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Disasterization: A Simple Way to Fix the Asset Pricing Properties of Macroeconomic Models

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This paper proposes a simple way to fix the asset pricing properties of traditional macroeconomic models by adding the possibility of disasters in those economies. I start from a traditional economy without disasters. I create a new economy which has the possibility that disasters will affect the capital stock and productivity, but is otherwise identical (except for the rate of time preference, for reasons that will become clear later). I call this process the "disasterization" of the original economy. The new disasterized economy behaves exactly like the original economy, except for its asset prices. This is to say that GDP, employment, savings, investment, etc., are the same except that they are scaled down if disasters have occurred. However, asset prices, which reflect the possibility of disasters, are different: the equity premium is higher, and, if inflation goes up in a disaster (an assumption that will be discussed), one obtains a positively sloped yield curve and positive bond risk premia.

Hence, this paper presents a simple, practical way to create economies enriched by risk premia: by adding disasters in the right places, i.e., by disasterizing economies.

This key idea is simply a starting point from which several useful directions can be taken. First, one can enrich the model with a reasonably realistic model of stocks and bonds—this is what I am doing in this paper. Second, one can study a case where the fall in capital and the fall in productivity are not exactly equal. This is the route chosen by Francois Gourio (2010), who studies Epstein-Zin preferences as well. Third, one could use this device to analytically study small changes in disaster risk and their macroeconomic impact, which is a route worth pursuing in future research. For instance, suppose that the disaster probability rises: what happens to the various economic quantities? Suppose that stock market valuations go up; what happens to investment and GDP? I note that these types of perturbation studies are possible only when one starts from a simple tractable model, such as the one proposed here.

The idea of disasterization gives a concrete and fairly general way to fix the asset pricing properties of macroeconomic models without losing tractability. Therefore, it brings us closer to the goal of a unified model of macro-finance.

Previous analyses have shown that it is hard to have both a stable interest rate and a varying equity premium. However, this task becomes easy with the disasterization procedure. The reason is that it glues two strands of modeling that are quantitatively reasonably successful: the real business cycle (RBC) (or New Keynesian) modeling of business cycles and the disaster literature. In its simplest incarnation (the one presented in this paper), it modifies the two parts just enough so that they fit together, but otherwise leaves them largely intact. Hence, it inherits the reasonably good fit from both the "pure" macro models (i.e., those that give up on asset prices) and the "pure" finance models (with endowment economies that give up on production). It thereby glues together asset prices and production in a seamless way.

This paper borrows from the disaster literature revived by Robert Barro (2006), who develops economies with a constant intensity of disasters, and incorporates a concrete disaster economy with time-varying disasters as in Gabaix (2010). There are arguably three paradigms for rational-representative-agent economies: external habits, long-run risks, and disasters. All three types of models are usually developed in endowment economies. However, it is hard to extend habits and long-run risk models to production economies, in part because they do not use the same CRRA utility function as the basic macro models. In contrast, the basic disaster model does, so it can easily be glued to macro models. This is what I do here.

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I. The Basic Idea

I start from an RBC model that can generate realistic macro dynamics, fix it, and yield a new economy with the same business cycle properties but different asset pricing properties. To illustrate the procedure, I use a simple example.

The Original Economy.—The utility function is a conventional $\mathbb{E}_0\left[\sum_{t=1}^{\infty} \rho_0^t \times (C_t^{1-\gamma}/(1-\gamma))\,\phi(L_t)\right]$. The production function is $Y = (AL)^{\beta} K^{1-\beta}$. Here, the aggregate productivity A is the Solow labor-augmenting one (so that in the long run K is proportional to A). I define it as $A_t = a_t e^{x_t}$, where a_t is the permanent part of productivity $a_{t+1} = e^g a_t$ and x_t is a transitory disturbance such that $x_{t+1} = \rho x_t + \eta_{t+1}$. The capital accumulation process is as follows: $K_{t+1} = (a_t e^{x_t} L_t)^{\beta} K_t^{1-\beta} - C_t + (1-\delta)K_t$.

Applying the Disasterization Procedure.—I construct a new economy that includes the possibility of disasters. If the disaster hits (which has probability p_t), the capital stock is multiplied by a factor B_{t+1} which can be thought of as less than one. For instance, if the disaster destroys 20 percent of the capital stock, $B_{t+1} = 0.8$. I use the notation $\Delta_{t+1} = 1$ if there is no disaster at $t+1$, and $\Delta_{t+1} = B_{t+1}$ if there is a disaster, for some $B_{t+1} > 0$. Hence, the law of motion for capital is: $K_{t+1} = [(a_t e^{x_t} L_t)^{\beta} K_t^{1-\beta} - C_t + (1-\delta)K_t] \times \Delta_{t+1}$. I will also propose that the permanent part of productivity falls by the same amount, $a_{t+1} = e^g a_t \times \Delta_{t+1}$, while the transitory disturbance is not affected, $x_{t+1} = \rho x_t + \eta_{t+1}$. In the original economy, $\Delta_{t+1} = 1$ at all dates.

A disaster affects the capital stock and productivity by the same amount. This makes the economics very convenient: suppose that the economy was on its balanced growth path before the disaster. After the disaster, both the capital stock and productivity are scaled down by 20 percent. Hence, right after the disaster, the economy is again on its new balanced growth path: it is the same as before, except that all "scale" variables (capital, productivity, wages) are permanently scaled down by 20 percent.

This assumption of a common shock to productivity and capital is largely for expediency, as the spirit of this paper is to highlight a particularly clean class of situations. I view the assumption as rather defensible, though. A drop in productivity can come from disruptions (e.g., political, institutional) resulting from disasters. Also, the extent of mean reversion after a disaster is disputed: Valerie Cerra and Sweta Chaman Saxena (2008) find no recovery after various financial crises. I submit that the cleanest serviceable benchmark is the one where capital and productivity fall by the same amount.

Finally, the utility function is the same as originally, except that the rate of time preference ρ is different: it is $\rho = \rho_0 \mathbb{E}\left[\Delta_t^{1-\gamma}\right]$, so that if $\gamma > 1$, agents in the disasterized economies are less patient than in the no-disaster economy (I assume $\mathbb{E}\left[\Delta_t^{1-\gamma}\right]$ constant, which is consistent with Gabaix 2010, though not with Jessica Wachter 2008). The reason is the following: because there are disasters, ceteris paribus, agents want to save more to create a buffer against disaster events. To undo this effect, I make them more impatient. Hence, the savings rate in the new and the old economy will be exactly the same, provided the impatience is judiciously chosen. The key result is the following.[1]

PROPOSITION 1: *The disasterized economy above can be solved using the following two-step procedure:* (*i*) *Solve for the "0" economy with no disasters, i.e., with* $\forall t, \Delta_t = 1$ *and a rate of time preference* ρ_0. *Call* $C_t^0, K_t^0, a_t^0, L_t^0, X_t^0$ *the solution for each t.* (*ii*) *Then, the solution of the disasterized economy* 1 *is* $(C_t, K_t, a_t) = \left(\mathcal{D}_t C_t^0, \mathcal{D}_t K_t^0, \mathcal{D}_t a_t^0\right)$ *and* $(L_t, X_t) = \left(L_t^0, X_t^0\right)$ *where* $\mathcal{D}_t = \Delta_1 \ldots \Delta_t$ *is the cumulative disaster. In other terms, the extensive variables* (C_t, K_t, a_t) *are scaled by* $\mathcal{D}_t$ *while the intensive variables* (L_t, X_t) *are left unchanged.*

The proposition also holds more generally, with $K_{t+1} = G(a_t e^{x_t} L_t, C_t, K_t, x_t)\Delta_{t+1}$, $a_{t+1} = a_t g\ (\eta_{t+1}, x_t)\Delta_{t+1}$, and $x_{t+1} = X(\eta_{t+1}, x_t)$, where x_t is a vector of "intensive" factors (e.g., a deviation from trend) and η_{t+1} is an i.i.d. vector of shocks. G is homogenous of degree 1 in (L_t, C_t, K_t).

"Macro" (capital, consumption, investment, labor) variables do not change at all (due to rescaling after disasters), but asset prices do change (as risk premia encode disaster risk).

[1] The result was in Gabaix (2007). It has also been formulated in a paper that started circulating in February 2009, Gourio (2010). That paper also contains a host of other material.

Thus, one can derive the yield curve, stock prices, etc., while keeping the macro side constant. I note that this "equivalence" result is in the spirit of Thomas D. Tallarini (2000).

Proof sketch: It is immediate to verify that $\left(C_t^0, K_t^0, a_t^0, L_t^0, X_t^0\right)$ satisfy the technological constraint of problem P^0 (for the "0" economy) if and only if $(C_t, K_t, a_t, L_t, X_t)$ satisfy that of problem P. Next, call $V(K_t, a_t, X_t)$ the value function of problem P and $V^0\left(K_t^0, a_t^0, X_t^0\right)$ the value function of problem P^0. Then, one has

$$V(K_t, a_t, X_t) = \max_{C,L} u(C_t, L_t) + \rho \mathbb{E}_t\left[V(K_{t+1}, a_{t+1}, X_{t+1})\right]$$

and similarly for V^0. V and V^0 are homogenous of degree $1-\gamma$ in (K^0, a^0).

Consider the value function for problem P, $v(K, a, X) = V^0(K, a, X)$, with policy $c = c_0$, $L = L_0$. I will show that v is the value function V, optimal in the P economy. Given

$$\begin{aligned} W &\equiv \rho \mathbb{E}_t\left[v(K', a', X')\right] \\ &= \rho \mathbb{E}_t\left[v(K_0'\Delta', a_0'\Delta', X_0')\right] \\ &= \rho \mathbb{E}_t\left[\Delta'^{1-\gamma} v(K_0', a_0', X_0')\right] \\ &= \rho \mathbb{E}\left[\Delta'^{1-\gamma}\right]\mathbb{E}\left[v(K_0', a_0', X_0')\right] \\ &= \rho_0 \mathbb{E}\left[v(K_0', a_0', X_0')\right] \end{aligned}$$

one sees that v satisfies the Bellman equation. By unicity of the solution, v is the optimal policy of problem P.

II. Applications

A. *Bonds: A Positive and Variable Yield Curve Slope*

Consider the price of a bond in such an economy. Assume that bond dynamics are as in Gabaix (2010): inflation is stochastic and will jump (on average up) during disasters. That makes nominal bonds riskier, so the yield curve slopes up. Thus, one has:

PROPOSITION 2 (*Bond prices*): *The price of a nominal zero-coupon bond of maturity T is: $Z_t^1(T) = Z_t^0(T) Z_{\$t}^*(T, I_t, \pi_t)$ where $Z_t^0(T)$ is the bond price in the original RBC economy and $Z_{\$t}^*(T, I_t, \pi_t)$ is the price in an endowment disaster economy given by Gabaix (2010, Theorem 2) with inflation I_t and bond risk premium π_t.*

Given that the disaster model has tractable expressions and realistic bond prices $Z_{\$t}^*(T, I_t, \pi_t)$, the new model has the same properties: a yield curve slope that is volatile and on average upward sloping, and bond risk premia. With little effort, I was able to fix the properties of the RBC (or New Keynesian) yield curve, which is flat or downward sloping, and contains no risk premium.

B. *Stock Prices: Why Is Q Volatile and Unrelated to Investment?*

In traditional macro models, stock prices are too stable. To make them more realistic, I incorporate some elements from Gabaix (2010). The consumption good in the economy is a Dixit-Stiglitz aggregate: $Y_t = \left(\int_0^1 Q_i^{1/\psi} di\right)^{\psi}$ with $Q_i = A K_i^{\alpha} L_i^{1-\alpha}$, $\psi > 1$, $A_{t+1} = A_t \times \Delta_{t+1}$. Each firm is a Dixit-Stiglitz monopolist with profits $(\psi - 1)Y_t$. I assume that corrective taxes and lump-sum rebates are in place to lead to the first-best allocation.

If a disaster takes place, there will be expropriation of *rents* (not capital) by F_t, so earnings are: $D_t = (\psi - 1) Y_t \Delta_0^{\pi} \ldots \Delta_t^{\pi}$, with $\Delta_t^{\pi} = 1$ in normal times, and $\Delta_t^{\pi} = 0$ with probability $1 - F_t$ and 1 otherwise in disasters.

Using the stochastic discount factor $M_t = \rho^t C_t^{-\gamma}$, the present value of future profits is: $V_t^{\pi} = \mathbb{E}_t\left[\sum_{s=0}^{\infty} (M_{t+s}/M_t) D_{t+s}\right]$. I model time-varying riskiness of a firm as captured by its resilience: $H_t \equiv p_t \mathbb{E}_t[B_{t+1}^{-\gamma} F_t - 1]$, which can be decomposed as $H_t = \mathrm{H}_* + \hat{H}_t$. $\hat{H}_t$, follows the linearity-generating (LG) process: $\hat{H}_{t+1} = (1 + H_*)/(1 + H_t)\, e^{-\phi_H} \hat{H}_t + \varepsilon_{t+1}^H$, where ϕ_H is the speed of mean-reversion and ε_{t+1}^H is uncorrelated with the disaster event.

Under these conditions (as in Gabaix 2010), the value of pure profits is: $V_t^{\pi} = (\psi - 1)\, Y_t / r_e \left(1 + \hat{H}_t/(r_e + \phi)\right)$, $r_e = R - \ln(1 + H_*)$, where $R = -\ln \rho + \gamma g_c$. The full value of the corporate sector is: $V_t = V_t^{\pi} + K_t$. The first part, V_t^{π}, is the capitalized present value of pure profits. The second, K_t, is the value of the physical capital in place. From this, one yields Tobin's $Q_t = V_t / K_t$.

A central paradox in macro-finance is that Q has a weak relationship with investment

(Thomas Philippon 2009). This is exactly what happens in this model. Investment is always equal to the RBC level of investment. The reason is that more physical investment leads to more K_t, which can be costlessly replaced. A higher stock valuation $\hat{H}_t$ simply signifies a higher present value of rents, but it is not correlated with investment as technology is in fixed supply here. There is a way to introduce a correlation with investment: high stock market valuations might spur entrepreneurship and the creation of new ideas.

Calibration.—I take $\psi = 1.2$, so the markup is about 20 percent. Because $r_e = 5$ percent (as in the Gabaix 2010 calibration), the typical P/D ratio is 20. The total firm value is $V_t = V_t^\pi + K_t = (\psi - 1)Y_t/r_e + K_t$. I find $V_t^\pi/Y_t = (\psi - 1)/r_e = 4$. By the usual calibration, $K/Y = 4$. So, half of the firm market valuation is for physical capital while the rest is for the present value of rents. With a debt/equity ratio of about 50 percent, one yields a volatility of the stock market of about 15 percent per year. I conclude that the model calibrates quite well.

C. *Disasterizing Other Economies*

The same idea can be used for other economies.

Economies with Habits.—Suppose the preferences are: $u(C_t, H_t, L_t) = (v(C_t, H_t)^{1-\gamma}/(1-\gamma))\phi(L)$, where H_t is a habit variable and $v(C, H)$ is homogenous of degree one. The habit process follows: $H_{t+1} = f(H_t, C_t)$ for a function f, e.g., $f(H, C) = \beta H + (1 - \beta)C$. One requires the disasterized economy to be a rescaling of the original economy: $H_{t+1} = f(H_t, C_t)\,\Delta_{t+1}$. Economically, after a disaster, people revise their habits ("aspiration levels") downward.

Monetary Economies with Sticky Prices.—With sticky prices, things remain the same, except that "level" variables have to adjust. For instance, after a disaster of 20 percent, the neoclassical wage and the money stock should fall by 20 percent. Formally, if w_t^0 and M_t^0 are the real wage and money stock at time t in the no-disaster economy, in the disasterized economy they become $w_t = \mathcal{D}_t w_t^0$ and $M_t = \mathcal{D}_t M_t^0$. Hence, the government removes some of the "excess liquidity" during the disaster.

III. Conclusion

This paper proposes a systematic way to enrich existing models by "disasterizing" economies, i.e., by adding disasters to the model in a careful way. A disaster of 20 percent should reduce the capital stock and productivity by the same 20 percent. The resulting economies are very easy to solve: they inherit useful business cycle properties from the macro model and asset pricing properties from the variable rare disaster model.

In the current framework, the macro side of the economy affects asset prices, but not vice versa. It would be insightful to extend the model to the case where finance affects the macroeconomy. The availability of a simple model where macro and finance are integrated surely takes us one step closer to that goal.

REFERENCES

Barro, Robert J. 2006. "Rare Disasters and Asset Markets in the Twentieth Century." *Quarterly Journal of Economics*, 121(3): 823–66.

Cerra, Valerie, and Sweta Chaman Saxena. 2008. "Growth Dynamics: The Myth of Economic Recovery." *American Economic Review*, 98(1): 439–57.

Gabaix, Xavier. 2007. "A Unified Theory of Ten Financial Puzzles." SSRN Working Paper 976436.

Gabaix, Xavier. 2010. "Variable Rare Disasters: An Exactly Solved Framework for Ten Puzzles in Macro-Finance." Unpublished.

Gourio, Francois. 2010. "Disaster Risk and Business Cycles." Unpublished.

Philippon, Thomas. 2009. "The Bond Market's Q." *Quarterly Journal of Economics*, 124(3): 1011–56.

Tallarini, Thomas D., Jr. 2000. "Risk-Sensitive Real Business Cycles." *Journal of Monetary Economics*, 45(3): 507–32.

Wachter, Jessica. 2008. "Can Time-Varying Risk of Rare Disasters Explain Aggregate Stock Market Volatility?" Unpublished.