NOTES CONCERNING REFERENCE DEPENDENT THEORIES AND THEIR USAGE FOR BEHAVIORAL EXPERIMENTS

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1. Introduction

One behavioral economics' central goals has been to predict a subject's choices under risk. The simplest case to consider is that of binary lottery such as the one given below.

$$L = \begin{cases} \mu + x, & p \\ \mu - y, & 1-p \end{cases} \tag{1}$$

with

$$px = (1-p)y \tag{2}$$

so that $\mu = \mathbb{E}[L]$.

This will be the prototypical lottery that the two main candidate theories will be evaluated by.

2. Koszegi & Rabin's Expectations Based Reference Dependence Theory (EBRD)

When we refer to EBRD, we will consider the following parameterizations:

Definition 1. *The utility of a stochastic consumption option, C, given a stochastic reference, R, is provided by:*

$$U(C|R) = \mathbb{E}\left[C + g(C-R)\right] \tag{3}$$

where we assume:

1. *$C \perp R$ (since the imagined reference point doesn't affect the stochastic outcome)*
2. *The gain-loss function is defined by:*

$$g(x) = \begin{cases} f(x), & x > 0 \\ -\lambda f(-x), & x \leq 0 \end{cases} \tag{4}$$

3. *$f : \mathbb{R}^+ \to \mathbb{R}^+$ is in C^2 and concave (diminishing sensitivity)*
4. *$f(0) = 0$ (gain-loss utility vanishes when an amount equals the reference)*
5. *$\lambda > 1$ (loss aversion)*

When a choice between a set of stochastic consumption options, $\{C_i\}$, the choice EBRD predicts depends on whether there exist exogenous reference or whether the references are generated endogenously. If we assume the existence of exogenous references, then given exogenous reference, R, one chooses option $C^* \in \{C_i\}$, by:

$$C^* = \text{argmax}_{C_i} U(C_i|R) \tag{5}$$

whereas if we assume one has endogenous references one's choice must be a *personal equilibrium*, meaning that if the person expects to receive that consumption option he prefers it over the other

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options. Otherwise, we assume that the person would then change their consumption choice. This condition is represented by:

$$C_i \in PE \quad \text{if} \quad U(C_i|C_i) \geq U(C_j|C_i), \forall j \neq i \tag{6}$$

If multiple personal equilibria are present then we assume that subject choose the personal equilibrium with highest utility.

$$C^* = \text{argmax}_{C_i \in PE} U(C_i|C_I) \tag{7}$$

and we denote this as a *preferred personal equilibrium.* We now prove the a few relevant propositions for understanding the theory's implications.

Proposition 1. *Consider the choice between μ and L when one is endowed μ. Denote the odds of winning the larger amount by:*

$$o := \frac{p}{1-p} \tag{8}$$

so that $y = ox$. EBRD predict that the subject chooses to keep the safe payment, μ, when:

$$\lambda > o\frac{f(x)}{f(ox)} \tag{9}$$

Proof. We first derive the condition for when the subject is endowed with μ.

$$U(\mu|\mu) > U(L|\mu) \tag{10}$$

$$\mu > \mu + pf(x) - \lambda(1-p)f(y) \tag{11}$$

$$\lambda > \frac{p}{1-p} \cdot \left(\frac{f(x)}{f(y)}\right) \tag{12}$$

$$\lambda > o\frac{f(x)}{f(ox)} \tag{13}$$

If the condition above is satisfied, then the condition for endogenous references is also satisfied when:

$$U(\mu|\mu) > U(L|L) \tag{14}$$

$$\mu > \mu + p(1-p)(1-\lambda)f(x+y) \tag{15}$$

$$\lambda > 1 \tag{16}$$

which is assumed so that the endogenous condition is equivalent to the exogenous condition when endowed with μ. □

Proposition 2. *If a subject has a priori fixed reference, r, then if $\alpha < 1$:*

(1) If the subjects chooses the safe option then $r < \mu + x$

(2) If the subject chooses the lottery then $r > \mu - y$

and if $f(x) = x$ the subject is indifferent between the two options in these two cases.

Proof. If $r > \mu + x$ then one chooses the safe option if:

$$\mu - \lambda f(r-\mu) > \mu - p\lambda f(r-l-x) - (1-p)\lambda f(r-l+y) \tag{17}$$

$$f(r-\mu) < pf(r-l-x) + (1-p)f(r-l+y) \tag{18}$$

which is false by Jensen's inequality for concave $f(x)$ and is an equality for $f(x) = x$.

Similarly, if $r < \mu - y$ the one chooses the safe option if:

$$\mu + f(\mu - r) > \mu + pf(l+x-r) + (1-p)f(l-y-r) \tag{19}$$

$$f(\mu - r) > pf(l+x-r) + (1-p)f(l-y-r) \tag{20}$$

which is true by Jensen's inequality for concave $f(x)$ and is an equality for $f(x) = x$.

□

In general, given Stevens' Law, we tend to assume that diminishing sensitivity follows a power law. We now prove the analogous proposition given this additional assumption.

Proposition 3. *Consider the choice between μ and L when one is endowed $e \in \{\mu, L\}$. Denote the odds of winning the larger amount by:*

$$o := \frac{p}{1-p} \tag{21}$$

so that $y = ox$. If a subject has diminishing sensitivity given by:

$$f(x) = x^\alpha, \quad \alpha \leq 1 \tag{22}$$

then the EBRD variants predict that the subject chooses the safe payment, μ, when:

(1) $\lambda > o^{1-\alpha}$ *(Exogenous EBRD with reference $e = \mu$)*
(2) $\lambda < \frac{o^{\alpha-1}-(1+o)^{\alpha-1}}{1-(1+o)^{\alpha-1}}, \quad \alpha < 1$ *(Exogenous EBRD with reference $e = L$)*
(3) $\lambda > o^{1-\alpha}$ *(Endogenous EBRD)*

Proof. We first derive the condition for when the subject is endowed with μ.

$$U(\mu|\mu) > U(L|\mu) \tag{23}$$
$$\mu > \mu + px^\alpha - \lambda(1-p)y^\alpha \tag{24}$$
$$\lambda > \frac{p}{1-p} \cdot \left(\frac{x}{y}\right)^\alpha = \frac{p}{1-p} \cdot \left(\frac{1-p}{p}\right)^\alpha \tag{25}$$
$$\lambda > o^{1-\alpha} \tag{26}$$

We next derive the condition for when the subject is endowed with L given $\alpha < 1$.

$$U(\mu|L) > U(L|L) \tag{27}$$
$$\mu + (1-p)y^\alpha - p\lambda x^\alpha > \mu + p(1-p)(1-\lambda)(x+y)^\alpha \tag{28}$$
$$(1-p)(y^\alpha - p(x+y)^\alpha) > \lambda p(x^\alpha - (1-p)(x+y)^\alpha) \tag{29}$$
$$o^\alpha - p(1+o)^\alpha > \lambda o(1 - (1-p)(1+o)^\alpha) \tag{30}$$
$$o^{\alpha-1} - p^{1-\alpha}o^{\alpha-1} > \lambda(1 - (1+o)^{\alpha-1}) \tag{31}$$
$$\lambda < \frac{o^{\alpha-1} - (1+o)^{\alpha-1}}{1-(1+o)^{\alpha-1}} \tag{32}$$

If $\alpha = 1$ then we would have equality across both sides.

Finally, if both conditions above are satisfied, then the condition for endogenous references is given by:

$$U(\mu|\mu) > U(L|L) \tag{33}$$
$$\mu > \mu + p(1-p)(1-\lambda)(x+y)^\alpha \tag{34}$$
$$\lambda > 1 \tag{35}$$

which is assumed so that the endogenous condition is equivalent to the exogenous condition when endowed with μ. □

It is worth considering the boundary cases without loss aversion and/or diminishing sensitivity.

Proposition 4. *Endogenous EBRD breaks down for all $\lambda \leq 1$.*

Proof. Given our exposition above, in the case of non-trivial $f(x)$, for there to be an equilibrium we require that either:

$$\lambda > o\frac{f(x)}{f(ox)}, \quad \text{or} \quad \lambda > \frac{f(ox)/o - f(x(o+1))/(o+1)}{f(x) - f(x(o+1))/(o+1)} \tag{36}$$

If $o = 1$ then both inequalities reduce to:

$$\lambda > 1 \tag{37}$$

so that if $\lambda \leq 1$ there is no equilibrium.

If $f(x) = x$ the first inequality is still violated as is the more general version of the second:

$$f(ox)/o - f(x(o+1))/(o+1) < \lambda\left(f(x) - f(x(o+1))/(o+1)\right) \tag{38}$$

□

Proposition 5. *If $\lambda < 1, f(x) = x$ then exogenous EBRD over wealth, in which utility over wealth is logarithmic, implies that a subject should be willing to wager fraction:*

$$\frac{(1-\lambda)w}{2+(1+\lambda)w}$$

of their wealth, w, on a fair coin toss.

Proof. The general utility given an amount wagered x is:

$$p\ln|w+x| + (1-p)\ln|w-x| + p\ln\left|\frac{w+x}{w}\right| - \lambda(1-p)\ln\left|\frac{w}{w-x}\right| \tag{39}$$

optimizing over x yields the amount:

$$x^* = w\left(\frac{(1-\lambda)w}{2+(1+\lambda)w}\right) \tag{40}$$

□

3. Third Generation Prospect Theory

We choose to compare EBRD with Sugden's Third Generation Prospect Theory (PT3).

Definition 2. *The utility of stochastic choice, C, given stochastic reference, R, is provided by:*

$$U(C|R) = \int_{\mathbb{R}^+} \pi(\mathbb{P}(G > x))dv(x) - \lambda\int_{\mathbb{R}^+} \pi(\mathbb{P}(-D > x))dv(x) \tag{41}$$

where we assume:

(1) $\pi \in C([0,1])$ is a monotonically increasing function with $\pi(0) = 0$ and $\pi(1) = 1$.
(2) $G := (C-R)^+$ the "Gain" sub-lottery
(3) $D := (C-R)^-$ the "Debt" sub-lottery (reflection principle)
(4) $v : \mathbb{R} \to \mathbb{R}$ is in C^2, $v(0) = 0$, and is concave (diminishing sensitivity)
(5) $\lambda > 1$ (loss aversion)

To incentivize this definition, recall that integration by parts yields the expected value of a function of the random variable, X, as:

$$\mathbb{E}[g(X)] = \int_{\mathbb{R}^+} (1 - F_X(x))dg(x) - \int_{\mathbb{R}^-} F_X(x_-)dg(x) \tag{42}$$

in which, $F_X(x)$ is the CDF, $F_X(x_-)$ its left limit, $g(0) = 0$, $g \in C^1(\mathbb{R})$ and $dg(x) = \frac{dg}{dx}dx$. We therefore consider the case in which g is the value function, v, in order to capture diminishing sensitivity. To include cumulative probability weighting–that is, subject over-sensitivty to extreme outcomes–we just apply π to the cumulative probabilities $1 - F_X(x)$ in our integrals. Since we assume $\pi \in C([0,1])$, satisfying $\pi(0) = 0, \pi(1) = 1$ this generates a new density/mass function through integration by parts averting any potential issues in which the subjective probabilities sum to a value greater than one.

For discrete lotteries with finite support we may rewrite the above definitions of the utility in a more practical form which we provide in the following two lemmas.

Lemma 1. *Let L be a non-negative discrete lottery with finite support over $n \geq 0$ strictly positive outcomes, $\{l_i > 0 \quad | i < j \to l_i < l_j\}$. Then the stochastic value of the lottery is given by:*

$$V(L) = v(l_1)\pi(\mathbb{P}(L > 0)) + \sum_{i=1}^{n-1} \left(v(l_{i+1}) - v(l_i)\right) \pi(\mathbb{P}(L > l_i)) \tag{43}$$

using the above lemma we proceed to find the utility for a general lottery and reference in the lemma below:

Lemma 2. *Let L, R be discrete lotteries with finite support, then the utility of the lottery, L, given reference lottery, R, is provided by:*

$$U(L|R) = V((L-R)^+) - \lambda V((R-L)^+) \tag{44}$$

in which the stochastic value function is defined by the previous lemma.

As an example, consider a lottery, L and reference, R, such that the random variable defined by their difference (using the the usual method of convolving their probabilities) has the following distribution:

$$L - R = \begin{cases} 3, & 1/3 \\ 1, & 1/3 \\ -1, & 1/3 \end{cases}$$

then the utility is found to be:

$$U(L|R) = (v(3) - v(1))\pi(\mathbb{P}(L - R > 1)) + v(1)\pi(\mathbb{P}(L - R > 0)) - \lambda v(1)\pi(\mathbb{P}(R - L > 0))$$

which may be simplified as:

$$U(L|R) = v(3)\pi(1/3) + v(1)(\pi(2/3) - \pi(1/3)) - \lambda v(1)\pi(1/3)$$

which, when $v(x) = x, \pi(p) = p$ yields: $U(L|R) = (4 - \lambda)/3$.

To interpret the theory we consider the following lemmas and propositions.

Lemma 3. *The utility scale given reference lottery, R, is centered at, R, so that positive utility implies improvement over the reference and negative utility implies relative diminishment. Stated formally,*

$$\forall R, \quad U(R|R) = 0 \tag{45}$$

Proof. The lemma trivially follows from the definition. □

Secondly, all value that exists in both the reference and consumption bundle of choice may be ignored. In other words, the theory is identical when defined over local changes in wealth and wealth itself.

Lemma 4. *Let there exist a background stochastic wealth, W, and induced additional expectations, R, as well as additional consumption option, C, then we have that:*

$$U(C + W|R + W) = U(C|R) \tag{46}$$

Proof. This trivially follows from Def. 2 (2), (3). □

Proposition 6. *PT3 predicts a gap between the amount one is willing to pay for an item and the amount one is willing sell it for. More generally, given items, L_1, L_2, if a person is willing to trade L_1 for L_2 then they are unwilling to make the reverse trade.*

Proof. If a person is willing to trade L_1 for L_2 then

$$U(L_2|L_1) > U(L_1|L_1) \tag{47}$$
$$> 0 \tag{48}$$

We then note that if we define G, D with respect to reference, L_2 that

$$U(L_1|L_2) + U(L_2|L_1) = (1-\lambda)\left(\int_{\mathbb{R}^+} \pi(\mathbb{P}(G > x))dv(x) + \int_{\mathbb{R}^+} \pi(\mathbb{P}(-D > x))dv(x)\right) \tag{49}$$
$$< 0 \tag{50}$$
$$U(L_1|L_2) < -U(L_2|L_1) \tag{51}$$

so that

$$U(L_1|L_2) < -U(L_2|L_1) \tag{52}$$
$$< 0 \tag{53}$$
$$= U(L_2|L_2) \tag{54}$$

implying that the person prefers to remain with L_2 in lieu of trading it for L_1.

□

Proposition 7. *Let $v(x) = x^\alpha, \alpha \in (0,1)$. Consider the lottery:*

$$M = \begin{cases} l_1, & p \\ l_2, & (1-p) \end{cases} \tag{55}$$

with $l_1 > l_2 > 0$. A person is willing to pay a maximum of:

$$x^{WTP} = \frac{\tilde{o}l_1 + \tilde{\lambda}l_2}{\tilde{o} + \tilde{\lambda}} \tag{56}$$

for the lottery and is willing to sell the lottery for a minimum of:

$$x^{WTA} = \frac{\tilde{\lambda}\tilde{o}l_1 + l_2}{\tilde{\lambda}\tilde{o} + 1} \tag{57}$$

in which:

$$\tilde{\lambda} = \lambda^{1/\alpha}, \qquad \tilde{o} = \left(\frac{\pi(p)}{\pi(1-p)}\right)^{1/\alpha} \tag{58}$$

In short, the willingness to pay and accept are weighted averages of the lottery's payoffs with the willingness to to accept placing greater weight on the higher payoff.

Proof. Firstly, we consider the willingness to pay.

$$U(x|x) = U(M|x) \tag{59}$$
$$0 = \pi(p) \cdot (l_1 - x)^\alpha - \lambda\pi(1-p) \cdot (x - l_2)^\alpha \tag{60}$$
$$\tilde{\lambda}(x - l_2) = \tilde{o}(l_1 - x) \tag{61}$$
$$x = \frac{\tilde{o}l_1 + \tilde{\lambda}l_2}{\tilde{o} + \tilde{\lambda}} \tag{62}$$

Likewise, the willingness to accept is given by:

$$U(x|M) = U(M|M) \tag{63}$$
$$\pi(1-p)\cdot(x-l_2)^\alpha - \lambda\pi(p)\cdot(l_1-x)^\alpha = 0 \tag{64}$$
$$(1/\tilde{o})(x-l_2) = \tilde{\lambda}(l_1-x) \tag{65}$$
$$x = \frac{\tilde{\lambda}\tilde{o}l_1 + l_2}{\tilde{\lambda}\tilde{o}+1} \tag{66}$$

□

In the context of our lottery, L, from earlier, this may be rewritten as:

$$WTP = \mu + x\frac{\tilde{o}-\tilde{\lambda}o}{\tilde{o}+\tilde{\lambda}} \tag{67}$$
$$WTA = \mu + x\frac{\tilde{\lambda}\tilde{o}-o}{\tilde{\lambda}\tilde{o}+1} \tag{68}$$

so that a person is only willing to pay less than the expected value when:

$$x\frac{\tilde{o}-\tilde{\lambda}o}{\tilde{o}+\tilde{\lambda}} < 0 \tag{69}$$
$$\tilde{o}-\tilde{\lambda}o < 0 \tag{70}$$
$$\lambda > \left(\frac{\tilde{o}}{o}\right)^\alpha \tag{71}$$

and, similarly, a person is only willing to accept a value greater than the expected value when:

$$x\frac{\tilde{\lambda}\tilde{o}-o}{\tilde{\lambda}\tilde{o}+1} > 0 \tag{72}$$
$$\tilde{\lambda}\tilde{o} > o \tag{73}$$
$$\lambda > \left(\frac{o}{\tilde{o}}\right)^\alpha \tag{74}$$

The results may be restated in the following proposition.

Proposition 8. *Let $v(x) = x^\alpha, \alpha \in (0,1)$. A person is willing to pay more than the expected value for a binary lottery if:*

$$\lambda < \left(\frac{\tilde{o}}{o}\right)^\alpha \tag{75}$$

and is willing to accept less than the expected value for a binary lottery if:

$$\lambda < \left(\frac{o}{\tilde{o}}\right)^\alpha \tag{76}$$

4. Calibrations

In calibrating and comparing EBRD and PT3, it is necessary to compare apples to apples. By this I mean to say that comparison between the two is only meaningful when they have the same number of degrees of freedom. Accordingly, when examining the two theories, if we assume $\pi(p) = p$ or $\alpha = 1$ in one theory we apply the same rule to the other. We now proceed to the comparisons.

4.1. **General Willingness to Exchange $\mathbb{E}[L]$ for L.** Consider a version of EBRD with probability weighting so that the expectation of the gain-loss utility is taken under the transformed probabilities. In other words, given the p-measure and π-measure, the EBRD utility is given by:

$$U(C|R) = \mathbb{E}_p[C] + \mathbb{E}_\pi[g(C-R)] \tag{77}$$

then if endowed with $\mathbb{E}[L]$ both theories predict that a subject will choose to keep their endowment instead of trading it for L if and only if:

$$\lambda > \left(\frac{\tilde{o}}{o}\right)^{\alpha}$$

For simplicity, in the next two subsections consider the lottery:

$$\tilde{L} = \begin{cases} l, & p \\ 0, & q = 1-p \end{cases}$$

4.2. **Simple WTP comparisons.** Assume that $\pi(p) = p$ and $\alpha = 1$, then the willingness to pay for $\tilde{L}$ under EBRD is:

$$x_{EBRD}^{WTP} = \frac{2pl}{1+p+\lambda q} \leq pl$$

whereas under PT3 it is:

$$x_{PT3}^{WTP} = \frac{pl}{p+\lambda q} \leq pl$$

Accordingly, we find that:

$$\Delta WTP := x_{EBRD}^{WTP} - x_{PT3}^{WTP} = \frac{2pl}{1+p+\lambda q} - \frac{pl}{p+\lambda q} \tag{78}$$

$$= \frac{(\lambda-1)pql}{(p+\lambda q)(1+p+\lambda q)} \tag{79}$$

$$> 0 \tag{80}$$

so that the simple EBRD agent is willing to pay more for a lottery than a simple PT3 agent.

The difference in implied probability weights is:

$$\frac{\Delta WTP}{l} = \frac{(\lambda-1)pq}{(p+\lambda q)(1+p+\lambda q)} \tag{81}$$

which is small.

4.3. **Simple WTA comparisons.** Assume that $\pi(p) = p$ and $\alpha = 1$, then the willingness to accept for $\tilde{L}$ under EBRD is:

$$x_{EBRD}^{WTA} = pl$$

whereas under PT3 it is:

$$x_{PT3}^{WTA} = \frac{\lambda pl}{q+\lambda p} \geq pl$$

Accordingly, we find that:

$$\Delta WTA := x_{EBRD}^{WTA} - x_{PT3}^{WTA} = pl - \frac{\lambda pl}{q+\lambda p} \tag{82}$$

$$= \frac{q(1-\lambda)}{q+\lambda p} pl \tag{83}$$

$$< 0 \tag{84}$$

so that the simple EBRD agent is willing to accept less for a lottery than a simple PT3 agent.

The difference in implied probability weights is:

$$\frac{\Delta WTA}{l} = \frac{pq(1-\lambda)}{q+\lambda p} > -\frac{1}{4} \tag{85}$$

and generally should not be observable given measurement error.

5. State Dependence in Simple PT3

5.1. Trading Lotteries. Here we consider the premium necessary to induce a person to exchange a lottery for another. To illustrate the behavior we consider two binary lotteries, X, Y with the person being endowed lottery, R.

In the case that $R = X$ we find that the subject chooses X when:

$$\mathbb{E}[X] - \mathbb{E}[Y] > \frac{\lambda - 1}{2}(y_1 - x_1) \tag{86}$$

whereas when $R = Y$ we find that the subject chooses X when:

$$\mathbb{E}[X] - \mathbb{E}[Y] > \frac{\lambda - 1}{2}(y_2 - x_2) \tag{87}$$

so that when both have the same expected value a subject prefers to stay with his reference lottery and requires a premium to exchange it. Let us denote this premium by, c_x, when the reference is X and c_y when the reference is Y. We can then derive the premium by solving the following:

$$0 = 0.5\lambda(y_1 + c_x - x_1) + 0.5(y_2 + c_x - x_2) \tag{88}$$

$$c_x = \frac{2}{\lambda+1}\left(\frac{\lambda-1}{2}(x_1 - y_1) + \mathbb{E}[X] - \mathbb{E}[Y]\right) \tag{89}$$

so that when $\lambda = 1$ the premium solely compensates for the difference in expected value whereas when we take $\lambda \to \infty$ the premium only serves to compensate for $x_1 - y_1$ which is the potential state in which there is loss. Similarly, we derive that:

$$c_y = \frac{2}{\lambda+1}\left(\frac{\lambda-1}{2}(y_2 - x_2) + \mathbb{E}[Y] - \mathbb{E}[X]\right) \tag{90}$$

6. Simple Insurance in PT3

We consider how PT3 would model a simplistic case of buying insurance (e.g. ignoring things such as moral hazard). Consider the negative lottery:

$$L = \begin{cases} -l, & p \\ 0, & (1-p) \end{cases} \tag{91}$$

which represents a chance of a fixed cost (e.g. car crash, house fire, etc.) occurring. How much would a person be willing to pay to insure this loss?

We consider this under three scenarios: the first is the case in which the person has long been aware of the risk and has internalized it so that their reference is L; the second case is one in which they hadn't though about the risk or insuring it until confronted with choice to buy insurance so that their reference is 0; the third is the case in which they assume that they'll have to acquire some sort of insurance (e.g. by laws requiring auto-insurance) so that the reference is some fixed amount, $-r$.

6.1. **Reference is L.** In this case we derive the maximum amount a person would be willing to pay, $y < l$, as:

$$0 = p(-y - -l) + \lambda q(-y) \tag{92}$$

$$y = \frac{pl}{p + \lambda q} < -\mathbb{E}[L] \tag{93}$$

so that the maximum a person is willing to pay is less than the expected value.

6.2. **Reference is 0.** In this case we derive the maximum amount a person would be willing to pay, $y < l$, as:

$$\lambda p(-l - 0) + q(0 - 0) = \lambda(-y - 0) \tag{94}$$

$$y = pl = -\mathbb{E}[L] \tag{95}$$

so that the maximum a person is willing to pay is the expected value.

6.3. **Reference is $-r$.** In this case to derive the maximum amount a person would be willing to pay, $y < l$, we consider two cases $\{-r \le -y,\}$ and $\{-r > -y\}$. When $\{-r \le -y\}$ we find that:

$$\lambda p(-l - -r) + q(0 - -r) = (-y - -r) \tag{96}$$

$$y = pl + (\lambda - 1)p(l - r) > -\mathbb{E}[L] \tag{97}$$

We impose the condition, $\{-r \le -y\}$ and see that this solution applies when:

$$r \ge y \tag{98}$$

$$\ge pl + (\lambda - 1)p(l - r) \tag{99}$$

$$\ge \frac{\lambda pl}{\lambda p + q} \tag{100}$$

When $\{-r > -y\}$ we find that:

$$\lambda p(-l - -r) + q(0 - -r) = \lambda(-y - -r) \tag{101}$$

$$y = pl + \frac{\lambda - 1}{\lambda} qr > -\mathbb{E}[L] \tag{102}$$

We impose the condition, $\{-r > -y\}$ and see that this solution applies when:

$$r < y \tag{103}$$

$$< pl + \frac{\lambda - 1}{\lambda} qr \tag{104}$$

$$< \frac{\lambda pl}{\lambda p + q} \tag{105}$$

so that

$$y = \begin{cases} pl + (\lambda - 1)p(l - r), & r \ge \frac{\lambda pl}{\lambda p + q} \\ pl + \frac{\lambda - 1}{\lambda} qr, & r < \frac{\lambda pl}{\lambda p + q} \end{cases} \tag{106}$$

so that the maximum a person is willing to pay is greater than the expected value.