On the Welfare Significance of Green Accounting as Taught by Parable

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"Green NNP" is a national accounting concept that subtracts off from GNP not just depreciation of capital, but also depletion of environmental assets. For a time-autonomous technology, Green NNP has a rigorous welfare interpretation as an exact measure of the economy's future power to consume. The present paper extends the standard framework to cover the case of labor-augmenting technological progress. For this special case of time dependence, a powerful conceptual parable applies. It is "as if" the conclusion from a simple-minded welfare story told by the standard neoclassical one-sector growth model covers a far more general situation involving any number of different types of capital, any convex production possibilities set, and fully optimizing behavior. Implications and applications are discussed. © 1997 Academic Press

1. INTRODUCTION

"Green NNP" is interpreted here to stand conceptually for the most inclusive possible measure of net national product, including net investments not just in traditional "produced means of production" like equipment and structures, but also in human capital, pools of natural resources, and environmental assets more generally—all evaluated at their respective efficiency prices.

Now it turns out that under certain assumptions a rather strong welfare interpretation can be given to Green NNP. The relevant assumptions include a perfectly competitive dynamic economy with constant rate of return on the single "ideal" consumption good and a stationary technology that does not depend explicitly on calendar time. In such a situation, Green NNP is proportional to the present discounted value of consumption that the economy is able to produce, with the constant of proportionality being exactly the rate of return on consumption. Thus, the *current* value of Green NNP exactly forecasts the "annuity-equivalent" of *future* consumption possibilities at the prevailing consumption interest rate.

While this result can serve as a powerful conceptual guide for indicating how to think about the welfare relationship between future consumption possibilities and current national income accounting, its practical applicability is somewhat limited by the assumptions of the model.

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The most restrictive assumption, by far, is the absence of technological progress. The result that a theoretically correct measure of welfare just exactly equals a theoretically correct measure of Green NNP relies completely on the time-autonomy of the system. But to the extent that whatever endogenous and exogenous factors thought to underlie technical change have been ignored, the situation is "as if" there exists a time-dependent residual shift factor that increases productivity but does not show up anywhere in national income accounts.

The consequences of technical change being absent from the standard time-autonomous model might be quite serious for the basic welfare interpretation of Green NNP. We know that future growth is largely driven by the rate of technological progress, however it is conceptualized. Since Green NNP theoretically equals annuity-equivalent future consumption possibilities *without* the residual, the proper measure of annuity-equivalent future consumption possibilities *with* the residual might conceivably call for a sizable upward adjustment of Green NNP.

This paper extends the existing standard framework to include labor-augmenting technological progress. The treatment of time dependency per se is not original to this paper, since there already exists a sizable literature on the subject. Weitzman [22] sketched the mathematical outlines of a corrective expression. Important formal contributions were made by Kemp and Long [8], Löfgren [9], Aronsson and Löfgren [1], Asheim [4], Hartwick [6], Nordhaus [13], Weitzman [23], and others. The paper closest to this one is Löfgren and Weitzman [10], which served as a kind of "spiritual parent" by deriving a similar formula for the familiar one-sector single-capital-good case with a linear production possibilities frontier. From the existing literature it emerges that there are several ways to express the effects of time dependency, each one having a somewhat different interpretation.

The main contribution of the present paper is to show that when technological progress is labor-augmenting at a constant exponential rate, then a conceptually insightful parable can be applied to interpret the welfare effects. The conclusion from telling a simple-minded non-optimizing welfare story based on the standard neoclassical one-sector growth model generalizes. Results are "as if" the parable extends to a far more complicated scenario involving any number of different kinds of capital-like goods, any convex constant-returns-to-scale production possibilities set, and fully optimizing behavior.

The parable yields a simple formula linking welfare to "Green NNP" and to a new concept called "Green Capital." Implications of the correction required to convert Green NNP and Green Capital into an appropriate welfare measure are discussed. A rough calculation based on reasonable values of the relevant parameters suggests that the required correction may be sizable—perhaps around 40% or more of conventionally measured net national product. This is commensurate with the estimates made previously by Weitzman [23]. A possible implication could be that long-term sustainability, like so much else about the future, is driven largely by projections of technological progress.

2. THE BASIC MODEL

To make the problem analytically tractable, we make the usual abstractions.

First of all, it is assumed that, in effect, there is just one composite consumption good. It might be calculated as an index number with given price weights, as a multiple of some fixed basket of goods, or, most generally, as a cardinal-utility-like

aggregator function possessing certain standard homogeneity properties. The important thing is that the consumption level in period t can be unambiguously registered by the single number $C(t)$. Thus, the paper assumes away all of the problems that might be associated with constructing an "ideal measure" of consumption.[1] Purging consumption of the index number problem will allow us to focus more sharply on the general meaning and significance of combining it with net investment when there is labor-augmenting technological progress.

The notion of "capital" used here is meant to be quite a bit more general than the traditional "produced means of production" like equipment and structures. Most immediately, pools of natural resources are considered to be capital. Human capital should also be included, to the extent we know how to measure it. Under a very broad interpretation, environmental assets generally might be treated as a form of capital.[2]

Suppose that altogether there are n capital goods, including stocks of natural resources. The stock of capital of type i ($1 \leq i \leq n$) in existence at time t is denoted $K_i(t)$, and its corresponding net investment flow is

$$I_i(t) \equiv \dot{K}_i(t) \equiv \frac{dK_i(t)}{dt}. \tag{1}$$

The n-vector $\mathbf{K} = \{K_i\}$ denotes all capital stocks, while $\mathbf{I} = \{I_i\}$ stands for the corresponding n-vector of net investments. Note that the net investment flow of a non-renewable natural capital like proved oil reserves could well be negative if the overall extraction rate exceeds the discovery and development of new fields.

Although a somewhat more general formulation is possible, we treat here the case of a single fixed factor, denoted L, and, for ease of exposition, called "labor." The $(n+1)$-dimensional production-possibilities set with capital stock $\mathbf{K}$ and labor L is denoted here $S(\mathbf{K}, L)$. Thus, the consumption–investment pair $(C, \mathbf{I})$ is producible if and only if

$$(C, \mathbf{I}) \in S(\mathbf{K}, L). \tag{2}$$

The production-possibilities frontier of S could be curved, as depicted in introductory economics textbooks, or linear, as in the standard neoclassical growth model, or some combination. The only restriction we impose is the following.

Assumption 1.   The production possibilities set $S(\mathbf{K}, L)$ exhibits convexity and constant returns to scale.

"Effective labor" is postulated to grow exponentially.

Assumption 2.   The fixed factor "labor" at time t can be written in the form

$$L(t) = L(0)e^{\nu t}. \tag{3}$$

Henceforth we will call the parameter ν the "growth rate of labor-augmenting technological change."

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[1] Nordhaus [13], in his section entitled "What is Consumption?", contains a relevant discussion of the basic issues involved.

[2] Mäler [11] includes a discussion of some of the relevant issues here.

Suppose the relevant discount rate for weighting consumption across time is r. A third key assumption of the "Basic Model" is the following:

Assumption 3.   The own-rate-of-return on consumption, r, is constant.

Now consider the optimal control problem of maximizing the expression

$$\int_0^\infty C(t)e^{-rt}\,dt \tag{4}$$

subject to the constraints

$$(C(t), I(t)) \in S(K(t), L(0)e^{\nu t}) \tag{5}$$

and

$$\dot{K}(t) = I(t) \tag{6}$$

and obeying the initial conditions

$$K(0) = K_0, \tag{7}$$

where $\mathbf{K}_0$ is the original endowment of capital available at starting time $t = 0$.

Note that, because the objective function (4) is linear in the consumption stream $\{C(t)\}$ with weights $\{e^{-rt}\}$, any solution of the above optimization problem will display a constant one-period rate of return on saving/consumption equal to r.

Let P_i represent the price of investment good of type i relative to a consumption-good numeraire price of one. Then $\mathbf{P}$ denotes the relevant n-vector of investment-good prices. A *Green-Net-National-Product Function* (expressed in real terms with consumption as numeraire) is defined as

$$G(K, P, L) \equiv \underset{(C, I) \in S(K, L)}{\text{maximum}} [C + P \cdot I]. \tag{8}$$

Expression (8) might legitimately be considered an "inclusive" or "Green" NNP function because the value of depleted natural resources, as well as capital depreciation, has been subtracted from GNP. While this paper could get by with weaker assumptions, for convenience it will be assumed that the Green NNP function $G(\cdot)$ is smooth in all of its arguments.

A *feasible* trajectory $\{C(t), \mathbf{K}(t)\}$ is any solution of (5), (6), (7).

With the assumptions that have been made thus far, a necessary and sufficient condition for a feasible trajectory $\{C^*(t), \mathbf{K}^*(t)\}$ to be optimal is that there exists an n-vector of investment prices $\{\mathbf{P}^*(t)\}$ such that, evaluated at any time $t \geq 0$ along the trajectory,

$$G(K^*(t), P^*(t), L(t)) = C^*(t) + P^*(t) \cdot \dot{K}^*(t) \tag{9}$$

and, for each i

$$\left.\frac{\partial G}{\partial K_i}\right|^* = rP_i^*(t) - \frac{dP_i^*}{dt} \tag{10}$$

and

$$\lim_{t \to \infty} P^*(t)K^*(t)e^{-rt} = 0. \tag{11}$$

Equations (9), (10), and (11) are precisely the competitive equilibrium conditions of a dynamic economy exhibiting a real interest rate of r on the numeraire consumption good.

Equation (9) just states that what is actually produced by the economy at any time maximizes its income—in other words, relative prices are equal to marginal rates of transformation. Equations (10) along with (11) are the well-known perfect foresight conditions of a competitive capital market.[3]

Let $Y^*(t)$ denote inclusive or Green NNP at time t as it would be measured by an ideal national income statistician in this model economy. Then

$$Y^*(t) = C^*(t) + P^*(t) \cdot \dot{K}^*(t). \tag{12}$$

Let the welfare value of an optimal policy be

$$W_\nu^* \equiv \int_0^\infty C^*(t) e^{-rt}\, dt. \tag{13}$$

The chief aim of this paper is to explain the relation between W_ν^* and "Green Accounting" in terms of a simple, easy-to-understand neoclassical growth parable that holds for all $\nu \geq 0$. The next section of the paper covers the already-known case $\nu = 0$. Then, the section after that deals with the not previously treated case $\nu > 0$.

3. A JELLY PARABLE FOR THE CASE $\nu = 0$

The primary novelty of this paper concerns the welfare significance of green accounting when there is labor-augmenting technological change—as taught by a specific parable. But in order to intuit better the logic of the parable for the case $\nu > 0$, we first indicate how it works for the case $\nu = 0$. There is nothing new of substance here, just a recasting of already-familiar results. The usefulness of retelling the known special case $\nu = 0$ in a somewhat different style consists of laying bare the structure of the basic analogy, the better to see development of the previously untreated case $\nu > 0$ as a natural extension of the same kind of underlying logic.

We start with the simplest possible model that can illustrate the basic principle. This artificial construct will be called the "jelly model."

A single homogeneous output is produced by a constant-returns-to-scale smooth neoclassical aggregate production function $F(J, L)$, where J stands for jelly capital and L stands for labor. At any time t, jelly output Y can be perfectly divided into consumption C and net investment I by the linear-trade-off formula:

$$C(t) + P \cdot I(t) = Y(t) \equiv F(J(t), L(t)). \tag{14}$$

In the above formula (14), P stands for the (exogenously given) price of the jelly-investment good relative to the consumption good as numeraire.

Because this first case deals with a time-independent technology, in this section we set $\nu = 0$ and treat L as fixed at the value $L(0)$.

[3] See, for example, Weitzman [22] footnote 5 for an explanation of the intertemporal efficiency condition (10), and, for example, Weitzman [21] for an explanation of the transversality condition (11).

The differential equation for jelly-capital formation is

$$\dot{J}(t) = I(t). \tag{15}$$

Now the basic question to be asked is the following: What is the simplest possible story that can be told to illustrate the power of such an economy to deliver future consumption?

This jelly-model economy is capable of delivering the constant consumption level

$$C_0(t) \equiv F(J(0), L(0)) \tag{16}$$

indefinitely, merely by setting net investment at

$$I(t)\big(= \dot{J}(t)\big) \equiv 0. \tag{17}$$

The next logical question is the following: What is the jelly-welfare value of the simple constant consumption stream $\{C_0(t)\}$? The answer depends on the discount rate r that is applied. For any given r, the consumption stream $\{C_0(t)\}$ yields welfare

$$W_0^j \equiv \int_0^\infty C_0(t)e^{-rt}\, dt. \tag{18}$$

For the simple constant consumption stream (16), formula (18) can be rewritten as

$$W_0^j \equiv \frac{Y}{r}, \tag{19}$$

where

$$Y \equiv F(J(0), L(0)) \tag{20}$$

is interpretable here as "jelly net national product" at initial time $t = 0$ for this parable model.

The essential connection linking the "Jelly Parable" with the "Basic Model" for the case $\nu = 0$ is obtained by interpreting Y from the jelly parable as standing, in this new context, for Green NNP $Y^*(0)$ from the basic model.

Result.[4]   In formula (19), which defines W_0^j, set

$$Y \equiv Y^*(0), \tag{21}$$

where $Y^*(0)$ is defined by (12). Then, for all r,

$$W_0^* = W_0^j. \tag{22}$$

4. A JELLY PARABLE FOR THE CASE $\nu > 0$

This section breaks some new ground by covering the previously untreated case $\nu > 0$. For guidance we try to parallel as closely as possible the logic of the parable for the previously treated case $\nu = 0$.

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[4] This "result" is essentially a translation of the main theorem from Weitzman [22] into the terminology and notation of the present paper.

Here we work with the same basic one-sector neoclassical jelly model:

$$C(t) + P \cdot I(t) = F(J(t), L(t)) \tag{23}$$

and

$$\dot{J}(t) = I(t). \tag{24}$$

The only difference is that here

$$L(t) = L(0)e^{\nu t}, \tag{25}$$

where $\nu > 0$.

We now ask the same basic question for the case $\nu > 0$ that we asked for the case $\nu = 0$: What is the simplest possible story that can be told to illustrate the power of such an economy to deliver future consumption?

Because $\nu > 0$, this economy has the potential to grow. What, then, is the simplest growth story that can be told in such a situation?

In this case, the jelly economy is able to deliver steady-state exponential growth at constant rate $\nu > 0$ forever, provided merely that the set-aside of net jelly investment is equal to

$$I(t)\big(= \dot{J}(t)\big) \equiv \nu \cdot J(t). \tag{26}$$

The corresponding value of consumption at time t is then given by

$$C_\nu(t) = \big[F(J(0), L(0)) - \nu PJ(0)\big]e^{\nu t}. \tag{27}$$

What is the welfare value of the simple exponentially growing consumption stream $\{C_\nu(t)\}$? The answer depends on the discount rate r that is applied. For any given r, the consumption stream $\{C_\nu(t)\}$ yields welfare

$$W_\nu^j \equiv \int_0^\infty C_\nu(t)e^{-rt}\, dt. \tag{28}$$

For the simple exponentially growing consumption stream (27), formula (28) can be rewritten as

$$W_\nu^j \equiv \frac{Y - \nu PJ}{r - \nu}, \tag{29}$$

where, in this case,

$$Y \equiv F(J(0), L(0)) \tag{30}$$

is interpretable here as "jelly NNP" at initial time $t = 0$ for the parable model, and

$$PJ \equiv PJ(0) \tag{31}$$

is interpretable here as the initial value of "jelly capital."

The essential result of this paper links the Jelly Parable with the Basic Model as follows:

THEOREM.   *In formula* (29), *which defines W_ν^j in the jelly parable, let Y now stand for $Y^*(0)$, which is current Green NNP from the basic model,*

$$Y \equiv Y^*(0) \tag{32}$$

while PJ now stands for the current value of the green capital stock

$$PJ \equiv P^*(0) \cdot K^*(0) \tag{33}$$

from the basic model. Then, for all r and for all ν such that $r > \nu$,

$$W_\nu^* = W_\nu^j. \tag{34}$$

Thus, although the jelly model might be quite flawed as a literal description of the world, its seemingly oversimplistic message about how to conceptualize the connection between current net production and future consumption possibilities can be rigorously defended.

The proof of the theorem is relegated to Appendix 1.

5. SUSTAINABILITY AND THE GREEN CAPITAL–OUTPUT RATIO

To make sense of the concept of sustainability, it must first be defined rigorously. The concept is amenable to several related interpretations. At the highest level of abstraction, the fundamental motivating idea is that sustainability should be some "good" aggregate measure of an economy's future prospects for consumption.

Here we choose what seems to us, overall, to be the best single measure of an economy's capacity to consume over time. In this paper *sustainability* is defined to be the hypothetical annuity-equivalent constant level of consumption that yields the same welfare as the economy actually has the potential to deliver—when evaluated at the intertemporal consumption tradeoff implicit in the economy's own competitive equilibrium rate of return on savings:

$$C_\nu^* \equiv rW_\nu^*. \tag{35}$$

An equivalent way of writing (35) is

$$C_\nu^* \equiv \frac{\int_0^\infty C^*(t)e^{-rt}\,dt}{\int_0^\infty e^{-rt}\,dt}. \tag{36}$$

As formula (36) indicates, this paper's definition of *sustainability*, C_ν^*, is the time-weighted average of future consumption possibilities, where the weight is just the economy's own rate of return on consumption.[5]

Note from (29) and (34) that the special case $\nu = 0$ reduces expression (35) to the well-known equality

$$C_0^* = Y^*(0). \tag{37}$$

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[5] To emphasize that we are using a specific index, our particular definition (36) is italicized in the text as *sustainability*. Note that expression (36) is not the highest actually attainable constant level of consumption, a Rawlsian max−min criterion that we, along with many other economists, find too rigid to be taken seriously as a useful index of sustainability. For more on this point, see, e.g., Solow [15].

In the situation where $\nu > 0$, however, Green NNP is not equal to the appropriately corresponding annuity-equivalent consumption level. What is the correction factor that should be applied to Green NNP to make it commensurate with this paper's measure of sustainability?

Define θ to be the "technological change premium" needed to convert $Y^*(0)$ into C_ν^*. By definition, the adjustment factor θ satisfies the condition

$$C_\nu^* = Y^*(0)[1 + \theta]. \tag{38}$$

Let the green capital–output ratio be defined in this model to be

$$\kappa \equiv \frac{P^*(0) \cdot K^*(0)}{Y^*(0)}. \tag{39}$$

The parameter κ stands conceptually for the ratio of the value of the most inclusive possible measure of capital to the value of the most inclusive possible measure of NNP.

Using formulas (29), (34), (35), and (39), expression (38) can be rewritten as

$$\theta = \frac{\nu(1 - r\kappa)}{r - \nu}. \tag{40}$$

Now let us try to make a ballpark estimate of θ, based on very rough data for the U.S. economy. The most unconventional parameter in Eq. (40) is κ, the green capital-output ratio. Green capital is taken here to be the sum of made assets plus natural assets plus human capital. Green NNP is taken here to be traditional net national product minus depletion of natural assets minus the cost of a clean environment.

As a very rough approximation, we estimate[6] $\kappa \sim 8$.

The "own rate of return on consumption" is a conceptual measure of how much *extra* consumption could be enjoyed next year from giving up a unit of consumption this year, other things being equal. The economic entity corresponding most closely to this concept is, arguably, the annual after-tax real return on capital, because it approximately defines the relevant intertemporal consumption tradeoff faced by the average citizen in deciding how much to save.

Using a figure of 5% for this interest rate,[7] and assuming population growth of 1% per year,[8] the own rate of return on *per capita* consumption is then estimated to be $r \sim 4\%$.

If the rate of growth of total factor productivity is about 1% per annum,[9] and the share of labor is taken to be the usual $2/3$, then the implied corresponding rate of labor-augmenting technological change[10] is $\nu \sim 1.5\%$.

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[6] See Appendix 2 for details.

[7] This round number of 5% could be justified by reference to Nordhaus [13] or Jorgenson [7].

[8] This is a decent approximation for annual U.S. population growth in the post-war period.

[9] This growth rate is consistent with BLS [17] and would change only inconsequentially whether output was measured by traditional NNP or Green NNP. See Appendix 2 for details of approximating Green NNP for the U.S. economy.

[10] The parameter ν is understood here to stand for the *net* growth rate of "as-if" labor-augmenting technological progress, after subtracting out environmental drag from possibly disproportionate growth of negative externalities. As indicated in footnote 9, at the current time in history this distinction makes no appreciable difference to the numerical "big-picture" exercise being conducted in the paper.

With such numbers, the correction factor indicated by formula (40) is $\theta \sim 41\%$.

It is interesting to note that the above point estimate of the technological change premium θ is very close to the point estimate made previously in Weitzman [23] using a somewhat different methodology.

6. CONCLUSIONS

There are two principal conclusions to be drawn from this paper—the first theoretical and the second empirical.

The theoretical conclusion is that there is a relatively simple way to envision the seemingly complicated impact of labor-augmenting technological progress on the welfare significance of green accounting.

The main theoretical result (34) is a kind of "as-if" dynamic aggregation theorem. The outcome from a fairly general basic growth model—involving labor-augmenting technological change, multiple types of capital, any production possibilities set that has a representation as a closed convex cone, and fully optimizing behavior—looks "as if" it were the outcome of a simple jelly parable. Note that the formal analogy goes through even when some of the capital goods may represent natural resources that are ultimately depleted over time and although the basic model need not at all be approaching steady-state growth at rate ν in the limit.

The jelly parable yields an exact expression (40) that indicates the appropriate upward correction required to convert Green NNP into the flow-like measure of sustainability that gives the right welfare-compatible weighted average of future consumption.

The empirical conclusion to be drawn from this paper repeats and, because it is based on a somewhat different approach, hopefully reinforces the earlier implications of Weitzman [23].

If we think generally of $[1 + \theta]$ as a parameter quantifying the ratio of sustainability to Green NNP, then how might this parameter best be estimated? However imperfect it might be, as a practical matter we have a better intuitive feeling for projecting future rates of technological progress than for forecasting the relevant future parameter values or functional forms from any existing model of endogenous growth theory. If we go the route of this paper, then we have a methodology for estimating $[1 + \theta]$ and can at least hope that it could be a decent approximation for what might also be derived from the "right" form of a more fully specified model where innovation and externalities are endogenously determined.[11]

Suppose that defensive environmental spending in an advanced industrial economy such as that in the United States can serve as a very rough measure of the welfare loss of the negative environmental externalities it is intended, in part, to offset.[12] If this is even approximately correct, then it is hard to argue that making

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[11] For an example of a model in the spirit of endogenous growth theory being used to address issues of social accounting and welfare measurement, see Aronsson and Löfgren [3].

[12] We realize that several important issues are being glossed over here, but it is only the approximate magnitude of the number that matters in the present context. One particular warning is is order, however. The current framework ignores the environmental doomsday scenario wherein pollution-like externalities are approaching a threshold level of potentially catastrophic damage, which is not signaled by any market-like indicator. In effect we are assuming for the sake of argument that defensive environmental spending more or less "restores" the environment and therefore keeps the economy at a reasonably safe distance away from any such hypothesized "environmental-reservoir" threshold.

all the proper adjustments for depleted natural resources and deteriorated environ-mental assets might bring conventionally measured NNP down by more than about a couple of percentage points when it is converted over to Green NNP.[13] What about the upward adjustment of NNP indicated by the "technological change premium"? Within the framework of this paper, formula (40) gives the theoretically appropriate relationship between (future) sustainability and (present) inclusive NNP.

Here, it seems from the calculations at the end of the previous section that the correction factor is considerably larger, perhaps an order of magnitude greater.

No one should feel fully at ease projecting the kind of crude numbers that lie behind the raw calculation of $\theta \sim 41\%$ onto the future, and, of course, θ will change with different assumed values of the underlying parameters.[14] Caution is therefore warranted when interpreting this kind of exercise at making a ballpark estimate of a "sustainability index," however such a measure may be defined. Yet, a reasonable parametric analysis done on (40)—based, admittedly, on present data reflecting present historical conditions—would appear to make the following conclusion difficult to contest:

Because it omits the role of technological progress, NNP, whether conventionally measured or green-inclusive, seems to understate an economy's sustainability, which, at least as of now, probably depends more critically on future projections of technical change than on the typical corrections undertaken in the name of green accounting.

The ultimate origins of the residual shift factors that economists tend to lump together under the high-sounding label of "labor-augmenting technological progress" are still poorly understood. But if future growth rates of this residual resemble those of the past, we are probably underestimating significantly an economy's future power to consume when we identify it with current NNP-like measures.

APPENDIX 1: PROOF OF THE MAIN PROPOSITION

Proof. (In the proof, algebraic manipulations are compressed to save space.) Define the Hamiltonian expression:

$$H(t) \equiv e^{-rt} Y^*(t). \tag{41}$$

To keep the proof simple, we assume that the function $H(t)$ is time-differentiable almost everywhere. Then, by differentiating expression (41), it is almost everywhere true that

$$\dot{H}(t) = e^{-rt}\left(\dot{Y}^*(t) - rY^*(t)\right). \tag{42}$$

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[13] See Appendix 2 for details of approximating Green NNP for the U.S. economy. Note that a 2% reduction represents a quite *high* estimate based on the calculations that are presented there, since Green NNP was actually *greater* than conventional NNP for the United States in 1987!

[14] A referee has asked us to note that a low-income natural-resource-intensive country might conceivably show half the rate of technological progress and twice the green capital–output ratio as we have assumed, implying a value of $\theta \sim 8\%$. We do not think that such extreme values deflect the general message given by our ballpark estimates of aggregate sustainability prospects for the world economy as a whole.

In what follows, every variable is evaluated along an optimal trajectory. Taking the total time derivative of $Y^*(t)$, from (12), (9), and (3), we have

$$\dot{Y}^*(t) = \sum_{i=1}^{n} \frac{\partial G}{\partial K_i} \dot{K}_i^* + \sum_{i=1}^{n} \frac{\partial G}{\partial P_i} \dot{P}_i^* + \frac{\partial G}{\partial L} \nu L(0) e^{\nu t}. \tag{43}$$

From (8), (9), and the theory of cost functions[15] it follows that along an optimal trajectory the following duality conditions must be satisfied for all i:

$$\frac{\partial Y}{\partial P_i} = \dot{K}_i^*. \tag{44}$$

Substituting from (44) and (10) into (43) and canceling out terms of the form $\sum \dot{P}_i^* \dot{K}_i^*$ yields along an optimal trajectory the equation

$$\dot{Y}^*(t) = r \sum_{i=1}^{n} P_i \dot{K}_i^* + \frac{\partial G}{\partial L} \nu L(0) e^{\nu t}. \tag{45}$$

By substituting from (12), expression (45) becomes equivalent to

$$\dot{Y}^*(t) = r(Y^*(t) - C^*(t)) + \frac{\partial G}{\partial L} \nu L(0) e^{\nu t}. \tag{46}$$

By the constant-returns-to-scale assumption 1, the green-NNP function $G(\cdot)$ is homogeneous of degree one in $\mathbf{K}$ and L. A standard application of Euler's theorem yields

$$Y^*(t) = \sum_{i=1}^{n} \frac{\partial G}{\partial K_i} K_i^*(t) + \frac{\partial G}{\partial L} L(0) e^{\nu t} \tag{47}$$

which can be rewritten as

$$\frac{\partial G}{\partial L} \nu L(0) e^{\nu t} = \nu \left[Y^*(t) - \sum_{i=1}^{n} \frac{\partial G}{\partial K_i} K_i^*(t) \right]. \tag{48}$$

Substituting from (10) and (12) into the right-hand side of (48) obtains

$$\frac{\partial G}{\partial L} \nu L(0) e^{\nu t} = \nu \left[\left(C^*(t) + P^*(t) \dot{K}^*(t) \right) - \left(r P^*(t) K^*(t) - \dot{P}^*(t) K^*(t) \right) \right]. \tag{49}$$

Substituting from (49) into (46) and collecting terms yields

$$\dot{Y}^*(t) = r Y^*(t) - (r - \nu) C^*(t)$$
$$+ \nu \left[P^*(t) \dot{K}^*(t) + \dot{P}^*(t) K^*(t) - r P^*(t) K^*(t) \right]. \tag{50}$$

Now substitute from (50) into (42), use basic differential calculus, and regroup terms to yield the expression

$$\frac{dH}{dt} = -(r - \nu) C^*(t) e^{-rt} + \nu \frac{d\left[e^{-rt} P^*(t) K^*(t) \right]}{dt}. \tag{51}$$

[15] See, e.g., footnote 9 in Weitzman [22].

When the differential equation (51) is integrated from zero to infinity, it is transformed into the expression

$$H(\infty) - H(0) = -(r - \nu)\int_0^\infty C^*(t)e^{-rt}\,dt$$

$$+ \nu\left[\lim_{t\to\infty} P^*(t)K^*(t)e^{-rt} - P^*(0)K^*(0)\right]. \qquad (52)$$

In a well-defined optimal control problem, the limiting value of the Hamiltonian must be zero[16]

$$H(\infty) \equiv \lim_{t\to\infty}\left[e^{-rt}Y^*(t)\right] = 0. \qquad (53)$$

Substituting from (53), (41), (11), and (13) into (52) yields, after some rearrangement, the desired expression:

$$W_\nu^* = \frac{Y^*(0) - \nu P^*(0)K^*(0)}{r - \nu}. \qquad (54)$$

This concludes the proof.   ∎

APPENDIX 2: ESTIMATION OF U.S. GREEN CAPITAL–OUTPUT RATIO IN 1987

Conventionally measured 1987 NNP is officially $4.029 trillion.[17]

On January 1, 1987, Made Assets were officially $11.566 trillion, while Developed Natural Assets were estimated to be $5.784 trillion.[18] The latter figure is obtained by adding together the subestimates of the values of land, subsoil assets (average estimate of value of proved reserves), and all other cited components of natural capital. Total Assets for 1987 are then estimated to be the sum, or $17.350 trillion.

Human Capital for 1987 is estimated to be $17.117 trillion.[19]

Green Capital is estimated to be the sum of Made Assets plus Developed Natural Assets plus Human Capital, or $34.467 trillion.

The "Cost of a Clean Environment" in 1987 is estimated to be $86 billion.[20] This number is taken here as a rough proxy for the defensive expenditures required to maintain the quality of air, water, and other environmental amenities or externalities.[21]

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[16] See, for example, Weitzman [21] or Michel [12].

[17] Survey of Current Business, July 1990, p. 43. Unless otherwise stated, all numbers reported in this Appendix are expressed in 1987 prices.

[18] U.S. Commerce Department [18], p. 41.

[19] Office of Management and Budget [20], p. 15. The 1985 and 1990 figures for the sum of Education Capital plus R & D Capital are linearly interpolated and then converted from 1993 fixed prices to 1987 prices by using the appropriate implicit GNP price deflator.

[20] U.S. Environmental Protection Agency [19], "Cost of Clean" for 1987 is estimated by linear interpolation to be 1.9% of GNP. In 1987, U.S. GNP is $4.5156 trillion. An alternative version based on marginal damages [5] makes no appreciable difference to the big picture. For a theoretical treatment of the proper correction required for externalities along an optimal path, see Aronsson and Löfgren [1].

[21] Even if this number is off by a factor of two or three, it will not change the "big picture."

Depletion of Natural Assets in 1987 is estimated to be $-\$328$ billion.[22]

Green NNP is estimated to be Conventional NNP minus "Cost of Clean" minus depletion of Developed Natural Assets, or \$4.271 trillion.

The green capital–output ratio is green capital divided by Green NNP, which here comes to about 8:1.

Although the estimate provided here of κ is crude, seemingly more refined procedures, possibly based on different data, and with a more sharp-looking, but ultimately false, sense of precision would be unlikely to alter substantively the basic conclusions of the numerical exercise performed in the paper.

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[22] U.S. Commerce Department [18], p. 15, taken equal to the difference between 1988 and 1987 average estimates of developed natural capital stock values, converted to 1987 prices. This number is negative in part because the stock of proved subsoil reserves increased over the year 1987, indicating positive net investment in the economic activity of {*discovery/development-minus-extraction*} of mineral stocks. Proved reserves often increase on average over time, although this does not occur in all years. There is scope for reasonable discussion about the appropriate way to enter changes in developed natural capital stocks into green national income accounts, but here it suffices to note that a different treatment or using different years would not appreciably change the basic empirical conclusions of the paper.

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