Temporal Resolution of Uncertainty and Recursive Models of Ambiguity Aversion

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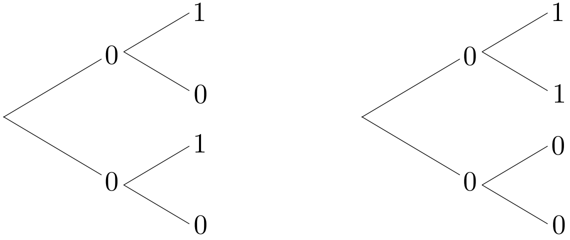
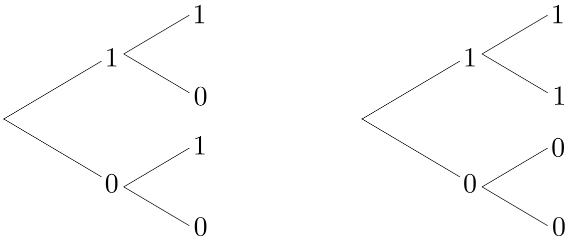
Preference for Earlier Resolution of Uncertainty



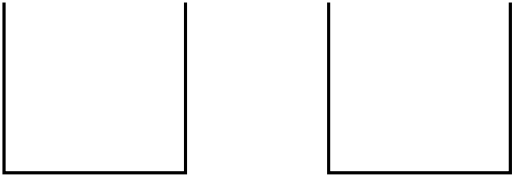
- instrumental value of information Spence Zeckhauser (1972)
- intrinsic value of information Kreps Porteus (1978)
- hidden actions Ergin Sarver (2012)

Aversion to Persistence/Long Term Risk



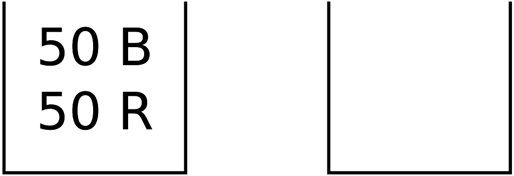
Duffie Epstein (1992)

Ambiguity



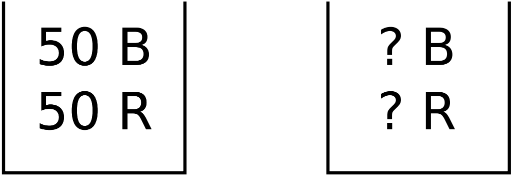
Kenes (1921), Ellsberg (1961)

Ambiguity



Kenes (1921), Ellsberg (1961)

Ambiguity



Kenes (1921), Ellsberg (1961)

Ambiguity

50 B
50 R

? B
? R

B

R

Kenes (1921), Ellsberg (1961)

Ambiguity

50 B 50 R	? B ? R

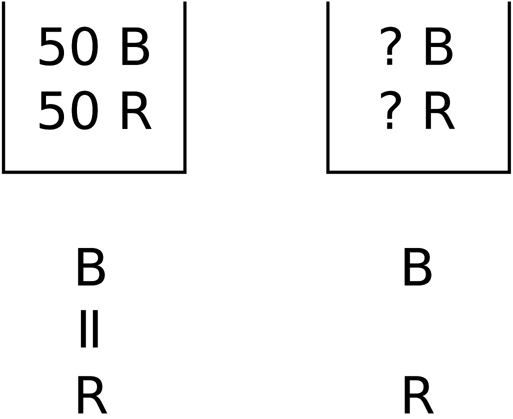
B

||

R

Kenes (1921), Ellsberg (1961)

Ambiguity



Kenes (1921), Ellsberg (1961)

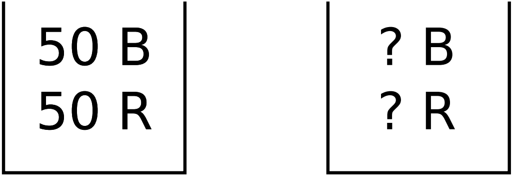
Ambiguity

50 B 50 R	? B ? R

B	B
$\|$	$\|$
R	R

Kenes (1921), Ellsberg (1961)

Ambiguity



B	>	B
=		=
R	>	R

Kenes (1921), Ellsberg (1961)

Main Message: modeling tradeoffs

The way we choose to model ambiguity aversion will impact:

- preference for earlier resolution of uncertainty
- aversion to long-run risk,

so the model ties these dimensions of preference together

Similar to the modeling tradeoff in Epstein Zin (1989), where

- risk aversion
- intertemporal elasticity of substitution
- preference for earlier resolution of uncertainty and aversion to long-run risk

are tied together

Preferences

$V_t = u(c_t) + \beta E(V_{t+1})$ Discounted Expected Utility

Preferences

$V_t = u(c_t) + \beta E(V_{t+1})$ Discounted Expected Utility

$V_t = W\big(u(c_t),\ E(V_{t+1})\big)$ Kreps–Porteus, Epstein–Zin

Preferences

$V_t = u(c_t) + \beta E(V_{t+1})$ Discounted Expected Utility

$V_t = W\big(u(c_t),\, E(V_{t+1})\big)$ Kreps–Porteus, Epstein–Zin

$V_t = u(c_t) + \beta I(V_{t+1})$ Discounted Ambiguity Aversion

Preferences

$V_t = u(c_t) + \beta E(V_{t+1})$ Discounted Expected Utility

this $\succsim$ is indifferent to timing and to long run risks

Preferences

$V_t = W\big(u(c_t), E(V_{t+1})\big)$ Kreps–Porteus, Epstein–Zin

$$\text{this } \succsim \text{ is } \begin{Bmatrix} \text{PERU} \\ \text{PLRU} \\ \text{IERU} \end{Bmatrix} \text{ iff } W(u, \cdot) \text{ is } \begin{Bmatrix} \text{convex} \\ \text{concave} \\ \text{linear} \end{Bmatrix}$$

Preferences

$$V_t = u(c_t) + \beta I(V_{t+1}) \qquad \text{Discounted Ambiguity Aversion}$$

- here the operator I replaces expectation
- captures ambiguity aversion
- question: how does PERU depend on I

Setting

Setting

$\mathcal{T} = \{0, 1, \ldots, T\}$ — discrete time, $T < \infty$

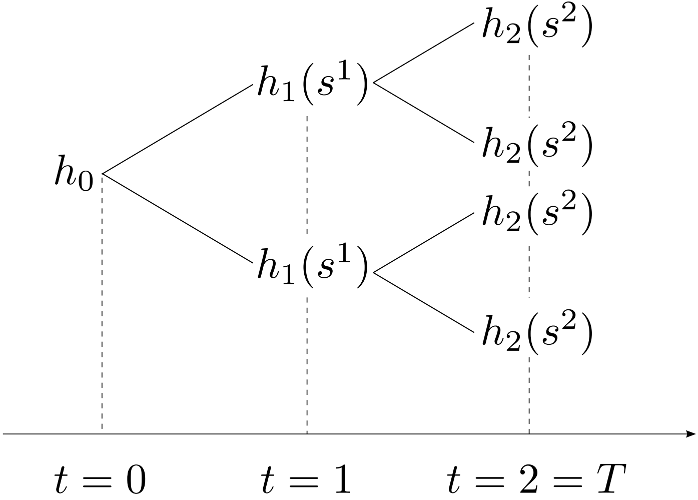
(S, Σ) — shocks, measurable space

$\Omega = S^{\mathcal{T}}$ — states of nature

X — consequences, convex subset of a real vector space

$h = (h_0, h_1, \ldots, h_T)$ — consumption plan, $h_t : S^t \to X$

Consumption Plan



IID uncertainty with EU

$$V_t(s^t, h) = u\big(h_t(s^t)\big) + \beta \int_S V_{t+1}\big((s^t, s_{t+1}), h\big)\, \mathrm{d}p(s_{t+1} \,|\, s^t)$$

IID uncertainty with EU

$$V_t(s^t, h) = u\big(h_t(s^t)\big) + \beta \int_S V_{t+1}\big((s^t, s_{t+1}), h\big)\, \mathrm{d}p(s_{t+1} \,|\, s^t)$$

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- what is IID is the underlying state process s^t

IID uncertainty with EU

$$V_t(s^t, h) = u\big(h_t(s^t)\big) + \beta \int_{S} V_{t+1}\big((s^t, s_{t+1}), h\big)\, \mathrm{d}p(s_{t+1} \,|\, s^t)$$

$$V_t(s^t, h) = u\big(h_t(s^t)\big) + \beta \int_{S} V_{t+1}\big((s^t, s_{t+1}), h\big)\, \mathrm{d}p(s_{t+1})$$

- what is IID is the underlying state process s^t
- consumption can have correlation

generalize beyond EU

$$V_t(s^t, h) = u\big(h_t(s^t)\big) + \beta \int_S V_{t+1}\big((s^t, s_{t+1}), h\big)\, \mathrm{d}p(s_{t+1})$$

generalize beyond EU

$$V_t(s^t, h) = u\big(h_t(s^t)\big) + \beta \int_S V_{t+1}\big((s^t, s_{t+1}), h\big)\,\mathrm{d}p(s_{t+1})$$

$$V_t(s^t, h) = u\big(h_t(s^t)\big) + \beta I\Big(V_{t+1}\big((s^t, \cdot), h\big)\Big)$$

where $\quad I : \mathbb{R}^S \to \mathbb{R}$

generalize beyond EU

$$V_t(s^t, h) = u\big(h_t(s^t)\big) + \beta \int_S V_{t+1}\big((s^t, s_{t+1}), h\big)\, \mathrm{d}p(s_{t+1})$$

$$V_t(s^t, h) = u\big(h_t(s^t)\big) + \beta I\Big(V_{t+1}\big((s^t, \cdot), h\big)\Big)$$

$$\text{where} \qquad I : \mathbb{R}^S \to \mathbb{R}$$

I constant over time—IID Ambiguity (Epstein and Schneider, 2003)

Uncertainty Averse Preferences

$I : \mathbb{R}^S \to \mathbb{R}$ is:

- continuous (supnorm)
- monotonic ($\forall_{s \in S}\ \xi(s) \geq \zeta(s) \Rightarrow I(\xi) \geq I(\zeta)$)
- normalized ($\forall_{r \in R}\ I(r) = r$)
- quasiconcave

$\rightarrow$ Uncertainty Aversion (Schmeidler, 1989)

Axiomatic foundations: Cerreia-Vioglio, Maccheroni, Marinacci, and Montrucchio (2011)

special cases

1. *Maxmin expected utility*: $I(\xi) = \min_{p \in C} \int \xi \, \mathrm{d}p$
2. *Variational*: $I(\xi) = \min_{p \in \Delta(\Sigma)} \left[\int \xi \, \mathrm{d}p + c(p) \right]$
3. *Multiplier*: $I(\xi) = \min_{p \in \Delta(\Sigma)} \left[\int \xi \, \mathrm{d}p + \theta R(p \| q) \right]$
4. *Confidence*: $I(\xi) = \min_{\{p \in \Delta(\Sigma) | \varphi(p) \geq \alpha\}} \left[\frac{1}{\varphi(p)} \int \xi \, \mathrm{d}p \right]$
5. *Second order expected utility*: $I(\xi) = \phi^{-1}\big(\int \phi(\xi) \, \mathrm{d}p \big)$
6. *Smooth ambiguity*: $I(\xi) = \phi^{-1}\big(\int_{\Delta(\Sigma)} \phi(\int \xi \, \mathrm{d}p) \, \mathrm{d}\mu(p) \big)$

special cases

1. *Maxmin expected utility*: Gilboa and Schmeidler (1989)
2. *Variational*: Marinacci, Maccheroni, and Rustichini (2006)
3. *Multiplier*: Hansen and Sargent (2001)
4. *Confidence*: Chateauneuf and Faro (2009)
5. *Second order expected utility*: Neilson (1993)
6. *Smooth ambiguity*: Klibanoff, Marinacci, and Muhkerhi (2005)

relations between them

- MEU $\subset$ Variational
- MEU $\subset$ Confidence
- Variational $\cap$ Confidence $=$ MEU
- Multiplier $\subset$ Variational
- SOEU $\cap$ Variational $=$ Multiplier
- SOEU $\subset$ KMM

two key properties

- $I(\xi + k) = I(\xi) + k$ for all $\xi \in \mathbb{R}^S, k \in \mathbb{R}$
  - shift invariance, Constant Absolute Ambiguity Aversion (Variational)
- $I(\beta\xi) = \beta I(\xi)$ for all $\xi \in \mathbb{R}^S$, $\beta \in (0, 1)$
  - scale invariance, Constant Relative Ambiguity Aversion (Confidence)
- so MEU has both

Discounted Uncertainty Averse Preferences

define by backward induction

$$V_T(s^T, h) := u\big(h_T(s^T)\big)$$
$$V_t(s^t, h) := u\big(h_t(s^t)\big) + \beta I\Big(V_{t+1}\big((s^t, \cdot), h\big)\Big);\ t = 0, \ldots, T-1$$

so Dynamically Consistent:

- Sarin and Wakker (1998)
- Epstein and Schneider (2003)
- Marinacci, Maccheroni, and Rustichini (2006)
- Klibanoff, Marinacci, and Muhkerhi (2009)

(without a fixed filtration, well known problems with DC)

Applications to macroeconomics and finance

Dow and Werlang (1992), Epstein and Wang (1994), Chen and Epstein (2002), Epstein and Schneider (2007, 2008), Drechsler (2009), Ilut (2009), Mandler (2013), Condie Ganguli (2014),

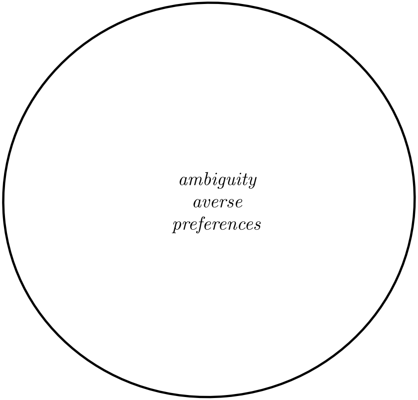
Maenhout (2004), Karantounias, Hansen, and Sargent (2009), Kleshchelski and Vincent (2009), Barillas, Hansen, and Sargent (2009)

Ju and Miao (2012), Chen, Ju, and Miao (2009), Hansen (2007), Collard, Mukerji, Sheppard, and Tallon (2011), Benigno and Nisticò (2009)

Results

Main Message

What we assume about I will have impact on Preference for Earlier Resolution of Uncertainty.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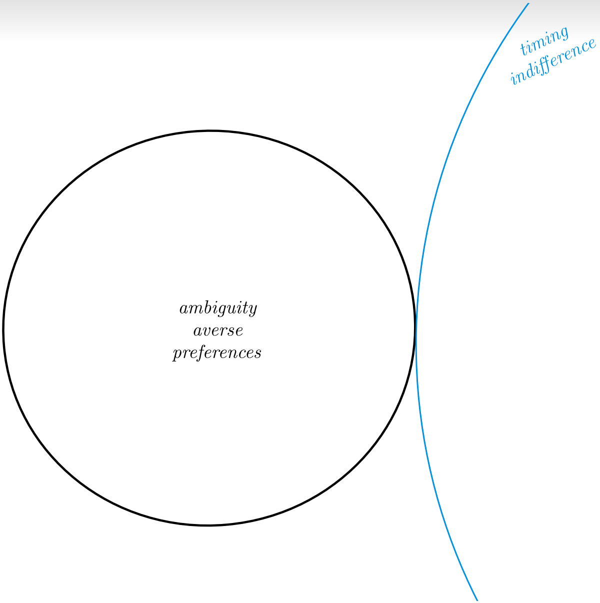

timing
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ambiguity
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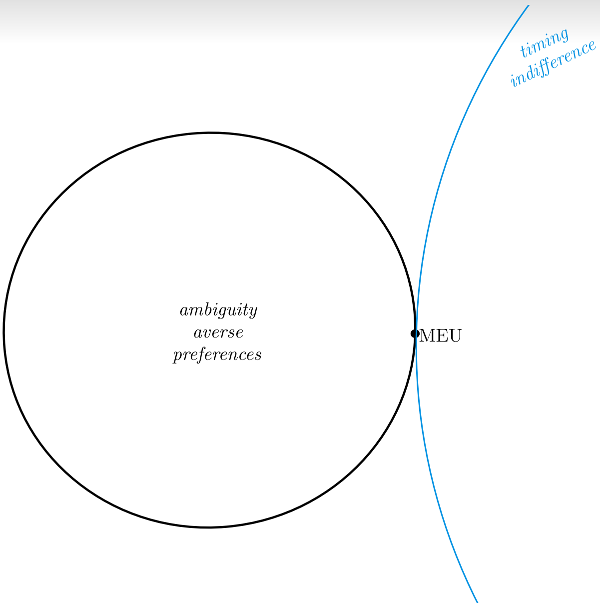

timing
indifference
ambiguity
averse
preferences
MEU

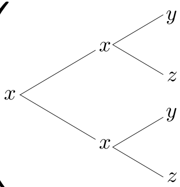
Theorem 1. A family of discounted uncertainty averse preferences satisfies indifference to timing of resolution of uncertainty if and only if $I(\xi) = \min_{p \in C} \int \xi \, \mathrm{d}p$.

Proof

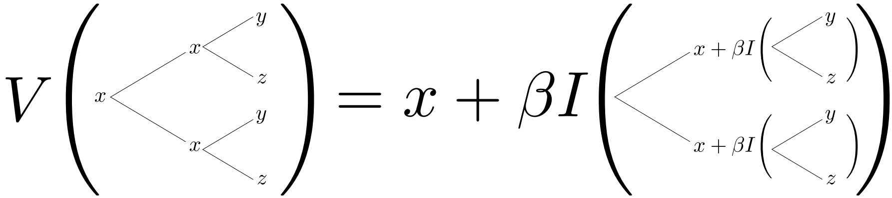
- indifference to timing $\Rightarrow$ shift-invariance **and** scale-invariance

- shift-invariance and scale-invariance $\Rightarrow$ MEU

Proof

$V\Bigg($  $\Bigg)$

Proof

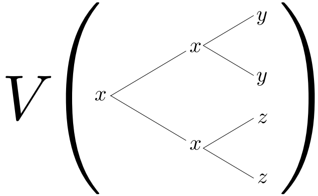


Proof

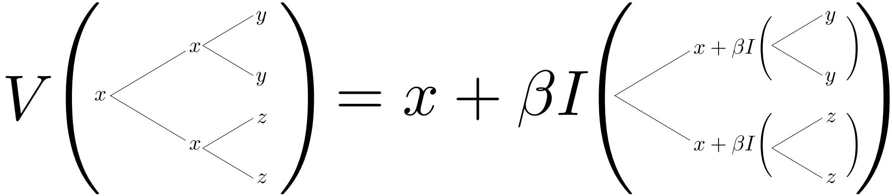
$$V\left(x \begin{array}{l} \nearrow x \begin{array}{l} \nearrow y \\ \searrow z \end{array} \\ \searrow x \begin{array}{l} \nearrow y \\ \searrow z \end{array} \end{array} \right) = x + \beta I \left(\begin{array}{l} \nearrow x + \beta I \left(\begin{array}{l} \nearrow y \\ \searrow z \end{array} \right) \\ \searrow x + \beta I \left(\begin{array}{l} \nearrow y \\ \searrow z \end{array} \right) \end{array} \right)$$

$$= x + \beta \left(x + \beta I \left(\begin{array}{l} \nearrow y \\ \searrow z \end{array} \right) \right)$$

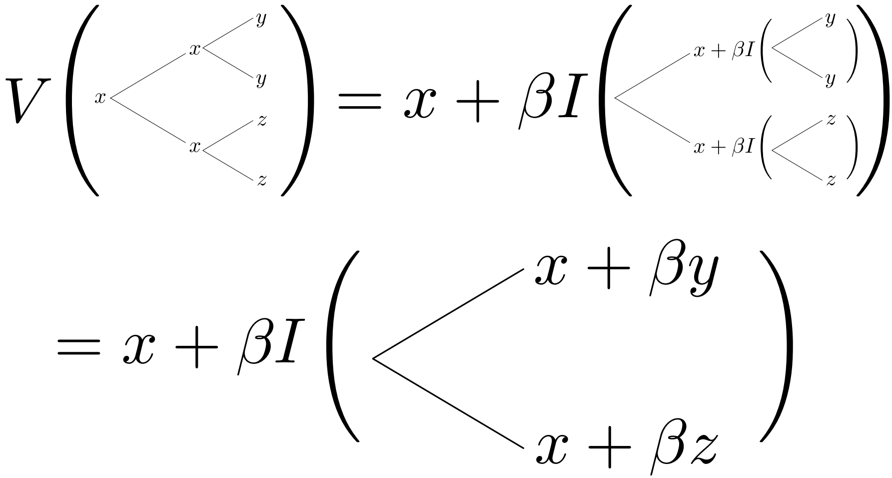
Proof



Proof



Proof



$$x + \beta\left(x + \beta I\left(\begin{array}{ll} & y \\ \langle & \\ & z \end{array}\right)\right)$$

$$x + \beta I\left(\begin{array}{ll} \nearrow & x + \beta y \\ & \\ \searrow & x + \beta z \end{array}\right)$$

Proof

$$x + \beta I\left(\begin{array}{l} \nearrow y \\ \\ \searrow z \end{array} \right)$$

$$I\left(\begin{array}{l} \nearrow x + \beta y \\ \\ \searrow x + \beta z \end{array} \right)$$

we have:

$$\exists_{\beta\in(0,1)}\forall_{x\in\mathbb{R}}\forall_{\xi\in\mathbb{R}^s}\quad x+\beta I(\xi)=I(x+\beta\xi)$$

Proof

we have:

$$\exists_{\beta \in (0,1)} \forall_{x \in \mathbb{R}} \forall_{\xi \in \mathbb{R}^s} \quad x + \beta I(\xi) = I(x + \beta \xi)$$

need to show:

$$\forall_{\beta \in (0,1)} \forall_{x \in \mathbb{R}} \forall_{\xi \in \mathbb{R}^s} \quad x + \beta I(\xi) = I(x + \beta \xi)$$

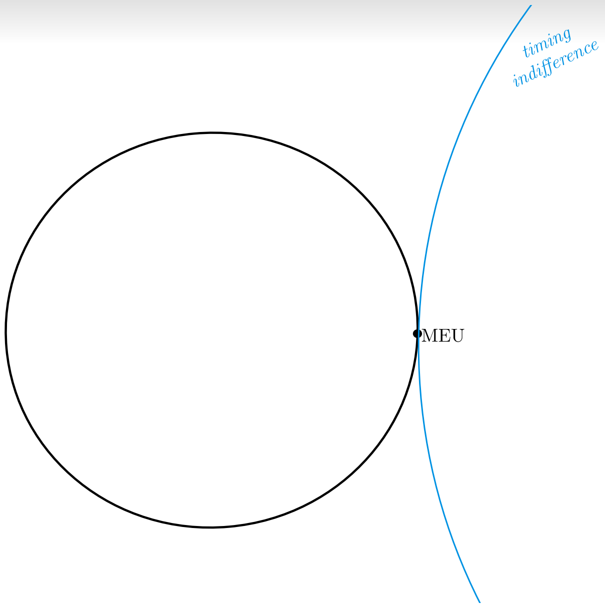
(details in the paper) $\square$

comments

- in some sense this argument could be used to axiomatize the recursive multiple priors model
- a related paper by Kochov (2012) axiomatizes MEU using a strong version of Stationarity, which has a flavor of IERU

Comparison to Risk

- Chew Epstein (1989) show IERU $\Rightarrow$ EU
- Grant Kajii Polak (2000) show (rank-dependent or betweenness) $+$ PERU $\Rightarrow$ EU
- so dispensing with objective probability makes more room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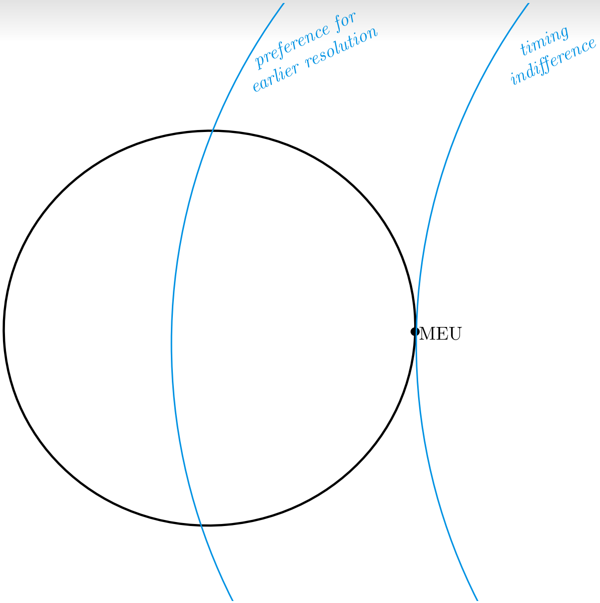
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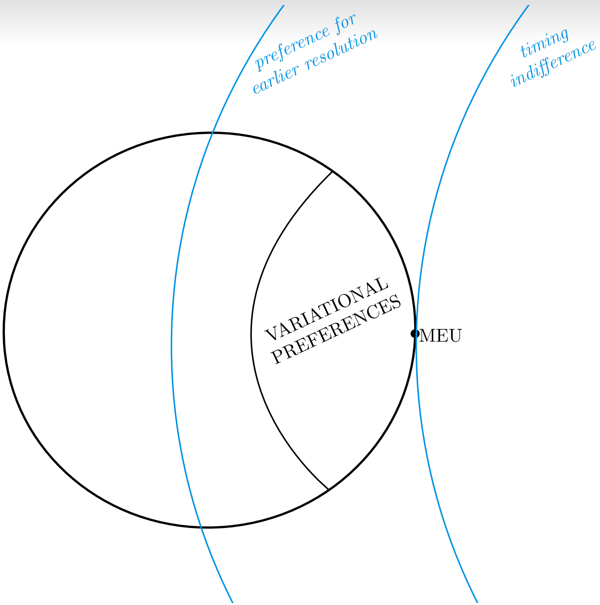
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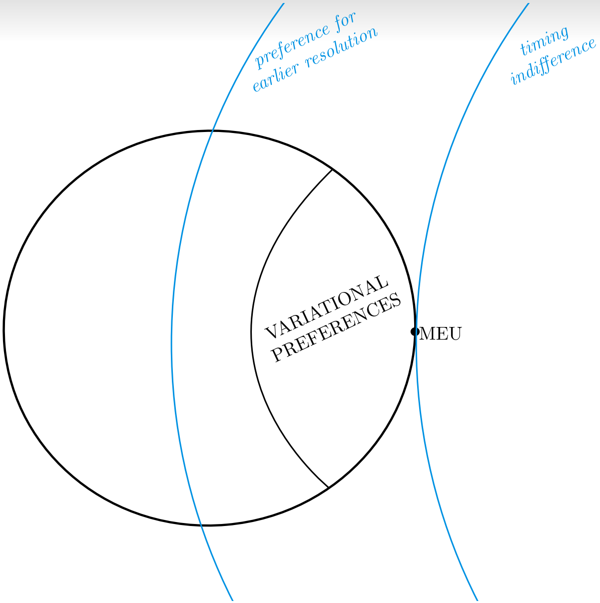
Theorem 2. A family of discounted variational preferences, $I(\xi) = \min_{p \in \Delta(\Sigma)} \int \xi \, \mathrm{d}p + c(p)$, always satisfies preference toward earlier resolution of uncertainty.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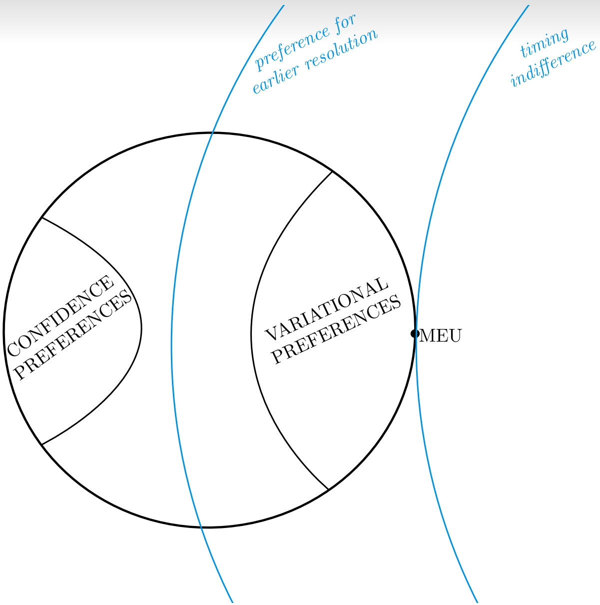
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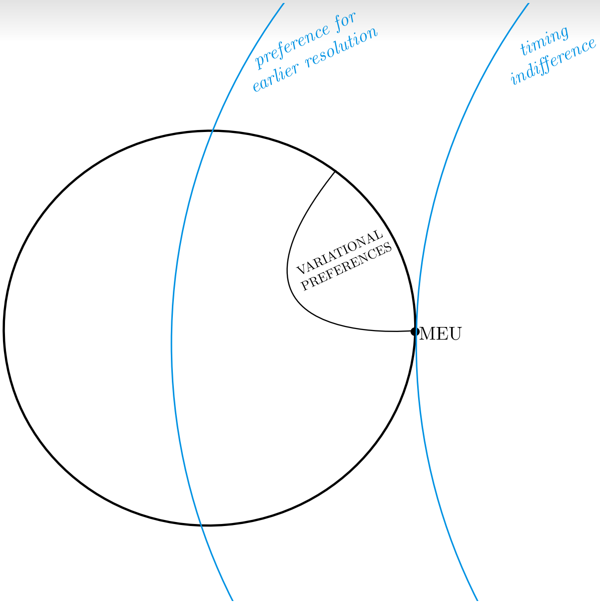
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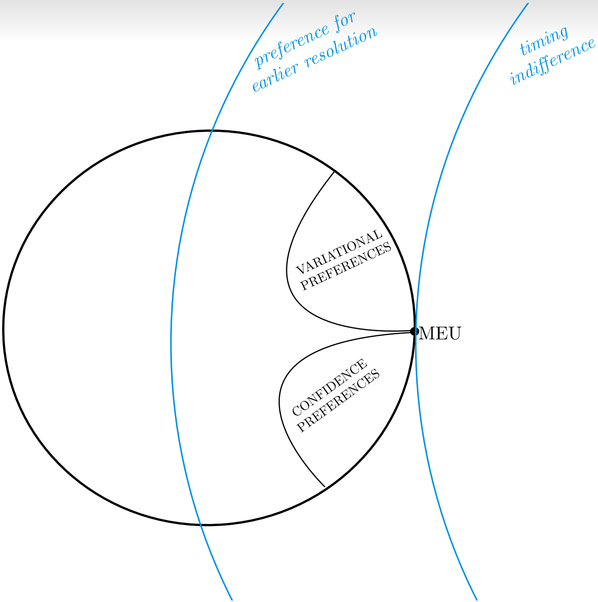
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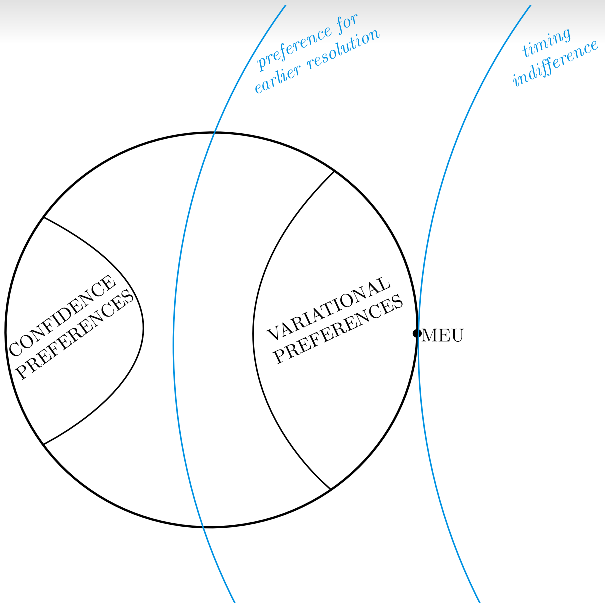
Theorem 3. A family of discounted confidence preferences, $I(\xi) = \min_{\{p \in \Delta(\Sigma) | \varphi(p) \geq \alpha\}} \frac{1}{\varphi(p)} \int \xi \, dp$, displays a preference for earlier resolution of uncertainty if and only if $I(\xi) = \min_{p \in C} \int \xi \, dp$.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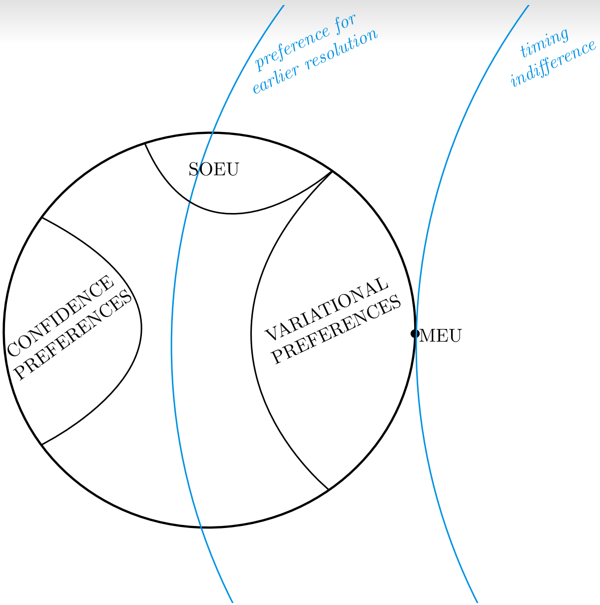
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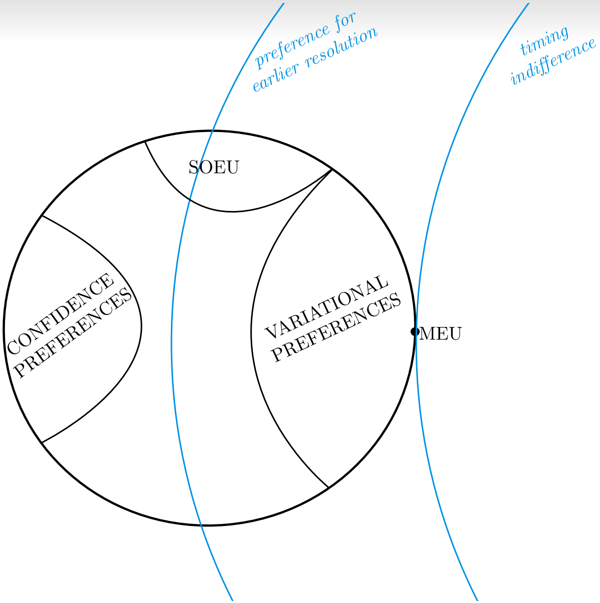
Theorem 4. A family of discounted second order expected utility preferences $I(\xi) = \phi^{-1}\big(\int \phi(\xi)\,\mathrm{d}p\big)$ with ϕ concave, strictly increasing and twice differentiable displays a preference for earlier resolution of uncertainty iff Condition 1 holds.

Condition 1. There exists a real number $A > 0$ such that $-\frac{\phi''(x)}{\phi'(x)} \in [\beta A, A]$ for all $x \in \mathbb{R}$

(this condition means that the curvature of ϕ doesn't vary too much)



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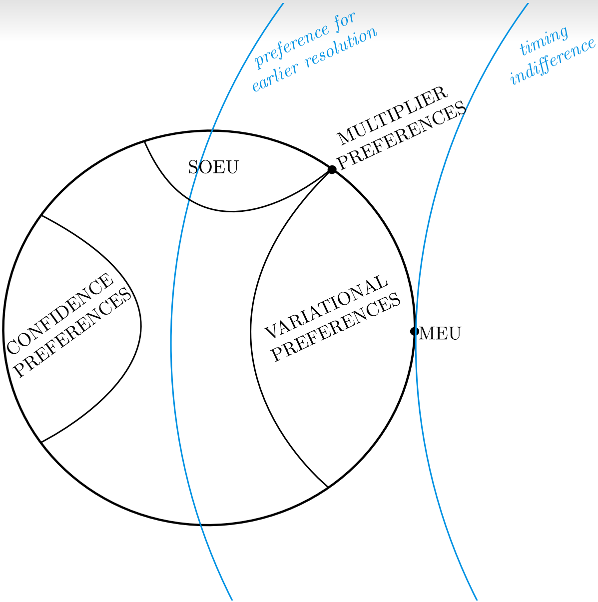
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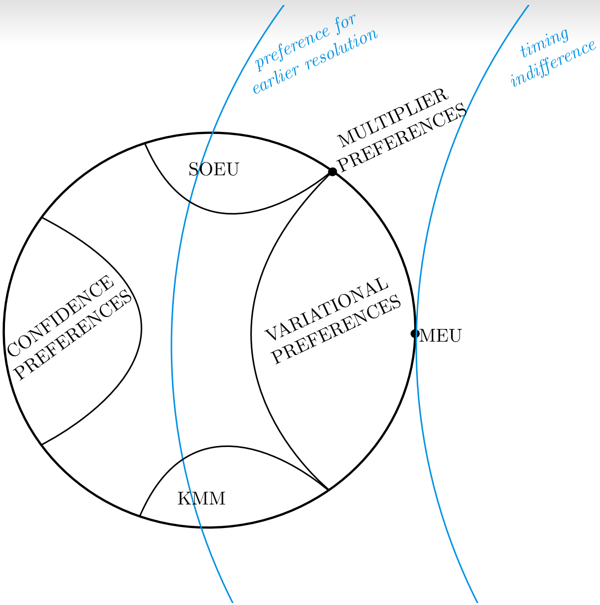
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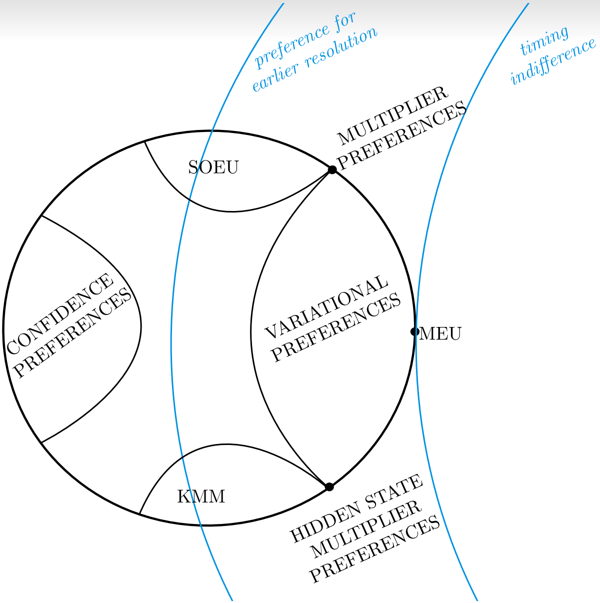
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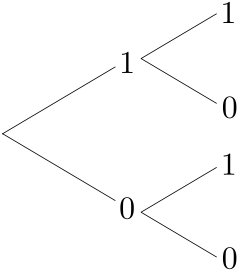
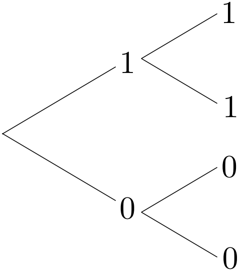
KMM
HIDDEN STATE
MULTIPLIER
PREFERENCES

Theorem 5. A family of discounted smooth ambiguity preferences $I(\xi) = \phi^{-1}\big(\int_{\Delta(\Sigma)} \phi(\int \xi \, \mathrm{d}p) \, \mathrm{d}\mu(p)\big)$ with ϕ concave, strictly increasing and twice differentiable displays a preference for earlier resolution of uncertainty if Condition 1 holds

Only if under additional assumption on the support of μ.

Persistence





Persistence

Theorem 6. A family of discounted variational preferences, $I(\xi) = \min_{p \in \Delta(\Sigma)} \int \xi \, \mathrm{d}p + c(p)$, always satisfies preference for iid.

A family of discounted confidence preferences, $I(\xi) = \min_{\{p \in \Delta(\Sigma) | \varphi(p) \geq \alpha\}} \frac{1}{\varphi(p)} \int \xi \, \mathrm{d}p$, always satisfies preference for iid.

In both cases, indifference to iid is satisfied if and only if $I(\xi) = \min_{p \in C} \int \xi \, \mathrm{d}p$.

(I do not know how to extend this result to all of I)

Aggregator

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$$V_t(s^t, h) = u\big(h_t(s^t)\big) + \beta I\big(V_{t+1}((s^t, \cdot), h)\big)$$

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$$V_t(s^t, h) = W\Big(h_t(s^t),\ I\big(V_{t+1}((s^t, \cdot), h)\big)\Big)$$

where $W : X \times \mathbb{R} \to \mathbb{R}$

Recursive Uncertainty Averse Preferences

$$V_T(s^T, h) = v\big(h_T(s^T)\big)$$

$$V_t(s^t, h) = W\left(h_t(s^t), I\left(V_{t+1}\big((s^t, \cdot), h\big)\right)\right); \;\; t = 0, \ldots, T-1$$

Question

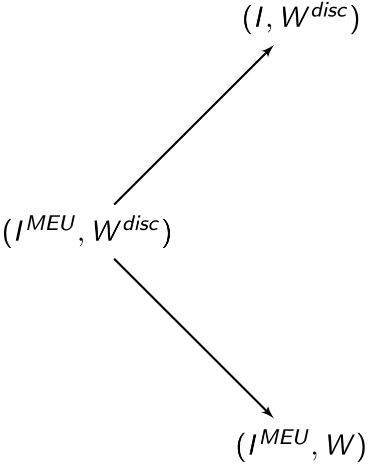
(I^{MEU}, W^{disc})

Question

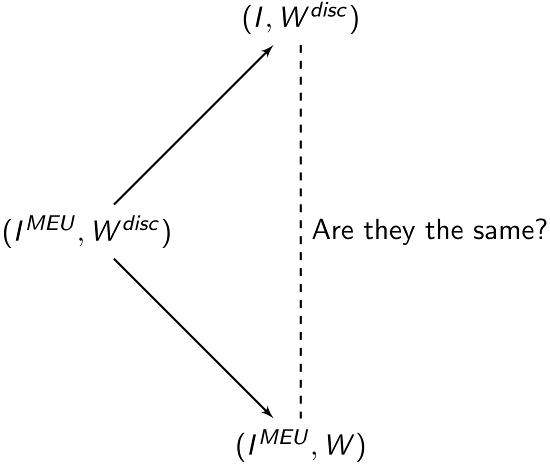
$$(I, W^{disc})$$

$$(I^{MEU}, W^{disc})$$

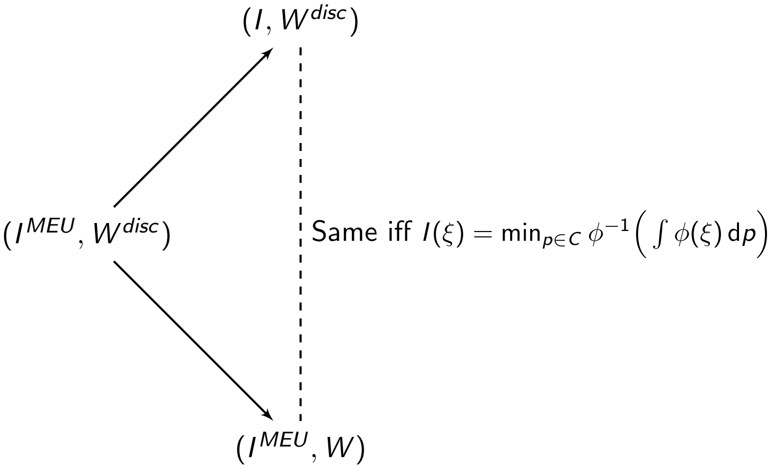
Question



Question



Question


(I, W^{disc})
(I^{MEU}, W^{disc})
Same iff $I(\xi) = \min_{p \in C} \phi^{-1}\Big(\int \phi(\xi)\, dp \Big)$
(I^{MEU}, W)

relation between the two models

- both W and I induce PERU
- but save for the case above, they are different $\succsim$
- do they fit the data in a different way?
- W—workhorse model of macrofinance (Epstein–Zin)
- what does I add?

recent work: Epstein, Farhi, Strzalecki (2014)

- suppose you are endowed with a consumption process h_t
- for what $\pi \in (0, 1)$ are you indifferent between

$$[h_t, \text{gradual resolution}] \sim [(1-\pi)h_t, \text{early resolution}]$$

- $\pi \in (20\%, 40\%)$ for workhorse models in finance using Epstein–Zin preferences (Bansal and Yaron 2004; Barro, 2009)
- how high is this number for models of ambiguity?

Conclusion:

interdependence of ambiguity and timing

MEU—only case of indifference

Questions:

theoretical: is this it? can we disentangle more?

empirical: how to measure this?

Thank you

Barillas, F., L. Hansen, and T. Sargent (2009): "Doubts