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A Reexamination of Friedman's Consumption Puzzle

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The different average and marginal consumption propensities estimated from time series data constitute a classic puzzle of the theory of consumption. This article argues that if consumption and income possess a common stochastic trend (and thus are cointegrated), both the average propensity to consume (APC) and the marginal propensity to consume (MPC) will be consistent but biased in small samples. Upon correcting for this small sample bias, the puzzling discrepancies between the APC and the MPC estimated using annual data for the United States from 1897 to 1949 become substantially smaller. This supports an alternative resolution of the puzzle based on the theory of cointegration.

KEY WORDS: Cointegration; Error correction; Finite-sample bias.

1. INTRODUCTION

One of the early empirical observations concerning the relation between aggregate consumption and aggregate income was that the marginal propensity to consume (MPC), measured by regressing time series data on consumption against disposable income, was substantially less than the average propensity to consume (APC), measured by dividing cumulative consumption over several years by cumulative disposable income. These different consumption propensities, taken together with the large "long-run" MPC estimated using Kuznets's (1946) decade-averaged data, constituted one of the important puzzles leading to the development of the permanent-income, relative-income, and life-cycle hypotheses of consumption. Indeed, as the treatments by Dornbusch and Fischer (1984), Hall and Taylor (1986), and Sargent (1979) bear out, posing and explaining this puzzle is still an important part of modern textbook treatments of the theory of consumption.

Friedman's (1957) famous answer to this puzzling difference between the APC and the "short-run" MPC is that permanent, not disposable, income is relevant for determining consumption and that disposable income measures permanent income, but only with error. Thus a regression of consumption on disposable income will encounter the "errors-in-variables" problem in regression analysis; the ordinary least squares (OLS) estimator of the long-run MPC will be inconsistent, understating the true long-run propensity. In contrast, the estimator of the APC will not be subject to this inconsistency, since the transitory component of income will be "averaged out" when comparing average consumption to average income over a long period. Thus the permanent-income hypothesis (PIH) was seen as resolving the puzzle of the differing MPC and APC estimates.

This article reexamines Friedman's explanation of the puzzle of the differing APC and MPC estimates obtained using time series data. The argument has two parts. First, both casual observation and formal tests indicate that disposable income is and has been a stochastically growing process; that is, disposable income contains a unit root in its autoregressive representation. If disposable income grows stochastically, then permanent income will grow stochastically as well. But transitory income, which under the PIH is a short-lived departure of disposable from permanent income, will nonetheless be stationary in the sense of not containing a stochastic or deterministic trend. It follows that permanent income, and thus consumption, will be nonstationary, sharing a common stochastic trend with disposable income. This common trend ensures that both the MPC and the APC will be consistent estimators of the propensity to consume out of permanent income; the differences between observations on disposable income at widely separated dates will overwhelm the contemporaneous short-run differences between permanent and transitory income.

Second, since the PIH implies that consumption and disposable income will share a *common* stochastic trend, these variables will be cointegrated as defined by Engle and Granger (1987). Although OLS estimators in levels regressions, such as a regression of consumption against income, are consistent in cointegrated systems, these estimators can exhibit a substantial bias in small samples. One important source of this bias is that consumption and income are in fact determined simultaneously with other macroeconomic variables, leading to correlation between changes in the regressor (income) and the error term. Because the variables are cointegrated, however, this small-sample bias can be estimated. Accordingly, I use Goldsmith's (1956) annual data from 1897 to 1949 to compute the MPC and APC estimators and their respective bias adjustments. After adjusting for bias, the MPC and APC estimates are substantially closer than are the unadjusted estimates. These results provide quantitative evidence that the PIH and the theory of cointegrated processes appear to resolve the APC–MPC puzzle, although the formal argument involves the small-sample bias of the MPC estimator rather than the inconsistency that is typically invoked in textbook treatments of the problem.

Section 2 reviews the APC–MPC puzzle and examines difficulties with the formal errors-in-variables explanation when income is growing. Section 3 presents the details of an alternative explanation of the puzzle when consumption and income contain a common stochastic trend. Section 4 reports bias-adjusted MPC and APC estimates and the results of formal tests of whether the APC and the MPC differ. The conclusions are summarized in Section 5.

2. FRIEDMAN'S PUZZLE AND DIFFICULTIES WITH HIS EXPLANATION

Before examining Friedman's explanation, I first briefly review the evidence constituting this consumption puzzle. Friedman's analysis of time series data drew on Goldsmith's annual real-consumption and real-disposable-income series from 1897 to 1949. The puzzling MPC's and APC's, computed using Goldsmith's real consumption and income data, are presented in Table 1. Letting C_t and Y_t denote real per capita consumption and real per capita disposable income, the MPC is the OLS slope coefficient obtained by regressing C_t against a constant and Y_t, and the APC is the ratio of average consumption ($\overline{C}$) to average income ($\overline{Y}$):

$$\text{MPC:} \quad \hat{\beta} = \sum_{t=1}^{T} (C_t - \overline{C})(Y_t - \overline{Y}) \Big/ \sum_{t=1}^{T} (Y_t - \overline{Y})^2 \tag{1}$$

and

$$\text{APC:} \quad \overline{\beta} = \sum_{t=1}^{T} C_t \Big/ \sum_{t=1}^{T} Y_t = \overline{C}/\overline{Y}. \tag{2}$$

Table 1. Estimated Average and Marginal Propensities to Consume Using Goldsmith's Data

Period	Consumption measure	APC	MPC
1. 1897–1949	D + ND	.89	.75
2. 1897–1949	ND	.88	.71
3. 1897–1906	ND	.89	.71
4. 1907–1916	ND	.89	.65
5. 1919–1929	ND	.88	.59
6. 1929–1941	ND	.94	.43
7. 1897–1914	ND	.89	.87
8. 1915–1929	ND	.87	.72
9. 1930–1949	ND	.88	.48

NOTE: Aggregate real consumption and real disposable income were computed using Goldsmith's (1956) data according to the note to Friedman's table 12 (1957, p. 126). The population series was taken from Friedman and Schwartz (1982, table 4.8). The subperiods in rows 3–9 are those of Friedman's table 12. Consumption of nondurables is denoted by ND, and consumption of durables is denoted by D. Discrepancies between these entries and those in Friedman's table 12 are typically .00 or .01, and never more than .03. These presumably arise from differences between Goldsmith's published data (used here) and prepublication data (which Friedman used), from differences in the population series, or from rounding errors in the early calculations.

The puzzle is why the MPC estimates are so much less than the APC estimates. Although the Keynesian consumption function $C_t = \alpha + \beta Y_t$ can account for a larger APC than MPC if α is positive, it does not explain the pattern of results in Table 1; were this consumption function correct, one would expect to find stable MPC's but APC's that decline as income rises [since $C_t = \alpha + \beta Y_t$ implies that $\overline{C}/\overline{Y} = (\alpha/\overline{Y}) + \beta$]. In fact, if anything the reverse pattern is found in Table 1. An additional puzzling feature is that the MPC's for each of the subperiods of approximately one decade are less than the MPC for the entire period; the average MPC over rows 3–6 is only .595, but the MPC computed using the entire sample is .71.

Friedman's explanation of the puzzle was, of course, that disposable income measures permanent income (Y_t^p) with error. Supposing the "true" consumption function to be linear with an intercept of 0, he assumed that observed disposable income and consumption can be represented by

$$Y_t = Y_t^p + \eta_{Yt} \tag{3}$$

and

$$C_t = \beta Y_t^p + \eta_{Ct}, \tag{4}$$

where η_{Yt} and η_{Ct} are transitory income and consumption, respectively. Because permanent income is measured with error, Friedman argued, the OLS estimate of the MPC will be biased toward 0. Early formal treatments of Friedman's formulation (e.g., Goldberger 1964) carried this argument one step further, stating that, if Y_t and Y_t^p have finite second moments and if permanent income and the transitory terms are mutually uncorrelated,then $\hat{\beta}$ will be inconsistent:

$$\hat{\beta} = \beta \left[\sum_{t=1}^{T} (Y_t^p - \overline{Y}^p)^2 \Big/ \sum_{t=1}^{T} (Y_t - \overline{Y})^2 \right] + o_p(1)$$
$$\xrightarrow{p} \beta[\operatorname{var}(Y_t^p)/\operatorname{var}(Y_t)] < \beta, \tag{5}$$

where $o_p(1)$ denotes the presence of random terms that tend to 0 in probability as the sample size increases.

This formal explanation of the puzzle provides a textbook example of the errors-in-variables problem in econometrics (e.g., Wonnacott and Wonnacott 1979). Unfortunately, if income is growing, this "inconsistency" explanation is wrong. The intuition behind this argument is simple. Adopt Friedman's model and substitute (3) into the first expression in (5) to obtain

$$\hat{\beta} \cong \beta \frac{T^{-1} \sum_{t=1}^{T} (Y_t^p - \overline{Y}^p)^2}{T^{-1} \sum_{t=1}^{T} (Y_t^p - \overline{Y}^p)^2 + T^{-1} \sum_{t=1}^{T} (\eta_{Yt} - \overline{\eta}_Y)^2}, \tag{6}$$

where the term in the denominator representing the sample covariance between Y_t^p and η_{Yt} has been dropped because this term has mean 0 by assumption. The sample variance of transitory income will tend (in probability) to its finite population variance. In contrast, suppose that the initial value of permanent income is fixed but that permanent income is growing stochastically. Then the sample variance of Y_t^p will grow without bound as the sample size increases. This growing variance of permanent income eventually dominates the sample variance of transitory income in (6); thus $\hat{\beta}$ is consistent for β.

The empirical evidence accords with the view that real per capita disposable income grew stochastically over this period. The OLS estimate of the first-order autocorrelation coefficient for disposable income, presented in the second column of Table 2, is .90. Since this estimator is biased downward in finite samples when the true value is 1 (see Dickey and Fuller 1979), this estimate would not be unexpected were there a unit root.

To provide some formal evidence concerning whether disposable income has a unit root, I followed a procedure suggested by Fuller (1976) and Dickey and Fuller (1979) and used by, for example, Nelson and Plosser (1982). In this framework, the null hypothesis is that income has an autoregressive integrated moving average (ARIMA) representation with no moving average terms [ARIMA(p, 1, 0)]; the alternative is that it has a stationary autoregressive representation [ARIMA (p + 1, 0, 0)] around a deterministic linear time trend. [Said and Dickey (1984) showed that, when the lag length increases with the sample size, the Dickey–Fuller procedure is in fact valid for an ARIMA(p, 1, q) null model, where p and q are of unknown order.] The test consists of estimating the regression equation given in Table 2 and examining the t statistic on δ_1. The asymptotic theory for this statistic is nonstandard under the null of a unit root, and the usual normal or t-distribution tables cannot be used to draw inferences; instead, the correct critical values were tabulated by Fuller (1976). The Dickey–Fuller t statistics testing for a unit root are reported in the final two columns of Table 2. When applied to disposable income, the tests fail to reject the unit-root hypothesis. This finding is consistent with Nelson and Plosser's (1982) inability to reject the hypothesis of a unit root in the logarithm of real per capita gross national product using annual data from 1909 to 1970. Moreover, as the results in the second and third rows of Table 2 indicate, the Dickey–Fuller tests also fail to reject the unit-root hypothesis for either total or nondurables consumption.

Table 2. Tests for Unit Roots in Real per Capita Series, 1897–1949: Tests for Unit Roots in Individual Series

		t statistics on $\hat{\delta}_1$	
Series	$\hat{\rho}_1$	$p = 1$	$p = 2$
Y_t	.90	−2.50	−2.61
C_t (D + ND)	.86	−2.80	−2.66
C_t (ND)	.88	−2.79	−2.59

NOTE: $\hat{\rho}_1$ denotes the first sample autocorrelation. The tests in Table 2 (in which time is included as a regressor) and in Table 3 (in which time is excluded) have different critical values. Rejections are based on the asymptotic critical values tabulated by Fuller (1976, table 8.5.2, p. 373). Regressions are $\Delta z_t - \delta_0 + \delta_1 z_{t-1} + \delta_2 t + \sum_{j=1}^{p} \delta_{2+j} \Delta z_{t-j}$ + error.

These results suggest that, since income appears to have grown stochastically, the "inconsistency" version of the errors-in-variables explanation does not solve the APC–MPC puzzle. Rather, since the PIH implies that permanent and disposable income will grow with a common stochastic trend, the MPC is consistent. This does not, however, mean that the PIH necessarily fails to explain the puzzle.

3. SMALL SAMPLE BIAS AS AN EXPLANATION

The observation that the PIH implies that consumption and income possess a common trend is at once the source of the failure of the formal errors-in-variables solution to the puzzle and the key to a modern restatement of Friedman's solution based on Engle and Granger's (1987) theory of cointegrated stochastic processes. Engle and Granger (1987) defined the vector time series variable X_t to be cointegrated if (a) each of its elements X_{it} has a unit root when considered individually, but (b) there is at least one linear combination of contemporaneous elements of X_t that is stationary in the sense of not containing a unit root. The practical import of cointegration is that one can derive asymptotic distributions of both the OLS estimator of the MPC and the ratio estimator of the APC under the assumption that these variables are cointegrated. These calculations indicate that although both estimators are consistent, they are also potentially biased in small samples. This raises the theoretical possibility that the PIH explains the APC–MPC puzzle, but that the explanation is based on the small-sample bias of estimators in cointegrated systems rather than on the inconsistency implied by the traditional errors-in-variables explanation.

The notion that the PIH implies that consumption and disposable income share a common (possibly stochastic) trend is certainly not new. Indeed, Ando and Modigliani (1963) noted, "When we deal with actual values, the movements of all the variables [aggregate consumption, income, and measures of wealth] are dominated by their common trend" (p. 65). Davidson, Hendry, Srba, and Yeo (1978) constructed a model of postwar consumption in the United Kingdom incorporating this view, and Engle and Granger (1987) provided empirical evidence (using postwar U.S. quarterly data) that consumption and income are cointegrated. Finally, Campbell (1987) used postwar data to test the cross-equation restrictions on the bivariate representation of consumption and income derived from the linear rational-expectations/permanent-income hypothesis, maintaining that consumption and income are cointegrated. None of this recent work, however, directly addresses the original puzzle of the differing MPC and APC estimates.

As Engle and Granger (1987) emphasized, the hypothesis that consumption and disposable income are cointegrated is testable. The first condition for cointegration—that C_t and Y_t individually contain a unit root—was tested in Section 2 by considering Dickey–Fuller tests for a unit root in the real per capita consumption and income series; the unit-root hypothesis could not be rejected for any of the series at the 10% level. The second condition for cointegration—that $C_t - \beta Y_t$ *not* contain a unit root—can be checked by supposing the contrary, that is, by testing the null hypothesis that $C_t - \beta Y_t$ contains a unit root; a rejection is consistent with cointegration. Several new methods have been proposed for performing this test when β is not known a priori; see, for example, Engle and Granger (1987, sec. 5) or Stock and Watson (1986). In the case at hand, however, a simpler approach is available. Specifically, since the values of the MPC implied by the PIH are large, I examine directly the hypothesis that $C_t - \beta Y_t$ is stationary, where β is arbitrarily set to .9, a value typical of those suggested for the long-run MPC. The results of this test, reported in Table 3, indicate that the unit-root hypothesis can be rejected at the 10% level using the Dickey–Fuller tests; that is, $C_t - .9Y_t$ appears not to have a unit root. [Note that this approach has the drawback that C_t and Y_t could be cointegrated with some $\beta \neq .9$, in which case the Dickey–Fuller tests in Table 3 would incorrectly fail to reject the null in large samples. The augmented Dickey–Fuller test for cointegration suggested by Engle and Granger (1987)—in which β is estimated—also rejects noncointegration for the total consumption and income data at the 10% level, however.] To summarize, the test results reported in Tables 2 and 3 are consistent with the hypothesis that real per capita consumption and disposable income are cointegrated, both using total consumption and consumption of nondurables.

I now turn to the task of deriving the asymptotic bias of $\hat{\beta}$ and $\overline{\beta}$. Engle and Granger (1987) presented a general representation of cointegrated variables, denoted jointly by the vector X_t. They assumed that ΔX_t (where $\Delta X_t = X_t - X_{t-1}$) has the vector moving average representation

$$\Delta X_t = F(L)\varepsilon_t, \tag{7}$$

where L is the lag operator, $F(L)$ is a matrix lag polynomial, and ε_t is serially uncorrelated with mean 0 and covariance matrix Σ_ε. Cointegration means that there is some vector α such that $\alpha' X_t$ is stationary, which implies that $\alpha' F(1) = 0$. This vector α is referred to as the cointegrating vector. In the consumption problem,

Table 3. Tests for Unit Roots in Real per Capita Series, 1897–1949: Tests for Cointegration

		t statistics on $\hat{\gamma}_1$	
Series	$\hat{\rho}_1$	$p = 1$	$p = 2$
$C_t - .9Y_t$ (D + ND)	.73	-3.16^a	-3.07^a
$C_t - .9Y_t$ (ND)	.77	-2.81^b	-2.70^b

NOTE: See the note to Table 2. Regressions are $\Delta z_t = \gamma_0 + \gamma_1 z_{t-1} + \sum_{j=1}^{p} \gamma_{1+j} \Delta z_{t-j}$ + error.

[a] Significant at the 5% level.

[b] Significant at the 10% level.

$X_t = (C_t\ Y_t)'$, and the cointegrating vector is $\alpha = (1\ -\beta)'$.

This formulation makes it possible to derive asymptotic representations for the MPC and APC estimators. The details of the derivation for the OLS estimator of the MPC were given in Stock (1987), and the derivation for the APC estimator proceeds along similar lines. These representations are in terms of partial sums of serially and contemporaneously uncorrelated transformations of ε_t. Let $v_t = \Sigma_\varepsilon^{-1/2} \varepsilon_t$, where $\Sigma_\varepsilon^{1/2} \Sigma_\varepsilon^{1/2\prime} = \Sigma_\varepsilon$, and let ξ_t be the integrated process formed by computing the partial sum of the v_t's,

$$\xi_t = \sum_{s=1}^{t} v_s. \tag{8}$$

Moreover, let T denote the sample size, and define the random variables

$$\Gamma_T = T^{-2} \sum_{t=1}^{T} \xi_t \xi_t', \qquad \Psi_T = T^{-1} \sum_{t=1}^{T} \xi_{t-1} v_t'$$

$$\Theta_T = T^{-1/2} \xi_T, \qquad \Lambda_T = T^{-3/2} \sum_{t=1}^{T} \xi_t. \tag{9}$$

The MPC estimator has an asymptotic representation in terms of the random vectors and matrices in (9). Under weak conditions on $F(L)$, the MPC estimator $\hat{\beta}$ will be consistent; when centered at β and scaled by the number of observations, it has the limiting representation

$$T(\hat{\beta} - \beta) = -[D_1'(\Psi_T - \Theta_T \Lambda_T')D_2 + M] \div [D_1'(\Gamma_T - \Lambda_T \Lambda_T')D_1] + o_p(1), \tag{10}$$

where

$$D_1 = -\Sigma_\varepsilon^{1/2\prime} F(1)' e_2,$$

$$D_2 = \Sigma_\varepsilon^{1/2\prime} F^*(1)' \alpha,$$

$$M = -D_1' D_2 - e_2' \sum_{i=0}^{\infty} F_i^* \Sigma_\varepsilon F_i^{*\prime} \alpha,$$

where $F_j^* = -\sum_{i=j+1}^{\infty} F_i$, $e_1' = (1\ 0)$, and $e_2' = (0\ 1)$.

The APC estimator also has a limiting representation in terms of the random variables in (9):

$$T(\overline{\beta} - \beta) = -(D_2' \Theta_T)/(D_1' \Lambda_T) + o_p(1). \tag{11}$$

As the sample size grows, $T(\hat{\beta} - \beta)$ and $T(\overline{\beta} - \beta)$ become well approximated by the respective expressions in (10) and (11). Moreover, these variables have well-defined random limits in terms of functionals of multivariate Brownian motion; for the technical details, related applications, and references, see Chan and Wei (1988), Phillips (1987), Solo (1984), Stock (1987), and Sims, Stock, and Watson (1986). The relevant implication is that (10) and (11) characterize the asymptotic behavior of $\hat{\beta}$ and $\overline{\beta}$.

From the perspective of the APC–MPC puzzle, a key feature of these results is the presence of the constant

M in the numerator of the representation of the MPC estimator—and the absence of any such "correction" term in the representation of the APC estimator. This correction arises because of correlation between (changes in) the regressor and the error term in the regression of C_t on Y_t. The early literature on the consumption function provided two important reasons why disposable income might be correlated with the error term in the MPC regression. The first possibility is simply Friedman's errors-in-variables argument. Specifically, suppose that permanent income is integrated but that it is measured with error as in (3) so that the discussion of Section 2 concerning errors-in-variables bias applies. Since an alternative interpretation of errors-in-variables bias is that the regressor is negatively correlated with the error term, when C_t and Y_t are cointegrated this gives rise to a negative correction term M. A second potential source of correlation is simultaneous-equations bias; since consumption and income are determined simultaneously with other aggregate economic variables, Y_t is not exogenous but rather is correlated with the regression error term. This point was first made by Haavelmo (1943) and was discussed in the context of a Keynesian IS–LM simultaneous-equations system by Ando and Modigliani (1963) and Goldberger (1964). In either of these explanations—errors-in-variables bias or simultaneous-equations bias—the key implication is that Y_t is correlated with the error in the OLS regression equation.

Whatever its source, when C_t and Y_t are cointegrated, this correlation between Y_t and the error introduces a small-sample bias—but not inconsistency—into the OLS estimator. It is thus theoretically possible that the PIH (by way of cointegration) might explain the puzzle not because of estimator inconsistency but because of small sample bias. Moreover, expressions (10) and (11) suggest that the bias will be larger in smaller samples, a feature qualitatively consistent with the subsample estimates of the MPC in Table 1 generally being lower than the estimates over the entire period.

4. EMPIRICAL RESULTS

The asymptotic representations (10) and (11) make it possible to estimate the asymptotic distributions of the MPC and APC and thus to compute bias-adjusted estimators. These asymptotic distributions are typically skewed, so there is some ambiguity about how to perform this bias adjustment. In this section, I calculate adjusted estimators that are asymptotically median-unbiased; that is, the adjusted estimators $\hat{\beta}^*$ and $\bar{\beta}^*$ have the property that $T(\hat{\beta}^* - \beta)$ and $T(\bar{\beta}^* - \beta)$ have asymptotic distributions with median 0.

4.1 Computational Issues

The calculation of the adjusted estimators has two stages. The first stage entails computing consistent estimators of the parameters D_1, D_2, and M that enter the asymptotic representations (10) and (11). Since these terms are functions of $\{F_j\}$ and Σ_ε in (7), estimating D_1, D_2, and M requires estimating (7) subject to the restriction that X_t be cointegrated. Although one can imagine estimating the moving average parameters in (7) directly, Engle and Granger (1987) suggested (and proved the validity of) a far simpler two-step procedure for estimating $F(L)$ and Σ_ε. In the first step, $\hat{\beta}$ [and thus $\hat{\alpha} = (1\ -\hat{\beta})'$] is computed by regression consumption on a constant and income. In the second step, the linear combination $\hat{\alpha}'X_{t-1} = C_t - \hat{\beta}Y_t$ is used as a regressor in estimating (by OLS, equation by equation) the vector error correction model (VECM)

$$\Delta X_t = \mu + A(L)\Delta X_{t-1} - \gamma\hat{\alpha}'X_{t-1} + \varepsilon_t, \quad (12)$$

where $A(L)$ is a matrix lag polynomial of order p, and where μ, γ, $A(L)$, and the covariance matrix of ε_t, Σ_ε, are unknown parameters. Given estimates of these parameters, $F(L)$ can be estimated by computing the response of X_t to unit impulses in ε_t implied by (12). The desired estimates of D_1, D_2, and M are then calculated from the resulting estimate of $F(L)$. In the empirical work reported hereafter, the VECM was estimated with $p = 1$ so that $A(L) = A$. [Conventional likelihood ratio tests failed to reject the null that $p = 1$ in (12) against the alternative that $p = 2$, even at the 50% significance level, using either total or nondurables consumption.]

The second stage in the calculation of $\hat{\beta}^*$ and $\bar{\beta}^*$ involves using the estimates of D_1, D_2, and M and the expressions (10) and (11) to calculate the respective asymptotic distributions of $T(\hat{\beta} - \beta)$ and $T(\bar{\beta} - \beta)$. The asymptotic distributions were computed by Monte Carlo integration using the limiting representations (8)–(11). Finally, the median biases are subtracted from $\hat{\beta}$ and $\bar{\beta}$, resulting in the asymptotically median-unbiased estimators $\hat{\beta}^*$ and $\bar{\beta}^*$.

This bias adjustment procedure involves two approximations concerning nuisance parameters in the process. First, recent investigations of the consumption function have employed logarithmic specifications, largely to correct for evident heteroscedasticity in residuals. The specifications considered here, however, are in levels so that the treatment parallels the historical statistical analyses that generated the APC–MPC puzzles. Second, if consumption and income have a nonzero drift, then $\hat{\beta}$ will be asymptotically normal (e.g., see Sims, Stock, and Watson 1986). The Monte Carlo results of Evans and Savin (1984), however, suggest that the drift needs to be large (relative to the standard deviation of the innovation in the univariate representation of the process) for the asymptotic normality to provide a satisfactory approximation; the smaller the drift, the larger the sample size must be. In the smallest case they consider (Evans and Savin 1984, table II, p. 1252), $T^{1/2}$ times the ratio of the drift to the standard deviation of the first difference of the process is 2.5, and the critical values are generally closer to those of the zero-drift case than to the Gaussian limit. This scaled ratio of 2.5 exceeds the corresponding ratios calculated for Goldsmith's consumption and income series (they are 1.6, 1.8, and 1.8, respectively, for total consumption, nondurables consumption, and disposable income), suggesting that the asymptotic approximation under the zero-drift assumption will be better than under the nonzero-drift assumption.

4.2 Bias-Adjusted Estimates

The APC and MPC estimates are presented in Table 4 for Goldsmith's data. The unadjusted estimates constitute Friedman's puzzle and are taken from the first rows of Table 1. The adjusted estimates ($\bar{\beta}^*$ and $\hat{\beta}^*$) are presented in the second block of Table 4. The adjusted estimates are in substantially better agreement then are the unadjusted estimates; for nondurables, the unadjusted MPC is .71 and the adjusted MPC is .86. Cointegration thus appears to provide an empirical as well as theoretical solution to the puzzle.

The theory of cointegrated processes suggests a third estimator of β that can be obtained by estimating α in a single equation of (12). Specifically, with $p = 1$ the consumption equation in (12) (using α rather than $\hat{\alpha}$) is

$$\Delta C_t = \mu_1 + a_{11}\Delta C_{t-1} + a_{12}\Delta Y_{t-1} - \gamma_1 C_{t-1} + \gamma_1 \beta Y_{t-1} + \varepsilon_{1t}. \quad (13)$$

Thus β can be estimated by the negative of the ratio of the coefficient on Y_{t-1} to the coefficient on C_{t-1} in an OLS regression of (13). Elsewhere (Stock 1987), I showed that this estimator (call it $\tilde{\beta}$) is consistent and, like $\hat{\beta}$ and $\bar{\beta}$, it has a limiting representation in terms of the random variables in (9) with a rate of convergence T. The estimates $\tilde{\beta}$ and their bias-adjusted counterparts $\tilde{\beta}^*$ are also reported in Table 4. The unadjusted estimates fall between $\bar{\beta}$ and $\hat{\beta}$. After bias adjustment, the APC estimates and $\tilde{\beta}^*$ are essentially the same.

The asymptotic representations in Section 3 also provide a way to test formally the null hypothesis that the APC and the MPC are equal against the alternative that they are not. Specifically, define the test statistic $q \equiv T|\hat{\beta} - \bar{\beta}|$. An asymptotic representation for q under the null hypothesis is obtained directly from (10) and (11). This asymptotic distribution in turn makes it possible to compute marginal significance levels (p values) and critical values for the test.

The test statistics and their p values are presented in Table 5 for the entire sample and for Friedman's subsamples. The p value for the test using the entire sample exceeds 45% both for total consumption and for nondurables consumption, presenting little evidence against the null hypothesis that the APC and the MPC are equal. In only one subsample (the Depression) can the hypothesis be rejected at even the 15% level.

Table 4. Bias-Adjusted Consumption Propensities, 1897–1949

Consumption measure	Unadjusted APC($\bar{\beta}$)	Unadjusted MPC($\hat{\beta}$)	Unadjusted $\tilde{\beta}$	Adjusted APC($\bar{\beta}^*$)	Adjusted MPC($\hat{\beta}^*$)	Adjusted $\tilde{\beta}^*$
D + ND	.89	.75	.78	.91	.85	.90
ND	.88	.71	.77	.91	.86	.88

NOTE: The bias adjustments reported in this table and the p values reported in Table 5 were computed using the asymptotic representations discussed in the text. The asymptotic distributions were integrated numerically using 6,000 draws of (Γ_T, Ψ_T, Θ_T, Λ_T) [given in (9) in the text], constructed using bivariate standard random walks with $T = 200$. The random walks were generated using a Gaussian pseudorandom-number generator. For further details concerning this technique, see Stock (1987, sec. 6).

4.3 The Random-Walk Implication of the PIH

These results suggest that, viewed over the entire sample period, cointegration provides an empirical as well as a theoretical explanation of the APC–MPC puzzle. But cointegration is an implication of many models of consumption; theories ranging from Duesenberry's (1952) relative-income hypothesis to Long and Plosser's (1983) real-business-cycle model suggest (at least informally) that if income is growing stochastically, then consumption and income will be cointegrated. Since the preceding empirical results do not distinguish among various theories that imply cointegration, it is of interest to test some stronger implications of the PIH as well. This section therefore concludes by testing Hall's (1978) proposition that, under a simple version of the PIH, consumption will follow a random walk. This hypothesis has received considerable attention using postwar U.S. data (e.g., see Bernanke 1985; Blinder 1981; Flavin 1981). As Engle and Granger (1987) pointed out, (13) provides a convenient framework for testing this hypothesis: If consumption follows a random walk, then the coefficients on lagged differences of consumption and income and on the lagged "error correction" term, $\hat{\alpha}'X_t$, will all be 0.

The coefficients, t statistics, regression F statistics, and selected regression diagnostics for (13) are presented in Table 6. Using either consumption series, the coefficient on the error correction term is negative and differs significantly from 0. In addition, regression F tests reject the hypothesis that the coefficients on all regressors are 0, providing evidence against the random-walk hypothesis. This "error-correction" coefficient has an intuitive economic interpretation; if C_t exceeds βY_t, then consumption in period $t + 1$ is reduced by an amount equal to this coefficient times the previous "overconsumption." For nondurables, this estimated

Table 5. Tests That APC = MPC for Different Sample Periods

Period	Consumption measure	q	Marginal significance level
1. 1897–1949	D + ND	7.39	.454
2. 1897–1949	ND	8.75	.468
3. 1897–1906	ND	1.74	.646
4. 1907–1916	ND	2.45	.668
5. 1919–1929	ND	3.24	.709
6. 1929–1941	ND	6.58*	.011
7. 1897–1914	ND	.39	.725
8. 1915–1929	ND	2.23	.994
9. 1930–1949	ND	7.84	.229

NOTE: See the note to Table 4.
* Significant at the 5% level.

Table 6. Estimated Error-Correction Models of Real per Capita Consumption, 1897–1949: Dependent Variable ΔC_t

Consumption measure	R^2	Regressor: $\hat{z}_{t-1}$	ΔC_{t-1}	ΔY_{t-1}	F(3, 47)	Q(6)
D + ND	.16	$-.383^a$ (−2.66)	−.031 (−.17)	−.077 (−.40)	2.96^b	5.94
ND	.21	$-.330^a$ (−2.61)	−.228 (−1.37)	.042 (.26)	4.14^b	6.70

NOTE: The reported regression is the ΔC_t equation in (12), estimated by OLS. The error correction term is $\hat{z}_t = C_t - \hat{\beta} Y_t$, where $\hat{\beta}$ is the OLS estimator of β. Q(6) denotes the Box–Ljung Q statistic evaluated using the first six sample autocorrelations of the regression residuals. t statistics are given in parentheses.

[a] Significant at the 1% level.

[b] Significant at the 5% level.

reduction is 33% of the previous overconsumption. It should be emphasized that the random-walk implication is derived under the assumption of fixed interest rates; more general specifications of the life-cycle or permanent-income hypotheses could in principle be consistent with the results in Table 6. In addition, the tests were performed using data that represent a time average, or flow, raising the theoretical possibility of spurious rejection as a result of temporal aggregation bias. Nonetheless, the point estimates of the error correction coefficients are large, suggesting a quantitatively important contribution of this lagged error-correction term in predicting future changes in consumption.

5. CONCLUSIONS

These results suggest that although Friedman's PIH provides an empirically consistent explanation of the APC–MPC puzzle, this explanation is more in keeping with Friedman's informal notion of errors-in-variables "bias" than with the formal "inconsistency" argument of the early econometricians. Using the theory of cointegrated processes, it is possible to compute bias-adjusted estimates of the MPC and APC estimators. Although the puzzling unadjusted MPC and APC estimates are, respectively, .71 and .88 for nondurables consumption, after adjusting for bias these estimates become a less puzzling .86 and .91.

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