"The Granular Origins of Aggregate Fluctuations": Supplementary Material

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This online appendix (i) presents some additional empirical robustness checks (ii) describes some econometric complements (on the attenuation bias in the R^2 of the granular regressions, and on why demeaning by industry-year averages can be better than demeaning by year averages) (iii) describes the time-series of the Granular Residual available online.

5. Robustness Checks for Granular Residual Regressions

I consider possible micro-level improvements in the construction of the granular residual. Perhaps the loading on the aggregate factors could depend on size. To explore that possibility, I consider models such as:

$$g_{it} = a\overline{g}_{I_i t} + b\left(\ln S_{i,t-1} - \overline{\ln S_{j,t-1;,j\in I_i}}\right)\overline{g}_{I_i t} + c\left(\ln S_{i,t-1} - \overline{\ln S_{j,t-1;,j\in I_i}}\right)^2 \overline{g}_{I_i t} + \varepsilon_{it} \qquad (36)$$

In this model, the loading of firm i can depend not only on the industry average $\overline{g}_{I_i t}$, but also on the size of the firm. To make coefficients easier to interpret, I recenter by $\overline{\ln S_{j,t-1}}$, the average log size, so that the mean of the second term is 0. Table VI reports the first stage of a variety of specifications (with $K = Q = 100$). Size controls are only sometimes significant, but their incremental explanatory power is very small compared to the earlier specification that simply controls for $\overline{g}_t$ and $\overline{g}_{I_i t}$. The second stage is reported in Table VII. The impact of the size control is very small. Hence, for practical purposes, one may recommend the two simplest versions of the granular residual, (33) and (34).

Still, one lesson from Table VI is that large firms have if anything a smaller loading on the common shocks than small firms (column 5). Hence, constructing the granular residual by removing a constant mean slightly overcorrects large firms, and, prima facie, works against finding a high explanatory power of the granular residual. This bias against the granular residual is, however, small, as Table VII shows.

Tables VIII and IX show some more regressions. In particular, I use $K = 100$ and $Q = 1000$. This is, it evaluates the "expected" behavior of the top 100 firms by examining the behavior of the top 1000 firms. Note that a difficulty is that in the early part of the sample there are less than 1000 firms; then I take the maximum number of firms.

The results are similar to Tables VI and VII, except that the R^2 of the first stage (explaining firm-level growth rate) and the second stage (explain GDP growth) are a bit lower. That may be

	(1)	(2)	(3)	(4)	(5)	(6)
(Intercept)	7.2e-16	-7.9e-17	7.2e-16	0.0014	7.2e-18	0.0019
	(0.002)	(0.0013)	(0.002)	(0.0024)	(0.0017)	(0.0021)
$\bar{g}_t$	1**		1**	0.9**	-0.00097	-0.083
	(0.076)		(0.076)	(0.093)	(0.068)	(0.081)
$\bar{g}_{I(i),t,}$		1**			1**	0.97**
		(0.02)			(0.021)	(0.022)
$(\ln S_{i,t-1} - \overline{\ln S_{t-1}})$			0.0041	0.0076	-0.0031	0.00012
			(0.0032)	(0.0048)	(0.0043)	(0.0051)
$(\ln S_{i,t-1} - \overline{\ln S_{t-1}}) \cdot \bar{g}_t$			-0.15	-0.42*	0.17	-0.041
			(0.12)	(0.18)	(0.11)	(0.16)
$(\ln S_{i,t-1} - \overline{\ln S_{t-1}})^2$				-0.0034		-0.0012
				(0.0035)		(0.0042)
$(\ln S_{i,t-1} - \overline{\ln S_{t-1}})^2 \cdot \bar{g}_t$				0.26*		0.17
				(0.13)		(0.12)
$(\ln S_{i,t-1} - \overline{\ln S_{I(i),t-1}})$					0.0068	0.0089
					(0.0046)	(0.0047)
$(\ln S_{i,t-1} - \overline{\ln S_{I(i),t-1}}) \cdot \bar{g}_{I(i),t}$					-0.31**	-0.4**
					(0.05)	(0.057)
$(\ln S_{i,t-1} - \overline{\ln S_{I(i),t-1}})^2$						-0.0053
						(0.0045)
$(\ln S_{i,t-1} - \overline{\ln S_{I(i),t-1}})^2 \cdot \bar{g}_{I(i),t}$						0.16**
						(0.05)
N	5700	5700	5700	5700	5700	5700
R^2	0.0292	0.305	0.0295	0.0302	0.31	0.312
Adj. R^2	0.029	0.305	0.029	0.0293	0.309	0.311

Table VI: Explaining Firm-Level Productivity Growth. This table presents the results of the different regressions of productivity growth g_{it} for the top 100 firms as defined by the previous year's sales on mean productivity growth measures at the global and industry level. The column numbers correspond to different specifications for the regressors. They are used in the next table. Standard errors in parentheses.

	(1)	(2)	(3)	(4)	(5)	(6)
(Intercept)	0.017**	0.017**	0.018**	0.019**	0.02**	0.019**
	(0.0025)	(0.0022)	(0.0024)	(0.0024)	(0.0022)	(0.0021)
Γ_t	2.5**	4.5**	2.8**	2.6**	4.2**	4.4**
	(0.69)	(0.82)	(0.68)	(0.7)	(0.83)	(0.84)
Γ_{t-1}	2.9**	4.3**	2.7**	2.9**	3.8**	4.6**
	(0.67)	(0.78)	(0.68)	(0.68)	(0.81)	(0.77)
Γ_{t-2}	2.1**	2.7**	2.1**	2.1**	2.9**	2.9**
	(0.71)	(0.79)	(0.7)	(0.72)	(0.82)	(0.81)
N	55	55	55	55	55	55
R^2	0.382	0.506	0.395	0.388	0.456	0.516
Adj. R^2	0.346	0.477	0.359	0.352	0.423	0.487

Table VII: Explanatory Power of the Granular Residual, under alternative specifications. This table reports the regressions of per capita GDP growth on the granular residual plus two lags, using the various specifications of Table VI to extract the idiosyncratic firm-level shocks. Standard errors in parentheses.

due to the fact that the sample is less homogenous, as we try to control the behavior of large firms by the behavior of quite smaller firms.

6. Econometric Complements

6.1. Attenuation Bias in the Granular Residual

I analyze the properties of the granular residual. The conclusion is that it suffers from attenuation bias, but the bias goes to 0 as the number of firms K becomes large. On the other hand, taking a large K (or Q) introduces new difficulties – the homogeneity assumption (37) is likely to be a less good approximation, as demonstrated in the previous section.

I consider first a one-factor model (no industry shocks). For firm i:

$$g_{it} = X_t + \varepsilon_{it} \tag{37}$$

where X_t is a common shock, and ε_{it} is an idiosyncratic shock. The granular residual is:

$$\Gamma_K = \frac{\sum_{i=1}^{K} S_i \left(g_i - \bar{g} \right)}{Y} \tag{38}$$

while the econometrician would like to know the "ideal" granular residual – a weighted mean of

	(1)	(2)	(3)	(4)	(5)	(6)
(Intercept)	3.2e-15	-1.3e-16	3.2e-15	-0.0021	2e-16	-0.0016
	(0.001)	(0.00079)	(0.001)	(0.0012)	(0.00096)	(0.0012)
$\bar{g}_t$	1**		1**	1**	-0.00053	0.0097
	(0.037)		(0.037)	(0.045)	(0.038)	(0.045)
$\bar{g}_{I(i),t},$		1**			1**	1**
		(0.012)			(0.013)	(0.015)
$(\ln S_{i,t-1} - \overline{\ln S_{t-1}})$			0.00019	-0.0011	-0.0026	-0.0037
			(0.00091)	(0.001)	(0.002)	(0.002)
$(\ln S_{i,t-1} - \overline{\ln S_{t-1}}) \cdot \bar{g}_t$			-0.051	-0.043	0.15**	0.16**
			(0.033)	(0.037)	(0.034)	(0.037)
$(\ln S_{i,t-1} - \overline{\ln S_{t-1}})^2$				0.0017**		0.00071
				(0.00057)		(0.00093)
$(\ln S_{i,t-1} - \overline{\ln S_{t-1}})^2 \cdot \bar{g}_t$				-0.0098		-0.0085
				(0.02)		(0.021)
$(\ln S_{i,t-1} - \overline{\ln S_{I(i),t-1}})$					0.0033	0.0034
					(0.002)	(0.002)
$(\ln S_{i,t-1} - \overline{\ln S_{I(i),t-1}}) \cdot \bar{g}_{I(i),t}$					-0.22**	-0.22**
					(0.014)	(0.015)
$(\ln S_{i,t-1} - \overline{\ln S_{I(i),t-1}})^2$						7e-04
						(0.00099)
$(\ln S_{i,t-1} - \overline{\ln S_{I(i),t-1}})^2 \cdot \bar{g}_{I(i),t}$						-2.4e-05
						(0.0093)
N	52895	52895	52895	52895	52895	52895
R^2	0.0134	0.113	0.0135	0.0137	0.117	0.117
Adj. R^2	0.0134	0.113	0.0134	0.0136	0.117	0.117

Table VIII: This table presents the results of the different regressions of productivity growth g_{it} for the top 1000 firms as defined by the previous years sales on mean productivity growth measures at the global and industry level. The column numbers represent correspond to different specifications for the regressors. They are used in the next Table. Standard errors in parentheses.

	(1)	(2)	(3)	(4)	(5)	(6)
(Intercept)	0.018**	0.018**	0.016**	0.023**	0.016**	0.023**
	(0.0028)	(0.0024)	(0.003)	(0.0029)	(0.0027)	(0.0027)
Γ_t	0.9	1.4*	1.3*	1.3*	1.3*	1.4*
	(0.54)	(0.53)	(0.57)	(0.58)	(0.57)	(0.58)
Γ_{t-1}	1.9**	2.8**	1.9**	1.7**	2.3**	2.1**
	(0.56)	(0.54)	(0.6)	(0.61)	(0.6)	(0.6)
Γ_{t-2}	0.51	0.81	0.55	0.47	0.7	0.64
	(0.56)	(0.53)	(0.59)	(0.6)	(0.59)	(0.6)
N	55	55	55	55	55	55
R^2	0.213	0.371	0.222	0.2	0.278	0.255
Adj. R^2	0.166	0.334	0.176	0.152	0.236	0.212

Table IX: Residual Productivity Growth Baseline Regressions, Q=1000, K=100. This table reports the results of regressions of the per capita GDP growth on the granular residual plus two lags where the granular residual is computed with $Q = 1000$ and $K = 100$ while using estimates of the residual productivity growth from the previous Table. Standard errors in parentheses.

the idiosyncratic shocks of the top K firms.

$$\Gamma_K^* = \frac{\sum_{i=1}^{K} S_i \varepsilon_i}{Y} \tag{39}$$

(the specific choice of the denominator does not matter here, as I investigate the R^2s, and R^2s do not change when one multiplies some variables by a constant).

GDP growth follows, as in the model of the NBER WP version of this paper:

$$y_t = \phi \Gamma_{Kt}^* + u_t \tag{40}$$

where u_t is a disturbance orthogonal to $(\varepsilon_{it})_{i=1...K}$. One would like to know how much R^2 of the idiosyncratic shocks of the top K firms explain, i.e., the R^2 of the *ideal* granular residual:

$$R_{\Gamma_K^*}^2 = \frac{cov\left(y_t, \Gamma_{Kt}^*\right)^2}{var\left(y_t\right) var\left(\Gamma_{Kt}^*\right)} \tag{41}$$

The empirical analysis only gives the R^2 of the granular residual Γ :

$$R_{\Gamma_K}^2 = \frac{cov\left(y_t, \Gamma_{Kt}\right)^2}{var\left(y_t\right) var\left(\Gamma_{Kt}\right)} \tag{42}$$

Econometrically, the situation is tricky, because economically, X_t is correlated with Γ_K^*.

A quantity of interest is the squared herfindahl of the top K firms:

$$H_K = \frac{\sum_{i=1}^K S_i^2}{\left(\sum_{i=1}^K S_i\right)^2} \tag{43}$$

Lemma 1 *The R^2 of the granular residual is a downward biased estimate of the R^2 of the ideal granular residual, by a factor $1 - \frac{1}{KH_K}$:*

$$R_{\Gamma_K}^2 = R_{\Gamma_K^*}^2 \left(1 - \frac{1}{KH_K}\right)$$

Proof. By rescaling, it is enough to analyze the case where $\sigma_\varepsilon = 1$. I call $\sum_{i=1}^K S_i = s$, and $\overline{X} = K^{-1} \sum_{i=1}^K X_i$ for a variable X. Then: $\Gamma = \sum_{i=1}^K (S_i/s - 1/K)\varepsilon_i$, which gives, dropping the K subscripts when there is no ambiguity:

$$\Gamma_t^* = \Gamma_t + \overline{\varepsilon_t} \text{ with } cov(\Gamma_t, \overline{\varepsilon_t}) = 0$$

which means that Γ is a noisy proxy for Γ^*. Also

$$cov(\Gamma_t^*, \Gamma_t) = var\Gamma_t = \left(H - \frac{1}{K}\right), \qquad var\Gamma_t^* = H$$

and

$$R_\Gamma^2 = \frac{cov(y, \Gamma_t)^2}{vary \cdot var(\Gamma_t)} = \frac{\phi^2 cov(\Gamma_t, \Gamma_t^*)^2}{vary \cdot var(\Gamma_t)} = \frac{\phi^2 (var\Gamma^*)^2 \left(1 - \frac{1}{HK}\right)^2}{vary \cdot var(\Gamma_t)} = \frac{cov(y, \Gamma_t^*)^2}{vary \cdot var\Gamma^*} \left(1 - \frac{1}{HK}\right)^2 \frac{var\Gamma^*}{var\Gamma}$$

$$= R_{\Gamma^*}^2 \left(1 - \frac{1}{HK}\right)^2 \frac{H}{H - \frac{1}{K}} = R_{\Gamma^*}^2 \left(1 - \frac{1}{HK}\right).$$

∎

Empirically, for the $K = 100$, firms, $\left(1 - \frac{1}{KH_K}\right) = 2/3$. Hence if empirically the $R_{\Gamma_K}^2 = 1/3$, the R^2 of the ideal granular residual is $R_{\Gamma_K^*}^2 = 1/2$. This bias is an attenuation bias, as the granular residual is a noisy proxy for the ideal granular residual.

If the distribution is very concentrated, then $H_K \gg 1/K$. Formally, the proof of Proposition 2 shows that if the Pareto exponent of the distribution is $1 \le \zeta < 2$, $KH_K \propto K^{2-2/\zeta}$, so $\lim_{K\to\infty} (KH_K)^{-1} = 0$, and as $K \to \infty$, $R_{Y,\Gamma_K}^2/R_{Y,\Gamma_K^*}^2 \to 1$. This is the sense in which, for large K, the granular residual identifies the explanatory power of the ideal granular residual.

The same reasoning applies, with messier expressions, with industry specific shocks, model: $g_{it} = x_t + x_{I_i} + \varepsilon_{it}$. The R^2 of the industry-demeaned Γ_t is a downward estimate R^2 of the ideal granular residual $\Gamma^{*,ind}$. The bias goes to 0 as the number of firms becomes large.

6.2. Why controlling for industry-year averages $\bar{g}_{I_i t}$ might be better than simply controlling for the year average $\bar{g}_t$

The paper controls for $\bar{g}_t$ and $\bar{g}_{I_i t}$. It justifies this by saying "The term $g_{it} - \bar{g}_{I_i t}$ may be closer to the ideal ε_{it} than $g_{it} - \bar{g}_t$, as $\bar{g}_{I_i t}$ may control better than $\bar{g}_t$ for industry-wide disturbances, e.g., industry-wide real price movements." In case this is useful, here is some more elaboration on this argument.

Indeed, take a production function $Y = F\left(A_1 L_1, ..., A_n L_n\right)$ with one factor, labor, and F fairly general but homogenous of degree 1. Then, the sales are $S_i = p_i A_i L_i$, hence the percentage changes (calling $\widehat{X} = dX/X$) are $\widehat{S}_i = \widehat{p}_i + \widehat{A}_i + \widehat{L}_i$, so the paper's construct g_i is:

$$g_i \equiv \widehat{S}_i - \widehat{L}_i = \widehat{A}_i + \widehat{p}_i$$

Thus, g_i is indeed TFP growth $\widehat{A}_i$, plus a "cost disturbance" $\widehat{p}_i$, which econometrically is akin to some measurement noise (except that, importantly, it can be correlated with $\widehat{A}_i$). If the $\widehat{p}_i$ are very correlated within an industry (that will be the case with a Dixit-Stiglitz structure inside the industry), $\widehat{p}_i \simeq \widehat{p}_I$, then $\bar{g}_{I_i t} \simeq \widehat{p}_I \simeq \widehat{p}_i$ (if there are many firms with uncorrelated shocks in the industry), and $g_i - \bar{g}_{I_i t}$ will be closer to $\widehat{A}_i$ than $g_i - \bar{g}_t$.

This general point being made, here is a clear example in which the demeaning by the industry $\bar{g}_{I_i t}$ is unambiguously better theoretically than the demeaning by $\bar{g}_t$. Consider an economy with m sectors where sector I produces: $Q_I = \left(\sum_{j \in I} (A_j L_j)^{1/\phi}\right)^{\phi}$ for $\psi > 1$, and GDP is $Y = \prod_{I=1}^{m} Q_I^{\alpha_I}$ with $\sum_{i=1}^{m} \alpha_I$. Then, the price of good Q_I satisfies $p_I Q_I = \alpha_I Y$, so

$$\widehat{p}_I + \widehat{Q}_I = \widehat{Y}$$

Also, (by optimization in the production of Q_I) the price of good $i \in I$ is $p_i = p_I \left(\frac{Q_i}{Q_I}\right)^{\frac{1}{\phi} - 1}$, so the sales of firm i are:

$$S_i = p_i Q_i = p_I Q_I \left(\frac{Q_i}{Q_I}\right)^{\frac{1}{\phi}}$$

so:

$$\widehat{S}_i = \frac{1}{\phi}\left(\widehat{Q}_i - \widehat{Q}_I\right) + \widehat{p}_I + \widehat{Q}_I$$

$$\widehat{S}_i = \frac{1}{\phi}\left(\widehat{Q}_i - \widehat{Q}_I\right) + \widehat{Y}$$

Note also that

$$\widehat{Q}_I = \frac{\sum_{i \in I} S_i \widehat{Q}_i}{\sum_{i \in I} S_i}$$

To simplify further, assume that labor cannot be reallocated in the short term (the more general

case is in the NBER WP of this paper), so that $\widehat{L}_i = 0$ and $\widehat{Q}_i = \widehat{A}_i$. Then, the measure growth rate $g_i \equiv \widehat{S}_i - \widehat{L}_i$ satisfies:

$$g_i = \frac{1}{\phi}\left(\widehat{Q}_i - \widehat{Q}_I\right) + \widehat{Y}$$

Next, consider the $\bar{g}$ term with many sectors and many firms (but a few big firms in a few big sectors)

$$\bar{g} = \widehat{Y}$$

while the industry average is

$$\bar{g}_{I_i} = \frac{-\widehat{Q}_I}{\phi} + \widehat{Y}$$

Denote by $s_i = S_i/Y$ the output share. The $\bar{g}$-demeaned granular residual (eq. 33 in the paper) is:

$$\Gamma^{\bar{g}} \equiv \sum_i s_i\left(g_i - \bar{g}\right) = \sum_i s_i \frac{1}{\phi}\left(\widehat{Q}_i - \widehat{Q}_I\right)$$

$$= \frac{1}{\phi}\sum_i s_i\widehat{A}_i - \frac{1}{\phi}\sum_I\left(\sum_{i\in I} s_i\right)\widehat{Q}_I$$

$$\Gamma^{\bar{g}} = 0 \tag{44}$$

The granular term, purging by $\bar{g}$, is just $\Gamma^{\bar{g}} \equiv 0$.

However, the industry-demeaned granular term (eq. 34 in the paper) is:

$$\Gamma^{ind} \equiv \sum_i s_i\left(g_i - \bar{g}_{I_i}\right) = \sum_i s_i\left(\frac{1}{\phi}\widehat{Q}_i\right) = \frac{1}{\phi}\sum_i s_i\widehat{A}_i = \frac{1}{\phi}\Gamma^*. \tag{45}$$

where $\Gamma^* = \sum_i s_i\widehat{A}_i$ is the ideal granular residual.

The key results are:

$$\Gamma^{\bar{g}} = 0, \qquad \Gamma^{ind} = \frac{1}{\phi}\Gamma^* \tag{46}$$

Hence, a finite sample, $\Gamma^{\bar{g}}$ will be pure noise, whereas Γ^{ind} will contain information on the ideal granular residual Γ^*. Thus, to proxy for the ideal granular residual Γ^*, the empirical granular residual $\Gamma^{ind} = \sum_{i=1}^{K}\frac{S_{i,t-1}}{Y_{t-1}}\left(g_{it} - \bar{g}_{I_it}\right)$ will be better suited than the empirical granular residual $\Gamma^{\bar{g}} = \sum_{i=1}^{K}\frac{S_{i,t-1}}{Y_{t-1}}\left(g_{it} - \bar{g}_t\right)$.

This example is of course not general (I just considered a tractable limit case), but it illustrates the point that controlling for g_{I_t} may control better for industry-wide disturbances, such as industry-wide real price changes.

7.   Description of the Granular Residual Time-Series

The online data contains the following series.

Series 1 ("GR, industry-demeaned, $K = 100$") is the granular residual Γ_t series, as used in the paper ($K = Q = 100$), using the industry-level demeaning.

Series 2 ("GR, $K = 100$") is the granular residual Γ_t series, as used in the paper ($K = Q = 100$), using the $\bar{g}_t$ demeaning.

Series 3 ("Narrative GR") is the "narrative granular residual" that corresponds solely to the firm-year events selected in the narrative. I define the Narrative Granular Residual Γ_t^N as being 0 if no event is selected, and equal to $\Gamma_t^N = S_{it-1} \left(g_{it} - \bar{g}_{I_i t} \right) / Y_{t-1}$ if a firm i is selected for that year (or I sum over the firms if several firms are selected).

Series 4 ("GR, $K = 5$") is the granular residual based on the top 5 firms only, Γ_t ($K = 5, Q = 100$), using the industry-level demeaning.