Prices versus Quantities across Jurisdictions

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Abstract: We extend the standard "prices versus quantities" framework for pollution control to cover multiple heterogeneous jurisdictions interacting strategically with each other. When multi-jurisdictional externalities are present and the uncertainties among jurisdictions are independent, the regulatory game exhibits a unique subgame perfect equilibrium. For any one jurisdiction, the equilibrium choice of instrument is given by the sign of the original prices versus quantities formula. Thus, it is an optimal strategy for a jurisdiction to choose a price instrument when the slope of its own marginal benefit is less than the slope of its own marginal cost and a quantity instrument when this condition is reversed. The result suggests that the original non-strategic criterion for the comparative advantage of prices over quantities may have wider applicability to determining instrument choice in a noncooperative strategic environment.

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WHAT IS THE BEST TYPE of policy instrument for controlling global pollution such as CO_2? Choosing the best instrument for controlling the negative externalities from national or global pollution has been a long-standing issue in environmental economics. Pigou (1920) introduced the central concept of placing a price charge on pollution (aka "Pigouvian tax") as an efficient way to correct a pollution externality. This

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Pigouvian-tax approach dominated economic thinking about the pollution externality problem for about the next half-century.

Dales (1968) introduced the idea of creating property rights in the form of tradable pollution permits (aka "allowances") as an efficient alternative to a Pigouvian tax. Montgomery (1972) proved the formal equivalence between a price on pollution and a dual quantity representing the total allotment of tradable permits. Henceforth, it became widely accepted that there is a fundamental isomorphism between a Pigouvian tax on pollution and the total quantity of caps allotted in a cap-and-trade system, with all permits trading at the same competitive-equilibrium market price as the Pigouvian tax. So far all the analysis has taken place in a deterministic context with full certainty.

Weitzman (1974) showed that there is no longer an isomorphism between price and quantity instruments when there is uncertainty in cost functions. With uncertainty, setting a fixed price stabilizes marginal cost while leaving the total quantity of pollution variable, whereas setting a fixed total quantity of tradable permits stabilizes total quantity while leaving marginal cost variable. The question then becomes: which instrument is better under which circumstances? Weitzman (1974) derived a relatively simple formula for the "comparative advantage of prices over quantities," denoted as Δ. The sign of Δ depends on the difference between the slope of abatement's marginal cost c and marginal benefit b, that is, $\text{sign}(\Delta) = \text{sign}(c - b)$. When the marginal benefit curve is flatter than the marginal cost curve ($b < c$), the sign of Δ is positive and prices are favored over quantities. Conversely, if the sign of Δ is negative, then quantities are preferred over prices.

There subsequently developed a sizable literature on the optimal choice of price versus quantity policy instruments under uncertainty.[1] Extensions have considered alternative forms of uncertainty (Adar and Griffin 1976; Fishelson 1976; Roberts and Spence 1976), various aspects of nonlinearities (Weitzman 1978; Yohe 1978; Kelly 2005), correlation between uncertain marginal costs and marginal benefits (Yohe 1978; Stavins 1996), investment behavior (Chao and Wilson 1993; Zhao 2003), and stock externalities (Hoel and Karp 2002; Pizer 2002; Newell and Pizer 2003; Fell et al. 2012; Goulder and Schein 2013; Kotchen 2018, among others). In these extensions, the results generally preserve the earlier insight that, all else held equal, flatter marginal benefits or steeper marginal costs tend to favor prices while steeper marginal benefits or flatter marginal costs tend to favor quantities.[2]

1. Given the number of published papers on "prices versus quantities," we have only included a subset that we subjectively judge to be most relevant to this paper. Similarly, as our analysis focuses on noncooperative equilibrium in a strategic setting, we have not included the literature about international cooperations and agreements.

2. The extensions to dynamic stock externality were phrased in terms of emission flows throughout a regulatory period (followed by a new regulatory period with new decision-relevant parameters). Moreover, note that combinations of instruments, such as a fixed price with a floor

Free riding at a global level lies at the heart of the suboptimal CO_2 problem: countries do not capture the full benefits of their abatement yet bear the full costs. To examine the optimal choice of policy instruments for managing global pollution, we extend the standard "prices versus quantities" framework for pollution control and cover multiple heterogeneous jurisdictions interacting strategically and noncooperatively with each other.[3]

We establish the result that the subgame perfect equilibrium choice of instrument for a jurisdiction is given by the sign of Δ in the context of our model. Country i's criteria for the choice of policy instrument are independent of what instruments the other jurisdictions choose when a jurisdiction's intensity of abatement maximizes its net benefit given the amount of other jurisdictions' abatement. Thus, on the equilibrium path of abatement, each jurisdiction has an optimal strategy: choose the price instrument when the slope of marginal benefit is less than the slope of marginal cost and the quantity instrument when this condition is reversed. The result here extends the standard "prices versus quantities" criterion to a setting with multiple countries and multilateral externalities. Our analysis suggests that the original nonstrategic criterion for the comparative advantage of prices over quantities may have wider applicability for determining instrument choice in a noncooperative strategic environment.

MODEL OF REGULATORY GAME

Suppose that there are n countries indexed $i = 1, 2, \ldots n$ and the set $N \equiv \{1, 2, \ldots, n\}$. Following standard convention, we consider a setting based on abatement q_i; identical results are obtained by focusing on pollution e_i.[4] Country i abates q_i and benefits from the global abatement of Σq_i, while incurring a private abatement cost of $C_i\ (q_i, \theta_i)$.

A country's abatement level is chosen by firms located in the country. From the viewpoint of a regulator in country i, the net benefit of abatement is described by

$$W^i = B_i\left(\sum q_i, \eta_i\right) - C_i(q_i, \theta_i), \tag{1}$$

where η_i is an independent reduced-form shock to benefits for i and θ_i is an independent reduced-form shock to cost for i. To facilitate comparability to the relevant literature, we assume quadratic functional forms:

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and ceiling on quantities, must supersede in expected welfare both a pure price and a pure quantity, because both of these pure instruments are special cases of such combinations of instruments. This insight traces back to Roberts and Spence (1976).

3. Endres and Finus (2002) study international agreements over uniform price and uniform quantity.

4. Since policies often cap pollution instead of fixing abatement, the focus may appear unrealistic in this dimension. Despite the appearance, no insight is lost as $q_i \equiv K - e_i$ for some constant K.

$$B_i\left(\sum q_i, \eta_i\right) \equiv [\beta_i + \eta_i]\left[\sum q_i\right] - \frac{b_i}{2}\left[\sum q_i\right]^2, \text{ and} \tag{2}$$

$$C_i(q_i, \theta_i) \equiv [\gamma_i + \theta_i]q_i + \frac{c_i}{2}q_i^2, \tag{3}$$

with β_i, γ_i, b_i, and c_i being positive parameters.

The timing of the regulatory game is as follows. First, each regulator chooses the type of regulatory policy instrument $\mathbb{P}_i \in \{\bar{p}_i, \bar{q}_i\}$, where abatement is regulated through price $\bar{p}_i$ or quantity $\bar{q}_i$. Second, countries simultaneously choose the intensity of the regulatory instrument chosen, after observing the outcome of $\{\mathbb{P}_i\}_{i=1}^n$ in the first stage. The choices in both stages are made under an informational constraint, that is, without regulators knowing beforehand the realization of the random variables (except that $\mathbb{E}[\theta_i] = \mathbb{E}[\eta_i] = 0$ and variances of $\sigma_{\theta_i}^2$ and $\sigma_{\eta_i}^2$). Third, nature reveals the independent values of θ_i and η_i for firms in each country. In the final stage, the representative firm in country i chooses abatement q_i given the type and intensity of regulatory instrument in place and the realization of the shocks. Finally, payoffs are realized for all countries. The analysis focuses on the subgame perfect equilibrium, henceforth equilibrium.

MAIN RESULT

We solve for the equilibrium backward: in the final stage, firms comply with regulation given the regulation in place and the realization of shocks. Next, regulators simultaneously choose the intensity of policy instruments. In the initial stage, regulators concurrently commit to a type of policy instrument before learning the realized values of shocks. Thus, the equilibrium involves both the qualitative type of policy instrument (i.e., price or quantity) and the quantitative intensity of the policy instrument (i.e., how much price or how much quantity).[5]

Given a regulatory constraint of either a quantitative quota $\bar{q}_i$ or a price per unit of abatement $\bar{p}_i$, a representative firm's optimal reaction function is

$$q_i(\bar{q}_i) \equiv \arg\min_{q_i \geq \bar{q}_i} C_i(q_i, \theta_i) \tag{4}$$

when abatement is regulated through quantity or

$$q_i(\bar{p}_i) \equiv \arg\max_{q_i}\{\bar{p}_i q_i - C_i(q_i, \theta_i)\} \tag{5}$$

when abatement is regulated through price. A firm's compliance balances the marginal benefit $\bar{p}_i$ to the marginal cost, $\gamma_i + \theta_i + c_i q_i$. Since firms observe the realization of

5. In the online appendix, we focus on the special case of two identical countries and present the economics behind the equilibrium outcome in a simpler setting.

θ_i prior to compliance, $\bar{p}_i$ incentivizes firms to incorporate the extra information about the shocks.

The equilibrium type of policy instrument for heterogeneous linear marginal cost and marginal benefit functions and independently distributed marginal cost and marginal benefit shocks is unique and given by the following proposition.

> **Proposition**: There exists a unique equilibrium for the regulatory game. In the first stage, a country i commits to the price instrument $\bar{p}_i$ if $c_i \geq b_i$ and commits to the quantity instrument $\bar{q}_i$ if $c_i \leq b_i$.

The proof of this proposition is long, and it is presented in the appendix. The proposition states that, in equilibrium, the quantitative formula for the comparative advantage of prices over quantities Δ^i, only the variance of θ_i changes, but the qualitative formula for the sign of Δ^i remains $\text{sign}(\Delta^i) = \text{sign}(c_i - b_i)$. The sign not only is independent of the policy instrument that other countries choose but also the same as the original "prices versus quantities" formula given in Weitzman (1974), so long as shocks are independent.

The intuition is easier to understand if one starts with one jurisdiction. For a single jurisdiction, the optimal rule is the one that balances the trade-off between the cost of abatement risk aversion and regulatory flexibility to take advantage of cost-saving technological shocks. The quantity instrument does not involve abatement risk yet fails to take advantage of socially useful cost savings. The price instrument is flexible and takes advantage of cost-saving shocks, yet it induces abatement volatility and higher cost of risk aversion. In a multi-country case, the intuition needs to be adjusted for price-induced externalities that spill over to all countries through the global abatement. When other countries provide positive abatement using the quantity instrument and the benefit function is quadratic, a country's marginal benefit curve shifts without changing the slope. However, when some countries provide positive abatement using the price instrument, the marginal benefit curve for all countries is subjected to random shifts, but without changing the slope. Despite the choice of the price instrument by other countries inducing abatement volatility in addition to the direct multi-country abatement externality, the solution to the regulatory flexibility versus abatement risk trade-off remains intact as the slopes do not change.

The model makes a number of simplifying assumptions. The critical assumptions are that uncertainty is independently distributed among countries and the marginal benefit and marginal cost functions are linear. The assumption of independently distributed uncertainty is crucial for the simplicity of our results. Without this assumption, results become combinatorial and are considerably messier. The assumption of linearity is made for tractability. These points taken, the result with heterogeneous countries demonstrates that the relative qualitative advantage of price over quantity for a given country is independent of the type of policy instrument other countries choose; and it is the same criterion as the original "prices versus quantities" formula. As can be seen from the proof, asymmetric policy environments in which some countries choose quantity and others choose price are thus captured by the general model. In an online appendix, we further assume completely symmetric "identical twin" jurisdictions with the same linear marginal cost and marginal benefit functions, differing only by independently and identically distributed (iid) marginal cost shocks and iid marginal benefit shocks. This simplification allows us to present the entire equilibrium outcome for two independent "identical twin" jurisdictions, which each set a price or quantity to maximize their own expected welfare conditional on the instrument choice and value chosen by the other jurisdiction, in the most accessible setting. Despite the simplifications, our result suggests that the original nonstrategic criterion for the comparative advantage of prices over quantities may still have wider applicability for determining instrument choice in a noncooperative strategic environment.

APPENDIX

Proof

Suppose that a country in the set N uses either a price or a quantity instrument, and not both. Without loss of generality, pick a country $i \in N$ and group the remaining $n - 1$ countries into disjoint sets M and R. Let the set M contain the m countries that use the price instrument, and the set R contain the remaining countries that use the quantity instrument, where $R \equiv N - M - \{i\}$. Note that M or R can be empty.

Country i's utility function is

$$u_i = [\beta_i + \eta_i][q_i + Q_{-i}] - \frac{b_i}{2}[q_i + Q_{-i}]^2 - [\gamma_i + \theta_i]q_i - \frac{c_i}{2}q_i^2,$$

where $\hat{Q}_m \equiv \Sigma_{j\in M} q_j(\bar{p}_j)$, $\bar{Q} \equiv \Sigma_{j\in R} q_j(\bar{q}_j)$, $Q_{-i} \equiv \hat{Q}_m + \bar{Q}$, and a global abatement is $q_i + Q_{-i}$.

Step 1: The loss formula. Country i's ex post welfare, for a given Q_{-i} and realizations of θ_i and η_i, is maximized when $q_i^* = \arg\max_{q_i} u_i$, that is,

$$q_i^* = \frac{\beta_i - \gamma_i}{b_i + c_i} - \frac{b_i}{b_i + c_i} Q_{-i} + \frac{\eta_i - \theta_i}{b_i + c_i}. \tag{A1}$$

For country i, any abatement q other than q_i^* for a given Q_{-i} results in an ex post welfare loss of

$$L^i(q, Q_{-i}) = \frac{b_i + c_i}{2}\left[q - q_i^*\right]^2. \tag{A2}$$

Step 2: Optimality in the abatement stage. Countries in the set R have total abatement of $\bar{Q}$. To find the total abatement of countries in M, note that for a $j \in M$ and a price $\bar{p}_j$, firms' optimal abatement solves $\max_{q_j}\{\bar{p}_j q_j - [[\gamma_j + \theta_j]q_j + (c_j/2)q_j^2]\}$:

$$q_j(\bar{p}_j) = \pi_j - \frac{\theta_j}{c_j}, \text{ where } \pi_j \equiv [\bar{p}_j - \gamma_j]/c_j \text{ and } \pi_M \equiv \sum_{j \in M} \pi_j. \tag{A3}$$

The total abatement of countries in the set M is

$$\hat{Q}_m = \pi_M - \sum_{j \in M} [\theta_j / c_j]. \tag{A4}$$

Step 3: Country i's loss from commitment to a price instrument.
Claim: The expected welfare loss from an optimally chosen price $\bar{p}_i$ is

$$\mathbb{E}L^i(q_i(\bar{p}_i), Q_{-i}) = \left[\sigma^2_{\eta_i} + \sum_{j \in M} [b_i/c_j]^2 \sigma^2_{\theta_j} + [b_i/c_i]^2 \sigma^2_{\theta_i}\right] / [2[b_i + c_i]]. \tag{A5}$$

Proof: With a price instrument $\bar{p}_i$, abatement becomes $q_i(\bar{p}_i) = \pi_i - (\theta_i/c_i)$. Substituting $q_i(\bar{p}_i)$ and (A4) into (A2), the ex post welfare loss from using $\bar{p}_i$ is

$$L^i(q_i(\bar{p}_i), Q_{-i}) = \frac{\left[[b_i + c_i]\pi_i - [\beta_i - \gamma_i] + b_i \pi_M + b_i \bar{Q} - \sum_{j \in M} [b_i/c_j]\theta_j - \eta_i - \frac{b_i}{c_i}\theta_i\right]^2}{2[b_i + c_i]}. \tag{A6}$$

Thus, the expected welfare loss from optimally chosen price becomes

$$\mathbb{E}L^i(q_i(\bar{p}_i), Q_{-i}) = \frac{[[b_i + c_i]\pi_i - [\beta_i - \gamma_i] + b_i \pi_M + b_i \bar{Q}]^2}{2[b_i + c_i]} + \frac{\sigma^2_{\eta_i} + \sum_{j \in M} \sigma^2_{\theta_j} [b_i/c_j]^2 + [b_i/c_i]^2 \sigma^2_{\theta_i}}{2[b_i + c_i]}. \tag{A7}$$

Minimizing $\mathbb{E}L^i(q_i(\bar{p}_i), Q_{-i})$ in (A7) with respect to π_i implies

$$\pi_i = \frac{\beta_i - \gamma_i}{b_i + c_i} - \frac{b_i}{b_i + c_i}\pi_M - \frac{b_i}{b_i + c_i}\overline{Q}.$$

(Note that if one solves for the optimal $\bar{p}_i$ instead, one would arrive at the same result, with more steps.) Substituting the value of π_i into (A7), one arrives at (A5). QED

Step 4: Country i's loss from commitment to a quantity instrument.
Claim: The expected welfare loss from an optimally chosen quota $\bar{q}_i$ is

$$\mathbb{E}L^i(q_i(\bar{q}_i), Q_{-i}) = \left[\sigma^2_{\eta_i} + \sum_{j \in M} [b_i/c_j]^2 \sigma^2_{\theta_j} + \sigma^2_{\theta_i}\right] / [2[b_i + c_i]]. \tag{A8}$$

Proof: Substituting (A1), (A4), and $\bar{q}_i$ into (A2), implies

$$L^i(q_i(\bar{q}_i), Q_{-i}) = \frac{\left[[b_i + c_i]\bar{q}_i - [\beta_i - \gamma_i - b_i[\bar{Q} + \pi_M]] - b_i \sum_{j\in M}\left[\theta_j/c_j\right] - \eta_i + \theta_i\right]^2}{2[b_i + c_i]}. \tag{A9}$$

The expected value of (A9) is

$$\mathbb{E}L^i(q_i(\bar{q}_i), Q_{-i}) = \frac{[[b_i + c_i]\bar{q}_i - [\beta_i - \gamma_i - b_i[\bar{Q} + \pi_M]]]^2}{2[b_i + c_i]} + \frac{\sigma^2_{\eta_i} + b_i^2 \sum_{j\in M}\left[1/c_j\right]^2 \sigma^2_{\theta_j} + \sigma^2_{\theta_i}}{2[b_i + c_i]}. \tag{A10}$$

The optimal value of $\bar{q}_i$ that minimizes the $\mathbb{E}L^i(q_i(\bar{q}_i)_i, Q_{-i})$ in (A10) is

$$\bar{q}_i = \frac{\beta_i - \gamma_i}{b_i + c_i} - \frac{b_i}{b_i + c_i}[\bar{Q} + \pi_M].$$

Substituting the value of this expression into (A10), one arrives at (A8). QED

Step 5: Country *i*'s optimal choice of policy instrument. Let the difference in expected welfare from price regulation versus quantity regulation be

$$\Delta^i \equiv \mathbb{E}L^i(q_i(\bar{q}_i), Q_{-i}) - \mathbb{E}L^i(q_i(\bar{p}_i), Q_{-i}) = \frac{c_i - b_i}{2c_i^2}\sigma^2_{\theta_i}. \tag{A11}$$

Thus, for any value of m such that $0 \le m \le n - 1$, country *i*'s choice over price versus quantity is dictated by the difference between c_i and b_i. QED

REFERENCES

Adar, Zvi, and James M. Griffin. 1976. Uncertainty and the choice of pollution control instruments. *Journal of Environmental Economics and Management* 3 (3): 178–88.

Chao, Hung-Po, and Robert Wilson. 1993. Option value of emission allowances. *Journal of Regulatory Economics* 5 (3): 233–49.

Dales, John H. 1968. *Pollution, property, and prices: An essay on policy-making and economics*. Toronto: University of Toronto Press.

Endres, Alfred, and Michael Finus. 2002. Quotas may beat taxes in a global emission game. *International Tax and Public Finance* 9 (6): 687–707.

Fell, Harrison, Ian A. MacKenzie, and William A. Pizer. 2012. Prices versus quantities versus bankable quantities. *Resource and Energy Economics* 34 (4): 607–23.

Fishelson, Gideon. 1976. Emission control policies under uncertainty. *Journal of Environmental Economics and Management* 3 (3): 189–97.

Goulder, Lawrence H., and Andrew R. Schein. 2013. Carbon taxes vs. cap and trade: A critical review. *Climate Change Economics* 4 (3): 1–28.

Hoel, Michael, and Larry Karp. 2002. Taxes versus quotas for a stock pollutant. *Resource and Energy Economics* 24 (4): 367–84.

Kelly, David L. 2005. Price and quantity regulation in general equilibrium. *Journal of Economic Theory* 125 (1): 36–60.

Kotchen, Matthew J. 2018. Which social cost of carbon? A theoretical perspective. *Journal of the Association of Environmental and Resource Economists* 5 (3): 673–94.

Montgomery, W. David. 1972. Markets in licenses and efficient pollution control programs. *Journal of Economic Theory* 5 (3): 395–418.

Newell, Richard, and William A. Pizer. 2003. Regulating stock externalities under uncertainty. *Journal of Environmental Economics and Management* 45 (2): 416–32.

Pigou, Arthur C. 1920. *The economics of welfare*. London: Macmillan.

Pizer, William A. 2002. Combining price and quantity controls to mitigate global climate change. *Journal of Public Economics* 85 (3): 409–34.

Roberts, Mark J., and Michael Spence. 1976. Effluent charges and licenses under uncertainty. *Journal of Public Economics* 5 (3–4): 193–208.

Stavins, Robert N. 1996. Correlated uncertainty and policy instrument choice. *Journal of Environmental Economics and Management* 30 (2): 218–32.

Weitzman, Martin L. 1974. Prices *vs*. quantities. *Review of Economic Studies* 41 (4): 477–91.

———. 1978. Optimal rewards for economic regulation. *American Economic Review* 68 (2): 683–91.

Yohe, Gary W. 1978. Towards a general comparison of price controls and quantity controls under uncertainty. *Review of Economic Studies* 45 (2): 229–38.

Zhao, Jinhua. 2003. Irreversible abatement investment under cost uncertainties: Tradable emission permits and emissions charges. *Journal of Public Economics* 87 (12): 2765–89.