# Identification of Dynamic Causal Effects in Macroeconomics

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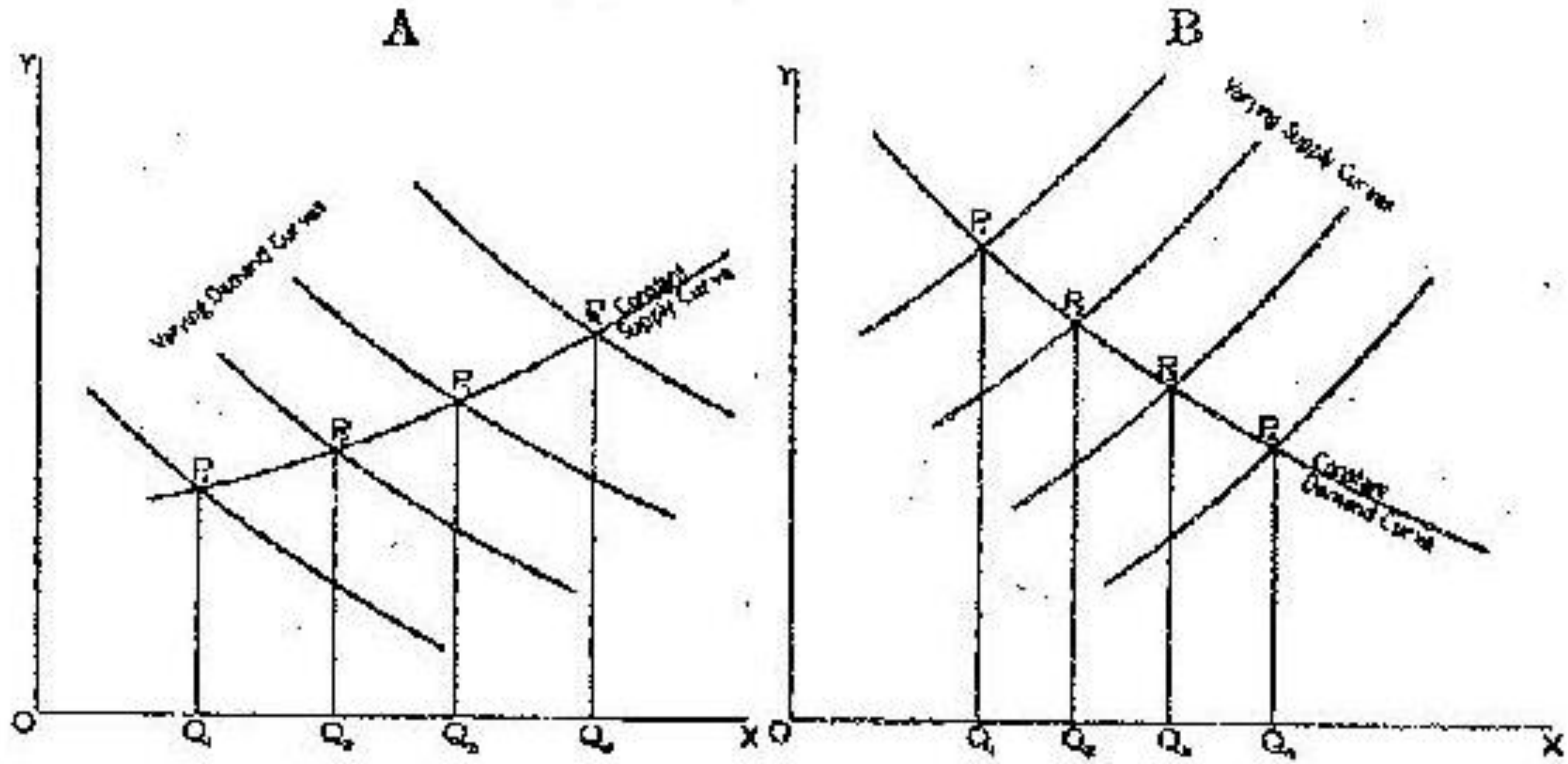
Joint work with Mark Watson, Princeton University

**Sargan Lecture**
**Royal Economic Society**
**Bristol, U.K.**

**April 11, 2017**

FIGURE 3. PRICE-OUTPUT DATA REVEAL—
(A) SUPPLY CURVE
(B) DEMAND CURVE




Source: P.G. Wright, (1928). *The Tariff on Animal and Vegetable Oils,* Appendix B.

Figure on left (and idea of simultaneity bias) appeared in P.G. Wright (*QJE*, 1915)

P.G. Wright, (1928), Appendix B, p. 314.

Supply equation:

$O = eP + S$

where:

- $O$ = output
- $P$ = price
- $S$ = supply disturbance
- $e$ = supply elasticity

$$eA.P = A.O - A.S_1$$

Suppose this multiplication to be performed for every pair of price-output deviations and the results added, then:

$$e\Sigma A.P = \Sigma A.O - \Sigma A.S_1 \quad \text{or} \quad e = \frac{\Sigma A.O - \Sigma A.S_1}{\Sigma A.P}$$

But A was a factor which did not affect supply conditions; hence it is uncorrelated with $S_1$; hence $\Sigma A.S_1 = 0$; and hence $e = \frac{\Sigma A.O}{\Sigma A.P}$.

Similarly if B is a factor, say, yield per acre, which does not affect demand conditions we shall have:

$$\eta = \frac{FH}{FG} = \frac{O - D_1}{P}; \ \eta P = O - D_1; \ \eta \Sigma B.P = \Sigma B.O - \Sigma B.D_1; \ \eta = \frac{\Sigma B.O - \Sigma B.D_1}{B.P}.$$

$$\text{But } \Sigma B.D_1 = 0 \quad \text{Hence } \eta = \frac{\Sigma B.O}{\Sigma B.P}$$

Success with this method depends on success in discovering factors of the type A and B. Several such factors of each type should be used if possible. Because of the slow adjustment of price to marginal cost five-year (or four-year or six-year) averages should be used for P′, O′,

**Philip Wright (1861-1934)**

*Economist, teacher, poet*

MA Harvard, Econ, 1887

Lecturer, Harvard, 1913-1917

**Sewall Wright (1889-1988)**

*genetic statistician*

ScD Harvard, Biology, 1915

Prof., U. Chicago, 1930-1954

# The Wrights’ letters, December 1925 - March 1926

March 4, 1926.

Dear Sewall:

It may interest you to see a very simple geometric demonstration which I have worked out for your method of estimating supply and demand curves without reference to the theory of path coefficients.

• • •

Supply equation:
$O = eP + S$

where:
- O = output
- P = price
- S = supply disturbance
- e = supply elasticity

A is factor uncorrelated with S

$$eP_1 = O_1 - S_1$$

$$eA_1P_1 = A_1O_1 - A_1S_1$$

$$eA_2P_2 = A_2O_2 - A_2S_2$$

$$eA_3P_3 = A_3O_3 - A_3S_3$$

$$\cdots$$

$$e\Sigma AP = \Sigma AO - \Sigma AS$$

$$= \Sigma AO$$ [since A is uncorrelated with S]

$$\therefore e = \frac{\Sigma AO}{\Sigma AP}$$

March 15, 1926

Dear Sewall:

...

| Supply | Real prices[1] [Money price ÷ index no.] | Output[2] (million bu.) | B Acreage[2] (millions) | Yield | B Rainfall[3] (inches) | B Ratio value flaxseed to spring wheat[4] | A Building permits (thousands) |
|---|---|---|---|---|---|---|---|
| 1903 | 126 | 27.3 | 3.23 | 8.4 | 3.40 | 93 | 128 |
| 4 | 153 | 23.4 | 2.26 | 10.3 | 2.19 | 75 | 140 |
| 5 | 123 | 28.5 | 2.53 | 11.2 | 4.27 | 95 | 186 |
| 6 | 126 | 25.6 | 2.51 | 10.2 | 3.30 | 93 | 181 |
| 7 | 133 | 25.9 | 2.86 | 9.0 | 2.66 | 119 | 187 |
| 8 | 157 | 25.8 | 2.68 | 9.6 | 3.38 | 76 | 175 |
| 9 | 204 | 19.7 | 2.08 | 9.5 | 3.10 | 95 | 213 |

...

[1] Average for crop year beginning Sep. 1. The Minneapolis price was divided by wholesale price-index all commodities to get "Real price".

[2] Figures are for calendar year.

[3] Figures are a simple average for rainfall (May, June, and July) for Duluth, Minn., Bismarck, N.D., Pierre, S.D.

[4] The ratios of the values of flaxseed per acre to spring wheat per acre lagged 1 year. i.e. the ratios for the year shown in the table are really the ratios for the preceding year.

## Modern (nonstructural) micro approach

Find a plausibly exogenous source of variation to identify the effect of interest (experiment, natural experiment):

$$Y_{2i} = \theta Y_{1i} + \gamma W_i + u_i$$

Instrument $z$: (i) Relevance: $\text{cov}(Y_1^{\perp}, z^{\perp}) \neq 0$, where $Y_1^{\perp} = Y_1 - \text{Proj}(Y_1 | W)$

(ii) Exogeneity: $E(u|W, z) = E(u| W)$

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## Modern (nonstructural) macro approach

Obtain impulse response function from a structural vector autoregression (SVAR).

$\text{A(L)}Y_t = \nu_t$, $\nu_t | \nu_{t-1}, \nu_{t-2}, ... \sim (0, \Sigma_\nu)$

$\nu_t = \text{H}\varepsilon_t$, $\varepsilon_t$ structural shocks

$y_t = \text{A(L)}^{-1}\text{H}\varepsilon_t$ (IRFs from SVAR)

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$\nu_t = H\varepsilon_t$, $\quad \varepsilon_t$ structural shocks

$y_t = A(L)^{-1}H\varepsilon_t$ $\quad$ (IRFs from SVAR)

## This lecture

- Pull together IV approach to macro shocks
  - Conditions on $z$ for identification of H
  - Conditions on $z$ for identification of dynamic causal effects without a SVAR
- Follow-on: tests of SVAR validity, IV odds & ends, time series odds & ends
- [Are there reasons to prefer local projections over SVARs?]

# Setup

**Structural MA:** $Y_t = \Theta(L)\varepsilon_t$

Structural shock: Define $\varepsilon_{1t}$ = autonomous, unexpected change in $Y_{1t}$

All disturbances: $\varepsilon_t = \begin{pmatrix} \varepsilon_{1t} \\ \varepsilon_{\bullet t} \end{pmatrix}$, $\varepsilon_t \mid \varepsilon_{t-1}, \varepsilon_{t-2}, ... \sim (0, \Sigma_\varepsilon)$ ($\bullet$ = "everything else")

The structural IRF *is* the dynamic causal effect of an autonomous change in $Y_{1t}$ on $Y_{2t+h}$:

$$\Theta_{h,21} = E\left(Y_{2t+h} \mid \varepsilon_{1t} = 1, \varepsilon_{\bullet t}, \varepsilon_s, s \neq t\right) - E\left(Y_{2t+h} \mid \varepsilon_{1t} = 0, \varepsilon_{\bullet t}, \varepsilon_s, s \neq t\right)$$

**SVAR MA**

Wold representation: $Y_t = C(L)\nu_t$, where $\nu_t = Y_t - Y_{t|t-1}$, $\nu_t \mid \nu_{t-1}, \nu_{t-2}, ... \sim (0, \Sigma_\nu)$

MA implied by SVAR: $Y_t = C(L)H\varepsilon_t$

**SVAR MA = structural MA if:** $C(L)H = \Theta(L) \Leftrightarrow$ **$H = C(L)^{-1}\Theta(L)$**

# Interpreting the condition $H = C(L)^{-1}\Theta(L)$

$$H = C(L)^{-1}\Theta(L) = (I + C_1L + \ldots)(\Theta_0 + \Theta_1L + \ldots) = \Theta_0 + \text{terms in } L, L^2, \ldots$$

(1) **Impact effect**: $H = \Theta_0$. *Typically called the SVAR identification condition.*

- Timing restrictions (Cholesky, etc.), long-run restrictions
- Heteroskedasticity
- Sign restrictions
- Direct measurement of shock of interest
- Method of external instruments

## Interpreting the condition **H = C(L)$^{-1}$Θ(L)**

$$\mathbf{H = C(L)^{-1}\Theta(L)} = (I + A_1L + \ldots)(\Theta_0 + \Theta_1L + \ldots) = \mathbf{\Theta_0 + \text{terms in } L, L^2,\ldots}$$

(1) **Impact effect**: $H = \Theta_0$. *Typically called the SVAR identification condition.*

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- Heteroskedasticity
- Sign restrictions
- Direct measurement of shock of interest
- Method of external instruments

(2) **No lagged terms**. $Y_t = C(L)\nu_t$ **and** $Y_t = \Theta(L)\varepsilon_t$, so $\nu_t = C(L)^{-1}\Theta(L)\varepsilon_t$

"No lags": $E(\nu_t \mid \varepsilon_{t-1},\varepsilon_{t-2},\ldots) = 0 \Leftrightarrow E(Y_t \mid Y_{t-1},Y_{t-2},\ldots,\varepsilon_{t-1},\varepsilon_{t-2},\ldots) = E(Y_t \mid Y_{t-1},Y_{t-2},\ldots)$

$\Leftrightarrow$ span($\nu_t$) = span($\varepsilon_t$)

$\Leftrightarrow$ Structural MA is invertible so $\varepsilon_t = \Theta_0^{-1}\nu_t$

- Interpretation: "no omitted variables"
- Called the "**invertibility**" or "**nonfundamentalness**" problem
- There are two main solutions to OVB:
  - Include OVs (large SBVARs, SDFMs, FAVARs, etc.); or
  - IV estimation

# The method of external instruments in SVARs ("SVAR-IV")

- Under the invertibility assumption, $v_t = \Theta_0\varepsilon_t$. The challenge is identifying $\Theta_0$.
- Suppose you have an instrument satisfying:

**Condition A** (i) $E\varepsilon_{1t}z_t = \alpha \neq 0$ (relevance)

(ii) $E\varepsilon_{\bullet t}z_t = 0$ (exogeneity w.r.t. other current shocks)

Then
$$Ev_t z_t = \Theta_0 E\varepsilon_t z_t = \Theta_0 E\begin{pmatrix}\varepsilon_{1t}z_t \\ \varepsilon_{\bullet t}z_t\end{pmatrix} = \Theta_0\begin{pmatrix}\alpha \\ 0\end{pmatrix} = \begin{pmatrix}\Theta_{0,11}\alpha \\ \Theta_{0,\bullet 1}\alpha\end{pmatrix} \qquad (1)$$

Adopt the:

**Unit effect normalization:** $\Theta_{0,11} = 1$

Then, from (1),
$$\frac{Ev_{2t}z_t}{Ev_{1t}z_t} = \frac{\Theta_{0,21}}{\Theta_{0,11}} = \Theta_{0,21}$$

$\Leftrightarrow$ IV estimator of $\Theta_{0,21}$ in: $v_{2t} = \Theta_{0,21}v_{1t} + u_t$ with IV $z_t$

***Unit effect vs. unit standard deviation normalization:*** $\Theta_{0,11} = 1$ or $\text{var}(\varepsilon_{1t}) = 1$?

# The method of external instruments (SVAR-IV), ctd.

1. Estimate VAR: $A(L)Y_t = v_t$

2. Estimate $\Theta_{0,21}$ by IV: $\hat{v}_{2t} = \Theta_{0,21}\hat{v}_{1t} + u_t$ using IV $z_t$

3. Estimate structural MA as $\hat{C}(L)\begin{pmatrix} 1 \\ \hat{\Theta}_{0,\bullet 1} \end{pmatrix}$, where $\hat{C}(L) = \hat{A}(L)^{-1}$

4. SEs by parametric bootstrap (or another method)

**References**

Stock (2008), Stock and Watson (2012), Mertens and Ravn (2012), Gertler and Karadi (2015), Montiel Olea, Stock, and Watson (2017),…

# Example: Gertler-Karadi (2015)

$Y_t = (\Delta \ln IP_t, \Delta \ln CPI_t, \text{1Yr Treasury rate}_t, \text{EBP}_t)$

$\text{EBP}_t$ = Gilchrist-Zakrajšek (2012) Excess Bond Premium

$z_t$ = "Announcement surprise"

= change in 4-week Fed Funds Futures around FOMC announcement windows

Sample period: 1990m1-2012m6 (monthly)

SVAR-IV

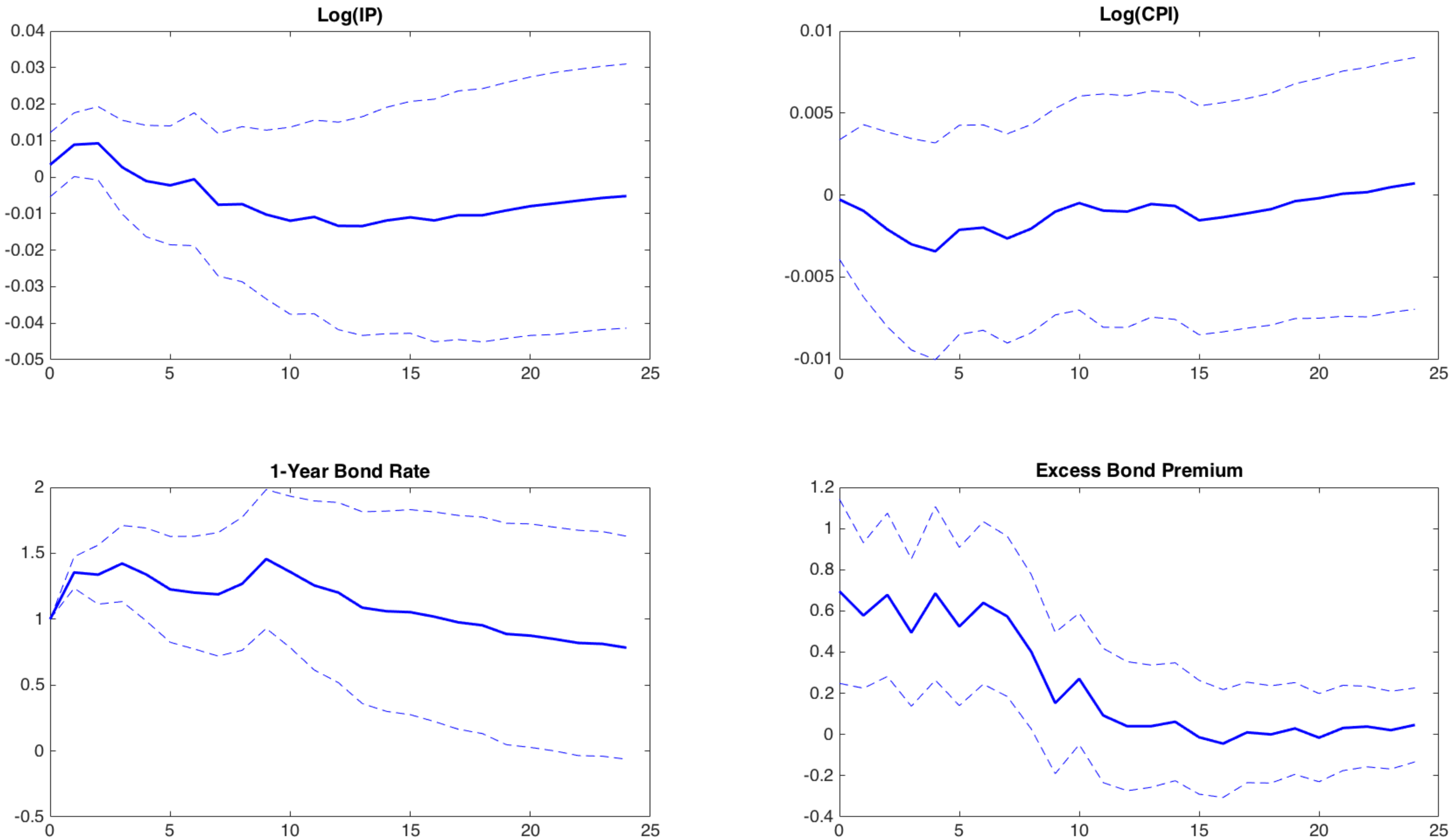
GK specification: 12 lag VAR

LP-IV

$W_t = Y_{t-1},\ldots, Y_{t-4}, z_{t-1},\ldots, z_{t-4}$

# Gertler-Karadi example, ctd.

Cumulative IRFs: **SVAR-IV** with ±1 SE bands



# Identification of structural MA without SVAR step

Structural MA: $Y_t = \Theta(L)\varepsilon_t$

Focus on variables 1 and 2:

$$Y_{1t} = \Theta_{0,11}\varepsilon_{1t} + \{\varepsilon_{\bullet t}, \varepsilon_{t-j}\} \quad (2)$$

$$Y_{2t+h} = \Theta_{h,21}\varepsilon_{1t} + \{\varepsilon_{\bullet t}, \varepsilon_{t+j}, \varepsilon_{t-j}\} \quad (3)$$

*Notation*:

$\{\varepsilon_{\bullet t}, \varepsilon_{t-j}\}$ = linear combination of $\varepsilon_{\bullet t}$ and lags of $\varepsilon$

$\{\varepsilon_{\bullet t}, \varepsilon_{t+j}, \varepsilon_{t-j}\}$ = linear combination of $\varepsilon_{\bullet t}$, lags of $\varepsilon$, and leads of $\varepsilon$

Again use the:

**Unit effect normalization:** $\Theta_{0,11} = 1$

Use (2) with the unit effect normalization to substitute $\varepsilon_{1t} = Y_{1t} - \{\varepsilon_{\bullet t}, \varepsilon_{t-j}\}$ into (3):

$$Y_{2t+h} = \Theta_{h,21}Y_{1t} + \{\varepsilon_{\bullet t}, \varepsilon_{t+j}, \varepsilon_{t-j}\} \quad (4)$$

OLS estimation of (4) suffers from simultaineity <u>and</u> OVB bias.

# Local Projections-IV

$$Y_{2t+h} = \Theta_{h,21} Y_{1t} + u_{t+h}^{(h)}, \text{ where } u_{t+h}^{(h)} = \{\varepsilon_{\bullet t}, \varepsilon_{t+j}, \varepsilon_{t-j}\} \tag{3}$$

Suppose the IV z satisfies:

**Condition B** (i) $E\varepsilon_{1t} z_t = \alpha \neq 0$ (relevance)

(ii) $E\varepsilon_{\bullet t} z_t = 0$ (exogeneity, other current shocks)

(iii) $E\varepsilon_{t+j} z_t = 0, j \geq 1$ ($\Leftarrow$ shocks are mds wrt past $z$, $\varepsilon$)

(iv) $E\varepsilon_{t-j} z_t = 0, j \geq 1$ ($\Leftarrow$ $z_t$ is mds wrt past shocks)

Conditions (ii) – (iv) imply that $Eu_{t+h} z_t = 0$, so with condition (i),

$$E\left(Y_{2t+h} z_t\right) = \Theta_{h,11} E\left(Y_{1t} z_t\right) \Rightarrow \Theta_{h,11} = \frac{E\left(Y_{2t+h} z_t\right)}{E\left(Y_{1t} z_t\right)}$$

- $\Theta_{h,11}$ can be estimated by IV regression of $Y_{2t+h}$ on $Y_{1t}$ using $z_t$ as an instrument
- Including control variables might reduce SEs, but isn't necessary for identification under condition A.

# LP-IV with control variables *W* to relax condition (iv)

$$Y_{2t+h} = \Theta_{h,21} Y_{1t} + \gamma W_t + u_{t+h}^{(h)} \tag{5}$$

where $W_t$ contains past variables (some past $Y$'s, $z$'s). Suppose $z$ satisfies:

**Condition C** (i) Relevance: $\text{cov}(Y_{2t}^{\perp}, z_t^{\perp}) = \alpha \neq 0$, where $Y_2^{\perp} = Y_2 - \text{Proj}(Y_2 | W)$

(ii) Exogeneity: $E\left(u_{t+h} | W_t, z_t\right) = E\left(u_{t+h} | W_t\right)$

Then $\Theta_{h,21}$ can be estimated in (5) by IV using instrument $z$ and control variables $W$.

References:

Local projections (LP)

Jordà (2005) for LP terminology

Local projections-IV (LP-IV)

Owyang, Ramey, and Zubairy (2013), Mertens (2016), Barnichon and Brownless (2017), Jordà, Schularick, and Taylor (2015), Ramey (2016), Ramey-Zubairy (forthcoming);

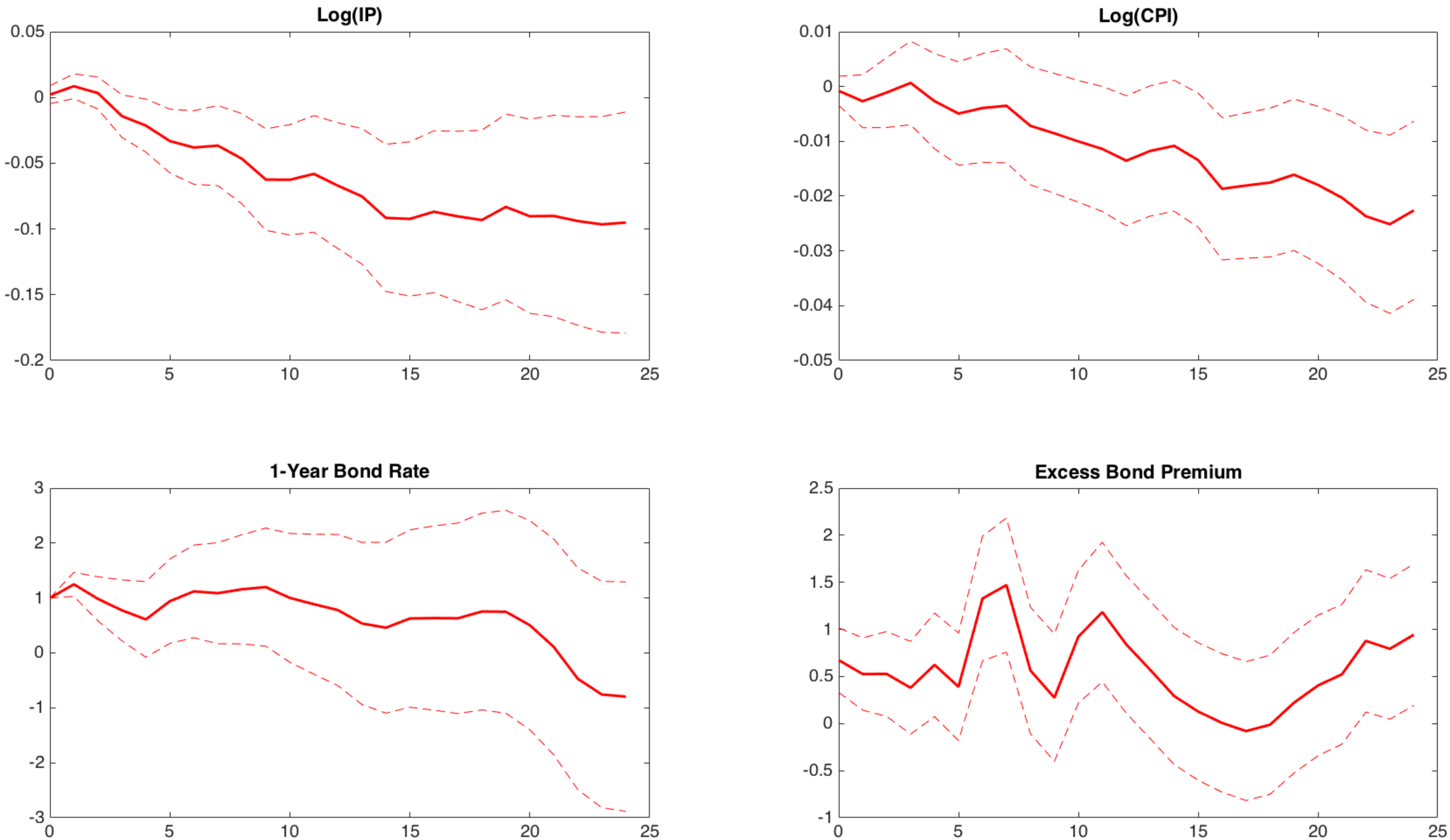
System estimation of structural MA without SVAR step

Plagborg-Møller (2016)

# Gertler-Karadi example, ctd.

Cumulative IRFs: **LP-IV** with ±1 SE bands

$W$ = 4 lags of $Y$, $z$



# SVAR-IV and LP-IV: Open questions and reminders

LP-IV: $Y_{2t+h} = \Theta_{h,21} Y_{1t} + \gamma W_t + u_{t+h}$ using IV $z_t$

Condition C (i) Relevance: $\text{cov}(Y_{2t}^{\perp}, z_t^{\perp}) \neq 0$

(ii) Exogeneity: $E\left(u_{t+h} \mid W_t, z_t\right) = E\left(u_{t+h} \mid W_t\right)$

## Open questions

1. If condition B(iv) fails, what are suitable control variables?
2. Is LP-IV IV robust to non-invertibility?
3. Can LP-IV and SVAR-IV be used to test for invertibility?
4. HAR inference – anything noteworthy?
5. LP-IV specification: Levels or first differences?
6. What if the instrument is weak?
7. How to handle news shocks?

## Reminders

1. IV estimation of distributed lag, AR-distributed lag specifications yields correct impact effect but incorrect dynamics in general
2. SVAR-IV is more efficient than LP-IV, if correctly specified
3. Potentially can improve LP-IV efficiency by imposing smoothness

## Q1. If condition A(iv) fails, what are suitable control variables?

$$Y_{2t+h} = \Theta_{h,21} Y_{1t} + \gamma W_t + u_{t+h}^{(h)}$$

**Condition C** (i) Relevance: $\text{cov}(Y_{2t}^{\perp}, z_t^{\perp}) = \alpha \neq 0$

(ii) Exogeneity: $E\left(u_{t+h} \mid W_t, z_t\right) = E\left(u_{t+h} \mid W_t\right)$

A sufficient condition for C(ii) is that Conditions B(ii) and B(iii) hold and that $W_t$ spans $\left\{\varepsilon_{t-1}, \varepsilon_{t-2}, \ldots\right\}$. Then

$$E\left(u_{t+h}^{(h)} \mid W_t, z_t\right) = E\left(\left\{\varepsilon_{t+h}, \ldots, \varepsilon_{t+1}, \varepsilon_{\bullet t}, \varepsilon_{t-1}, \ldots\right\} \mid \varepsilon_{t-1}, \ldots, z_t\right)$$

$$= E\left(\left\{\varepsilon_{t+h}, \ldots, \varepsilon_{t+1}\right\} \mid \varepsilon_{t-1}, \ldots, z_t\right) + E\left(\left\{\varepsilon_{\bullet t}\right\} \mid \varepsilon_{t-1}, \ldots, z_t\right) + E\left(\left\{\varepsilon_{t-1}, \ldots\right\} \mid \varepsilon_{t-1}, \ldots, z_t\right)$$

$$= E\left(\left\{\varepsilon_{t-1}, \ldots\right\} \mid \varepsilon_{t-1}, \ldots\right) = E\left(u_{t+h}^{(h)} \mid W_t\right)$$

Remarks

1. This (perhaps) suggests using generic instruments – e.g. factors from a DFM
   - But assuming condition C(ii) is satisfied using $W_t = Y_{t-1}, \ldots$ is equivalent to assuming span($\varepsilon_t$) = span($\nu_t$) – that is, the SVAR is invertible.
     - If invertibility fails, then LP-IV using $W_t = Y_{t-1}, \ldots$ will be inconsistent.
     - And if you *can* span $\varepsilon_t$, you might as well use SVAR-IV!

# Q1. If condition A(iv) fails, what are suitable control variables? (ctd)

Remarks, ctd.

2. In some cases, it should be possible to construct valid control variables using application-specific knowledge.
   - Announcement-day monetary shocks
   - Political disruptions (wars) as oil supply shocks
   - Legislation on fiscal policy

Toy example (shock that drags out over two perioids)

Observe $z_t = \zeta_t + b\zeta_{t-1}$, where $\zeta_t$ satisfies condition *B*.

Then $z_t$ violates condition B(iv):

$$E\left(\varepsilon_{1t-1} z_t\right) = E\left[\varepsilon_{1t-1}(\zeta_t + b\zeta_{t-1})\right] = bE\left(\varepsilon_{1t-1}\zeta_{t-1}\right) = b\alpha$$

But if $(1 + bL)$ is invertible, then Condition C(ii) holds with $W_t = (1+bL)^{-1} z_{t-1}$

Implications:

1. Looking for generic instruments only leads you back to SVAR-IV
2. The instrument mds condition – or something close – is critical to valid inference

# Q2. Is LP-IV robust to non-invertibility?

Yes, under Conditions B or C.

Under Condition A:

$$\hat{\Theta}_{h,1}^{SVAR-IV} = \hat{C}_h \frac{\sum y_t^{\perp} z_t^{\perp}}{\sum \hat{v}_{1t} z_t^{\perp}} \xrightarrow{p} C_h \Theta_{0,1}, \text{ where } C(L) = A(L)^{-1}$$

whereas under condition B or C,

$$\hat{\Theta}_{h,1}^{LP-IV} = \frac{\sum Y_{t+h}^{\perp} z_t^{\perp}}{\sum Y_{1t}^{\perp} z_t^{\perp}} \xrightarrow{p} \Theta_{h,1}$$

In general $C_h \Theta_{0,1} \neq \Theta_{h,1}$ if $\Theta$(L) is not invertible.

## Q3. Can LP-IV and SVAR-IV be used to test for invertibility?

Yes, under condition B or C.

Consider a **near-invertible local alternative**: $C(L)^{-1}\Theta(L) = \Theta_0 + T^{-1/2}\delta(L)L$

so

$$\nu_t = \Theta_0\varepsilon_t + T^{-1/2}\delta(L)\varepsilon_{t-1} \text{ and } \Theta_h = C_h\Theta_0 + d_h/\sqrt{T} \;.$$

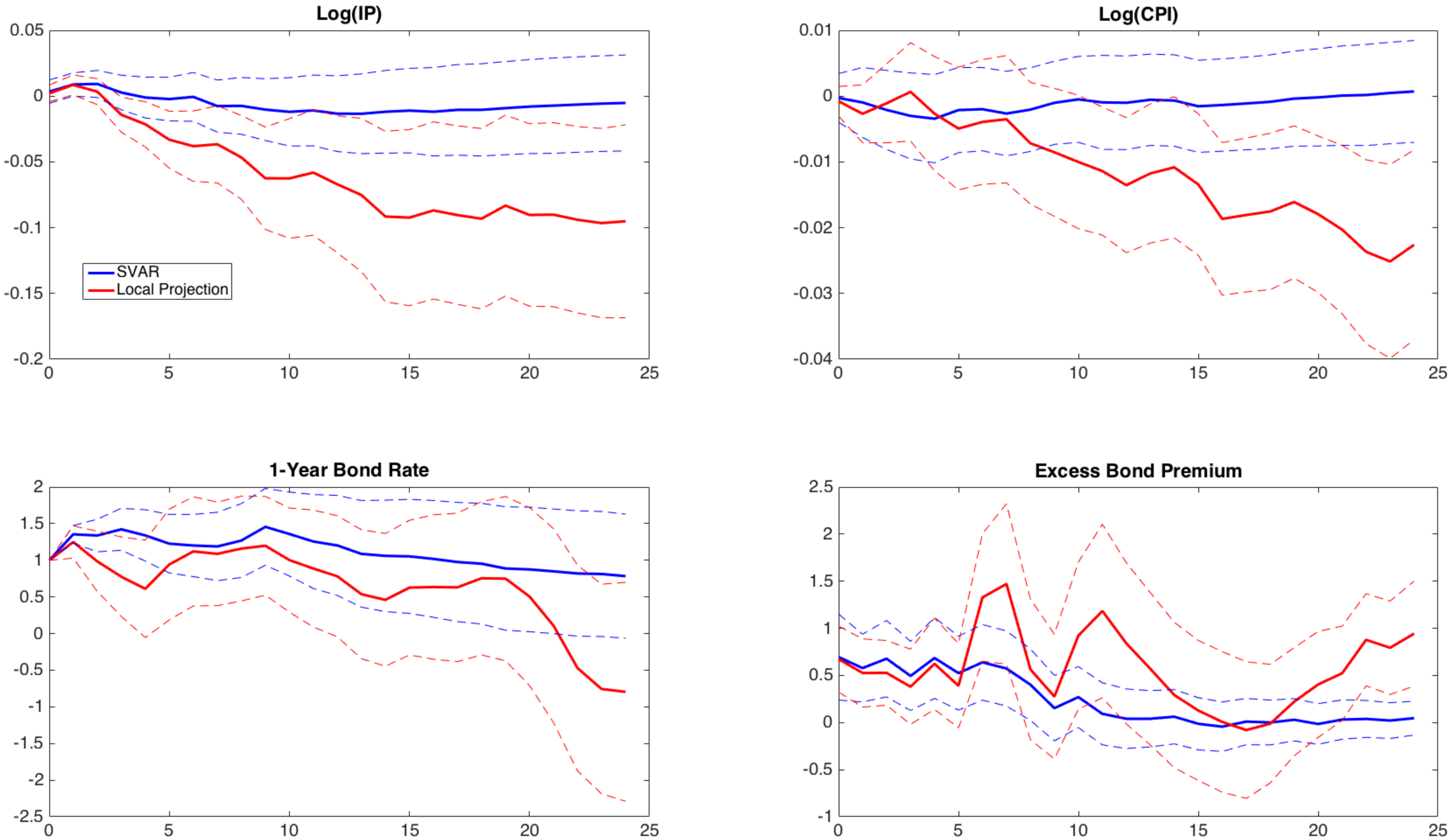
Then

$$\Psi_T = \sqrt{T}\left(\hat{\Theta}_{h,1}^{SVAR-IV} - \hat{\Theta}_{h,1}^{LP-IV}\right) = \frac{\frac{1}{\sqrt{T}}\sum\left[Y_{t+h}^{\perp} - \hat{C}_h Y_t^{\perp}\right]z_t^{\perp}}{\frac{1}{T}\sum \hat{\nu}_{1t} z_t^{\perp}} \xrightarrow{d} N\left(d^h\alpha, V_h\right)$$

- Test for mis-specification of VAR, in the spirit of a Hausman test
- This test based on $\Psi_T$ differs from other invertibility tests in the literature, which test predictability of VAR forecast errors.
- This tests both predictability and multistep v. direct forecast coefficients, and does not require an invertible SVAR to exists (just a structural MA).

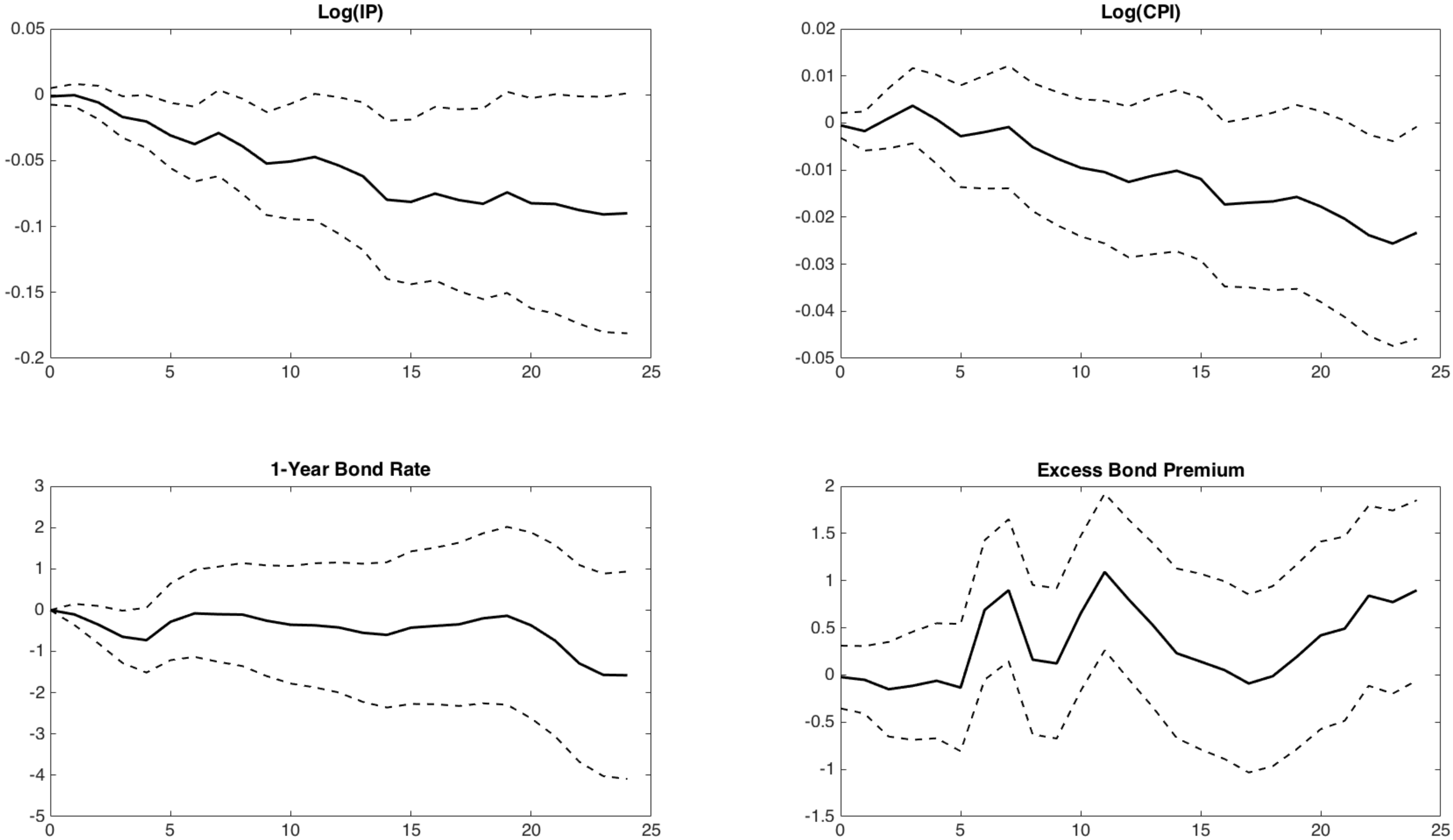
# Gertler-Karadi example, ctd.

Cumulative IRFs: **SVAR-IV** and **LP-IV** and ±1 SE bands (parametric bootstrap)



# Gertler-Karadi example, ctd.

Test statistics by horizon by variable: entries are *t*-statistics $\Psi_T / \sqrt{\hat{V}_h}$



## Q4. HAR inference – anything noteworthy?

- HAR inference is needed for LP-OLS (standard LP method) – standard direct multiperiod ahead regression problem.

- But HAR isn't needed for SVAR-IV under condition B (mds property of $z_t$)

## Q5. LP-IV specification: Levels or first differences?

Consider estimation of cumulative causal effect:

In levels: $$Y_{2t+h} = \Theta_{h,21} Y_{1t} + \gamma W_t + u_{t+h}^{(h)}$$

In first differences: $$\Delta Y_{2t+h} + \ldots + \Delta Y_{2t+1} = \Theta_{h,21} Y_{1t} + \gamma W_t + u_{t+h}^{(h)}$$

Suppose $Y_{1t}$ and $Y_{2t}$ are persistent (e.g. local to unit root) and Condition B holds:

- If there is no $W_t$,, then for both the levels and first differences specifications:
  - Nonstandard distributions at all horizons
  - Not resolved by including linear time trend

- If $W_t$ includes $Y_{i\text{-}1}$,…:
  - Levels and cumulated differences specifications of $Y_{2t}$ are equivalent
  - For $h$ s.t. $h/T \to \lambda > 0$, distribution of LP-IV is mixture of normals, with mean zero (heavy tailed)

## Q6. What if the instrument is weak?

$$Y_{2t+h} = \Theta_{h,21} Y_{1t} + \gamma W_t + u_{t+h}^{(h)}$$

We have a rich set of tools to handle weak instruments.

- Weak IV biases towards OLS – which here is bias towards Cholesky with shock 1 ordered first!
  - This true in both SVAR-IV and LP-IV (Montiel-Olea, Stock, and Watson)

- Single horizon weak-instrument robust inference
  - Single instrument: Anderson-Rubin (efficient if homoskedastic)
  - Multiple instruments: CLR (nearly efficient if homoskedastic)

The literature is aware of the weak IV possibility

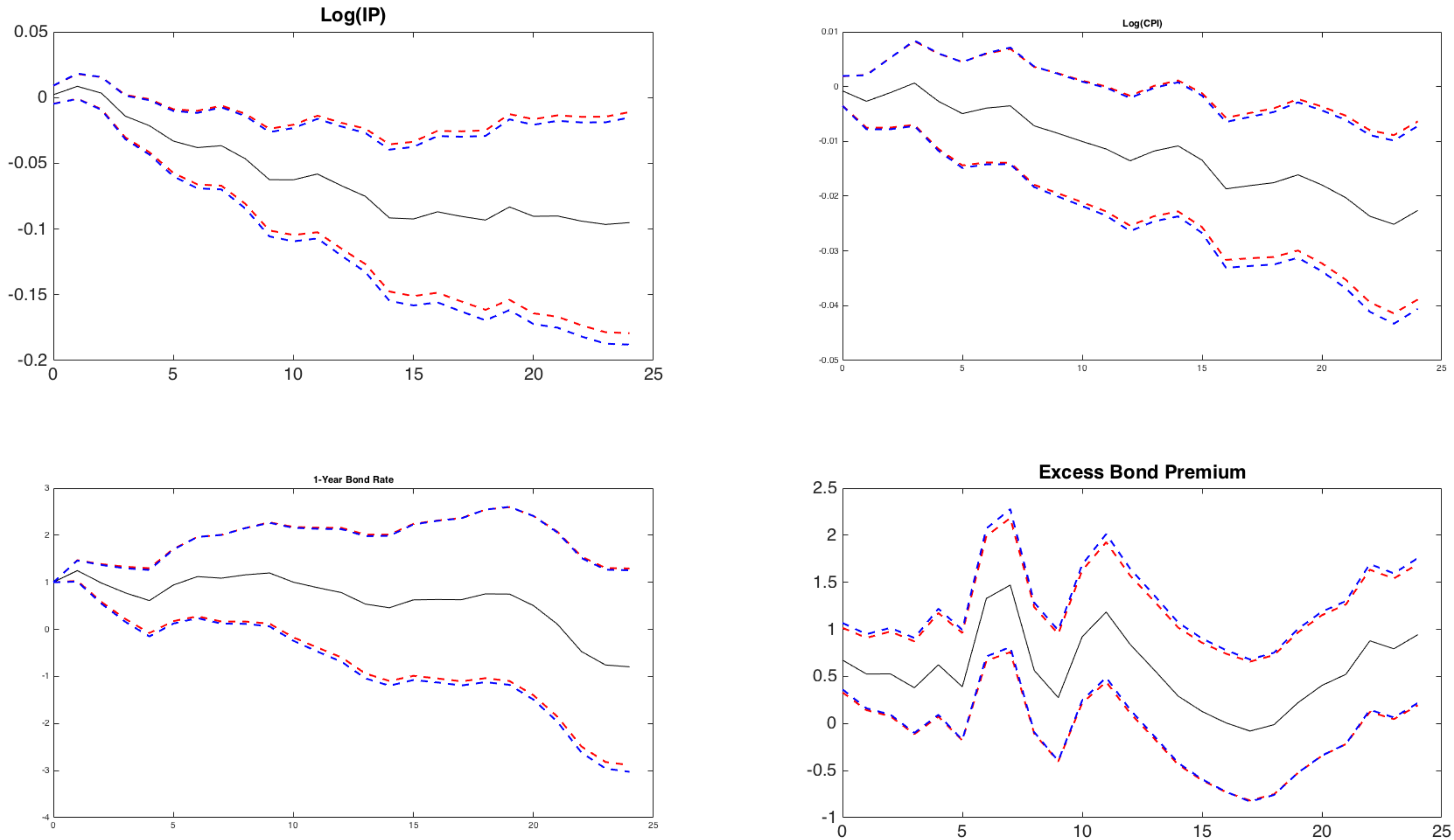
Stock and Watson (2012), Gertler-Karadi (2015); Ramey (2016)

Gertler-Karadi example

- First stage $F$ = 15.9 (SVAR-IV) and $F$ = 23.7 (LP-IV)
- Anderson-Rubin confidence intervals…

# Gertler-Karadi example, ctd.

LP-IV 68% bands: **±1 SE** and **Anderson-Rubin Confidence Interval**



## Q7. How to handle news shocks?

Essentially this just requires a change to the unit effect normalization.

<u>Example</u>

$\varepsilon_{1t}$ is a productivity shock (invention)
$z_t$ is news about that invention
$\varepsilon_{1t}$ affects observed TFP with a lag
$\varepsilon_{1t}$ affects consumption today via present value of future output

$Y_{1t} = \Delta\ln TFP_t = \Theta_{1,12}\varepsilon_{1t\text{-}1}$ + lags and other shocks
$Y_{2t} = \Delta\ln Consumption_t = \Theta_{0,12}\varepsilon_{1t} + \Theta_{1,12}\varepsilon_{1t\text{-}1}$ + lags and other shocks

The unit effect normalization fails (impact effect on TFP growth is 0), and $z_t$ is an irrelevant (weak) instrument for $\eta_{1t}$.

A 1-lag unit effect normalization succeeds: $\Theta_{1,12} = 1$

- A unit shock to $\varepsilon_{1t}$ increases TFP next period by 1 unit.
- All parts of conditions B and C still hold.
- The scaling for the IV regression is $E\eta_{1t+1}z_t$
- The MA need not be invertible (news shock literature)

# Reminders

1. IV estimation of distributed lag, AR-distributed lag specifications generally yields correct impact effect but incorrect dynamics.

Distributed lag: $Y_{2t} = \Theta_{21}(L)Y_{1t} + \{\varepsilon_{\bullet t}, \varepsilon_{t-j}\}$

ADL: $Y_{2t} = \Theta_{21}(L)Y_{1t} + \rho(L)Y_{2t-1} + \{\varepsilon_{\bullet t}, \varepsilon_{t-j}\}$

- Even under condition B, $z_{t-j}$ is correlated with $\varepsilon_{t-j}$, so $Eu_t z_{t-j} \neq 0$.

2. SVAR-IV is more efficient than LP-IV, if correctly specified

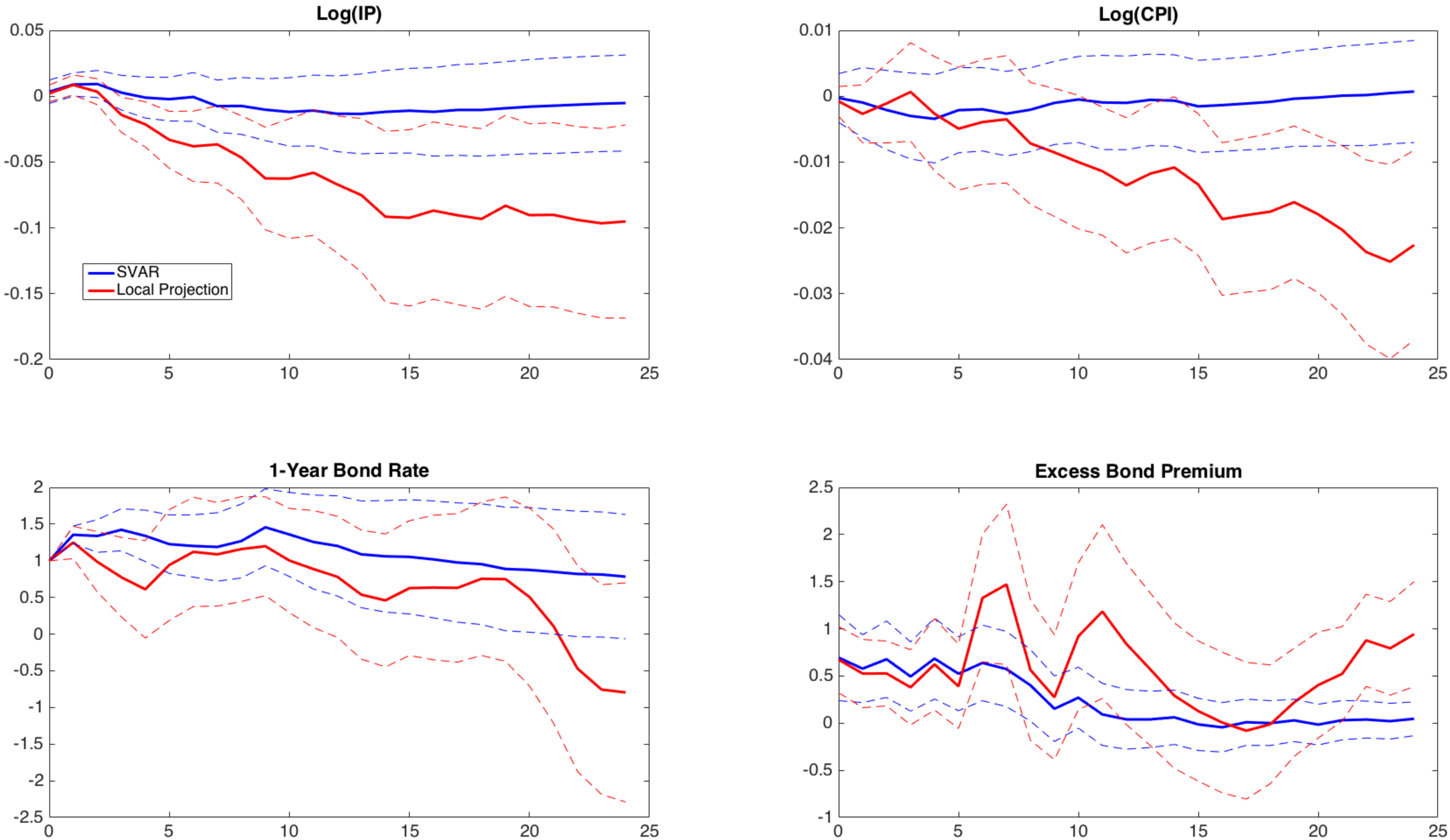
Reference: Kim and Kilian (2011) for simulations; standard IV and VAR results for first-order asymptotics (e,g, Lütkepohl (2005))

3. Potentially can improve LP-IV efficiency by imposing smoothness

References: Barnichon and Brownless (2017), Plagborg-Møller (2016)

# Gertler-Karadi example, ctd.

Cumulative IRFs: **SVAR-IV** and **LP-IV** and ±1 SE bands (parametric bootstrap)



Microeconometric IV methods carry over to macro

- arguably yielding more credible inference on (dynamic) causal effects;

The "dynamic" part requires some additional restrictions (e.g. $z_t$ mds);

Well-known lessons about IVs from microeconometrics also carry over; and

These lessons aren't new…

# The first IV regression (March 15, 1926)

| | | | B | | B | B | A |
|---|---|---|---|---|---|---|---|
| Supply | Real prices [1] [Money price ÷ index no.] | Output [2] (million bu.) | Acreage [2] (millions) | Yield | Rainfall [3] (inches) | Ratio value Flaxseed to Spring wheat [4] | Building permit (thousands) |
| 1903 | 126 | 27.3 | 3.23 | 8.4 | 3.40 | 93 | 128 |
| 4 | 153 | 23.4 | 2.26 | 10.3 | 2.19 | 75 | 140 |
| 5 | 123 | 28.5 | 2.53 | 11.2 | 4.27 | 95 | 186 |
| 6 | 126 | 25.6 | 2.51 | 10.2 | 3.30 | 93 | 181 |
| 7 | 133 | 25.9 | 2.86 | 9.0 | 2.66 | 119 | 187 |
| 8 | 157 | 25.8 | 2.68 | 9.6 | 3.38 | 76 | 175 |
| 9 | 204 | 19.7 | 2.08 | 9.5 | 3.10 | 95 | 213 |

…

1/ Average for crop year beginning Sep. 1. The Minneapolis price was divided by wholesale price-index all commodities to get "Real price".

2/ Figures are for calendar years.

3/ Figures are a simple average for rainfall (May, June, and July) for Duluth, Minn., Bismarck, N.D., Pierre, S.D.

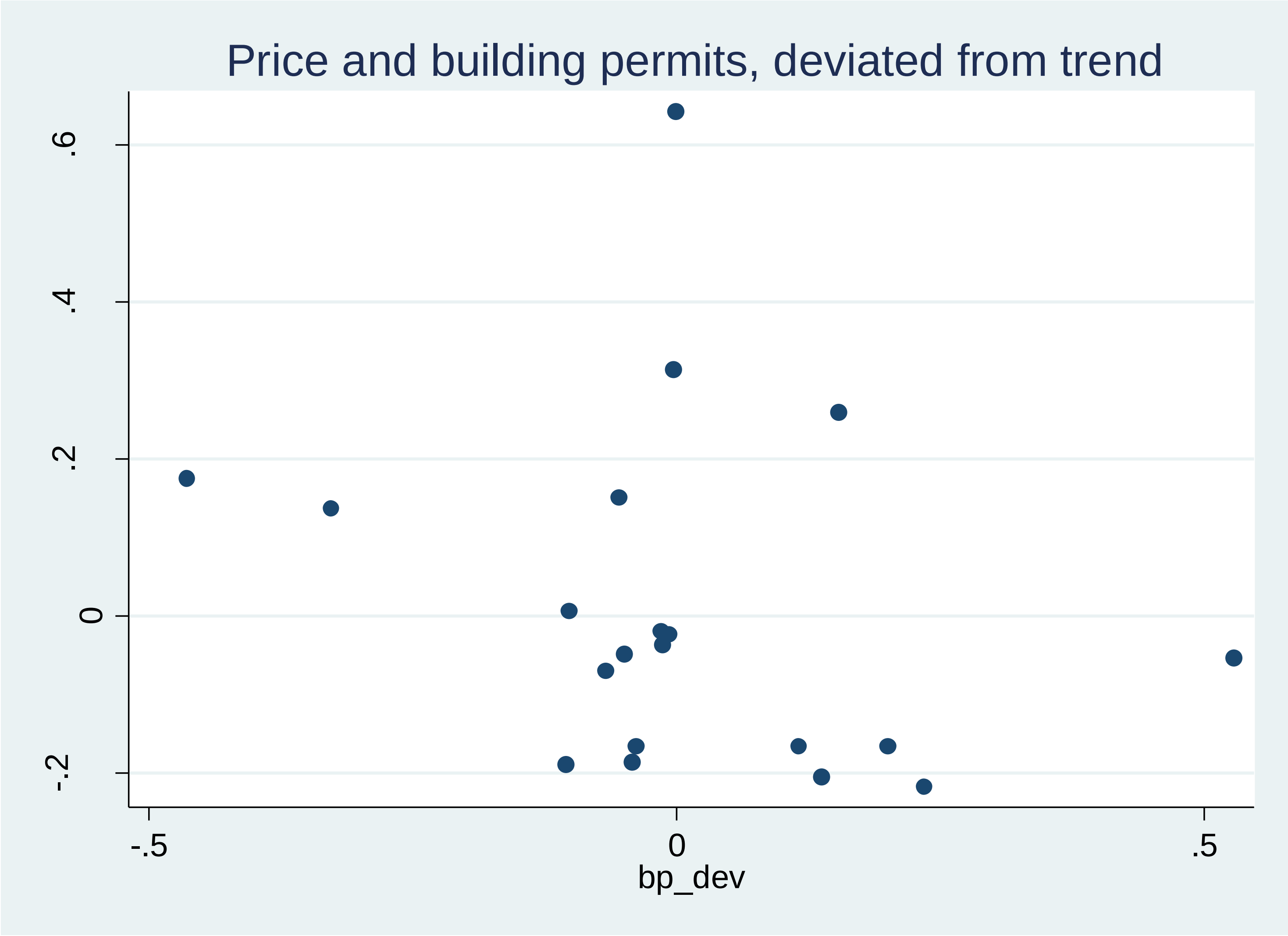
4/ The ratios of the values of flaxseed per acre to spring wheat per acre lagged 1 year. i.e. the ratios for the year shown in the table are really the ratios for the preceding year.

# P.G. Wright's flaxseed price and output data

- Prices are Minneapolis fall prices; annual data, 1904-1923, % deviation from trend
- *z* = building permits on East coast

Estimated **supply** elasticity = -0.76

First stage *F* = 1.25



# PG Wright to Sewall Wright, March 15, 1925

of the economic "universe".

The problem, therefore, boils down to this: In the case of any specific commodity is it possible to find factors which have such distinct causal relations with ~~output~~ or demand conditions that the values of e and y computed from them can be accepted with any confidence as having any relation with actuality. Such factors, I fear, especially in the case of demand conditions, it is not easy to find. I have been experimenting with flaxseed and so far have arrived at no results in which I can place much confidence.

The most likely data which I have been able to secure

MARCH 15 1926 →

## *The IV regression he never computed…*

**Wright 1925 data: demand estimation using rainfall in upper Midwest**

$z$ = rainfall in Minnesota, Wisconsin, North Dakota

IV estimate of demand elasticity = -0.52 (SE = 0.15)

First stage $F$ = 12.8