Price Dispersion in Stationary Networked Markets*

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Abstract

Different sellers often sell the same good at different prices. Using a strategic bargaining model, I characterize how the equilibrium prices of a good depend on the interaction between its sellers' costs, its buyers' values, and a network capturing various frictions associated with trading it. In contrast to the standard random-matching model of bargaining in stationary markets, I allow agents to strategically choose whom to make offers to, which qualitatively changes how the network shapes prices. As in the random-matching model, the market decomposes into different submarkets, and—in the limit as bargaining frictions vanish—the law of one price holds within but not across them. But strategic choice of partners changes both *how* the market decomposes into different submarkets and the determinants of each submarket's price.

Price dispersion is pervasive in both online and offline markets.[1] Several frictions—which effectively restrict who can trade with whom—are behind this widespread phenomenon. Perhaps most prominent among these are informational frictions. As George Stigler put it (Stigler, 1961, p. 214),

> Price dispersion is a manifestation—and, indeed, it is the measure—of ignorance in the market.

In this paper, I take the trading frictions associated with trading a good as given, and I investigate exactly how they translate into price dispersion in a decentralized bargaining

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[1] See for example Kaplan, Menzio, Rudanko, and Trachter (2017).

setting. For simplicity, and in contrast to the search-theoretic approach pioneered by Stigler (1961), I take these frictions to be binary: Two agents can either trade with no frictions (e.g., because they know each other), or they cannot trade at all.[2]

I consider different buyers and sellers of a homogeneous good (with different values and costs, respectively) who strategically bargain over both whom to trade with, and their terms of trade. An exogenous network describes which buyer-seller pairs can trade with each other. The focus is on situations in which the agents enter the market over time in such a way that their trading opportunities are stationary.[3] This is intended to approximate the predominant economic forces in large markets where the relevant trading opportunities are roughly constant over time.

The main innovation with respect to the literature on bargaining in stationary networks (e.g., Manea 2011, Nguyen 2015, Polanski and Vega-Redondo 2018) is that I allow the agents to strategically choose whom to make offers to. Strategic choice of partners is a natural element of bargaining, and—as I discuss below—it substantially affects how the network shapes agents' bargaining power. In particular, in contrast to the random-matching model of Manea (2011), trading frictions are *not* sufficient to explain price dispersion: Only the *interaction* between heterogeneities in values or costs and the incompleteness of the network structure can generate deviations from the law of one price.

I start by showing that—as in the canonical bilateral alternating-offers model of Rubinstein (1982)—the process of iterated conditional dominance determines the terms of trade in each part of the network.[4] In particular, the model admits an essentially unique subgame-perfect equilibrium: Each agent has a *cutoff price* such that she always accepts trades at prices that are at least as good for her as her cutoff price. The cutoff price of a buyer (seller) is determined by her value (cost) and the price at which she can trade in equilibrium when she is the proposer—that is, the lowest (highest) cutoff price among her partners. This implies that the bargaining power of an agent does not depend on the bargaining power of all of her partners. Rather, it is determined by the minimum or the maximum among the cutoff prices of all of her partners.[5] This contrasts with the prediction of random-matching models,

[2]See Manea (2016) for an excellent recent survey of related models of bilateral trade in networks.

[3]The literature on non-cooperative bargaining in stationary markets goes back to Rubinstein and Wolinsky (1985). See for example Gale (1987), Binmore and Herrero (1988), de Fraja and Sákovics (2001) and Lauermann (2013).

[4]Iterated conditional dominance similarly pins down the terms of trade in the analogous random-matching model of Manea (2011) (see Manea 2017a).

[5]This is a manifestation of the *outside option principle* (e.g., Sutton 1986), which states that only credible outside options matter in bargaining. Indeed, in the present model, any threat by a buyer (seller) that involves

and suggests that the economic forces that determine the terms of trade in each part of the network are more local in nature than these models suggest.[6]

The result that drives the equilibrium characterization is that each agent's cutoff price can be bounded using a local network statistic—her *best price*—which is her equilibrium cutoff price in the hypothetical situation in which she can choose one of her partners and bargain bilaterally with her *in isolation*.[7] While this bound is typically not tight, I show that it is tight at the extremes: The best price of the buyer with the highest best price is her cutoff price and, analogously, the best price of the seller with the lowest best price is her cutoff price. This implies that all the agents' cutoff prices converge to the same price in the limit as bargaining frictions vanish if and only if the buyer with the highest limit best price is connected to the seller with the lowest limit best price. In particular, the condition that the buyer with the highest value is connected to the seller with the lowest cost is necessary, but not sufficient, for the law of one price to hold.

Unlike the extreme market prices, the intermediate prices cannot be identified at a glance from the agents' best prices. However, I describe an algorithm that decomposes the trading network into different components (or *submarkets*), with the property that the law of one price holds—in the limit as bargaining frictions vanish—within submarkets but not across them. This algorithm works by first identifying the buyer with the highest limit best price (who determines the price of the submarket with the highest price), and then identifying the set of all the traders who, in equilibrium, also end up trading in the submarket with the highest price. Repeated application of this algorithm (after removing from the network all the agents that trade in the submarkets previously identified) finds the price and all the traders in each submarket.[8]

The network decomposition algorithm that I describe in this paper uncovers the role that each trading relationship plays in shaping price dispersion in decentralized markets. For

making offers to anyone but one of her partners with the lowest (highest) cutoff price are not credible—and hence do not affect anyone's bargaining power. Binmore, Shaked, and Sutton (1989) provide experimental evidence that is consistent with this principle.

[6]Under random matching, the fact that an agent is exogenously matched to bargain with positive probability with each of her partners implies that the number of partners with whom she can profitably trade in equilibrium affects her bargaining power.

[7]The idea of characterizing the equilibrium by considering these hypothetical situations is inspired by Elliott and Nava (forthcoming), who show that an efficient Markov-perfect equilibrium exists in their (thin-market) setting for sufficiently small bargaining frictions if and only if the payoffs that result from bilateral bargaining between the efficiently matched pairs are in the core of the market.

[8]Repeated application of the analogous algorithm for sellers provides the same decomposition (from the submarket with the lowest price up instead of the submarket with the highest price down).

instance, using this algorithm, I illustrate that, while a reduction in trading frictions (i.e., adding new links to the trading network) cannot increase the gap between the highest and the lowest market price, it can increase the number of different prices at which the good is traded at in equilibrium. This suggests that the relationship between price dispersion and ignorance is not as straightforward as Stigler's (1961) quote above suggests.

The rest of this paper is organized as follows. After presenting the model in section 1, I describe the essentially unique subgame-perfect equilibrium for arbitrary bargaining frictions in section 2, and I characterize the limit equilibrium terms of trade in section 3. I further discuss the contribution of this paper in the context of the related literature in section 4, and I conclude in section 5. I relegate the details of some of the proofs to the appendix.

1 Model

1.1 The Market $\mathcal{M}$

There is a finite set B of types of buyers and a finite set S of types of sellers of a homogenous good. The type i of a buyer is determined by her value $v_i \geq 0$ for the good and the set of sellers that she can trade with. Similarly, the type j of a seller is determined by her cost $c_j \geq 0$ of producing the good and the set of buyers that she can trade with. I let N denote the set of all types of agents—that is, the union of the set B and the set S—and I let n denote the cardinality of the set N.[9] For simplicity, I assume that each buyer only values one unit of the good, that each seller can only produce one unit of the good (and has no consumption value for this good), and that the following *generic* assumption holds.[10]

Assumption 1.1. Each type i of buyer has a different value v_i, and each type j of seller has a different cost c_j. Moreover, the prices that emerge from bilateral Nash bargaining (with zero threat points) between any two different buyer-seller pairs are different (that is $\frac{v_i+c_j}{2} \neq \frac{v_{i'}+c_{j'}}{2}$ unless $i = i'$ and $j = j'$), and none of these prices coincide with the value of any buyer or the cost of any seller.

Fix an undirected buyer-seller network g with node set N.[11] If two types share an edge in the network g, I say that they are *connected*, or that they are *partners*. Without loss of

[9] I often refer to a buyer of type i as "buyer i," and to a seller of type j by "seller j."

[10] Throughout this paper, I say that a property holds *generically* if it holds with probability one when values and costs are independently drawn from a continuous and atomless distribution.

[11] A (directed) network g is a pair (N, E), where N is a set of nodes, and E is a set of edges between the nodes; that is, $E \subseteq \{(i,j) \mid i,j \in N\}$. A network (N,E) is *undirected* if (i,j) is in E implies that (j,i) is in E.

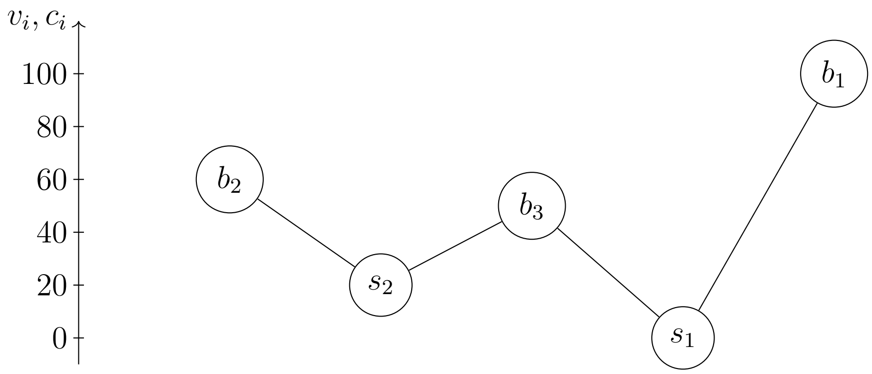


Figure 1: Market $\mathcal{W}$ of Example 1.1, comprised of two types of sellers (s_1 and s_2, with costs 0 and 20, respectively) and three types of buyers (b_1, b_2, and b_3, with values 100, 60, and 50, respectively). The height of each node reflects its associated value or cost. An edge between two nodes indicates that the associated types can trade with each other.

generality, I assume that each buyer i has a connection to a seller j whose cost c_j is lower than her value v_i and, analogously, that each seller j has a connection to a buyer i whose value v_i is higher than her cost c_j. Letting v and c be the profiles of buyers' values and sellers' costs, I refer to the set $\mathcal{M} = (N, v, c, g)$ as the *market*.

Example 1.1. Throughout this paper, I illustrate the main concepts and results using the market $\mathcal{W}$ depicted in Figure 1. It consists of two types of sellers, s_1 and s_2, and three types of buyers, b_1, b_2 and b_3. Sellers of type s_1 have a lower cost than sellers of type s_2. Both types of sellers can trade with the buyers of type b_3—who place relatively low value on the good. In addition, the sellers of type s_2 can trade with the buyers of type b_2—who value the good more than the buyers of type b_3 do—and the sellers of type s_1 can trade with the buyers of type b_1—who value the good the most.

1.2 The Game $\Gamma(\mathcal{M}, \delta)$

Given the market $\mathcal{M}$ and a common discount factor $\delta > 0$, I study the following infinite-horizon complete-information bargaining game $\Gamma(\mathcal{M}, \delta)$, which is intended to capture the predominant strategic forces in a steady state of this market.[12] In each period $t = 0, 1, \ldots$, one agent of each type is *active*. More precisely, for each type i, there exists a sequence $i_0, i_1, \ldots, i_\kappa, \ldots$ of agents of type i. In the first period ($t = 0$), the set of active agents is $\{i_0\}_{i \in N}$ and, for each $\kappa \geq 0$, when the active agent i_κ reaches an agreement and exits the

[12] This game is similar to the one considered by Rubinstein and Wolinsky (1985) and the subsequent literature studying bargaining in stationary markets.

market in a given period, agent $i_{\kappa+1}$ becomes active in the next period.

In each period $t = 0, 1, \ldots$, one active agent is selected uniformly at random to be the *proposer*. The proposer chooses one of her connections j to be the *receiver*, and makes the active agent of type j a take-it-or-leave-it offer to trade at a certain price p. If the receiver accepts this offer, the proposer and the receiver trade at the specified price and exit the market. Otherwise, they stay in the market for the next period.[13] Buyer b's period-T utility of trading at price p in period $T + \tau$ is $\delta^\tau(v_b - p)$. Analogously, seller s's period-T utility of trading at price p in period $T + \tau$ is $\delta^\tau(p - v_s)$.

2 Equilibrium: Arbitrary Bargaining Frictions

In this section, I investigate the subgame-perfect equilibrium of the game $\Gamma(\mathcal{M}, \delta)$ for an arbitrary discount factor δ.[14] After describing the sense in which this game admits an essentially unique equilibrium, I investigate what determines the equilibrium terms of trade in each part of the network. The main finding is that equilibrium prices are driven by pairs of types that essentially ignore the presence of other types in the market and engage in bilateral bargaining, and that some such pairs—those with extreme terms of trade—can be identified by computing one local network statistic for each type. This provides the basis for characterizing, in section 3, the equilibrium prices in the limit as the discount factor δ goes to 1.

2.1 Unique Equilibrium Prices

Proposition 2.1 below shows that each type has a *cutoff price* such that, in every subgame-perfect equilibrium of the game $\Gamma(\mathcal{M}, \delta)$, she always accepts trades at prices that are at least as good for her as her cutoff price. This implies that, in every equilibrium, each buyer i that has access to at least one seller whose cutoff price is lower than her value v_i proposes to trade at price ρ_j with one of her *preferred partners* j—that is, one of her connections j with the lowest cutoff price ρ_j—and all such offers are always accepted.[15] Similarly, in every

[13] For simplicity, when the proposer is buyer i, I restrict the proposed price p to be weakly lower than v_i. Analogously, when the proposer is seller j, I restrict the proposed price p to be weakly higher than c_j.

[14] The strategy profile $(\sigma_i)_{i \in N}$ is a *subgame-perfect equilibrium* of the game $\Gamma(\mathcal{M}, \delta)$ if it induces a Nash equilibrium in each of its subgames. I often refer to the notion of subgame-perfect equilibrium simply as an "equilibrium."

[15] If such offers were accepted with probability less than one, buyer i would not have a best response. This is because buyer i can always ensure that seller j accepts her proposal with probability one by proposing the

equilibrium, each seller j that has access to at least one buyer whose cutoff price is higher than her cost c_j proposes to trade at the price ρ_i with one of her preferred-partners i—that is one of her connections i with the highest cutoff price ρ_i—and all such offers are always accepted. I say that a buyer i that has no access to any seller j whose cutoff price ρ_j is strictly lower than her value v_i is *her own preferred partner* and, analogously, that a seller j that has no access to any buyer i whose cutoff price ρ_i is strictly higher than her cost c_j is her own preferred partner.[16]

Proposition 2.1. *Each buyer i has a cutoff price ρ_i such that, in every subgame-perfect equilibrium of the game $\Gamma(\mathcal{M}, \delta)$, she always accepts trades at price $p < \rho_i$, and she always rejects trades at price $p > \rho_i$. Analogously, each seller j has a cutoff price ρ_j such that, in every subgame-perfect equilibrium of the game $\Gamma(\mathcal{M}, \delta)$, she always accepts trades at price $p > \rho_j$, and she always rejects trades at price $p < \rho_j$.*

The statement of Proposition 2.1 can be strengthened, in the sense that the process of *iterated conditional dominance*—which, in the present context, is a weaker solution concept than subgame-perfect equilibrium[17]—determines behavior in the same way. This process, however, does not provide much insight into how the interaction between sellers' costs, buyers' values and the underlying network g capturing the relevant trading frictions determines agents' bargaining power. For this reason, Proposition 2.1 is only the starting point of the equilibrium characterization. Its proof is analogous to the one given by Fudenberg and Tirole (1991) in the context of the canonical bilateral bargaining model of Rubinstein (1982), and follows the one given by Manea (2017a) in the context of a random-matching version of this model, so I defer it to Appendix A.

2.2 Cutoff Price is Determined by Preferred Partner's Cutoff Price

One of the main difficulties in understanding the sources of bargaining power in networked markets is that, in general, the bargaining power of an agent depends on the bargaining power of all of her partners, which in turn depends on the bargaining power of all of their partners, and so on, so bargaining power is a complex function of the network structure.

price $\rho_j - \epsilon$, for any $\epsilon > 0$.

[16]Naturally, a type that is her own preferred partner never trades in equilibrium (see Lemma B.1).

[17]In games of perfect information—like the bargaining game $\Gamma(\mathcal{M}, \delta)$—the notion of iterated conditional dominance is weaker than the concept of subgame-perfect equilibrium in the following sense: Every subgame-perfect equilibrium survives the process of iterated conditional dominance (Theorem 4.3 in Fudenberg and Tirole, 1991).

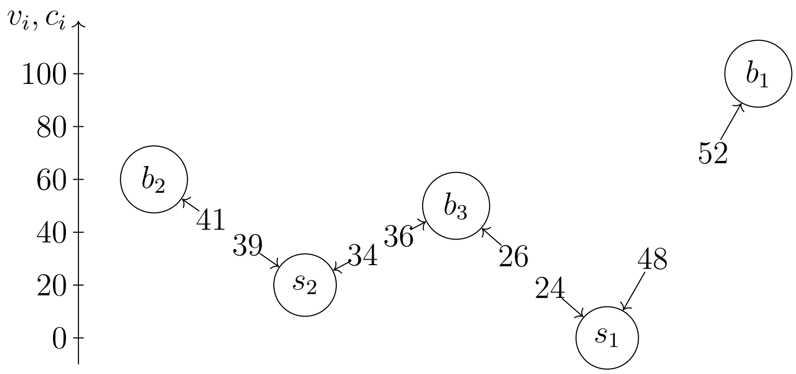


Figure 2: The Rubinstein prices p_{ij} in the game $\Gamma(\mathcal{W}, .98)$. The number next to the arrow that points from type j to type i is the ij-Rubinstein price.

However, I now show how, in the game $\Gamma(\mathcal{M}, \delta)$, the bargaining power of an agent does not depend on the bargaining power of all of her connections. Rather, is is determined by the minimum or the maximum of the cutoff prices among all of her connections. In particular, Proposition 2.2 below describes how the cutoff price of each type is a weighted average of her own cost or value and the cutoff price of her preferred partners. Interestingly, this implies that the cutoff prices of mutually-preferred partners are determined independently of others' values, costs and connections. This is especially relevant because, as I discuss below, mutually-preferred partners essentially determine all the equilibrium prices.

Proposition 2.2. *If buyer i's preferred partner is type j, then letting $\alpha := \frac{\delta}{\delta+(1-\delta)n}$, their cutoff prices ρ_i and ρ_j satisfy*

$$\rho_i = (1-\alpha)v_i + \alpha\rho_j. \tag{1}$$

Analogously, if seller j's preferred partner is type i, then their cutoff prices ρ_i and ρ_j satisfy

$$\rho_j = (1-\alpha)c_j + \alpha\rho_i.$$

Proof. Let j be the preferred partner of buyer i (an analogous argument works for sellers). The cutoff price of buyer i is the price ρ_i that leaves her indifferent between trading at this price at a given period t, which gives her a time-t utility of

$$v_i - \rho_i, \tag{2}$$

and waiting for period $t+1$, which gives her a time-t expected utility of

$$\delta\left(\frac{1}{n}\underbrace{(v_i-\rho_j)}_{i\text{'s utility when proposer}} + \frac{n-1}{n}\underbrace{(v_i-\rho_i)}_{i\text{'s expected utility when not proposer}}\right). \tag{3}$$

since, in period $t+1$, if she is the proposer (which occurs with probability $1/n$) she trades at the cutoff price ρ_j of her preferred partner j and, otherwise, she either trades at price ρ_i or does not trade (note that, by definition, buyer i is indifferent between trading at her cutoff price ρ_i and waiting for the next period, so her expected payoff in a period in which she is not the proposer is $v_i - \rho_i$ independently of whether she trades or not). Equating the expressions (2) and (3) gives Equation 1. □

Corollary 2.1. *If buyer i and seller j are mutually-preferred partners, then their cutoff prices ρ_i and ρ_j are given by*

(4) $$\rho_i = \frac{v_i + \alpha c_j}{1+\alpha} =: p_{i,j} \text{ and } \rho_j = \frac{c_j + \alpha v_i}{1+\alpha} =: p_{j,i}.$$

The prices $p_{i,j}$ and $p_{j,i}$ are exactly the cutoff prices of buyer i and seller j when bargaining in the random-proposer version of the classical bilateral alternating-offers model of Rubinstein (1982), in which, in each period, each agent has a probability of $1/n$ of being selected to be the proposer. For this reason, I refer to these prices as the *ij-Rubinstein price* and the *ji-Rubinstein price*, respectively.[18] Note that both the ij-Rubinstein price and the ji-Rubinstein price converge—as δ goes to 1—to the price that emerges from the Nash bargaining solution (with zero threat points) between i and j; that is, $\frac{v_i+c_j}{2}$.[19] Figure 2 depicts the relevant Rubinstein prices p_{ij} in the game $\Gamma(\mathcal{W}, .98)$.

2.3 Mutually-Preferred Partners Determine All Prices

The observation that one's cutoff price only depends on the cutoff price of her preferred partners suggests that—in order to understand how cutoff prices are determined—it is useful to trace out who is the preferred partner of whom. To this end, I define the *preferred-partner network G* to be a directed network with node set N that has an edge from type i to type j if j is a preferred partner of i. For instance, Figure 3 depicts the preferred-partner network in the game $\Gamma(\mathcal{W}, .98)$ of Example 1.1. This network has two maximal connected subnetworks (when viewed as an undirected network)[20]—or *components*. One component contains

[18]These notional prices are related to—but different from—Elliott and Nava's (forthcoming) *Rubinstein payoffs*, which they define in their matching setting as the limit payoffs that would arise if all pairs in the efficient match bargained bilaterally. In particular, the ij-Rubinstein price in the present setting is defined for all bargaining frictions—and not only in the limit as bargaining frictions vanish.

[19]This close connection between the strategic bargaining model of Rubinstein (1982) and the Nash bargaining solution was first discussed by Binmore (1987) (see also Binmore, Rubinstein, and Wolinsky 1986).

[20]A network (N', E') is a *subnetwork* of network (N, E) if $N' \subseteq N$ and $E' \subseteq E$. A *walk in* (N, E) *from i to j of length m* is a list $(i_1, i_1, \ldots, i_m)$ of nodes in N with $i_1 = i$ and $i_m = j$ such that, for each $1 \leq k \leq m-1$, (i_k, i_{k+1})

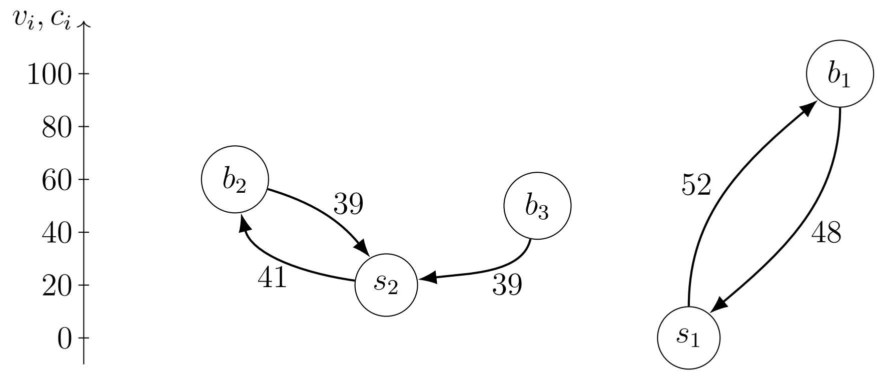


Figure 3: Preferred-partner network in the in the game $\Gamma(\mathcal{W}, .98)$ of Example 1.1. An edge from i to j indicates that j is a preferred partner of i (that is, that type i offers to trade with type j when she is the proposer) and the number associated to it is (approximately) the corresponding price offered (equivalently, j's cutoff price).

the mutually-preferred partners b_1 and s_1 and has relatively high cutoff prices. The other component contains the mutually-preferred partners b_2 and s_2, and has relatively low cutoff prices.

Note that each component of the preferred-partner network depicted in Figure 3 has exactly one pair of mutually-preferred partners, whose terms of trade are determined by bilateral bargaining (as described by Corollary 2.1 above) and essentially determine all the prices in it.[21] Proposition 2.3 shows that this is generically the case, which implies that mutually-preferred partners drive the equilibrium prices in the game $\Gamma(\mathcal{M}, \delta)$.

Proposition 2.3. *Generically, each type has a unique preferred partner, and each component of the preferred-partner network with at least two nodes contains exactly one pair of mutually-preferred partners.*

I defer the details of the proof of Proposition 2.3 to Appendix B. Here, I describe the main idea behind it, which is that, generically, *the preferred-partner network G does not contain cycles with more than two nodes*.[22] Since each type has at least one preferred partner and that—as shown by Lemma B.1 in the appendix—a type who is her own preferred partner is not

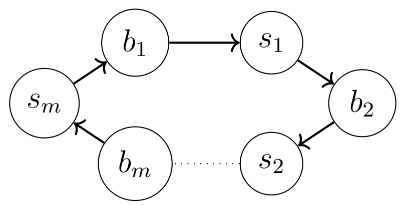
is in E. A network (N', E') is a *maximal connected subnetwork* of the network (N, E) if there is no subnetwork $(N'', E'') \neq (N', E')$ of network (N, E) with $E' \subset E''$ such that, for every $i, j \in N''$, there is a walk in (N'', E'') from i to j.

[21]Indeed, when the discount factor δ is close to 1, each agent's cutoff price is essentially the cutoff price of her preferred partner (see Equation 1), so the cutoff price of each type that is in a component of the preferred-partner network G with mutually-partners (i, j) is essentially the price that emerges from bilateral bargaining between i and j.

[22]A *cycle in G* is a walk that begins and ends at the same node.

the preferred partner of any other type, this implies that each component of the preferred-partner network with at least two nodes contains at least one pair of mutually-preferred partners. The fact that, generically, there is exactly one such pair then follows from the observation that—as shown by Lemma B.4 in the appendix—generically, each type has a unique preferred partner.

Suppose for contradiction that there exists a cycle $(b_1, s_1, b_2, s_2, \dots, b_m, s_m)$ with $m \geq 2$ such that, for each i in $\{1, 2, \dots, m\}$, the preferred partner of buyer b_i is seller s_i and the preferred partner of seller s_i is buyer b_{i+1} (indices are modulo m), as depicted below.



For each i in $\{1, 2, \dots, m\}$, seller s_{i-1} is connected to buyer b_i, since b_i is the preferred partner of s_{i-1}. Hence, the fact that the preferred partner of b_i is s_i implies that $\rho_{s_i} \leq \rho_{s_{i-1}}$ for all i in $\{1, 2, \dots, m\}$, which is only possible if all these weak inequalities are actually equalities. Analogously, $\rho_{s_i} = \rho_{s_{i-1}}$ for all i in $\{1, 2, \dots, m\}$. Hence, for all i and j in $\{1, 2, \dots, m\}$, buyer i and seller j are mutually-preferred partners. By Corollary 2.1, this implies that $c_{s_i} = c_{s_{i-1}}$ and $v_{b_i} = v_{b_{i-1}}$ for all i in $\{1, 2, \dots, m\}$, which is not generic.

2.4 Who are the Mutually-Preferred Partners?

The discussion so far implies that the trading network g endogenously decomposes into different components, with the prices in each component essentially determined by its unique pair of mutually-preferred partners. This suggests that understanding what determines who are the mutually-preferred partners is the key to characterizing the equilibrium prices. As a first step, I now show how the mutually-preferred partners *with extreme cutoff prices* can be easily identified using local network statistics. This is the basis for the recursive procedure (which I describe in section 3) that finds all the mutually-preferred partners (for sufficiently high discount factors δ). To do this, I first describe, in subsubsection 2.4.1, a personalized bound—based on local information—on each type's cutoff price, and then I show, in subsubsection 2.4.2, that this bound is tight at the extremes.

2.4.1 Best Prices Bound Cutoff Prices

The observation that the cutoff price of mutually-preferred partners is essentially determined by bilateral bargaining between them, and that i's preferred partner can always choose to reciprocate i's offers, suggests that the cutoff price of each type is weakly less beneficial for her than the price that emerges from bilateral bargaining between her and any of her partners. I now formalize this idea. In particular, I show how the cutoff price of each type can be bounded using only *local information*—that is, information about her value or cost and that of her partners. This will later be useful to provide tight bounds on the highest and the lowest cutoff prices in the game $\Gamma(\mathcal{M}, \delta)$ from these local network statistics.

To determine this bound for buyer i, for instance, first identify her *best partner* j—that is, the seller that she is connected to with the lowest cost. Second, identify her *best price*, defined as her ij-Rubinstein price p_{ij}. In other words, buyer i's best price is her cutoff price in the hypothetical situation in which she can (i) choose one of the sellers that she has access to, and (ii) bargain bilaterally with her as in the random-proposer version of the alternating-offers model of Rubinstein (1982) (with proposer probabilities given by $1/n$). Proposition 2.4 shows that no buyer does better in equilibrium than how she would do in this hypothetical case. Intuitively, one's partners' bargaining power only increases when they can trade with others. For instance, in the game $\Gamma(\mathcal{W}, \delta)$ of Example 1.1, while buyer b_3's best price is relatively low due to her access to the low-cost seller s_1, her cutoff price ends up being higher than her best price because s_1 can bargain with the high-cost buyer b_1 instead.

Definition 2.1. Buyer i's *best partner* is the seller j that she is connected to with the lowest cost c_j, and her *best price* is the ij-Rubinstein price p_{ij}. Analogously, seller j's *best partner* is the buyer i that she is connected to with the highest value v_i, and her *best price* is the ji-Rubinstein price p_{ji}.

Proposition 2.4. *A buyer's best price is a lower bound on her cutoff price. Analogously, a seller's best price is an upper bound on her cutoff price.*

Proof. Suppose that buyer i's preferred partner is seller j (a similar argument proves the analogous statement for sellers). It is enough to show that $\rho_i \geq p_{i,j}$ (where $p_{i,j}$ is defined in Equation 4). Denoting seller j's preferred partner by k (see Figure 4), we have that $\rho_i \leq \rho_k$ and, using Proposition 2.2, that $\rho_i = f_{i,j}(\rho_k)$ where $f_{i,j}(x) := (1-\alpha)(v_i + \alpha v_j) + \alpha^2 x$. By definition, $p_{i,j} = f_{i,j}(p_{i,j})$. As illustrated in Figure 5 below, this implies that $\rho_i \geq p_{i,j}$. $\square$

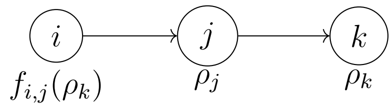


Figure 4: Illustration of the situation described in the proof of Proposition 2.4.

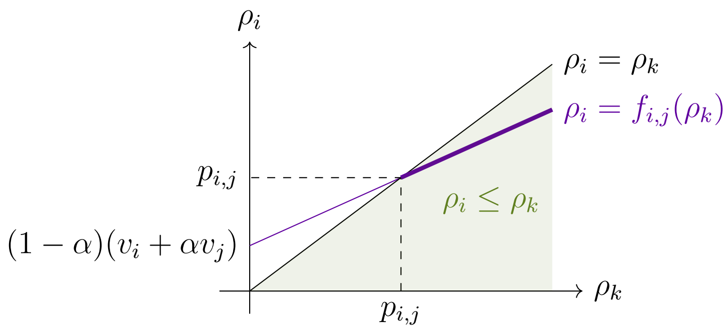


Figure 5: Illustration of the argument in the proof of Proposition 2.4. The fact that $\alpha < 1$ implies that the slope of the function $f_{i,j}$ is strictly less than one. The fact that $p_{i,j} = f_{i,j}(p_{i,j})$ and that $\rho_i \leq \rho_k$ implies that we must be in the thicker portion of the image of $f_{i,j}$. In other words, that $\rho_i \geq p_{i,j}$.

2.4.2 The Extreme Best Prices are the Extreme Cutoff Prices

A type's best price is—in general—not the relevant measure of her bargaining power. For instance, in the market $\mathcal{W}$ of Example 1.1, buyer b_3 has a significantly lower best price than buyer b_2, but these two types end up trading at the same price in equilibrium (when they are the proposers). However, Proposition 2.5 below shows that the best price is the relevant measure of bargaining power of those types with extreme cutoff prices. Intuitively, since a buyer with the highest cutoff price is—naturally—the preferred partner of all of her partners, it is as if she was in the hypothetical situation that defines her best price: Namely, it is as if she could choose any one of her partners and bargain bilaterally with her. And she chooses her best partner—so her cutoff price is exactly her best price (and analogously for sellers).

Proposition 2.5. *The cutoff price of a buyer with the highest cutoff price is her best price. Analogously, the cutoff price of a seller with the lowest cutoff price is her best price.*

Proof. A buyer h with the highest cutoff price is, by definition, a preferred partner of all the sellers that she is connected to. Hence, the cutoff price ρ_j of each seller j that she is connected to is $(1-\alpha)c_j + \alpha\rho_h$, so buyer h's preferred partner is her connection with the lowest cost; that is, her best partner. The result then follows from the fact that buyer h's best price is defined to be her cutoff price when she and her best partner are mutually-preferred partners. The analogous argument proves the analogous statement regarding sellers. □

Corollary 2.2 below highlights how computing each type's best price is sufficient to bound the set of all the cutoff prices from above and below. This result follows from Proposition 2.4 and Proposition 2.5, since the former implies that the highest best price among buyers is a *lower bound* on the highest cutoff price, and the latter implies that the highest best price among buyers is an *upper bound* on the highest cutoff price (and analogously for sellers).

Corollary 2.2. *The highest best price among buyers is the highest cutoff price, so the buyer with the highest best price and her best partner are mutually-preferred partners. Analogously, the lowest best price among sellers is the lowest cutoff price, so the seller with the lowest best price and her best partner are mutually-preferred partners.*

For instance, in the game $\Gamma(\mathcal{W}, .98)$, the buyer with the highest best price is b_1, and the seller with the lowest best price is s_2. In this case, Corollary 2.2 implies that the best prices of these two types (52 and 39, respectively) are the highest and the lowest cutoff prices in this game, respectively.

While Corollary 2.2 identifies what determines the highest and the lowest cutoff price in the game $\Gamma(\mathcal{M}, \delta)$, it falls short of identifying all the cutoff prices. The existence of bargaining frictions makes this task challenging, because these frictions create heterogeneities in the cutoff prices within components of the preferred-partner network, which prevent us from ranking these components in terms of their prices. For this reason, I now turn to studying the equilibrium of this game in the limit as bargaining frictions vanish. In this limit, the heterogeneities in cutoff prices within components vanish, and this facilitates the description of a procedure that characterizes all the equilibrium prices.

3 Equilibrium: Arbitrarily-Small Bargaining Frictions

In this section, I characterize the equilibrium prices in the game $\Gamma(\mathcal{M}, \delta)$ in the limit as the discount factor δ goes to 1. The only trading frictions that remain in this limit are those generated by the underlying trading network g. Hence, this characterization isolates the effect of the trading network g on the equilibrium terms of trade. After showing that the limit in question is well defined, I leverage the results of the previous section to (i) provide a necessary and sufficient joint condition on the primitives (the exogenous trading network as well as values and costs) for the law of one price to hold, (ii) describe an algorithm that identifies each agent's terms of trade, and (iii) illustrate how price dispersion can increase after adding new connections in the underlying trading network, which emphasizes the subtlety of the relationship between price dispersion and trading frictions.

3.1 The Limit Preferred-Partner Network

Proposition 3.1 below ensures that the equilibrium prices in the game $\Gamma(\mathcal{M}, \delta)$ are well defined in the limit as the discount factor δ goes to 1: In particular, it establishes that the preferred-partner network G is the same for all sufficiently high discount factors δ. Combined with Proposition 2.2 above, this implies that the equilibrium prices converge as δ converges to 1. The proof is similar to that of the analogous result in the case of random matching (Manea 2011, Proposition 1), so I defer it to Appendix C.

Proposition 3.1. *There exists $\underline{\delta} < 1$ and G^* such that $G = G^*$ for all $\delta > \underline{\delta}$.*

I refer to the network G^* in Proposition 3.1 as the *limit preferred-partner network*, and to each of its connected components (when viewed as an undirected network) as a *submarket*. By Corollary 2.1, the cutoff prices of mutually-preferred partners i and j in the limit preferred-partner network converge—as δ goes to 1—to the one that results from Nash bargaining between them; that is, if i is a buyer and j is a seller, their cutoff prices converge to $\frac{v_i+c_j}{2}$. Moreover, it follows from Proposition 2.2 that the cutoff price of every type in the submarket of the mutually-preferred partners i and j also converges to this price; I refer to this price as the *submarket's price*. Intuitively, in the limit as bargaining frictions vanish, the cutoff price of each type (the price at which she indifferent between trading and not trading) converges to the cutoff price of her preferred partner (the price at which she can trade when she is the proposer), which implies that all the cutoff prices in any submarket converge to the same price.[23]

In the case of the market $\mathcal{W}$ of Example 1.1, the preferred-partner network depicted in Figure 3 is also the limit preferred-partner network. The price of the submarket with the highest price (which contains b_1 and s_1) is the price that emerges from Nash bargaining between its mutually-preferred partners b_1 and s_1—namely, 50. Similarly, the price of the submarket with the lowest price (which contains b_2, b_3 and s_2) is the price that emerges from Nash bargaining between its mutually-preferred partners b_2 and s_2—namely, 40.

[23] I often refer to all the submarkets' prices as the *market prices*.

3.2 The Law of One Price

Say that the *law of one price holds in the market* $\mathcal{M}$ if all the cutoff prices in $\Gamma(\mathcal{M}, \delta)$ converge to the same value as the discount factor δ goes to 1. Corollary 3.1 is the analog of Corollary 2.2: It describes how to identify the mutually-preferred partners in the submarkets with the highest and the lowest price, respectively. In particular, it implies that the law of one price holds in the market $\mathcal{M}$ if and only if the highest limit best price among buyers is equal to the lowest limit best price among sellers.

Definition 3.1. The *limit best price* of buyer i is the price $\frac{v_i+c_j}{2}$ that emerges from bilateral Nash bargaining between her and her best partner j. Analogously, the *limit best price* of seller j is the price $\frac{v_i+c_j}{2}$ that emerges from bilateral Nash bargaining between her and her best partner i.

Corollary 3.1. *The buyer with the highest limit best price and her best partner are mutually-preferred partners in the submarket with the highest price. Analogously, the seller with the lowest limit best price and her best partner are mutually-preferred partners in the submarket with the lowest price.*

Corollary 3.1 implies that a necessary condition for the law of one price to hold is that the highest-value buyer is connected to the lowest-cost seller. Indeed, denoting by $\overline{v}$ the highest value among buyers and by $\underline{c}$ the lowest cost among sellers, $\frac{\overline{v}+\underline{c}}{2}$ is simultaneously a lower bound on the highest limit best price among buyers and an upper bound on the lowest limit best price among sellers. However, this condition is not sufficient for the law of one price to hold: It must also be the case that the highest-value buyer has the highest limit best price among buyers, and that the lowest-cost seller has the lowest limit best price among sellers. For instance, in the market $\mathcal{W}$ of Example 1.1, the highest-value buyer (b_1) and the lowest-cost seller (s_1) are connected, but the fact that the seller s_2 has a lower best price than the seller s_1 (despite having a higher cost), implies that her neighbors (b_2 and b_3) can trade with her at a price lower than the price at which s_1 trades at. In particular, the law of one price fails in this example.

Corollary 3.2 below highlights that reducing the trading frictions of a market never increases the gap between the highest and its lowest market price. Moreover, it identifies the conditions under which adding a connection between a buyer and a seller reduces this gap.

Corollary 3.2. *Adding an edge in the underlying network g does not increase the gap between the highest and the lowest market price, and it reduces this gap if and only if it either (i) connects the buyer b with the highest best price to a seller whose cost is smaller than the one of b's best partner, or* *(ii) connects the seller s with the lowest best price to a buyer whose value is higher than the one of s's best partner.*

For instance, in the market $\mathcal{W}$ of Example 1.1, connecting s_1 and b_2 does not decrease the gap between the highest and the lowest market price (since it neither changes b_1's nor s_2's limit best price), but connecting s_2 and b_1 does (since s_2's best partner becomes b_1, her limit best price becomes 60 and, hence, s_1 becomes the seller with the lowest limit best price).

3.3 Equilibrium Price Dispersion

The discussion above shows how (i) in order to characterize the market prices, it is sufficient to identify the mutually-preferred partners in each submarket, and (ii) the mutually-preferred partners of the submarket with the highest and the lowest price can be identified by computing the limit best price of each agent. While it is impossible to identify at a glance the mutually-preferred partners in the submarkets with intermediate prices, I now describe an algorithm that identifies both the mutually-preferred partners and the rest of the types in each submarket. This algorithm exploits the fact that—in contrast to the case with arbitrary bargaining frictions—the cutoff prices in each submarket converge to the same price in the limit as bargaining frictions vanish.

The key of the characterization of all the market prices is the description of algorithm $\mathcal{B}$ (Definition 3.3 below), which identifies the set of all the types in the submarket with the highest price. Once this algorithm has identified the submarket with the highest price, it can be applied again to the rest of the market to find the types in the submarket with the second-highest price, and so on, until it has identified all the types in each submarket—and the corresponding prices.[24]

The idea that algorithm $\mathcal{B}$ exploits to identify all the types that trade in the submarket with the highest price is intuitive: All the sellers that can, do trade in the submarket with the highest price p. And the only buyers that trade in the submarket with the highest price p are those that, if they were not to do so, they would find themselves in such a weak bargaining position that they would be forced to trade at a price higher than p. This might occur, for example, when all the low-cost sellers that she has access to trade in the submarket with the highest price. In order to formalize this idea, Definition 3.2 below defines the agents' best partners and limit best prices in subsets of the market $\mathcal{M}$.

[24]A procedure analogous to algorithm $\mathcal{B}$ can be defined to identify the types in the submarket with the *lowest* price, then the types in the submarket with the second-lowest price, and so on.

Definition 3.2. Let $M \subseteq N$. If buyer i is connected to a seller in M with a strictly lower cost than her value, then her *best partner in* M is the seller j in S that she is connected to with the lowest cost c_j, and her *limit best price in* M is $\frac{v_i+c_j}{2}$. Otherwise, she is her own best partner in M, and her *limit best price in* M is her own value v_i. Analogously, if seller j is connected to a buyer in S with a strictly higher value than her cost, then her *best partner in* M is the buyer i in M that she is connected to with the highest value v_i, and her *limit best price in* M is $\frac{v_i+c_j}{2}$. Otherwise, she is her own best partner in S, and her *limit best price in* S is her own cost c_j.

Definition 3.3 (Algorithm $\mathcal{B}$)**.** Let $X^0 = \{b, s\}$, where b is the buyer with the highest limit best price p, and s is her best partner. Proceed inductively as follows: In step $k \geq 1$, let X^k be the union of X^{k-1}, the set of all the sellers whose cost is lower than p and that have a connection in X^{k-1}, and the set of all the buyers whose limit best price in $N - X^{k-1}$ is higher than p. End in the first step κ for which $X^\kappa = X^{\kappa-1}$, and let $X := X^\kappa$.

For instance, in the context of the market $\mathcal{W}$ in Example 1.1, algorithm $\mathcal{B}$ proceeds as follows: In step 0, $X^0 = \{b_1, s_1\}$, since b_1 is the buyer with the highest limit best price (50), and s_1 is her best partner. In step 1, no sellers are included in X^1, since s_2 cannot trade with b_1, and no buyers are included in X^1 either, since b_3's limit best price in the restriction of market $\mathcal{W}$ to $\{b_2, s_2, b_3\}$ is $35 = \frac{v_{b_3}+c_{s_2}}{2} < \frac{v_{b_1}+c_{s_1}}{2} = 50$. Hence, Theorem 3.1 below implies that the submarket with the highest price consists only of the buyer b_1 and the seller s_1. For brevity, I defer the details of the proof of Theorem 3.1 to Appendix D.

Theorem 3.1. *The set X defined by algorithm $\mathcal{B}$ is the set of all the types in the submarket with the highest price.*

As discussed in subsection 3.2 above, the law of one price holds if and only if the highest limit best price among buyers is equal to the lowest limit best price among sellers. In particular, given the generic Assumption 1.1, when the highest limit best price among buyers is equal to the lowest limit best price among sellers, it must be that all the types are in the same submarket. I now describe an independent proof of this result, which clarifies the connection between Theorem 3.1 and the earlier discussion.

Corollary 3.3. *The set X defined by algorithm $\mathcal{B}$ is equal to the set of all types N if and only if the highest limit best price among buyers is equal to the lowest limit best price among sellers.*

Proof. Suppose first that $X = N$. Suppose for contradiction that the highest limit best price p among buyers is strictly higher than the lowest limit best price among sellers. Let s denote the seller with the lowest limit best price. Let l denote the step of algorithm $\mathcal{B}$ in which seller

s is included in X^l. By definition, there is a buyer b that s is connected to that is in X^{l-1}. Using Assumption 1.1, this implies that $\frac{v_b+v_s}{2}$ is strictly higher than p, which implies that the limit best price of s is strictly higher than p, a contradiction.

In the other direction, suppose that the highest limit best price p among buyers is equal to the lowest limit best price among sellers. Suppose for contradiction that $X \neq N$. Then, using Assumption 1.1, the limit best price in $N - X$ of each buyer in $N - X$ is strictly lower than p, while the limit best price in $N - X$ of each seller in $N - X$ is strictly higher than p. In other words, for each buyer i in $N - X$ we can find a seller j in $N - X$ that she is connected to with $\frac{v_i+c_j}{2} < p$, and similarly, for each seller j in $N - X$ we can find a buyer i in $N - X$ that she is connected to with $\frac{v_i+c_j}{2} > p$, a contradiction. $\square$

Remark 3.4. *The assumption maintained throughout this article that agents are equally likely to be the proposers and that they have a common discount factor is not important for any of the results of this paper. In particular, the analogs of Corollary 3.1 and Theorem 3.1 hold if, letting q_i and r_i denote agent i's proposer probability and discount rate, respectively, we define buyer i's* limit best price *as the minimum—over all her partners j—of $\frac{r_i/q_i}{r_i/q_i+r_j/q_j}v_i + \frac{r_j/q_j}{r_i/q_i+r_j/q_j}c_j$, and the corresponding seller j as her best partner (and analogously for sellers).[25] Intuitively, in general, the bargaining power of agent i when bargaining bilaterally with agent j depends not only on their relative values and costs, but also on their relative impatience and proposer probabilities.[26]*

3.4 Comparative Statics

Stigler (1961) asserted that price dispersion is a measure of ignorance in the market (see the quote in the introduction above). If we think of the trading network g as describing this ignorance, Stigler's quote would suggest that adding edges to the network g would tend to ameliorate price dispersion. However, Corollary 3.5 below highlights that price dispersion can increase when new edges are added in the trading network g (in the sense that the number of different prices at which the good is traded at increases).[27]

Corollary 3.5. *The number of different submarkets can increase after adding edges in the underlying network g.*

[25] In this case, denoting Δ the length of time periods, the discount factor of each type i is $e^{-r_i\Delta}$, and the limit is taken as Δ goes to zero.

[26] The analog of Assumption 1.1 in this case is that for every two buyer-seller pairs (i, j) and (i', j'), $\frac{r_{i'}/q_{i'}}{r_{i'}/q_{i'}+r_{j'}/q_{j'}}v_{i'} + \frac{r_{j'}/q_{j'}}{r_{i'}/q_{i'}+r_{j'}/q_{j'}}c_{j'} \neq \frac{r_i/q_i}{r_i/q_i+r_j/q_j}v_i + \frac{r_j/q_j}{r_i/q_i+r_j/q_j}c_j$.

[27] As emphasized by Corollary 3.2, however, adding edges to the underlying network g cannot increase the difference between the highest and the lowest market price.

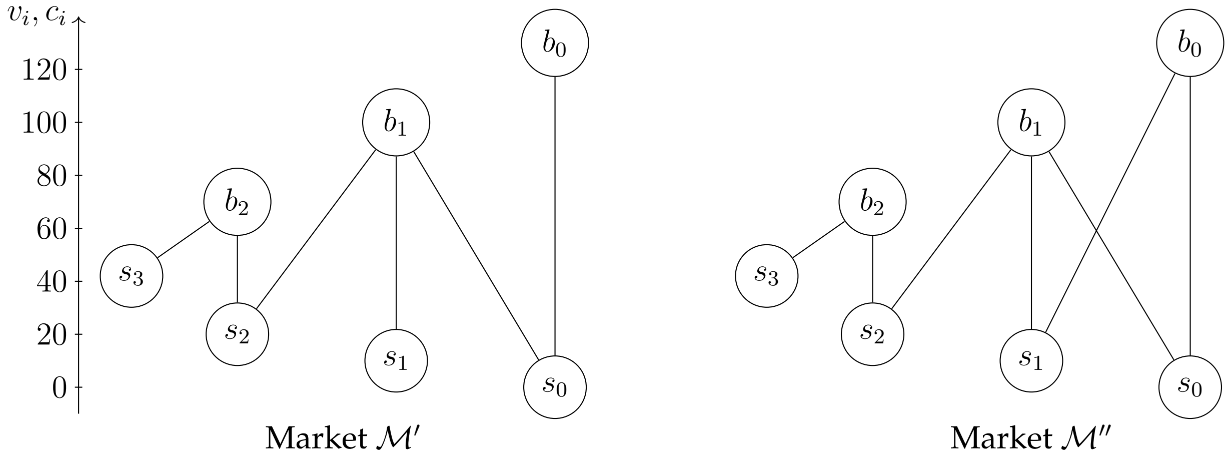


Figure 6: The market $\mathcal{M}'$ (right) is the same as the market $\mathcal{M}$ (left), except for the extra edge between buyer b_0 and seller s_1. The values of the buyers b_0, b_1 and b_2 are 130, 100 and 72, respectively. The costs of the sellers s_0, s_1, s_2 and s_3 are 0, 10, 20 and 40, respectively.

I illustrate Corollary 3.5 using markets $\mathcal{M}'$ and $\mathcal{M}''$ depicted in Figure 6. Market $\mathcal{M}''$ only differs from market $\mathcal{M}'$ in that buyer b_0 and seller s_1 are connected in the former, but not in the latter. Figure 7 depicts the prices that emerge from Nash bargaining between any two pairs of agents that can trade with each other in these markets. In both markets, b_0 is the buyer with the highest limit best price (65), so the set X^0 defined by algorithm $\mathcal{B}$ is $\{b_0, s_0\}$. Algorithm $\mathcal{B}$ ends with $X = X^0$ in market $\mathcal{M}'$, because the highest limit best price of buyer b_1 in $N - X^0$ (55) is lower than b_0's limit best price (65). In contrast, the added connection allows seller s_1 in market $\mathcal{M}''$ to enter the submarket with the highest price. Even in this case, however, buyer b_1 does not join this submarket, since her limit best price with seller s_2 (60) is still lower than b_0's limit best price.

The fact that seller s_1 joins the submarket with the highest price in market $\mathcal{M}''$ leads to an increase in the second highest price (from 55 to 60), which in turn leads to an increase in the number of submarkets: While the submarket with the second highest price in $\mathcal{M}'$ contains all the agents except for b_0 and s_0, it contains only b_1 and s_2 in market $\mathcal{M}''$. As a result, connecting b_0 and s_1 leads to the formation of three submarkets (with prices 65, 60 and 56) instead of two submarkets (with prices 65 and 55). Note, however, that seller s_1 is the seller with the lowest limit best price in $\mathcal{M}'$, and adding the edge (b_0, s_1) increases her limit best price. As a consequence, the addition of this edge reduces the difference between the highest and the lowest market price (from $65 - 55$ to $65 - 56$).

Continuing with the interpretation of the trading network g as measuring the "ignorance in the market," Corollary 3.6 shows that no agent ever benefits from her ignorance.

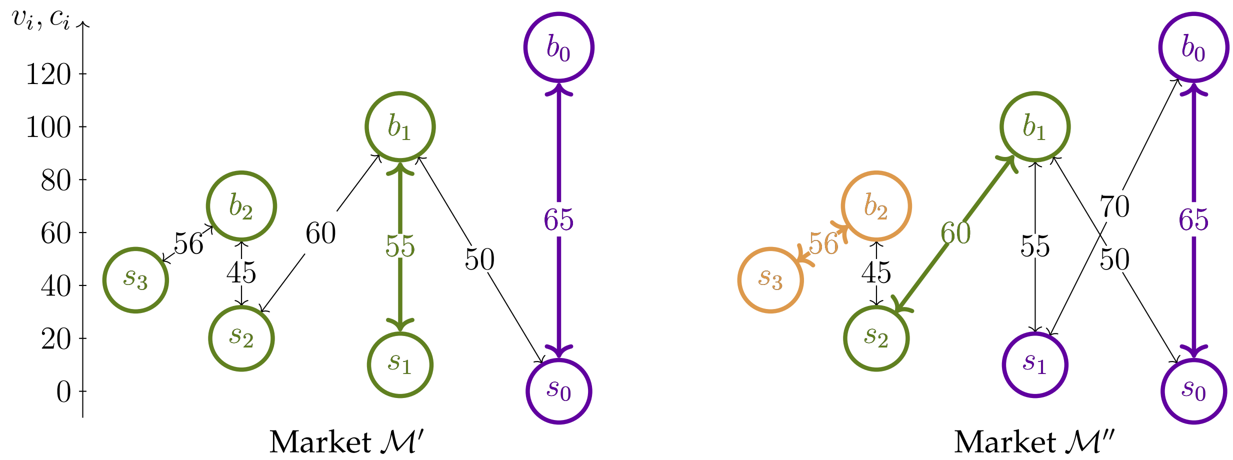


Figure 7: Prices that emerge from Nash bargaining between any two pairs of agents that can trade with each other in markets $\mathcal{M}'$ and $\mathcal{M}''$. Market $\mathcal{M}'$ decomposes into two submarkets: $\{s_0, b_0\}$ and $\{s_1, b_1, s_2, b_2, s_3\}$ with prices 65 and 55, respectively (left). Market $\mathcal{M}''$ decomposes into three submarkets: $\{s_0, b_0, s_1\}$,$\{b_1, s_2\}$ and $\{b_2, s_3\}$ with prices 65, 60 and 56, respectively (right).

Corollary 3.6. *A buyer's limit cutoff price does not increase when the set of types that she can trade with expands. Analogously, a seller's limit cutoff price does not decrease when the set of types that she can trade with expands.*

Proof. Suppose that we add the connection between buyer i and seller j in the network g, where i is a buyer and j is a seller. Note that if, before adding the edge (i, j), buyer i is not in the set X defined by algorithm $\mathcal{B}$, then X does not change after adding this edge. Hence, adding the edge (i, j) can only allow buyer i to be part of a submarket with a lower price. The analogous argument shows that adding this edge can only allow seller j to be part of a submarket with a higher price. $\square$

4 Related Literature

This article contributes to the emerging literature on strategic bargaining in networked markets.[28] An important part of this literature studies the conditions under which decentralized bargaining in markets without inflows of traders can be efficient and/or feature the law of

[28]See for example Kranton and Minehart (2001), Calvó-Armengol (2001), Corominas-Bosch (2004) and Polanski (2007) for important early contributions to this literature.

one price.[29] In this paper, I focus instead on markets in which the inflows of traders into the market balance its outflows, and I provide a framework to investigate how different frictions affect the terms of trade in these markets.

Most related to this paper is Manea (2011), who also characterizes the equilibrium prices in the limit as bargaining frictions vanish in a non-cooperative model of bargaining in a stationary networked market.[30] The present paper differs from Manea (2011) in two important ways. First, it allows agents to strategically choose whom to make offers to—instead of being randomly matched to bargain. Second, it allows heterogeneities in buyers' values and sellers' costs—instead of assuming that each potential trading relationship has the same value—which, as already emphasized in the introduction, are essential for the emergence of price dispersion in this setting.

Strategic choice of partners is a natural element of bargaining, and the contrast between the results of this paper and those obtained in the context of random-matching models illustrates how it can fundamentally alter the determinants of bargaining power in decentralized markets. For example, as Polanski and Vega-Redondo (2018) show, under random matching, the condition for the law of one price to hold in the limit as bargaining frictions vanish has to do with the bargaining power of each *subset* of buyers: Informally, each subset of buyers must collectively have enough edges to low-cost sellers, or, equivalently, each subset of seller must collectively have enough edges to high-value buyers. In contrast, I show that under strategic choice of partners, this condition involves computing one local statistic for each agent—her best price—and comparing it across agents: The highest best price among buyers must be equal to the lowest best price among sellers. Also, under random matching, the buyer-to-seller ratio in each submarket plays a crucial role in determining each submarket's price (Manea 2011). In contrast, I show that, under strategic choice of partners, not only the market decomposes into a different set of submarkets, but also the price in each submarket instead corresponds to Nash bargaining between two of its members.

The contrast between the determinants of prices under random and strategic matching has an intuitive explanation: Under random matching, the ratio of the probability that a seller in a given submarket is the proposer relative to that of a buyer is directly proportional to its buyer-to-seller ratio. This implies that the bargaining power of the sellers in a given submarket is proportional to its buyer-to-seller ratio—and, as a consequence, so is its price.

[29] See for example Abreu and Manea (2012a,2012b), Elliott (2015), Donna, Schenone, and Veramendi (2017), Agranov and Elliott (2017) and Elliott and Nava (forthcoming).

[30] See also Nguyen (2015), who uses convex-programming techniques to characterize the essentially-unique stationary subgame-perfect equilibrium of a model that nests the one in Manea (2011).

But, when the agents strategically choose whom to make offers to, the connection between the buyer-to-seller ratio in a given submarket and the relative proposer probabilities of each side of the submarket is lost. Indeed, even if we choose the buyers' proposer probabilities to be proportional to the market's buyer-to-seller ratio—so that equilibrium prices are a function of this ratio— each submarket's price in the present setting is independent of *its* buyer-to-seller ratio (unless only one submarket forms in equilibrium).

Nguyen (2015) provides a useful characterization of the essentially-unique stationary subgame-perfect equilibrium of a model that nests the one in Manea (2011). In contrast to both Manea (2011) and the present paper, his characterization relies on convex-programming techniques. In an extension of the model of Manea (2011) that allows for heterogenous sellers' costs and buyers' values, Polanski and Vega-Redondo (2018) use the characterization in Nguyen (2015) to identify the conditions under which the law of one price holds in the limit as bargaining frictions vanish. In contrast to the present paper, however, Polanski and Vega-Redondo (2018) do not characterize the equilibrium prices in markets in which the law of one price does not hold.

5 Conclusion

The widespread phenomenon of price dispersion is the result of different frictions that prevent the traders who buy at high prices from reaching out to the traders that sell at low prices. In this paper, I take these frictions as given—in the form of a buyer-seller network that determines which agents can trade with each other—and I show how they shape the prices that emerge when buyers and sellers engage in decentralized strategic bargaining in a stationary market.

The main distinction between the present paper and previous work on strategic bargaining in stationary markets is that—instead of being randomly matched in each period—agents strategically choose whom to make offers to. Strategic choice of partners qualitatively changes both how the market decomposes into different submarkets and how each submarket's price is determined. In particular, when agents strategically choose their partners, the price in each submarket does not depend on its buyer-to-seller ratio, but it is instead the price that emerges from bilateral bargaining between two of its members. Leveraging this observation, I describe an algorithm that characterizes how the interaction between sellers' costs, buyers' values, and the network structure determines the prices in decentralized stationary markets.

The assumption that the exogenous flow of traders into the market exactly matches the endogenous flow of traders out of the market makes the analysis of the determinants of steady-state prices in networked markets tractable. I leave for future research the investigation of the extent to which the models featuring this simplifying assumption capture the main economic forces present in the steady states of more general models with endogenous inflows and outflows of traders. By providing foundations for the replica assumption in random-matching bargaining models, Manea (2017b) constitutes an important step in this direction.

Appendix

A Proof of Proposition 2.1

I prove Proposition 2.1 using a slightly more general framework than the one described in section 1. In subsection A.1 I describe this framework. In subsection A.2 I state the analog of Proposition 2.1 in this framework (Proposition A.1) and I prove it.

A.1 Framework

The set of agents is N. Fix an $n \times n$ symmetric matrix s. I refer to the weighted network s as the *surplus network*, and I assume that agents i and j can produce $s_{i,j} \in \mathbb{R}$ units of surplus. Letting δ_i denote type i's discount factor, I consider the following infinite-horizon bargaining game $\Gamma(s, d, q)$ generated by the network s, the discount-factor profile d, and a probability distribution q on N. In each period, one agent is selected to be the proposer (agent i is selected with probability q_i). The proposer p then chooses an agent r and offers her a wage $w \leq s_{p,r}$ to help her produce $s_{p,r}$ units of surplus. If r accepts the offer, p and r exit the game with payoff $s_{p,r} - w$ and w, respectively; in period $t+1$, agents p and r are replaced by replicas.[31] If r rejects the offer, the two agents remain in the game for the next period. The informational and knowledge assumptions, histories and strategies are analogous to those described in section 1.

A.2 Iterated Conditional Dominance

Following Fudenberg and Tirole (1991, page 128), I define iterated conditional dominance on the class of multi-stage games with observed actions as follows.

Definition A.1. Action a_i^t available to some agent i at information set H_t is *conditionally dominated* if every strategy of agent i that assigns positive probability to action a_i^t in the information set H_t is strictly dominated. *Iterated conditional dominance* is the process that, at each round, deletes every conditionally-dominated action given the strategies that have survived all the previous rounds.

Fudenberg and Tirole (1991) show how iterated conditional dominance solves the alternating-offers bilateral model of Rubinstein (1982). Manea (2017a) shows how iterated conditional

[31] If the proposer p makes an offer to herself, her payoff is $s_{p,p}$ and she is replaced by a replica.

dominance also solves a wide class of models similar to the one considered in this article. I prove Proposition A.1 using the techniques developed in Manea (2017a).

Proposition A.1. *Every agent i has a wage w_i such that—after the process of iterated conditional dominance—she always accepts (rejects) an offer that gives her strictly more (less) than w_i.*

Proof. The proof consists of two steps. First, I define recursively two sequences $(m_i^k)_{i\in N}$ and $(M_i^k)_{i\in N}$, and show by induction on k that after every step s of iterated conditional dominance (see below for a formal definition of such a step), each agent i always rejects every offer that gives her strictly less than $\delta_i m_i^s$ and always accepts every offer that gives her strictly more than $\delta_i M_i^s$. Second, I show that both sequences $(m_i^k)_{i\in N}$ and $(M_i^k)_{i\in N}$ converge to the same point $(w_i)_{i\in N}$.

(i) Iterated Conditional Dominance Procedure

Let me start by reviewing how the process of iterated conditional dominance works in $\Gamma(r, s, q, \Delta)$. For simplicity, I break up the procedure into steps $0, 1, \ldots$, with each step containing three rounds.

Step 0.

Round 0a. Note that a strategy that ever accepts with positive probability a negative share is strictly dominated by the strategy *reject all offers and make only offers that give me a positive share.* These are all the actions that are eliminated in Round 0a. Hence, after this round *every agent i always rejects every offer that gives her strictly less than $\delta_i m_i^0$*, where

$$m_i^0 := 0. \tag{5}$$

Round 0b. Given the actions that survive round 0a, *agent i has an expected payoff (at the beginning of the period, before the proposer has been chosen) of at most M_i^0*, where

$$M_i^0 := \max_j \{s_{i,j}\}. \tag{6}$$

because, by assumption, no agent j can ever offer agent i a payoff higher than $s_{i,j}$, and, by the actions eliminated in round 0a, no agent ever accepts a negative payoff. Hence, every strategy S of agent i that ever rejects with positive probability an offer a that gives her strictly more than $\delta_i M_i^0$ is strictly dominated by a similar strategy S' that specifies *accept a with probability π* in every instance in which S specifies *reject a with probability π.* These are all the actions that are eliminated in Round 0b; so after this round *every agent i always accepts every offer that gives her strictly more than $\delta_i M_i^0$.*

Round 0c. Given the actions that survive rounds 0a and 0b, every strategy S of agent i that ever makes an offer with positive probability that gives $y > \delta_j M_j^0$ to agent j is strictly dominated by a similar strategy S' that specifies *offer* $y - \epsilon > \delta_j M_j^0$ *to agent* j *with probability* π in every instance in which S specifies *offer* y *to agent* j *with probability* π, since agent j must accept both y and $y - \epsilon$. These are all the actions that are eliminated in round 0c; after this round *no agent ever makes an offer giving* $y > \delta_j M_j^0$ *to any agent* j.

Proceeding inductively imagine that, after step $s = k \in \mathbb{Z}_{\geq 0}$, we have concluded (as we have just done for the case $s = 0$) that every agent i:

1. rejects every offer that gives her strictly less than $\delta_i m_i^s$,
2. has an expected payoff (at the beginning of each period) of at most M_i^s,
3. accepts every offer that gives her strictly more than $\delta_i M_i^s$, and
4. does not make offers that give strictly more than $\delta_j M_j^s$ to any agent j.

I now show that points (1) to (4) also hold at step $s = k + 1$.

Step $k + 1$.

I refer to strategies that assign positive probability only to actions that have survived all previous rounds of iterated conditional dominance as "surviving strategies."

Round (k+1)a. Given the surviving strategies, it is conditionally dominated for agent i to ever accept an offer that gives her a surplus strictly lower than $\delta_i m_i^{k+1}$, where m_i^{k+1} is defined as follows:

$$m_i^{k+1} := q_i \max \left(\max_{j \in N} (s_{i,j} - \delta_j M_j^k), \delta_i m_i^k \right) + (1 - q_i)\delta_i m_i^k \tag{7}$$

To see this, consider a period-t subgame where agent i has to respond to an offer $x < \delta_i m_i^{k+1}$. I argue that, for sufficiently small $\epsilon > 0$, accepting this offer is conditionally dominated by the following plan of action—which is designed to give her a time-t expected payoff that approaches $\delta_i m_i^{k+1}$ as ϵ goes to 0: *Reject all offers received at dates* $t' \geq t$. *When selected to be the proposer at time* t', *offer* $\delta_j M_j^{k+t+1-t'} + \epsilon$ *if* $t' \in [t + 1, t + k + 1]$ *and* $\max_{j \in N} (s_{i,j} - \delta_j M_j^{k+t+1-t'}) > \delta_i m_i^{k+t+1-t'}$, *and make an unacceptable offer otherwise (e.g., offer a negative amount to some agent).*

Note that since $t' \geq t + 1$, we have that $k + t + 1 - t' \leq k$. Hence, by the induction hypothesis, all agents j accept the offer $\delta_j M_j^{k+t+1-t'} + \epsilon$ at period $t' \in [t + 1, t + k + 1]$. Moreover, note that

Equation 7 can be written as

$$(8)\qquad m_i^{k+1} = \begin{cases} \delta_i m_i^k & \text{if } \max_{j\in N}\,(s_{i,j} - \delta_j M_j^k) \leq \delta_i m_i^{k+t+1-t'} \\ q_i \max_{j\in N}\,(s_{i,j} - \delta_j M_j^k) + (1-q_i)\delta_i m_i^k & \text{otherwise} \end{cases}$$

and an analogous equation can be used to expand the term m_i^k in Equation 8, and then m_i^{k-1} in the resulting equation, and so on until reaching $m_i^0 = 0$. It is clear from the resulting formula for m_i^{k+1} that, under the surviving strategies, the strategy constructed above generates an expected period-t payoff for i of $\delta_i m_i^{k+1}$ as $\epsilon \to 0$. Hence, letting $\epsilon > 0$ be sufficiently small, this strategy conditionally dominates accepting x in period t. These are the actions eliminated in round (k+1)a; after this round *no agent i ever accepts any offer that gives her a surplus lower than $\delta_i m_i^{k+1}$.*

Round (k+1)b. Given the surviving strategies, it is conditionally dominated for agent i to reject an offer that gives her strictly more than $\delta_i M_i^{k+1}$, where M_i^{k+1} is defined by

$$(9)\qquad M_i^{k+1} := q_i \max\left(\max_{j\in N}\,(s_{i,j} - \delta_j m_j^k), \delta_i M_i^k\right) + (1-q_i)\delta_i M_i^k$$

To prove this, I show that for each agent i, all surviving strategies deliver expected payoffs of at most M_i^{k+1} at the beginning of period t. First, consider a period-t subgame where i is the proposer. Note that i cannot make an offer that generates an expected payoff greater than

$$\max\left(\max_{j\in N}\,(s_{i,j} - \delta_j m_j^k), \delta_i M_i^k\right).$$

To see this note that, under the surviving strategies, all agents j reject all offers lower than $\delta_j m_j^k$, and when j rejects an offer, i can expect a period-$(t+1)$ payoff of at most M_i^k. Second, consider a period-t subgame where i is not the proposer; under the surviving strategies, i can expect a period-t payoff of at most M_i^k. Therefore, *agent i has an expected payoff (at the beginning of each period) of at most M_i^{k+1}*. These are all the actions that are eliminated in round (k+1)b; after this round, *no agent ever offers strictly more than $\delta_j M_j^{k+1}$ to agent j.*

Round (k+1)c. Given the surviving strategies, every strategy S of agent i that ever makes an offer that gives $y > \delta_j M_j^{k+1}$ to agent j is strictly dominated by a similar strategy S' that specifies *offer $y - \epsilon > \delta_j M_j^{k+1}$ to agent j with probability π* in every instance in which S specifies *offer y to agent j with probability π*, since agent j must accept both y and $y - \epsilon$. These are all the actions that are eliminated in round (k+1)c; after this round *no agent ever makes an offer giving $y > \delta_j M_j^{k+1}$ to any agent j.*

(ii) The sequences $(m_i^k)_{i\in N}$ and $(M_i^k)_{i\in N}$ converge to the same limit.

First, we prove by induction on k that for all $i \in N$, the sequence $(m_i^k)_{k\geq 0}$ is increasing in k, the sequence $(M_i^k)_{k\geq 0}$ is decreasing in k, and $\max_{j\in N}(s_{i,j}) \geq M_i^k \geq m_i^k \geq 0$ for all $k \geq 0$. This implies that both sequences $(m_i^k)_{i\in N}$ and $(M_i^k)_{i\in N}$ converge.

Note that $m_i^0 = 0$ and $M_i^0 := \max_j \{s_{i,j}\}$, and that Equation 7 and Equation 9 imply that $m_i^1 \geq 0$ and $M_i^1 \leq \max_j \{s_{i,j}\}$, so $m_i^1 \geq m_i^0$ and $M_i^1 \leq M_i^1$. Now suppose that for some $l \in \mathbb{N}$:

$$m_i^l \geq m_i^{l-1} \text{ and } M_i^l \leq M_i^{l-1}.$$

I show that

$$m_i^{l+1} \geq m_i^l \text{ and } M_i^{l+1} \leq M_i^l.$$

Note that, by the induction hypothesis, every summand in Equation 7 when $k = l+1$ is smaller than when $k = l$, which implies that $m_i^{l+1} \leq m_i^l$. Similarly, every summand in Equation 9 when $k = l+1$ is bigger than when $k = l$, which implies that $M_i^{l+1} \geq M_i^l$. Hence, the sequence $(m_i^k)_{k\geq 0}$ is increasing in k and the sequence $(M_i^k)_{k\geq 0}$ is decreasing in k, which, implies that

$$\max_{j\in N}(s_{i,j}) \geq M_i^k \geq m_i^k \geq 0 \text{ for all } k \geq 0.$$

since $\max_{j\in N}(s_{i,j}) = M_i^0 > m_i^0 = 0$.

Second, I show that the sequences $(m_i^k)_{i\in N}$ and $(M_i^k)_{i\in N}$ converge to the same limit. Let D^k be $\max_{i\in N}(M_i^k - m_i^k)$. I show that

$$D^k \leq \left(\max_{j\in N} \delta_j\right)^k D^0 = \left(\max_{j\in N} \delta_j\right)^k \max_{j,j'\in N}(s_{j,j'})$$

for all $k \geq 0$; that is, that D^k converges to 0 as k grows large. Indeed,

$$\begin{aligned}
D^{k+1} &= \max_{i\in N} \quad [M_i^{k+1} - m_i^{k+1}] \\
&= \max_{i\in N} \quad \Bigg[q_i \max\left(\max_{j\in N}(s_{i,j} - \delta_j m_j^k), \delta_i M_i^k\right) + (1-q_i)\delta_i M_i^k \\
&\qquad\quad -q_i \max\left(\max_{j\in N}(s_{i,j} - \delta_j M_j^k), \delta_i m_i^k\right) + (1-q_i)\delta_i m_i^k\Bigg] \\
&= \max_{i\in N} \quad \Bigg[q_i\Big[\max\left(\max_{j\in N}(s_{i,j} - \delta_j m_j^k), \delta_i M_i^k\right) - \max\left(\max_{j\in N}(s_{i,j} - \delta_j M_j^k), \delta_i m_i^k\right)\Big] \\
&\qquad\quad +(1-q_i)\Big[\delta_i M_i^k - \delta_i m_i^k\Big]\Bigg] \\
&= \max_{i\in N} \quad \Bigg[q_i\Big[\max\left(s_{i,j^*} - \delta_{j^*} m_{j^*}^k, \delta_i M_i^k\right) - \max\left(s_{i,j^*} - \delta_{j^*} M_{j^*}^k, \delta_i m_i^k\right)\Big] \\
&\qquad\quad +(1-q_i)\Big[\delta_i M_i^k - \delta_i m_i^k\Big]\Bigg] \\
&\leq \max_{i\in N} \quad \Big[q_i \max\left(\delta_{j^*}(M_{j^*}^k - m_{j^*}^k), \delta_i(M_i^k - m_i^k)\right) + (1-q_i)\delta_i D^k\Big] \\
&\leq \max_{j\in N} \quad \delta_j D^k
\end{aligned}$$

where j^* is any element of $\underset{j\in N}{\text{argmax}}(s_{i,j} - \delta_j M_j^k)$, and the second inequality is a consequence of Lemma A.1 below. □

Lemma A.1 (Manea 2017a)**.** *For all* $w_1, w_2, w_3, w_4 \in \mathbb{R}$,

$$|max(w_1, w_2) - \max(w_3, w_4)| \leq \max(|w_1 - w_3|, |w_2 - w_4|).$$

B Proof of Proposition 2.3

Proposition 2.3 follows from Lemma B.2, Lemma B.3 and Lemma B.4 below. Indeed, when each type has a different cutoff price, each type has a unique preferred partner, so each component of the preferred-partner network G has at most one cycle. Hence, Lemma B.2, Lemma B.3 and Lemma B.4 together imply that, generically, each type that is not her own preferred partner can reach exactly one pair of mutually-preferred partners in G.[32]

Lemma B.1 provides a preliminary observation that is useful to prove Lemma B.2.

Lemma B.1. *A type who is her own preferred partner is not the preferred partner of any other type.*

Proof. Let buyer i be her own preferred partner (an analogous argument works for sellers), let j be one of her connections, and suppose for contradiction that i is one of j's preferred partners. Since $c_j < v_i = \rho_i$, we get $\rho_j = \alpha c_j + (1-\alpha)\rho_i = \alpha c_j + (1-\alpha)v_i < v_i$, a contradiction. □

Lemma B.2. *Each type that is not her own preferred partner can reach at least one cycle of length at least two in the preferred-partner network G.*

Proof. Each type can reach at least one cycle in the network G, because there are finitely many types, each with at least one preferred partner. Moreover, no type that is not her own preferred partner can reach a cycle of length 1. This is because, as shown by Lemma B.1, if a type is her own preferred partner, she is not anyone else's preferred partner. □

Lemma B.3. *Generically, the preferred-partner network does not contain cycles with more than two nodes.*

Proof. See the text below Proposition 2.3. □

[32]A node *can reach a pair (i, j) of mutually-preferred partners in G* if there is either a walk in G from it to i or a walk in G from it to j.

Lemma B.4. *Generically, no two types have the same cutoff price.*

Proof. Consider two different buyers i and i' that have the same cutoff price. Given Corollary 2.1, this is not generic if each of i and i' have a mutually-preferred partner. So suppose without loss of generality that buyer i does not have a mutually-preferred partner, and let seller j be one of her preferred partners. Also, let seller j' be one of the preferred partners of buyer i'. Using Proposition 2.2, we get that

$$\alpha v_i + (1-\alpha)\rho_j = \rho_i = \rho_{i'} = \alpha v_{i'} + (1-\alpha)\rho_{j'}. \tag{10}$$

Generically, ρ_j does not depend on v_i since, by assumption, i is not one of j's preferred partners and there are no cycles in the preferred-partner network G with more than two nodes (Lemma B.3). Moreover, $\rho_{j'}$ either does not depend on v_i or—in case there is a walk in G from j' to i—is linear in v_i, with coefficient $\alpha(1-\alpha)^\kappa$, for some $\kappa \geq 1$. Hence, generically, Equation 10 does not hold. □

C Proof of Proposition 3.1

For each agent i, let $f_i^\delta(x)$ denote i's cutoff price when her preferred-partner's cutoff price is x and the discount factor is δ; that is, using Proposition 2.2, $f_i^\delta(x) = \frac{\delta}{\delta+(1-\delta)n}x + \frac{(1-\delta)n}{\delta+(1-\delta)n}v_i$ for each buyer i, and $f_j^\delta(x) = \frac{\delta}{\delta+(1-\delta)n}x + \frac{(1-\delta)n}{\delta+(1-\delta)n}c_j$ for each seller j. Also, given a walk $W = (1, 2, \ldots, k-1, k, k-1)$, let $f(W,\delta)$ denote 1's cutoff price when W is a walk of the preferred-partner network and the discount factor is δ. That is, $f(W,\delta) = f_1^\delta \circ f_2^\delta \circ \cdots \circ f_{k-2}^\delta\left(p_{k-1,k}(\delta)\right)$ where $p_{i,j}(\delta)$ is as in Corollary 2.1.

Suppose for contradiction that there is no $\overline{\delta} < 1$ such that the preferred-partner network is fixed for all $\delta \in (\overline{\delta}, 1)$. Given that there are finitely many different walks in g, and that both $f(W,\delta)$ and each agent's cutoff price are continuous in δ,[33] this implies that there exist two different walks W and W' in g with the same first node, and a strictly increasing sequence $\mathcal{S} = (\delta_z)_{z\in\mathbb{N}}$ converging to one, such that (i) when $\delta \in \mathcal{S}$, $f(W,\delta) = f(W',\delta)$, and (ii) for all $z \in \mathbb{N}$ and all $\delta > \delta_z$ sufficiently close to δ_z, $f(W,\delta) > f(W',\delta)$. But $f(W,\delta) = f(W',\delta)$ is a polynomial equation in δ, so it either holds for all δ or for at most finitely many values of δ, a contradiction. □

[33]To see that each agent's cutoff price is continuous in δ, note that, for each agent i, $\{m_i^k\}_{k\in\mathbb{N}}$ defined in the proof of Proposition A.1 is a sequence of continuous functions in δ, which converges uniformly to w_i on any interval $I \subset (0,u)$ with $u < 1$. Hence, by the uniform convergence theorem, w_i is also continuous in δ.

D Proof of Theorem 3.1

The proof of Theorem 3.1 follows from Lemma D.1 and Lemma D.2 below. Lemma D.1 shows that all the types in X are in the submarket with the highest price p. This is intuitive: All the sellers that can, do trade in this submarket. Similarly, a buyer who—if she did not enter this submarket—would find herself in such a weak bargaining position that she would have to pay a higher price than p, also has no better option but to trade in this submarket.

Definition D.1. For any set $S \subseteq N$, I refer to the market $\mathcal{M}^S = (N, v, g^S, r)$, where g^S denotes the network g after removing from it every edge involving any agent in $N - S$ as *the restriction of the market* $\mathcal{M}$ *to* S.

Lemma D.1. *The set X defined by algorithm $\mathcal{B}$ is a subset of the set H of all the types in the submarket with the highest price.*

Proof. I prove by induction on k that, for all $0 \leq k \leq \kappa$, $X^k \subseteq H$. The base step follows from Corollary 3.1. The induction step is as follows. Let $0 < k \leq \kappa$ be such that all types in X^{k-1} are in H. The fact that all the sellers in X^k are in H is clear. Indeed, a seller in X^k whose preferred partner in G^* is not in H is connected to a buyer in H, which is a contradiction, since, by definition, the cutoff prices of all the types in H converge to a higher price than the cutoff prices of all the types who are not in H.

It only remains to show that the preferred partner of a buyer whose best price among the types in $N - X^{k-1}$ is higher than p is in H. To see this, suppose otherwise. Then Corollary 3.1 applied to the restriction of the market $\mathcal{M}$ to $N - H$ implies that the second-highest market price is higher than p, a contradiction. $\square$

Lemma D.2 shows that X contains all the types in the submarket with the highest price.

Lemma D.2. *The set H of all the types in the submarket with the highest price is a subset of the set X defined by algorithm $\mathcal{B}$.*

Proof. I show that there exists $\epsilon > 0$ such that, for all small-enough bargaining frictions, the cutoff price of every agent $i \in N - X$ is lower than $p - \epsilon$, which implies that $H - X$ is empty, since all the cutoff prices of the types in H converge to p.

For each discount factor $\delta \in (0, 1)$, let $\sigma'(\delta)$ and $\sigma''(\delta)$ be subgame-perfect equilibria of $\Gamma(\mathcal{M}', \delta)$ and $\Gamma(\mathcal{M}'', \delta)$, respectively, where $\mathcal{M}'$ and $\mathcal{M}''$ denote the restriction of the market

$\mathcal{M}$ to $N - X$ and X, respectively. By Corollary 3.1, it is enough to show that $(\sigma_i(\delta))_{i\in N}$, defined by $\sigma_i(\delta) = \sigma_i''(\delta)$ if $i \in X$ and $\sigma_i(\delta) = \sigma_i'(\delta)$ otherwise, is a subgame-perfect equilibrium of $\Gamma(\mathcal{M}, \delta)$ for all discount factors δ sufficiently close to 1.

Note that $(\sigma_i(\delta))_{i\in N}$ is a subgame-perfect equilibrium of $\Gamma(\mathcal{M}, \delta)$ if and only if no agent has an incentive to deviate when she is the proposer.[34] First, note that, by construction, no agent (not) in X has an incentive to deviate and make a proposal to another agent (not) in X. Second, note that, by definition, no seller in $N - X$ has any connection in X.

Hence, it only remains to show that, when δ is close enough to 1, no buyer in $N - X$ has an incentive to deviate by making offers to a seller in X, and that no seller in X has an incentive to deviate by making offers to a buyer in $N - X$. But this is clear, since, by Corollary 3.1 and Lemma D.1 applied to the market $\mathcal{M}''$, the cutoff prices of all the types in X converge to p, and, by Corollary 3.1 and Lemma D.1 applied to the market $\mathcal{M}'$, the cutoff price of each agent in $N - X$ converges to a price strictly smaller than p. $\square$

[34]Indeed, as shown by Proposition 2.2, the cutoff price of an agent's preferred partner determines her own cutoff price.

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