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Free Access vs Private Ownership as Alternative Systems for Managing Common Property

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1. INTRODUCTION

One case of an external diseconomy is so often cited in the verbal tradition that it has become almost the classical example. This is the "problem of the common" in which no rent is imputed to a scarce fixed factor such as land, fishing grounds, or highways. As is well known, the resulting free access equilibrium is inefficient because what tends to get equated among alternative uses is the *average* product of the variable factor instead of its *marginal* product.

In this paper a formal model is developed which is used to characterize and compare the alternative static allocations of resources which occur under conditions of free access and of private property ownership. There turns out to be a definite limitation on the amount of inefficiency that can be introduced into a competitive situation when property is freely accessible. That bound has an interesting welfare interpretation. The variable factor will always be better off with (inefficient) free access rights than under (efficient) private ownership of property. This proposition is the main result of the present paper.

2. THE ECONOMIC FRAMEWORK

Suppose there are n pieces of property. Each property is a fixed factor having the potential of producing output when the variable factor works with it. Both output and variable input are treated as homogeneous and uniform. Such an assumption will allow us to talk about whether variable factor units as a whole are made better or worse off by differing institutional arrangements.

The index i will stand for any integer from 1 to n. Let $x_i \geqslant 0$ be the amount of variable input (quantified in equivalence or efficiency units) applied to the ith property. This results in total product

$$y_i = f_i(x_i)$$

as described by the production function $f_i(x_i) \geqslant 0$ with $f_i(0) = 0$.

For all $x_i > 0$ the average product of x_i is defined as

$$A_i(x_i) \equiv f_i(x_i)/x_i\ .$$

It is assumed that average products cannot increase with input,

$$0 < x \leqslant x' \Rightarrow A_i(x) \geqslant A_i(x'). \tag{1}$$

For completeness the average product of zero input is defined as the limit

$$A_i(0) \equiv \lim_{x \to 0^+} A_i(x).$$

This limit must exist (although it might be infinite) because $A_i(x)$ is monotonic in x.

If $w \geqslant 0$ is the return to the variable factor, the latter is offered up in total amount $S(w) \geqslant 0$, so that

$$S(w) = \sum_1^n x_i\ .$$

The supply schedule $S(w)$ reflects the willingness of variable input units to commit themselves to active work as a function of the return they receive. One case of special interest is where supply is fixed. It is assumed that the supply curve is upward sloping,

$$w \leqslant w' \Rightarrow S(w) \leqslant S(w'). \tag{2}$$

Total output $y = \sum_1^n y_i \geqslant 0$ commands a market price of $p = g(y) > 0$. The inverted demand curve $g(y)$ is assumed to be normally sloped

$$y \leqslant y' \Rightarrow g(y) \geqslant g(y') > 0. \tag{3}$$

3. Some Examples

The model outlined in the previous section can be used to describe in a static way certain competitive economic activities which could be and often are operated on a common property basis. Possible applications are quite diverse, but they all exhibit that same basic structure described in Section 2. Four kinds of examples are considered below. However the classification is somewhat arbitrary and the list could be extended without difficulty.

(i) *Gathering natural harvests.* Included under this category might be fishing, hunting, grazing, primitive agriculture, etc. As a specific example, consider fishing. Property i might be the ith lake. Variable input x_i could be the number of effective fishing units operating on lake i. Output y_i would be the total catch of fish on lake i, perhaps measured by weight. Presumably (1) holds because the yield per effective fishing unit decreases as the number of fishing units goes up. The interpretation of (2), (3) is obvious.

(ii) *Shared resources from common pools.* Here are classified such activities as petroleum extraction or the use of water from common sources. The different properties are just the various pools. The variable input might be the number of taps, appropriately standardized. Average output declines with an increase in use because of, e.g., the adverse effects on fluid pressure. Or, if the water is being employed as an industrial coolant or carrier of wastes, its quality may be impaired by increased usage. A somewhat more fanciful example of a shared resource shows up in some forms of pure research. Considering the stock of potential results in a specific research area as a pool of knowledge which is less than directly proportional to the research effort being applied due to decreasing returns, certain research activities can be cast in the prototype mold of Section 2.

(iii) *Commercial transportation lines.* Examples include freight or communication routes of various sorts, such as highways, pipelines, etc. For the commercial trucking industry, property i might be the ith highway route connecting two given cities. Variable input x_i could be the number of effective units of trucking capacity hauling freight between one city and the other via route i. Output y_i would be the total tonnage delivered each day which was transported from one city to the other along highway i. In this case $A_i(x_i)$ is the delivered tonnage per truck day using highway i, which would presumably decline when the road started to become congested. Here p is the competitive price of shipping a ton of freight between one city and the other and w is the return to an effective unit of trucking capacity engaged in that activity.

(iv) *Queueing situations.* A wide variety of potential applications can be included under this heading. Consider for example the problem of commuter congestion. All commuters are assumed to be identical. Property i, envisioned here as a queue, could be the ith highway route connecting a given residential center with a particular work place. The variable input x_i would be the number of commuter-units using highway i to go back and forth to work each day. The average output per commuter on i would be the free time delivered to a user of this queue, i.e., twenty-four hours minus the transit time lost in going between home and work using route i. Naturally transit time increases with the number of commuters on any road, due to congestion. The "demand curve" would simply reduce to the form $g(y) = \bar{p}$, with $\bar{p}$ the social value of an extra unit of free time (all commuters are assumed to be indifferent on the margin between spending an extra minute commuting and being paid $\bar{p}/60$ dollars). The supply curve $S(w)$ just gives the total supply of highway commuters as a function of the free time it leaves them, measured in dollar equivalents. Another queueing situation which can be put into the framework of Section 2 is the probabilistic job search model. In any period a job seeker applies to one of a set of work opportunities, each of which pays a fixed wage if he is hired, but the wages differ among jobs. On average, the typical applicant can expect to receive the going remuneration for a given job reduced by the ratio of the number of job openings to job seekers if the number of applicants for a job exceeds the number of openings.

4. Free Access Equilibrium

One of the most ancient and basic forms of property management is communal ownership. The essence of this economic system is that the community denies to any group or individual the prerogative to block usage of communally owned property. There are no private or governmental property rights and therefore no institutional arrangement exists for collecting rents. As a result of free access, competitive variable input units can and will move freely to that property which offers them the highest product per unit. An external diseconomy is typically created because independent units of the variable factor ignore the effects of their actions on the average products of others in considering only the product *they* stand to gain or lose by a proposed change.

Free access competitive equilibrium thus allocates the variable factor so that its average product is equalized on all properties that are used. This allocation system is denoted FA and the equilibrium values of entities in it are capped by a tilde.

$$\begin{aligned}
&\tilde{x}_i > 0 \Rightarrow \frac{\tilde{p} f_i(\tilde{x}_i)}{\tilde{x}_i} = \tilde{w} \\
&\tilde{x}_i = 0 \Rightarrow \tilde{p} A_i(0) \leqslant \tilde{w} \\
&S(\tilde{w}) = \sum_1^n \tilde{x}_i \\
&\tilde{p} = g\left(\sum_1^n f_i(\tilde{x}_i)\right).
\end{aligned} \tag{4}$$

We assume that there is an *FA* equilibrium. (The issue of existence is not of interest for its own sake in the present paper and anyway it is not difficult to give supplementary conditions which would insure it.)

5. Private Ownership Equilibrium

The economic system of private ownership sanctions the property rights of a certain class (or group) who owns the property and determines its usage. Under perfect competition the variable factor can be hired at a common competitive price and self interested rentiers will hire that amount of variable input which maximizes their profits. Perfectly competitive private ownership equilibrium therefore equates the value marginal product of the variable factor with its price on all property in competitive use. Such an allocation system is denoted *PO*, and in *PO* equilibrium variables are capped by a circumflex.

$$\begin{aligned}
&\text{for all } i,\ \hat{p} f_i(\hat{x}_i) - \hat{w}\hat{x}_i = \max_{x \geqslant 0} \hat{p} f_i(x) - \hat{w}x \\
&S(\hat{w}) = \sum_1^n \hat{x}_i \\
&\hat{p} = g\left(\sum_1^n f_i(\hat{x}_i)\right).
\end{aligned} \tag{5}$$

A solution of the above equations is assumed to exist with $\sum_1^n \hat{x}_i > 0$.

Note in *PO* that $\hat{w}/\hat{p}$, the "marginal product" of x_i, is tangent to $f_i(x_i)$ at $x_i = \hat{x}_i$. The competitive rental

$$R_i \equiv \hat{p} f_i(\hat{x}_i) - \hat{w}\hat{x}_i \geqslant 0$$

will be collected on property i.

With *PO* it is conceptually equivalent to think of rentiers as hiring variable input at competitive prices to maximize profits or as charging efficiency tolls for the use of their property and then allowing the competitive variable factor to allocate itself with otherwise unimpaired access *a la FA*. In the latter arrangement variable inputs hire property at the going toll rate and then receive the *net* average value product (after payment of tolls) which thus tends to get equalized on property in use. Equilibrium quantities and prices under both arrangements will be identical if the tolls are correctly reckoned. The proper efficiency toll for property *i*, denoted τ_i, would be defined as follows

$$\tau_i = \begin{cases} 0 & \text{if} \quad \hat{x}_i = 0 \\ R_i/\hat{x}_i & \text{if} \quad \hat{x}_i > 0. \end{cases}$$

This is a competitive toll schedule because any other toll on property *i* would not yield greater revenue to that property.

There is even a way of envisioning *PO* in terms of producer cooperatives which take a lease on property at the competitive rental price and determine their membership size by maximizing the dividend of net revenue (after payment of rent) per variable factor member. The solution is the same as before if rentals have been accurately determined.

It is also conceptually irrelevant to the determination of an optimal allocation whether *PO* is regarded as based on competitive *private* ownership of property or on efficiently organized government *public* ownership, so long as problems of income distribution have been abstracted away. The social task facing planners having complete control of income transfers is to maximize the difference between the social value of output and the opportunity cost of the variable factor

$$\int_0^{y'} g(y)\, dy - \int_0^{w'} w\, dS(w),$$

subject to

$$S(w') = \sum_i^n x_i$$

$$y' = \sum_1^n f_i(x_i).$$

The solution of this problem is of course the competitive solution $x_i = \hat{x}_i$. The only difference between the two cases is in who gets the surplus—private rentiers or the government.

A way of restating this duality is to point out, as economists are fond of doing, that *PO* is an efficient economic system. If it were *not*, for some set of $x_i' \geqslant 0$ with

$$\sum_1^n x_i' \leqslant \sum_1^n \hat{x}_i$$

we would have

$$\sum_1^n f_i(x_i') > \sum_1^n f_i(\hat{x}_i).$$

This would mean that for some j

$$\hat{p}f_j(x_j') - \hat{w}x_j' > \hat{p}f_j(\hat{x}_j) - \hat{w}\hat{x}_j$$

which contradicts (5).

Thus, for the model building purposes of theoretically characterizing efficient allocation, who owns property and what factor is thought of as hiring the other in the economic system we are calling *PO* is somewhat arbitrary. Which arrangement is in fact to be employed would largely depend on institutional considerations and on tradition. For example we usually think of optimal highway management as a public ownership problem involving efficiency tolls, whereas farming is typically envisioned by us as an arrangement involving private land ownership and hired labor.

6. Overcrowding under Free Access

A notion commonly held about *FA* is that in some sense property will be overcrowded by comparison with the efficient *PO* system. In the present section this characterization of *FA* allocation is quantified and demonstrated to be true.

From the theorem of the next section (which does not in any way depend on the results of this section for its proof) and from (2),

$$\sum_1^n \hat{x}_i \leqslant \sum_1^n \tilde{x}_i \,. \tag{6}$$

Inequality (6) demonstrates the presence of *general* overcrowding in *FA* since it shows that more of the variable factor is used in that system than under *PO*. But what about the allegation that *individual* pieces of better quality property will be overused in *FA* as compared with *PO*?

The classical story is frequently told in terms of labor crowding and overworking the more fertile land when no rents or tolls are collected.

To investigate this proposition, we must start by defining what is to be meant by the quality of a given piece of property. A natural measure might be the efficiency toll which would be charged for user services to a unit of variable input in *PO*. The *average* toll charged in *PO* is

$$\bar{\tau} = \sum_1^n \tau_i \hat{x}_i \Big/ \sum_1^n \hat{x}_i .$$

It would be natural to say that if $\tau_j > \bar{\tau}$, property j is in some sense of better than average quality. The following theorem shows that the notion of superior property being overcrowded in *FA* can be given a rigorous justification.

THEOREM. *If* $\tau_j > \bar{\tau}$, *then* $\tilde{x}_j > \hat{x}_j$.

Proof. Since $\tau_j > 0$, it follows that $\hat{x}_j > 0$. We then have

$$\frac{f_j(\hat{x}_j)}{\hat{x}_j} = \frac{\hat{w}}{\hat{p}} + \frac{\tau_j}{\hat{p}} > \frac{\hat{w}}{\hat{p}} + \frac{\bar{\tau}}{\hat{p}} = \frac{\hat{w}}{\hat{p}} + \frac{\sum_1^n R_i}{\hat{p} \sum_1^n \hat{x}_i}$$

$$= \frac{\hat{w} \sum_1^n \hat{x}_i + \sum_1^n R_i}{\hat{p} \sum_1^n \hat{x}_i} = \frac{\sum_1^n f_i(\hat{x}_i)}{\sum_1^n \hat{x}_i} .$$

Combining,

$$f_j(\hat{x}_j)/\hat{x}_j > \sum_1^n f_i(\hat{x}_i) \Big/ \sum_1^n \hat{x}_i . \tag{7}$$

Using (5) and summing over all properties,

$$\hat{p} \sum_1^n f_i(\hat{x}_i) - \hat{w} \sum_1^n \hat{x}_i \geqslant \hat{p} \sum_1^n f_i(\tilde{x}_i) - \hat{w} \sum_1^n \tilde{x}_i .$$

From (6), dividing the (nonnegative) left hand side of the above inequality by (positive) $\sum_1^n \hat{x}_i$ and the right side by $\sum_1^n \tilde{x}_i$ yields, after simplification,

$$\sum_1^n f_i(\hat{x}_i) \Big/ \sum_1^n \hat{x}_i \geqslant \sum_1^n f_i(\tilde{x}_i) \Big/ \sum_1^n \tilde{x}_i \tag{8}$$

Substituting from (4) into the right hand side of (8),

$$\sum_1^n f_i(\hat{x}_i) \Big/ \sum_1^n \hat{x}_i \geqslant \frac{\tilde{w}}{\tilde{p}} .$$

Combining the above inequality with (7) yields

$$\tilde{p} f_j(\hat{x}_j)/\hat{x}_j > \tilde{w}.$$

It follows from (1) and (4) that

$$\tilde{x}_j > \hat{x}_j\,,$$

which concludes the proof.

7. Efficiency and Distribution in the Two Systems

Suppose that we take the side of the variable factor and ask which system of property management is "better" from its point of view. As already noted, *PO* must be efficient whereas *FA* will generally be inefficient. This observation tends to bias most economists in favor of *PO*. After all, no matter whose welfare is at stake the efficient *PO* situation must be preferable because even if variable factor units don't themselves own the properties, rent or toll income collected under *PO could* always be redistributed to make them at least as well off as they were under the inefficient *FA* situation. Unfortunately such enlightened textbook transfer policies rarely take place in the real world. Certainly of no *less* interest from a practical standpoint is the polar opposite case where those rents or tolls being collected are siphoned off to elsewhere in the economy and do not flow back to the variable factor in significant amounts. With no redistribution, it is not easy to see offhand under which allocation system the variable factor does better. In *FA* there is a smaller sized distribution pie than there otherwise might be, due to inefficiency. On the other hand the variable factor gets *all* of the pie instead of just a slice, as with *PO*.

The following theorem shows that the distribution effect is always stronger than the efficiency effect. When the variable factor units have parted with the tolls or rents needed to make them "efficient" they will find themselves worse off than they were under inefficient average product equalizing. In the polar case of no redistribution, the variable factor cannot fare better under *PO* than under *FA*. This is in contrast to the perfect transfer case usually analyzed by economists where *PO* always turns out to be best. What happens with in-between cases would naturally depend on the degree of redistribution.

Theorem. $\tilde{w} \geqslant \hat{w}$.

Proof. Suppose by contradiction that $\tilde{w} < \hat{w}$. From monotonicity of $S(w)$, $S(\tilde{w}) \leqslant S(\hat{w})$ or

$$\sum_1^n \tilde{x}_i \leqslant \sum_1^n \hat{x}_i .$$

Because *PC* is efficient

$$\sum_1^n f_i(\tilde{x}_i) \leqslant \sum_1^n f_i(\hat{x}_i),$$

so that

$$g\left(\sum_1^n f_i(\tilde{x}_i)\right) = \tilde{p} \geqslant \hat{p} = g\left(\sum_1^n f_i(\hat{x}_i)\right).$$

Since $\sum_1^n \tilde{x}_i \leqslant \sum_1^n \hat{x}_i$ and $\sum_1^n \hat{x}_i > 0$, there must be at least one $\hat{x}_j > 0$ with $\tilde{x}_j \leqslant \hat{x}_j$. Because $R_j \geqslant 0$,

$$\hat{w} \leqslant \hat{p} A_j(\hat{x}_j).$$

Thus,

$$\hat{w} \leqslant \hat{p} A_j(\hat{x}_j) \leqslant \tilde{p} A_j(\hat{x}_j) \leqslant \tilde{p} A_j(\tilde{x}_j) \leqslant \tilde{w}. \qquad \text{Q.E.D.}$$

By way of summarizing, there may be a good reason for propertyless variable factor units to be against efficiency improving moves toward marginalism like the introduction of property rights or tolls unless they get a specific kickback in one form or another.