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Asymmetries in Price and Quantity Adjustments by the Competitive Firm

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Focusing on the crucial role of inventory carry-overs in the production and sales decision, we describe the profit maximizing behavior of a dynamic competitive firm facing random prices. Each firm's behavior is incorporated into a stochastic equilibrium model of the competitive industry with uncertain demand. The industry model exhibits asymmetric cyclical fluctuations of the "Keynesian" sort: when demand is weak, output contracts while price holds at a fixed floor; when demand is strong, price increases as output is constrained by a ceiling. Even in a pure world of constant returns, without increasing costs, the inability to instantaneously coordinate production and sales along with the existence of inventories is sufficient to yield a "backward L" shaped supply curve for the short run. *Journal of Economic Literature* Classification Numbers: 020, 021, 022.

INTRODUCTION

This paper starts with the premise of a lag between the decision to produce and the actual appearance of output. Given such an assumption, we construct a simple partial equilibrium model of a competitive industry which focuses on the role of inventories as a mechanism for the optimal intertemporal allocation of sales and production. Firms in the industry "smooth" production and sales over time by adjusting both the output price and the inventory of final goods. Price and quantity adjustments are viewed as complementary ways of maintaining equilibrium.

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The "stylized facts" about short run cyclical behavior are that during periods of unexpectedly low demand, producers respond by cutting sales, output, and employment rather than relaying exclusively on price as a mechanism for maintaining equilibrium. On the other hand, during periods of unexpectedly high demand there is greater pressure on price, while output does not respond as rapidly. The contrasting adjustment roles of prices and quantities might be broadly termed a "Keynesian asymmetry" because they have been a significant part of unemployment and inflation theories in the Keynesian tradition of macro-economics.

Certain features of Keynesian asymmetry might appear to be somewhat inconsistent with standard Walrasian analysis. For example, the idea that prices should never fall below costs of production, however weak is demand and however large is the unanticipated accumulation of inventories, would seem to be a contradiction with the usual supply and demand analysis.

In this paper we argue that there are *intrinsic* (as contrasted with *ad hoc*) reasons for such a Keynesian asymmetry, even in competitive industries where individual firms have no control over price. In a world of production lags, inventories act as a crucial link between present sales, present production, and future stocks. Inventories provide profit maximizing firms with the means to "spread" the effects of unanticipated demand fluctuations over time, so diminishing the impact of weak demand that it never pays to lower prices below production costs because it is more profitable to withhold sales and cut next period's output. The opportunity for firms to make intertemporal substitutions of this sort results in an upward sloping convex short-run supply curve even in the absence of increasing costs.

Although there are other ways to generate upward sloping convex short-run supply curves (by relying, for example, on increasing production costs, imperfect information, or adjustment costs) we believe that the emphasis placed here on the importance of inventory adjustments as a means of "spreading" the effects of shocks over time has not received the attention it deserves. The aim of this paper is to explain how inventories play a crucial "smoothing" role which results in an upward sloping, convex supply curve even in the simplest model of a competitive industry with constant production costs.

The Model

The highly stylized model developed in this section captures the notion that the responsiveness of market price to fluctuations in market demand is smaller when inventory holdings remain positive than when they are drawn down to zero and there is an industry-wide stock-out. We treat a competitive industry composed of n independent firms producing a homogeneous product. For each firm there is a production lag of one period from the time inputs are committed to the time output is forthcoming. The one period discount factor is

$$\beta$$

If firm i markets q_t^i units in period t, aggregate industry sales are

$$Q_t = \sum_{i=t}^{n} q_t^i. \tag{1}$$

Demand in each period is uncertain until the period begins. We assume that in period t the market price P_t can be written in the functional form

$$P_t = F(Q_t, \varepsilon_t), \tag{2}$$

where ε_t is the realization of an independently, identically distributed random variable denoting the state of demand prevailing at time t but unknown before.

The inverted demand function $F(Q, \varepsilon)$ has the properties:

$$F_1(Q, \varepsilon) < 0, \tag{3}$$

$$F(\infty, \varepsilon) = 0, \tag{4}$$

$$F(0, \varepsilon) = \infty, \tag{5}$$

for every possible realization of ε.

In period t, firm i will have available for marketing a stock of

$$s_t^i$$

consisting of production y_{t-1}^i commissioned in period $t-1$ but forthcoming in period t, plus unsold inventory stocks

$$s_{t-1}^i - q_{t-1}^i$$

held over from period $t-1$.

In period t, after the market price P_t is known, firm i decides *sales*:

$$q_t^i, \tag{6}$$

and *production*:

$$y_t^i. \tag{7}$$

Production y_t^i will become available for use in period $t+1$, but only after the firm commits funds

$$cy_t^i \tag{8}$$

in period t. The coefficient c represents the unit cost of production, assumed constant for all firms. Constant production cost is perhaps not a bad assumption for firms purchasing all inputs in competitive markets, and in any case it can serve as a point of departure for more complicated specifications.

Sales and production decisions satisfy

$$0 \leqslant q_t^i \leqslant s_t^i,$$
$$0 \leqslant y_t^i,$$

which guarantee that next period's available stock

$$s_{t+1}^i = s_t^i + y_t^i - q_t^i \tag{9}$$

will be non-negative.

Suppose firm i believes that

$$\Phi(P) = Pr\{P_t \leqslant P\} \tag{10}$$

is the cumulative distribution function of prices in every period t. Under this belief, define

$$V(P_t, s_t^i)$$

to be the expected present discounted value of following an optimal policy when the price is currently P_t and the available stock is s_t^i.

If the sales and production decisions (6), (7) are expected profit maximizing for the stationary price distribution $\Phi(P)$, they must satisfy the dynamic programming condition:

$$\begin{aligned} V(P_t, s_t^i) &= P_t q_t^i - cy_t^i + \beta EV(P, s_t^i + y_t^i - q_t^i) \\ &= \max_{\substack{0 \leqslant q \leqslant s_t^i \\ 0 \leqslant y}} \{P_t q - cy + \beta EV(P, s_t^i + y - q)\}. \end{aligned} \tag{11}$$

When each firm, perceiving that $\Phi(P)$ represents the "true" distribution of prices, reacts in a way which confirms the distribution $\Phi(P)$, we have what is called a stochastic equilibrium, or a rational expectations equilibrium.

More formally, a stochastic competitive equilibrium is a stationary price distribution (10), a sales decision (6), and a production decision (7), which in every period satisfies conditions (1), (2), (9), and (11). This formulation results in the concept of market equilibrium relevant in the present context.

Although the theory does not explain how a stochastic equilibrium comes about, hopefully there is some valid, if unspecified, dynamic mechanism at work. We deliberately do not specify the mechanism whereby quantity is allocated among producers in equilibrium because our results are robust to different allocation mechanisms.

The basic economic intuition behind this market equilibrium concept is that if firms have incorrect price expectations, profit maximizing behavior would cause them to change their reactions altering the distribution of prices, until eventually each firm has expectations about the distribution of next period's price which are consistent with the "true" distribution. This is the usual dynamic story used to justify a rational expectations equilibrium, and we believe that it is convincing in this particular context.

A stochastic competitive equilibrium in this model is compatible with a multiplicity of individual reactions because of the knife edge behavior induced by constant costs. (The situation here is no different from any other competitive equilibrium with constant returns.) However, in its aggregate or industry variables, which are the relevant ones for our analysis, the present model has a unique stochastic equilibrium. The aggregate industry variables are:

$$S_t = \sum_i s_t^i,$$

$$Q_t = \sum_i q_t^i,$$

$$Y_t = \sum_i y_t^i,$$

and P_t.

Aside from (2), the only other condition of aggregate balance is

$$S_{t+1} = S_t + Y_t - Q_t. \tag{12}$$

The basic result of the present paper is that the stochastic market equilibrium has the following simple characterization.

Define S^* to be the solution of the equation

$$c = \beta E \max\{F(S^*, \varepsilon), c\}. \tag{13}$$

Define $\hat{Q}(\varepsilon)$ to be the quantity demanded at price c in state ε

$$c = F(\hat{Q}(\varepsilon), \varepsilon). \tag{14}$$

THEOREM. *The unique aggregate equilibrium* (*) *is*:

$$S_t^* = S^*, \tag{15}$$

$$Q_t^* = \min\{\hat{Q}(\varepsilon_t), S^*\}, \tag{16}$$

$$Y_t^* = Q_t^*, \tag{17}$$

$$P_t^* = \max\{F(S^*, \varepsilon_t), c\} = F(Q_t^*, \varepsilon_t). \tag{18}$$

INTERPRETATION

A rigorous proof is presented in the next section. However, a heuristic explanation may provide some insight into the intuition behind the theorem.

Instead of thinking about the industry as a whole, consider the decisions being made by a single firm. If market price is less than unit cost, $P_t < c$, a profit maximizing firm would be unwilling to sell anything. The opportunity cost of withholding one unit of stock from sale is the market price, P_t. The marginal gain from withholding one unit of stock from sale and reducing production by one unit is just the unit cost of production, c. Clearly, the loss of revenue is more than offset by the savings from the reduction in current production. It is profit miximizing for the firm to supply succeeding periods by holding over stock from the current period rather than producing. Since $F_t(0, \varepsilon) = \infty$, we conclude that in equilibrium the market price is greater than or equal to unit cost, $P_t \geqslant c$.

On the other hand, if market price is strictly greater than unit cost, $P_t > c$, the firm would maximize profits by dumping all of its available stock on the market. The opportunity cost of holding over a unit of stock is the revenue that the firm would lose by not selling it this period, P_t. The gain from not selling the unit of stock is the unit cost, c, which would allow the firm a one unit decrease in current production and still maintain the same available stock for sale in the following period. In other words, the laying-over of any current stock is not profit maximizing, because demand in future periods can be met more economically by scheduling greater production today rather than holding over existing stocks.

Thus, price is the maximum of the unit cost of production and what the price would be if all stock is sold. If the stock in period t is S_t,

$$P_t = \max\{F(S_t, \varepsilon), c\}.$$

In period $t-1$ it costs c to change production by a unit, which is worth an average value EP_t the following period, or βEP_t discounted back to the present. In a stochastic competitive equilibrium, therefore

$$c = \beta EP_t = \beta E \max\{F(S_t, \varepsilon), c\},$$

which holds only for $S_t = S^*$ by the definition (13).

The above argument is not rigorous, and, indeed, there are several serious flaws in the reasoning. But hopefully the explanation conveys some intuitive feeling for why the theorem is true and, thereby, is at least suggestive of the robust nature of some of the results under more general circumstances. The germ of truth which would remain in more complicated models is that inventory holdovers provide a mechanism for intertemporal substitution and production "smoothing" which essentially creates a price floor for "low" levels of demand.

Further insight may be gained into the industry short-run supply schedule that is generated in this model by examining diagrammatically the stochastic equilibrium (13)–(18). The right-angle supply curve SS is drawn in Fig. 1. Each value of ε determines a demand curve of the form $P = F(Q, \varepsilon)$. Different realizations of ε represent shifts in demand. Depicted in Fig. 1 are two demand curves corresponding to realizations of ε yielding "weak" demand D_1D_1, and "strong" demand D_2D_2. In each period the stochastic aggregate equilibrium is the point where supply equals demand.

The short-run supply curve of Fig. 1 can only be interpreted as a highly stylized approximation to reality. It does focus sharply on the asymmetry in the firms' use of inventories to "smooth" price fluctuations when demand is "weak" as opposed to "strong." The basic asymmetry of price and quantity adjustments is clear from the geometry. It is the assumption that unit costs

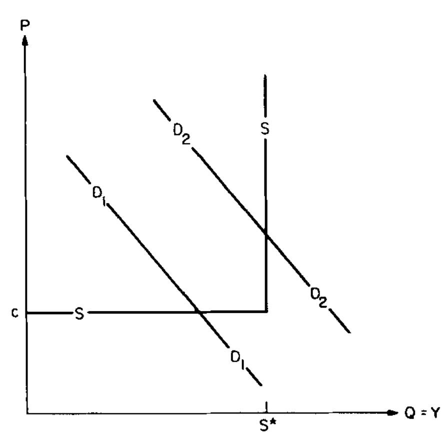

FIG. 1. Supply and demand.

are constant which gives rise to the right-angled nature of the short-run supply schedule as well as other simple features. While we could not expect such crisp results in a more general model, we do expect the basic message to carry over into more realistic settings.

Using the diagrammatic representation, it is instructive to compare the stochastic equilibrium (13)–(18) with two other, related, equilibrium models; one in which there is no production lag and production can adjust instantaneously, the other in which there are no inventory holdovers. If there were no production lag, the decision to produce could be made after uncertainty is resolved, and the competitive supply curve would be the horizontal line $P = c$ which is appropriate for a constant cost industry. On the other hand, with a production lag but no inventory carry-overs, the supply curve in each period would be vertical because all available stock must be sold that period. This last situation corresponds to the newsboy problem or the Marshallian fishmarket parable common in economic folklore.

The supply curve relevant to a situation with lagged production *and* inventories is composed of a vertical *and* a horizontal section. The key role of inventories is to introduce a horizontal section into the supply curve. If the producer sees price falling below cost it is more profitable to cut sales, increase inventories, and lower production, rather than dumping the full stock on the market. Inventories provide the link between present sales, present production, and future stocks which make it "as if" there were instantaneous production in the horizontal range of the supply curve.

Proof of the Form of a Stochastic Equilibrium

That (13), (14) give unique solutions follows from (3)–(5).

The main part of the proof consists of showing that if a stochastic equilibrium exists, it must be of the unique form (13)–(18).

Consider the classical search theory problem about when to sell an item in stock. Price in every period is an independently, identically distributed random variable with cumulative distribution function $\Phi(P)$. In period t, the owner has the option of selling the item at the realized price P_t or rejecting the offer and waiting for another.

Let z be the expected present discounted value of following an optimal selling policy as of the period before any offers are made (or just after an offer is foregone). A standard result of search theory (applied to the present context) is that z is the unique solution of the equation

$$z = \beta E \max\{P, z\}, \tag{19}$$

and the optimal policy is of the form:

if $P_t > z$, sell the entire stock;

if $P_t < z$, sell nothing;

if $P_t = z$, indifferent.

Because of (5), in profit maximizing equilibrium,

$$Q_t > 0 \tag{20}$$

for all t. Thus, $P_t < z$ is impossible, whereas if $P_t > z$ the entire stock is sold and $P_t = F(S_t, \varepsilon_t)$. In stochastic equilibrium, then,

$$P_t = \max\{F(S_t, \varepsilon_t), z\}. \tag{21}$$

Using (21), Eq. (19) becomes

$$z = \beta E \max\{F(S_t, \varepsilon), z\}. \tag{22}$$

Whenever $P_{t-1} > z$, which occurs with positive probability from (19), all stock must be sold. If there were no production at such times, the next period's sales would have to be zero, a contradiction with the equilibrium condition (20). Thus, there is a non-zero probability of positive production in each period. The expected present discounted value of a unit of output is z, an invariant number, whereas its cost of production is c, so that the existence of a positive profit maximizing production level implies

$$z = c. \tag{23}$$

Substituting (23) into (22), we have

$$c = \beta E \max\{F(S_t, \varepsilon), c\} \tag{24}$$

which can only hold for

$$S_t = S^*, \tag{25}$$

where S^* is defined by (13). Equation (25) is condition (15).

Condition (18) is derived by substituting (23) and (25) into (21).

Condition (16) follows from (18).

Condition (17) follows from (15) and (12).

This concludes our proof of the form of a stochastic competitive equilibrium, provided it exists.

It is easy to demonstrate that a stochastic equilibrium exists in the present model. For example,

$$s_t^i = \frac{S^*}{n},$$

$$q_t^i = \frac{Q_t^*}{n},$$

$$y_t^i = \frac{Y_t^*}{n},$$

$$P_t = P_t^*,$$

is a stochastic equilibrium, as can readily be verified.

Actually, certain basic elements of the story told by Fig. 1 remain true almost no matter how anticipations are formed and almost no matter what is the structure of demand uncertainty. Under extremely general conditions which essentially guarantee positive production and sales, the competitive supply curve SS is valid except that current industry stock S_t replaces the stationary equilibrium value S^*. The supply and demand picture of Fig. 1 determines P_t and Q_t under minimal assumptions. Left out of this analysis is the determination of current production Y_t (and future industry stock S_{t+1}) which in the general case depends on anticipations and the structure of uncertainty. With "rational expectations," a condition like (24) could be used to close the model.

Prices versus Quantities as Adjustment Mechanisms

Inventories are not the only type of "quantity" adjustment that profit maximizing firms can utilize to maintain equilibrium. Backorders, the intensity with which a given labor force is utilized, and capacity utilization are other types of "quantity" adjustment. Of course, the best known candidate for an application of the principle that there is an asymmetry in the use of "price" and "quantity" adjustment is the idea that recession and cyclical unemployment are caused in part by insufficient aggregate demand. In this regard the Keynesian asymmetry, analyzed for an individual market in this paper, is at least suggestive of a possible equilibrium explanation for a short-run aggregate supply curve roughly resembling a "backward L."

In our model, we took input costs as constant and derived the behavior of competitive output prices. But nothing except the interpretation changes in the model if costs vary over time, even stochastically, so long as: the general price level changes in the same proportions, demand in real terms is unaltered, and real profits are maximized. The same results obtain, only in this context they would be interpreted as a theory of *relative* price changes. This brings up the interesting question of whether a partial equilibrium theory of asymmetric adjustments in relative input–output prices has implications for a theory of absolute price changes. These are questions for future research.

Conclusion

Although the present model is highly stylized, we believe it gives accurate insights into more general and realistic situations. As we have already noted, the contrasting role of "prices" and "quantities" is a potentially important feature of macro-economic dynamics, as well as industry dynamics. In our opinion, the role of inventory carry-overs is crucial to an understanding of this asymmetry. In most realistic environments the primary adjustment to downswing demand shocks would be made by "quantities" because it is more profitable to withhold sales and cut future production than to sell now at below-cost prices. With upswing demand shocks, on the other hand, the primary real-world adjustment is through "prices." We believe the spirit of this conclusion, appropriately modified, should remain for models of more complicated environments.

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