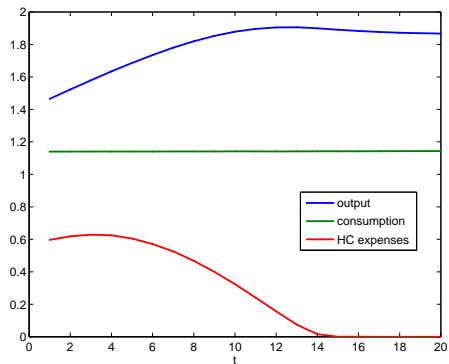
# Variance of Consumption Growth



# Allocations: Consumption, HC, and output



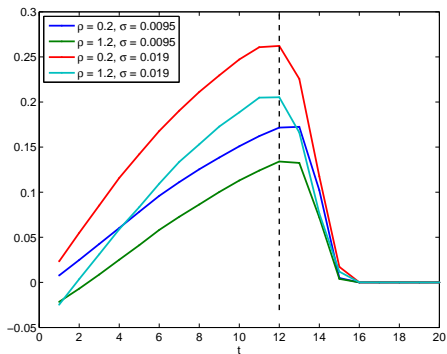
(a)  $\rho_{\theta_s} = 0.2$



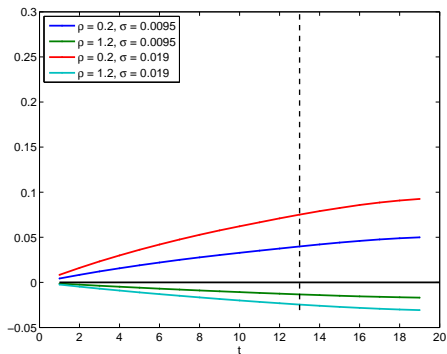
(b)  $\rho_{\theta_s} = 1.2$



# Effects of volatility (I)



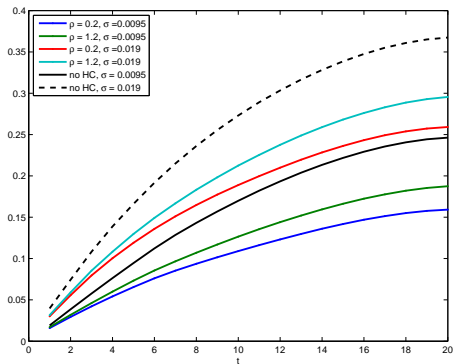
(a) Gross Wedge  $\tau_S$



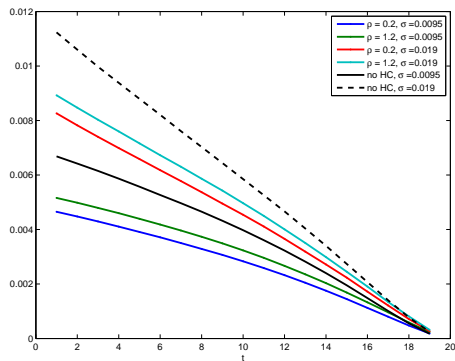
(b) Net Wedge  $t_S$

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## Effects of volatility (II)



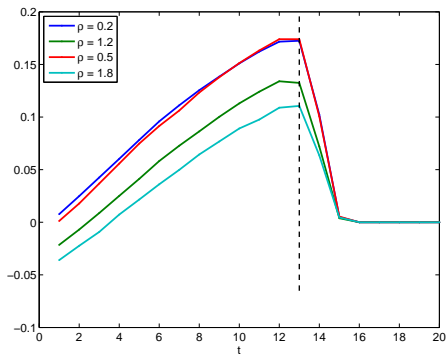
(c) Labor Wedge  $\tau_L$



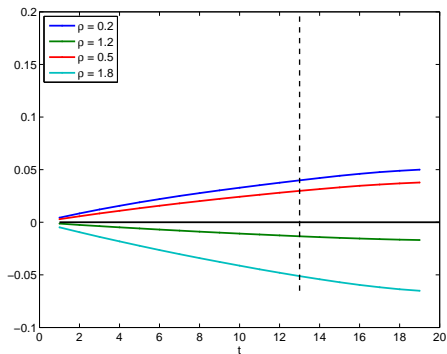
(d) Capital Wedge  $\tau_K$

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# Effects of $\rho_{\theta_s}$ (I)

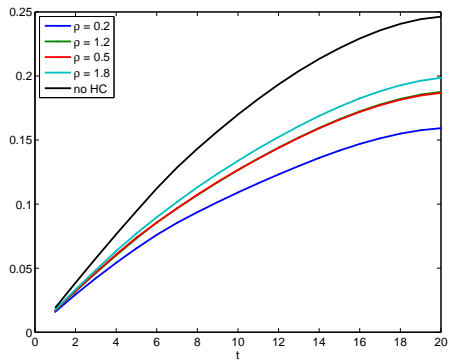


(a) Gross Wedge  $\tau_S$

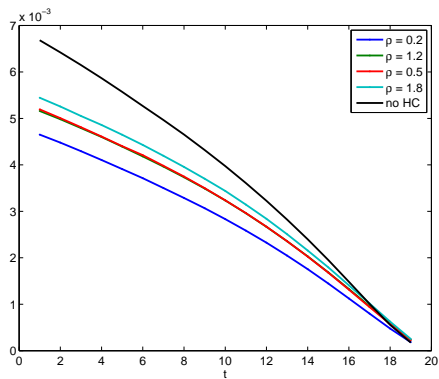


(b) Net Wedge  $t_S$

## Effects of $\rho_{\theta_s}$ (II)



(c) Labor Wedge  $\tau_L$



(d) Capital Wedge  $\tau_K$

## Agent's program with ICLs

$$V_1(b_0, \theta_0) = \sup \sum_{t=1}^T \int \left[ u_t(c_t(\theta^t)) - \phi_t \left( \frac{y_t(\theta^t)}{w_t(\theta_t, s_{t-1}(\theta^{t-1}) + e_t(\theta^t))} \right) \right] P(\theta^t) d\theta^t$$

$$\begin{aligned} & c_t(\theta^t) + \frac{1}{R} b_t(\theta^t) + M_t(e_t(\theta^t)) - b_{t-1}(\theta^{t-1}) - L_t(e_t(\theta^t)) \\ & \leq y_t(\theta^t) - D_t(L^{t-1}(\theta^{t-1}), y^{t-1}(\theta^{t-1}), e_t(\theta^t), y_t(\theta^t)) - T_Y(y_t(\theta^t)) - T_K(b_t) \end{aligned}$$

$$s_t(\theta^t) = s_{t-1}(\theta^{t-1}) + e_t(\theta^t)$$

$$s_0 \text{ given, } e_t(\theta^t) \geq 0, b_0 = 0, b_T \geq 0$$

$$L_t(e_t) = M_t(e_t) \quad \forall t, \forall e_t$$

$$\begin{aligned} & D_t(L^{t-1}, y^{t-1}, e_t^*(\theta^{t-1}, \theta), y_t^*(\theta^{t-1}, \theta)) + T_Y(y_t^*(\theta^{t-1}, \theta)) \\ & = y_t^*(\theta^{t-1}, \theta) - c_t^*(\theta^{t-1}, \theta) \end{aligned}$$

for all  $(L^{t-1}, y^{t-1})$  such that  $\theta^{t-1} \in \Theta^{t-1} \left( \left\{ M_1^{-1}(L_1), \dots, M_{t-1}^{-1}(L_{t-1}) \right\}, y^{t-1} \right) \neq \emptyset$ ,  
and all  $\theta \in \Theta$ , where the history of education  $e^{t-1}$  is inverted from  $L^{t-1}$ . [▶ Back to Main](#)

## Deductibility Scheme with Linear cost

$$-\frac{\partial T_t}{\partial e_t} = \underbrace{(1 - \beta) \sum_{j=1}^{T-t} \beta^{j-1} E_t \left( \frac{u'_{t+j-1}}{u'_t} \frac{\partial T_{t+j-1}}{\partial y'_{t+j-1}} \right) + \beta^{T-t} E \left( \frac{u'_T}{u'_t} \left( \frac{\partial T_T}{\partial y_T} \right) \right)}_{\text{Staggered deductions}} - \underbrace{\sum_{j=1}^{T-t} \beta^j E \left( \frac{u'_{t+j}}{u'_j} \left( \frac{\partial T_{t+j}}{\partial b_{t+j-1}} - \frac{\partial T_{t+j}}{\partial s_{t+j-1}} \right) \right)}_{\text{No arbitrage with Physical capital}}$$

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## General Deductibility Scheme with nonlinear cost

$$\begin{aligned}
 -\frac{\partial T_t}{\partial e_t} = & \sum_{j=1}^{T-t} \beta^{j-1} E_t \left( \frac{u'_{t+j-1}}{u'_t} \frac{\partial T_{t+j-1}}{\partial y_{t+j-1}} \left( M'_{t+j-1} - \frac{1}{R} M'_{t+j} \right) \right) + \beta^{T-t} E_t \left( \frac{u'_T}{u'_t} \left( \frac{\partial T_T}{\partial y_T} \right) \right) \\
 & - \sum_{j=1}^{T-t} \beta^j E_t \left( \frac{u'_{t+j}}{u'_t} \left( (1 - \zeta'_{M',t+j}) E_{t+j-1} (M'_{t+j}) \frac{\partial T_{t+j}}{\partial b_{t+j-1}} - \frac{\partial T_{t+j}}{\partial s_{t+j-1}} \right) \right)
 \end{aligned}$$

- Marginal cost not constant: deduction in period  $t + j$  occurs at dynamic marginal cost effective then  $(M'_{t+j} - \frac{1}{R} M'_{t+j+1})$ , not at “historic” marginal cost at time of the purchase  $M'_t$ .
  - ▶ purchase of  $\Delta e$  at time  $t$  is deducted as  $(M'_{t+j} - \frac{1}{R} M'_{t+j+1})\Delta e$  from  $y_{t+j}$  at  $t + j$ .
- “No-arbitrage” term takes into account differential tax increases from physical capital versus human capital
  - ▶ risk adjusted:
 
$$\zeta'_{M',t+1} \equiv -\text{Cov} \left( \frac{\beta u'_{t+1}}{u'_t} - 1, M'_{t+1} \right) / \left( E_t \left( \frac{\beta u'_{t+1}}{u'_t} - 1 \right) E_t (M'_{t+1}) \right).$$