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Harvesting versus Biodiversity: An Occam's Razor Version[1]

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Abstract. The point of departure for this paper is the familiar prototype fisheries model where a fictitious sole owner harvests a fish population to maximize present discounted profits. The paper answers analytically the following question. "*What happens to a policy when the sole owner also values biodiversity, as well as profits?*" It turns out that the size of the steady-state stock and the number of species preserved are both higher, when species diversity is positively valued. This paper provides a sharp characterization of the optimal policy in terms of the usual economic parameters and an exogenously introduced willingness-to-pay function for species preservation.

Key words: biodiversity, optimal policy, parameters

JEL classification: Q2, Q3

1. Introduction

In this paper we take a fresh look at the trade-off between harvesting and biodiversity. Stressing the Aristotelian principle, today referred to as Occam's Razor, that "entities must not be multiplied beyond what is necessary",[2] we strive to keep the discussion as simple and intuitive as possible. The core analysis is conducted in three steps. We start by introducing the classical linear harvesting problem. We then introduce completely symmetric species, which each have a preservation value. Typically, species loss is irreversible, but to understand better the irreversible case, we solve a hypothetical problem where species loss is assumed to be reversible, and where the creation of new species is costless.

Both the classical linear harvesting problem and the augmented linear harvesting problem have a solution which involves a most rapid approach (MRA) towards a steady state. Introducing irreversible species loss in the third step is shown to imply a solution, which, depending on the initial number of species, resembles either the linear harvesting problem, its augmented version, or a combination of the two. The intuition behind the solution is procured with the help of a simple diagram. The formal proof of the main theorem is relegated to the appendix.

In the final section of the paper, we show that the structure of the optimal policy is robust with respect to the introduction of uncertainty about the minimum viable population level.

2. The Model

THE CLASSICAL LINEAR HARVESTING PROBLEM

A classical economic resource management problem is the optimal harvest of a renewable resource, say a fish population. A simple and well-defined solution to this problem emerges when the Hamiltonian of the optimization problem is linear in the harvest (the control variable). To derive the first best solution, it is convenient to adopt the fiction of a "sole owner", who has the complete rights to the exploitation of a given fish population. He is assumed to act to maximize profit, subject to the initial stock of the fish population and its growth dynamics. To be more specific, the optimal management problem is characterized as the following:

$$\underset{\{h(t)\}}{\text{Max}} \int_0^\infty B(x(t))h(t)e^{-\delta t}\mathrm{d}t \tag{1}$$

subject to

$$\dot{x} = F(x(t)) - h(t) \qquad x(0) = x_0$$

$$h_{\min} \leq h(t) \leq h_{\max}$$

where δ is the discount rate, and $B(x)$ is the average *net* benefit function, i.e. the price minus average harvesting cost. In other words, the social planner maximizes the present value of the harvest, $h(t)$, subject to the growth function $F(x(t))$ and the initial condition $x(0) = x_0$. The harvest level is constrained by lower and upper limits, $h_{\min}$ and $h_{\max}$, respectively. The average net benefit function, $B(x)$, is assumed to be twice continuously differentiable, strictly concave, and non-decreasing in x. The growth function $F(x)$ is twice continuously differentiable with $F(0) = 0$, $F'(0) = \infty$, $F''(x) \leq 0$, and $F(x) \geq 0$ for $0 \leq x \leq \kappa$, where κ is the carrying capacity of the environment with $F(\kappa) = 0$.

Note that both the net benefit function and the growth function are autonomous, i.e. they do not explicitly depend on the time variable t. Then, following Clark (1990), it is straightforward to show that the singular path is a steady state defined by a real number $\hat{x}_c$ satisfying the following equation:

$$\phi(\hat{x}_c; \delta) = F'(\hat{x}_c) + B'(\hat{x}_c)F(\hat{x}_c)[B(\hat{x}_c)]^{-1} - \delta = 0. \tag{2}$$

This equation indicates that, in the steady state, the marginal productivity of the resource stock plus the capital gain of an increase in the resource stock is equal to the discount rate. It is easily shown that the assumptions about $F(x)$ and $B(x)$

guarantee the existence of a steady state, since $\lim_{x \to \kappa} \phi(x; \delta) < 0 < \lim_{x \to 0} \phi(x; \delta)$, and $\phi(x; \delta)$ is continuous.

If $\phi'(\hat{x}_c; \delta) < 0$ at any steady state, then the steady state is unique as a direct consequence of the fact that the number of steady states is odd. This is true if $\hat{x}_c > \bar{x}$, where $\bar{x}$ is the golden rule resource stock defined by $F'(\bar{x}) = 0$, and generally true if the product $B(x)F(x)$ is strictly concave in x. Under uniqueness, the steady state stock will be a decreasing function of the discount rate.

From standard optimal control theory (cf. Clark 1990), we also know that the optimal path towards the steady state is a most rapid approach (MRA), i.e.,

$$h^*(t) = \begin{cases} h_{\max} \textit{ whenever } x > \hat{x}_c \\ F(\hat{x}_c) \textit{ whenever } x = \hat{x}_c \\ h_{\min} \textit{ whenever } x < \hat{x}_c. \end{cases} \tag{3}$$

THE AUGMENTED LINEAR HARVESTING PROBLEM

We are now ready to reinterpret the augmented linear harvesting model in terms of an optimal trade-off between harvesting and biodiversity preservation. Let $n(t)$ denote the number of species[3] and $x(t)$ the total resource stock (biomass) at time t. To highlight the structural aspects of the trade-off, we assume extreme symmetry, such that the resource stock of each species is determined by $x_i(t) = x(t)/n(t)$, where $x_i(t)$ is the total biomass of species i. Each species is also assumed to have a common minimum viable population of k_0.

Now let $w(n)$, with $w'(n) < 0$, be the willingness to pay for preserving a species when n species exist. It may be thought of as an answer to an “as if” question in an ideal contingent valuation study. We now define

$$W(n) = \int_0^n w(s)\, \mathrm{d}s. \tag{4}$$

Hence, $W(n)$ is the total willingness to pay for preserving n species. The function is strictly concave, twice differentiable, and increasing in n. Note that $W'(n) = w(n) > 0$, and $W''(n) = w'(n) < 0$.

It is most reasonable to treat the loss of species as irreversible, but let us for the sake of argument assume that species are perfectly, and costlessly reversible. In such a world, since the marginal species has a positive value, along an optimal program, one will always choose

$$n(t) = x(t)/k_0, \tag{5}$$

i.e. any given total biomass will be used to create the maximum number of viable species. To get most easily to the essence of the problem, we will assume that all species are completely symmetric in everything, including reproductive dynamics.

This means that we can, provided that each species is as large as the minimum viable population, work with an aggregate macro form of the biomass equation,[4] i.e.,

$$\dot{x} = n\dot{x}_i = ng(x_i) - nh_i = F(x) - h. \tag{6}$$

In other words, the growth dynamics equation $\dot{x} = F(x) - h$ can be given a reduced form macro interpretation. Each symmetric species has a constant return to scale-like mini-version of it, which can be aggregated by multiplication with the number of species. When the species is completely reversible, the continuously differentiable relationship between x and n given by equation (5) determines the dynamics of n as soon as the dynamics of x is given.

Now, the optimization problem can be written in the following manner:

$$\underset{\{h(t)\}}{\text{Max}} \int_0^\infty [B(x(t))h(t) + W(x(t)/k_0)]e^{-\delta t}\,\mathrm{d}t \tag{7}$$

subject to

$$\begin{aligned} &\dot{x} = F(x(t)) - h(t) \qquad x(0) = x_0 \\ &h_{\min} \leq h(t) \leq h_{\max}. \end{aligned} \tag{8}$$

The harvesting versus biodiversity problem is now formulated in a manner which, again, implies that the singular solution, is a steady state, $\hat{x}_a$, determined by the following condition:

$$\Phi(\hat{x}_a;\delta) = F'(\hat{x}_a) + \frac{B'(\hat{x}_a)}{B(\hat{x}_a)}F(\hat{x}_a) + \frac{W'(\hat{x}_a/k_0)}{k_0 B(\hat{x}_a)} - \delta = 0. \tag{9}$$

The interpretation is that in the steady state the marginal productivity of the resource stock plus the capital gain of an increase in the resource stock plus the "marginal benefit" from the resource stock *per se* is equal to the discount rate. The optimal policy is a MRA to the steady state $\hat{x}_a$.

It is straightforward to show that the steady state stock is higher when there is a preservation value involved, i.e. $\hat{x}_a > \hat{x}_c$. Under assumptions analogous to those introduced in connection with the classical linear harvesting problem, the steady state stock and the number of species will be unique and it is a decreasing function of the discount rate. An exogenous negative shift in the marginal preservation value of species $W'(\hat{x}_a/k_0)$ has the same qualitative effect on the steady state as the change in the discount rate δ. This is also reflected in the relationship between the two steady state stocks $\hat{x}_c$ and $\hat{x}_a$, the former implying a marginal preservation value equal to zero. It is also a simple exercise to show that an increase in the minimum viable population, k_0, decreases the steady state biomass as well as the number of species.

THE LINEAR HARVESTING PROBLEM UNDER IRREVERSIBLE SPECIES LOSS

If species loss is irreversible, we will have to modify the setup slightly, and the new setup will change the optimal solution. As we will show, however, the solution can be related to the two steady states above, in a rather simple manner.

The most important change in relation to the reversible case, is that the number of species will now be determined by the following condition:

$$n(t) = \min[x(t)/k_0, n(0)]. \tag{10}$$

This means that the number of species is bounded from above by the initial number of species, provided that this entity is less than the maximum number of viable species at time t. In other words, it is expliticly assumed that one cannot create new species. If the initial number of species is greater than the maximum number of viable species at time t, the latter determines the existing number of species. Using equation (10), we can define the preservation value function as:

$$U(x(t)) \equiv W\{\min[x(t)/k_0, n(0)]\} \tag{11}$$

which has the following properties:

$$\begin{aligned} &\textit{If } x(t)/k_0 < n(0),\ \textit{then } U'(x) = U'_+(x) = U'_-(x) = W'(x/k_0)/k_0 \\ &\textit{If } x(t)/k_0 > n(0),\ \textit{then } U'(x) = U'_+(x) = U'_-(x) = 0 \\ &\textit{If } x(t)/k_0 = n(0),\ \textit{then } U'_+(x) = 0,\ \textit{and } U'_-(x) = W'(x/k_0)/k_0. \end{aligned} \tag{12}$$

In other words, the preservation value function has a kink at $x(t) = k_0 n(0)$. The optimization problem can now, in its reduced form macro version, be rewritten as:

$$\underset{\{h(t)\}}{\text{Max}} \int_0^\infty [B(x(t))h(t) + U(x(t))]e^{-\delta t}\mathrm{d}t \tag{13}$$

subject to the growth dynamics equation and the harvest constraint in Equation (8). The optimal policy will now, in a very natural way, depend on the initial number of species. To understand the intuition, it is worthwhile presenting the reasoning behind the optimal policy with the help of a simple diagram (Figure 1).

In the diagram, the main relationship is a one between the number of species and the stock of the total biomass, $n(t) = x(t)/k_0$, which will hold true at each instant of time when species loss is reversible and the creation of new species is costless. This is the line OA shown in Figure 1. The steady state in the classical linear harvesting problem, $\hat{x}_c$, as well as the steady state of the augmented reversible harvesting problem, $\hat{x}_a$, and their corresponding steady state number of species have their loci on this diagonal.

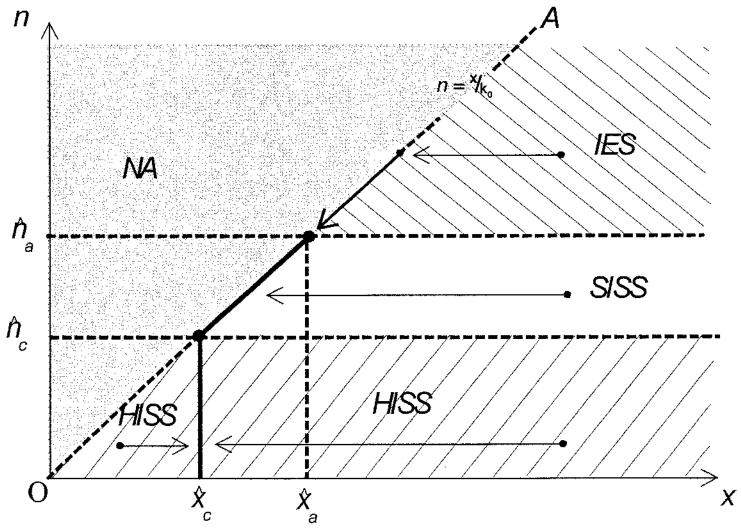

Figure 1. An illustration of the optimal harvest policy.

For starting points above the "OA-line", we have $n(0) > x(0)/k_0$. The symmetry assumption implies that all species would have a lower stock level than the minimum viable population level with $x(0)/n(0) < k_0$ and thereby would become extinct. In other words, no harvesting policy can be sustainable in the non-applicable (NA) region within the region above the OA-line. Thus, in the following analysis, we will focus on the case below the OA-line with $n(0) < x(0)/k_0$.

We have split up the vertical axis into three regions, HISS (heavy initial shortage of species), SISS (slight initial shortage of species), and IES (initial excess of species). These are related to the steady state number of species that is implied by the previous two linear harvesting problems. To be more specific, all equilibrium resting points lie on the heavy shaded kinked line segments in Figure 1, which represent a sort of "combination" of the two simple cases considered previously. Based on the equilibrium number of species $\hat{n}_c = \hat{x}_c/k_0$ and $\hat{n}_a = \hat{x}_a/k_0$ for the two typical cases, we define the regions by the following inequalities:

$$
\begin{aligned}
&n(0) < \hat{n}_c \text{ HISS} \\
&\hat{n}_c \leq n(0) \leq \hat{n}_a \text{ SISS} \qquad (14) \\
&\hat{n}_a < n(0) \text{ IES.}
\end{aligned}
$$

For feasible $n(0)$ in SISS region, it is true that $U'(x) = 0$, and it is costless in terms of preservation value to decrease the biomass while keeping the number of species intact. Hence, from the linearity of the Hamiltonian in the harvest, it is optimal to approach the OA-line by a MRA. Once on the OA-line, it would be desirable to move to $(\hat{x}_a, \hat{n}_a)$, but this is not feasible when species loss is irreversible. The best one can do is to remain on the OA-line by harvesting the growth of each species. In the IES case it is feasible, and also optimal, to follow the same policy as in the reversible case and approach the steady state $(\hat{x}_a, \hat{n}_a)$ by a MRA policy. This is achieved by the extinction of species along OA down to the steady state. Note that, since the number of species as such is valued, and the rate of interest is positive, it is not optimal to start harvesting by depleting species. It is always better to deplete the biomass of existing species.

For $n(0)$ in the HISS region there are two cases. The total biomass is either larger or smaller than $\hat{x}_c$. In the former case the optimal policy is MRA with $h = h_{\max}$, without depleting a single species. When the biomass is equal to $\hat{x}_c$, the harvest is put equal to growth and the system remains at $(\hat{x}_c, n(0))$.

In the latter case, for starting points to the left of $\hat{x}_c$, and below OA, the biomass is below the optimal steady state stock of the linear harvesting problem. In this situation, it is feasible and optimal to move to its steady state as quickly as possible, by increasing the biomass of each existing species as fast as possible by setting $h = h_{\min}$.

We can sum up the above intuitive discussion of the optimal solution in the following theorem:

THEOREM: *if $n(0) > \hat{n}_a$ (IES), the optimal policy is MRA of $x(t)$ to $\hat{x}_a$, with extinction of $n(0) - \hat{x}_a/k_0$ species. There are two potential optimal ways to do this. If, $x(0)/k_0 = n(0)$, the population of ecah species is the minimum viable by definition, and the number of species is reduced to $\hat{n}_a$. If $n(0) < x(0)/k_0$, each species is harvested by MRA down to its minimum viable population, thereafter species are harvested until the number equals $\hat{n}_a$.*

If $n_0 \leq \hat{n}_c$ (HISS), the optimal policy is MRA of $x(t)$ to $\hat{x}_c$ with all species preserved.

If $\hat{n}_c \leq n(0) \leq \hat{n}_a$ (SISS), the optimal policy is MRA of $x(t)$ to $n(0)k_0$, with all species preserved.

Proof: The optimality of the most rapid approach for all starting points follows from a slightly modified proof of Proposition 3 in Spence and Starrett (1975). The details are available in the appendix.

Clearly, the steady state harvests in the three possible cases are $h = F(\hat{x}_a)$, $h = F(\hat{x}_c)$ and $h = F(n(0)k_0)$, respectively.

In the HISS case, the steady state stock $\hat{x}_c$ cannot be reached by keeping the stock of individual species at their minimum viable levels, due to the heavy shortage of species with $n(0) < \hat{n}_c$. the best that can be done is to compensate for the lack of species by raising the stock of each species over the minimum viable level. In the SISS case, once on the OA line, one “would like to follow” the optimal policy under reversible species, and move to $(\hat{x}_a, \hat{n}_a)$. This is no longer feasible, and the best thing to do is to stop at $x = k_0 n(0)$ with all initial species preserved. The IES case involves too many species, and depending on whether one starts above or at the minimum viable population, one either brings the stock down to the minimum viable level, and then decreases the number of species or, in the latter case, the abundance of species is harvested from the start.

The reason why the populations, in the former case, are brought down to their minimum viable levels, is that it is possible in this manner, to temporarily enjoy the same level of harvest together with a larger number of species, than under any other program that takes the system to the steady state at maximum speed.

While the HISS case is most akin to the classical linear harvesting problem, the IES case resembles the augmented linear problem with starting points above the optimal stock. The SISS case has the MRA property from both the classical and augmented linear harvesting problems. It has, however, a special property, which it shares with the HISS case, in that the initial number of species determines its steady state stock of each species.

3. The Basic Model Extended to Uncertainty

Note that the basic model presented above has a steady state stock per species at the knife edge which is the minimum viable population level. It may be argued that a small negative perturbation from such a solution would drive the ecosystem to become functionally unstable. In this section, we extend the basic model by taking into account an uncertainty measure associated with the minimum viable population level, and show that the structure of the solution to the basic model remains relevant.

Assume that the social planner does not know the exact size of the minimum viable population, and thus treats such a threshold as a random variable z. Let the probability density of this variable be $\omega(z)$ with $z \in [a, b]$, then the probability for the actual stock per species over the threshold can be described by the c.d.f $\Omega(x/n) = \int_a^{x/n} \omega(z)\,\mathrm{d}z$, where x is the total resource stock and n the number of species. In case the actual stock per species falls down below the threshold, we assume a loss of instantaneous utility $C < 0$. With these assumptions, the optimization problem (cf. Cropper 1976) becomes

$$\max_{\{h(t),n(t)\}} \int_0^\infty [\Omega(x(t)/n(t))(B(x(t))h(t) + U(n(t))) \\ + (1 - \Omega(x(t)/n(t)))C]e^{-\delta t}\mathrm{d}t \tag{15}$$

subject to the resource dynamics and harvest constraints in Equation (8).

In the case of reversible species stocks, a straightforward application of the maximum principle leads to a singular path (the steady state solution) $(\hat{x}, \hat{n})$, and the optimal path would be a MRA to the steady state. With irreversible species loss, if $n_0 > \hat{n}$ (IES), then the optimal policy would be to deplete the $n_0 - \hat{n}$ redundant species by the MRA, whereas, if $n_0 < \hat{n}$, the number of species is constrained by the initial number. In the latter case, the solution is similar to the HISS case described in the previous section. Note that under endogenous risk, we obtain an isolated steady state, and the SISS case disappears.

It can be readily shown that the steady state stock per species $\hat{x}/\hat{n}$ increases with the loss C, meaning that an increase in the penalty of functional instability induces a higher steady state level of resource stock per species. This is intuitively clear and consistent with the basic result derived by Cropper (1976).

4. Concluding Comments

Every resource economist is familiar with the standard model of optimal fisheries management. This simple model of renewable-resource harvesting begins by postulating a fictitious "sole owner," who can be envisioned as either a private company or a government agency. The sole owner possesses complete property rights over the exploitation of a given fish population, and the aim is to determine a harvesting policy that maximizes present discounted profits. As is also well known in this model, under the standard assumptions the optimal harvesting policy is simple and intuitive, taking the form of a most-rapid-approach to the profit maximizing steady state. Simple as it is, the standard model is evidently considered to be useful overall, for in practice it is employed as a starting point for most articles and books on the optimal management of renewable resources. We propose also to use the standard model here as a point of departure. Like others who build upon this model, we are trying to understand what happens when the model is tweaked in a particular direction. The direction in which we are interested to tweak the model is toward understanding analytically what happens to an optimal policy when society, in the form of the sole owner, values biodiversity per se, as well as profits.

Naturally, the answer to the question of "what happens" depends upon *how much* is biodiversity valued relative to profits. Our method for capturing this aspect of the problem is by way of introducing exogenously a "willingness-to-pay-for-species-diversity" function. The enhanced model yields a relatively sharp solution that indicates clearly how willingness to pay for species diversity interacts with the more usual economic profitability considerations to determine an optimal policy.

The analysis shows that when biodiversity is positively valued, for whatever reason, then the message of the standard fisheries model changes somewhat. In the standard model, the fish population is harvested to the level where the marginal economic rate of return on investment in the fish stock equals the prevailing real interest rate. Now, when biodiversity is valued along with profits, an interior solution corresponds to the fish population being harvested at the level where the economic rate of return equals the prevailing interest rate *minus* the willingness to pay for preserving the marginally threatened species, expressed as a fraction of economic benefits. Additionally, there exists here a conceptually interesting corner solution, caused essentially by the fact that species extinction is irreversible. It is quite possible to be "stuck" in an initial corner situation where the relevant factor limiting harvests is the unwillingness to have another species become extinct, which predominates over narrow economic profitability considerations (the HISS and SISS regions). This paper explains and interprets the exact connection between initial conditions and the emergence of an interior or a corner solution.

Some features of the model will, of course, disappear in a more general setting, such as the MRA towards the steady state. However, it is our feeling that this basic insight of the model will survive the introduction of a more realistic framework.[5]

Notes

1. The authors would like to thank the two anonymous referees for valuable comments and suggestions.
2. In Latin: "Non sunt multiplicanda entia praeter necessitatem". William of Ockham (1284–1349) was an English philosopher and theologian.
3. For convenience we treat the number of species as a continuous variable, but everything goes through for an integer-valued *n*. In the latter case, the relevant value will be either the integer just-above or just-below the value of *n* we use here.
4. This can be rigorously done by assuming that each species has a constant return to scale growth function $\dot{x}_i = g(x_i, \kappa_i)$, where $x_i = x/n$, and $\kappa_i = \kappa/n$ are, respectively, the stock of species *i* and species *i*'s share of the total carrying capacity κ. From this we obtain $ng(x_i, \kappa_i) = G(x, \kappa) = F(x)$. As one referee pointed out, there is a body of literature that argues that biodiversity is important because of its impact on productivity over a range of environmental conditions (see Holling et al. 1995). To reflect these resilience arguments the growth function would have to be written as $F = F(x, n)$. This represents a seemingly trivial extension of the model, but it introduces a much more complicated dynamics.
5. For a comparison with corresponding result in a more elaborated framework the reader is referred to a recent paper by Li and Löfgren (1998).

Appendix: Proof of Theorem

In this appendix we give a formal proof of our main result. The proof relies on a general result in Spence and Starrett (1975).

Proof: Define $M(x) \equiv U(x) + B(x)F(x)$ *and* $N(x) = -B(x)$. Our problem equations (7)–(8) can now be translated into:

$$\text{Max} \int_0^\infty [M(x) + N(x)\dot{x}]e^{-\delta t}\, \mathrm{d}t$$

subject to

$$h_{\min} \le h \le h_{\max}$$

$$x(0) = x_0.$$

Now define

$$S(x) = \int_0^x N(z)\,\mathrm{d}z$$

$$V(x) = M(x) + \delta S(x)$$

Intergrating the objective function by parts, using the above definitions, translates it into:

$$\mathrm{Max} \int_0^\infty V(x) e^{-\delta t}\,\mathrm{d}t.$$

Our underlying assumption assures that $V(x)$ is concave and everywhere differentiable except at the point $x = n(0)k_0$, where the following left and right hand derivatives apply:

$$\begin{aligned} V'_{-}(n(0)k_0) \;=\; & W'(n(0)k_0) + B'(n(0)k_0)F(n(0)k_0) + \\ & B(n(0)k_0)F'(n(0)k_0) - rB(n(0)k_0) \end{aligned}$$

$$V'_{+}(n(0)k_0) = B'(n(0)k_0)F(n(0)k_0) + B(n(0)k_0)F'(n(0)k_0) - rB(n(0)k_0)$$

It is also easily confirmed that: $\lim_{x\to\infty} V'(x) > 0$, *and* $\lim_{x\to\infty} V'(x) < 0$.

The above conditions guarantee that $V(x)$ has a unique maximum, x^*, on $(0, \infty)$. Our problem now satisfies all the conditions of Proposition 3 in Spence and Starrett (1975). This means that the optimal policy is a MRA to x^*.

It remains to characterize x^*. There are three possible cases to consider.

Case 1 : $n(0)k_0 < \hat{x}_c$ HISS

In this case it is easily confirmed that $V'_{+}(n(0)k_0) > V'(\hat{x}_c) = 0$, and hence that the optimal steady state $x_1^* > n(0)k_0$. Thus, the relevant first order condition is $V'_{+}(x_1^*) = 0$, which yields $x_1^* = \hat{x}_c$.

Case 2 : $n(0)k_0 > \hat{x}_a$ IES

Here it holds that $V'_{-}(n(0)k_0) < V'(\hat{x}_a) = 0$ and hence that $x_2^* < n(0)k_0$. Thus, the relevant first order condition is $V'_{-}(x_2^*) = 0$ and $x_2^* = \hat{x}_a$.

Case 3 : $\hat{x}_c \le n(0)k_0 \le \hat{x}_a$ SISS

In this case we have $V'_{+}(n(0)k_0) \le 0 \le V'_{-}(n(0)k_0)$, and $x_3^* = n(0)k_0$.

The rest of the proof follows directly from the definition of a MRA policy, as applied to the above three cases.

Q.E.D.

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