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Behavior of quantum coherence of Ξ-type four-level atom under bang–bang control

Yan-Hui Wang [a,b], Liang Hao [a], Xiang Zhou [a], Gui Lu Long [a,c,*]

[a] Key Laboratory for Atomic and Molecular NanoSciences and Department of Physics, Tsinghua University, Beijing 100084, PR China
[b] Department of Physics, Hunan University of Science and Technology, Xiangtan, Hunan 411201, PR China
[c] Tsinghua National Laboratory for Information Science and Technology, Beijing 100084, PR China

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ABSTRACT

In this paper, we have studied the bang–bang (BB) decoupling scheme to suppress the phase decoherence, the amplitude decoherence and the general decoherence in a four-level Ξ-type atom system. The corresponding dynamical decoupling groups are given for designing the decoupling pulse sequences to suppress these three kinds of decoherence, respectively. Results show that in a proper time scale, the decoupling operations suppress the decoherence effectively. Especially in the ideal limits, it can suppress the decoherence completely. We also give the time scale in which the BB control works well. Numerical simulations show that, the larger cycle times N, the better effect of the BB decoupling operations under a fixed time scale.

1. Introduction

Decoherence is one of the major difficulties to quantum computation and quantum information [1,2]. Decoherence destroys the coherence of the quantum superposition states of systems in the process of unitary evolution and noise is detrimental to quantum algorithms [3–6]. As is well known, coherence and entanglement are the prerequisite conditions for quantum computation, and they are sources of power that enable quantum computer to surpass classical computer.

Controllability of quantum system has been studied for quite a long time [7], and methods were invented to suppress decoherence in quantum systems. Three categories of schemes to suppress the decoherence have been developed, namely error-avoiding codes [8–11], error-correcting codes [12] and quantum bang–bang control [13–15]. Error-avoiding codes encode information into the degenerate subspace of the error operators so that the information will not be affected by the error operators, while error-correcting codes restore the loss of information due to decoherence or quantum dissipation by monitoring the system and conditionally carrying on suitable feedback control. The quantum bang–bang (BB) control technique is based on the pioneering of coherent averaging effects by Haeberlen and Waugh using tailored pulse sequences to manipulate the effective Hamiltonian [16], and it has been developed into a solid traditional decoupling and refocusing techniques in nuclear magnetic resonance (NMR) [17].

Four-level system and higher dimensional system, especially the four-level atom system [18–21], have attracted much interest for their application in quantum control and quantum computation. Recently, the suppression of decoherence in quantum information processing have become a hot topic [22–29], and many works have been done to study the decoherence and construct the decoupling sequences in the spin-based qubits systems [30–35]. The dynamical decoupling methods to suppress the decoherence of the three-level systems have been discussed in [36,37]. In recent years, there have been much interest in Ξ-type four-level atom system [38]. Therefore we are motivated to generalize the BB control scheme in the four-level atom systems, generalizing the work for three level atoms in Refs. [36,37]. Here in this work, three types of decoherence models in the four-level atomic systems, the phase decoherence, the amplitude decoherence, and the general decoherence in which the phase and amplitude damping present simultaneously, have been studied. And we give the corresponding decoupling BB control group, and used them to design the decoupling pulse sequences to suppress these three kinds of decoherence. Our numerical simulations show that the decoupling scheme can suppress the decoherence effectively. We find that the smaller the interval between the adjacent BB operations, the better the suppression effect.

This paper is organized as follows: in Section 2 we give a brief account of the dynamical decoupling mechanism. In Section 3 we study the BB decoupling scheme in the ideal limits and give the concrete BB decoupling operation sequence to suppress the adiabatic decoherence, thermal decoherence and the general

* Corresponding author. Address: Key Laboratory for Atomic and Molecular NanoSciences and Department of Physics, Tsinghua University, Beijing 100084, PR China. Tel./fax: +86 10 62772692.
E-mail address: gllong@tsinghua.edu.cn (G.L. Long).

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decoherence. Then in Section 4 we numerically simulate the effect of the suppression of the decoherence in the non-ideal limits. At the end of this paper, we give a brief summary in Section 5.

2. Dynamical decoupling mechanism

Dynamical decoupling control uses the tailored unitary pulses, the impulsive full-power operations, which are termed as bang–bang controls and can be turned on/off in negligible amount of time τ_p ($\tau_p \ll \Delta t$ where Δt is the inter-operation period) with ideally arbitrarily large strength, to filter out the unwanted interactions present in the full Hamiltonian. Consequently the control over the dynamical behavior of a quantum system can be realized and the suppression of decoherence is achieved. The active dynamical control in the bang–bang limit proves to be a good tool for engineering the evolution of coupled quantum system.

A decoupler on the whole system S_{ab} composed of the atom and the bath is realized by the repeated cycle of sequences of BB control operations $\mathscr{G} = \{g_k\},\ (k = 0, \ldots, |\mathscr{G}| - 1)$ with free evolutions sandwiched between and the Hamiltonian for the free evolution is $H' = H + H_I$ where H is the Hamiltonian for the system S_a of the atom and the laser fields, and H_I is the interaction Hamiltonian between the baths and the system S_a.

We now consider a single cycle and let $\delta U = \exp(-i\Delta tH')$ denote the free evolution in the cycle which lasts for a time $\Delta t = t(N|\mathscr{G}|)^{-1}$. Here, N is the number of cycles. The decoupling operations $\{g_k\}$ are selected so as to satisfy the condition

$$\sum_{k=0}^{|\mathscr{G}|-1} g_k^\dagger H_I g_k = 0. \tag{1}$$

The evolution of the composite system S_{ab} in a single cycle time T_c under the decoupler can be described as $U(T_c) = \prod_{k=0}^{|\mathscr{G}|-1} g_k^\dagger \delta U g_k = \prod_{k=0}^{|\mathscr{G}|-1} g_k^\dagger e^{-iH'\Delta t} g_k = \exp(-i\Delta t \sum_{k=0}^{|\mathscr{G}|-1} g_k^\dagger H' g_k) = e^{-iT_c H_{\text{eff}}}$, where

$$H_{\text{eff}} = \frac{1}{|\mathscr{G}|} \sum_{k=0}^{|\mathscr{G}|-1} g_k^\dagger H' g_k. \tag{2}$$

So in the ideal limits $T_c \to 0$ and $N \to \infty$, after the application of the decoupling operators $\{g_k\}$, the evolution of the system can be described as $U(t) = \lim_{N\to\infty}[U_N(t)]^N = e^{-itH_{\text{eff}}}$ which means that we have successfully eliminated the unwanted Hamiltonian H_I from the total Hamiltonian H' and decoupled the interaction between the system S_a and its environmental bath. Therefore the quantum qubits can be immune from the environment completely by using BB control operators. In order to fulfill the goal of decoupling, the main task is to design the finite bang–bang control operators $\{g_k\}$ which satisfy the decoupling condition Eq. (1).

3. BB decoupling scheme in the ideal limits

We consider the atom in the Ξ-type as is shown in Fig. 1, where three upper levels namely $|3\rangle$, $|2\rangle$ and $|1\rangle$ are coupled to the corresponding adjacent lower levels $|2\rangle$, $|1\rangle$ and $|0\rangle$ via fields with resonance frequencies ω_{32}, ω_{21} and ω_{10}, respectively. The corresponding eigenvalues are labeled as E_3, E_2, E_1 and E_0, and we have $\omega_{32} = \frac{E_3 - E_2}{\hbar}$, $\omega_{21} = \frac{E_2 - E_1}{\hbar}$, and $\omega_{10} = \frac{E_1 - E_0}{\hbar}$. Similarly we define ω_{20}, ω_{30} and ω_{31} to be the resonant frequency for the atom and the laser field though they do not correspond to an available transition here. An example of such Ξ-type four-level scheme can be found in the energy levels of the Rubidium system, which consists of $5s_{1/2}$, $5p_{3/2}$, $5d_{3/2}$ and $5d_{5/2}$ levels where the transitions of the ladder system are $5s_{1/2} \to 5p_{3/2}(12817\ \text{cm}^{-1})$, $5p_{3/2} \to 5d_{3/2}(12885\ \text{cm}^{-1})$, $5d_{3/2} \to 5d_{5/2}(2.96\ \text{cm}^{-1})$, respectively [38].

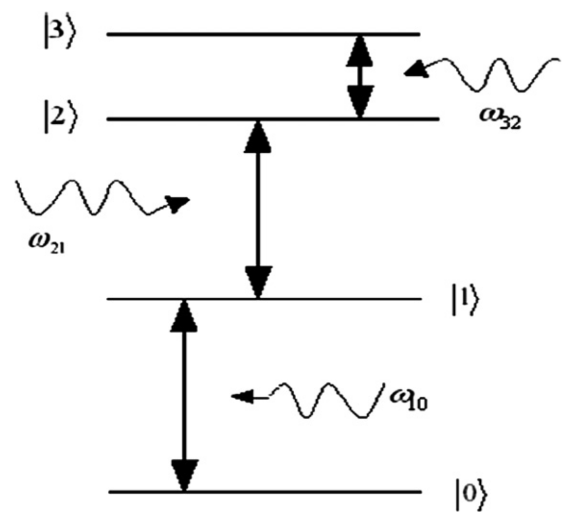

Fig. 1. Four-level atom in the Ξ-configuration shone with three fields of frequencies ω_{10}, ω_{21} and ω_{32}.

We define the following notations

$$\begin{aligned}
\sigma_z^{(n,n-1)} &= |n\rangle\langle n| - |n-1\rangle\langle n-1|,\\
\sigma_x^{(n,n-1)} &= |n\rangle\langle n-1| + |n-1\rangle\langle n|,\\
\sigma_y^{(n,n-1)} &= i(|n\rangle\langle n-1| - |n-1\rangle\langle n|),\\
\sigma_-^{(n,n-1)} &= |n-1\rangle\langle n|,\\
\sigma_+^{(n,n-1)} &= |n\rangle\langle n-1|,
\end{aligned} \tag{3}$$

where i is the the square root of minus one and n runs from 1 to 3.

The total Hamiltonian for such system can be expressed as $H = H_0 + H_{D.F.}$. Here H_0 is the four-level atom Hamiltonian and can be expressed as

$$H_0 = \sum_{n>m=0}^{3} \frac{\hbar\omega_{nm}}{4} \sigma_z^{(n,m)}. \tag{4}$$

were we have ignored the constant energy term $\sum_{n=0}^{3} E_n/4$. If we assume that the transition dipole moment for the linearly polarized electric fields are real, namely $g_{nm} = g_{mn}^*$ for simplicity, and let $\mathscr{E}_n$ be the amplitude for the electric moment, the three decoupling fields can be written as

$$H_{\text{D.F.}} = -\sum_{n=1}^{3} g_{n,n-1} \sigma_x^{(n,n-1)} \mathscr{E}_n \cos(\omega_{nn-1} t). \tag{5}$$

When the atom is exposed to reservoirs, we can describe reservoirs by a large number of uncoupled bosonic modes, i.e. a reservoir of simple harmonic oscillators, and write its Hamiltonian as

$$H_E = \sum_{i=1}^{3} \sum_{ki} \hbar\omega_{ki} a_{ki}^\dagger a_{ki}, \tag{6}$$

where a_{ki} and $a_{ki}^\dagger$ are the bosonic annihilation and creation operators of the baths. This model has had a long history of use for researching the problem of decoherence in quantum computers [10,13,31–34].

The coupling interaction Hamiltonian between the four-level system and the baths which leads to the quantum decoherence is H_I and

$$H_I = \hbar \sum_{i=1}^{3} \sum_{ki} (\alpha_{ki}\sigma_z^{(n,n-1)} + \beta_{ki}\sigma_x^{(n,n-1)}) \cdot (r_{ki} a_{ki}^\dagger + r_{ki}^* a_{ki}) \tag{7}$$

where α_{ki} and β_{ki} are the coefficients of the relative magnitude of the phase decoherence and the amplitude decoherence, respectively, and $\{r_{ki}\}$ are the coupling constants for virtual exchanges of excitations with the thermal reservoirs. We see that when $\alpha_{ki} \neq 0$ and $\beta_{ki} = 0$, the reservoir is an adiabatic reservoir which results in the phase damping; when $\alpha_{ki} = 0$ and $\beta_{ki} \neq 0$, the reservoir is a thermal reservoir which results in the amplitude damping; and when $\alpha_{ki} \neq 0$ and $\beta_{ki} \neq 0$, the reservoir is a general reservoir which results in the amplitude and phase damping simultaneously.

Then the Hamiltonian for the whole system S_{ab} takes the form

$$H' = H_0 + H_{\mathrm{D.F.}} + H_E + H_I. \tag{8}$$

According to the discussion in Section 2, in order to fulfill the goal of decoupling, the main task is to design the finite bang–bang control operators $\{g_k\}$ which satisfy the condition $\sum_{k=0}^{|\mathscr{G}|-1} g_k^\dagger H_I g_k = 0$. We find $\{g_k\}$ has the form $\mathscr{G} = \{g_0 = I, g_1 = \alpha f_1, g_2 = \alpha f_2, g_3 = \alpha f_3, g_4 = \beta v_1, g_5 = \alpha\beta f_1 v_1, g_6 = \alpha\beta f_2 v_1, g_7 = \alpha\beta f_3 v_1\}$, where $\alpha = 1$ when $\alpha_{ki} \neq 0$, otherwise $\alpha = 0$; $\beta = 1$ when $\beta_{ki} \neq 0$, otherwise $\beta = 0$. More explicitly, for the general decoherence, the decoupling operation set is $\mathscr{G} = \{g_k k = 0, 1, 2, 3, 4, 5, 6, 7\}$; for the phase damping, the decoupling operation set has another simpler form $\mathscr{G} = \{g_k k = 0, 1, 2, 3\}$; for the amplitude damping, the correspond simpler form is $\mathscr{G} = \{g_k k = 0, 4\}$. Here I is the identity operator and

$$f_1 = \begin{pmatrix} 0 & 0 & 0 & -i \\ i & 0 & 0 & 0 \\ 0 & i & 0 & 0 \\ 0 & 0 & i & 0 \end{pmatrix}, f_2 = \begin{pmatrix} 0 & -i & 0 & 0 \\ 0 & 0 & -i & 0 \\ 0 & 0 & 0 & -i \\ i & 0 & 0 & 0 \end{pmatrix},$$
$$f_3 = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{pmatrix}, v_1 = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}. \tag{9}$$

They satisfy the decoupling condition

$$\alpha \sum_{k=0}^{|\mathscr{G}|-1} g_k^\dagger \sigma_z^{(n,n-1)} g_k = 0,$$
$$\beta \sum_{k=0}^{|\mathscr{G}|-1} g_k^\dagger \sigma_x^{(n,n-1)} g_k = 0, \tag{10}$$

where i runs from 1 to 3. Combining Eqs. (7) and (10), we see that $\sum_{k=0}^{|\mathscr{G}|-1} g_k^\dagger H_I g_k = 0$. Therefore the evolution Hamiltonian for the composite system in the ideal limits of $T_c \to 0$ and $N \to \infty$ is $e^{-iH_{\mathrm{eff}} t}$, which means that the decoherence interaction between the system and its environment is completely averaged out. Therefore the mentioned decoupling operation sets can suppress their corresponding kinds of decoherence completely in the ideal limits.

The BB decoupling operators in Eq. (9) can be rewritten as

$$\begin{aligned}
f_1 &= \exp\left(i\frac{\pi}{2}\sigma_x^{(1,0)}\right)\exp\left(i\frac{\pi}{2}\sigma_x^{(2,1)}\right)\exp\left(i\frac{\pi}{2}\sigma_x^{(3,2)}\right),\\
f_2 &= \exp\left(-i\frac{\pi}{2}\sigma_x^{(3,2)}\right)\exp\left(-i\frac{\pi}{2}\sigma_x^{(2,1)}\right)\exp\left(-i\frac{\pi}{2}\sigma_x^{(1,0)}\right),\\
f_3 &= \exp\left(i\frac{\pi}{2}\sigma_x^{(1,0)}\right)\exp\left(i\frac{\pi}{2}\sigma_x^{(2,1)}\right)\exp\left(i\frac{\pi}{2}\sigma_x^{(3,2)}\right)\\
&\quad\cdot\exp\left(i\frac{\pi}{2}\sigma_x^{(1,0)}\right)\exp\left(i\frac{\pi}{2}\sigma_x^{(2,1)}\right)\exp\left(i\frac{\pi}{2}\sigma_x^{(3,2)}\right),\\
v_1 &= \exp(i\pi\sigma_z^{(1,0)})\exp(i\pi\sigma_z^{(2,1)}).
\end{aligned} \tag{11}$$

So the decoupling operators $\{g_k\}$ can be physically realized by consecutive D.F. fields that interact with state transitions $|3\rangle \leftrightarrow |2\rangle$, $|2\rangle \leftrightarrow |1\rangle$ and $|1\rangle \leftrightarrow |0\rangle$, respectively. For example, g_1 can be realized by a $\frac{\pi}{2}$-pulse at frequency ω_{32} which is followed by a $\frac{\pi}{2}$-pulse at frequency ω_{21} and another $\frac{\pi}{2}$-pulse at frequency ω_{10}. The other decoupling elements in $\mathscr{G}$ can be constructed likewise.

As is discussed in Section 2, the iteration of the sequence of BB operations with intermediate free evolution $\prod_{k=0}^{|\mathscr{G}|-1} g_k^\dagger U_0 g_k$ can be used to effectively average out the interaction Hamiltonian H_I. With the full expressions for these operators $\{g_k\}$ and the knowledge of their physical realization, we demonstrate the exact sequences of operations in a cycle in Figs. 2–4.

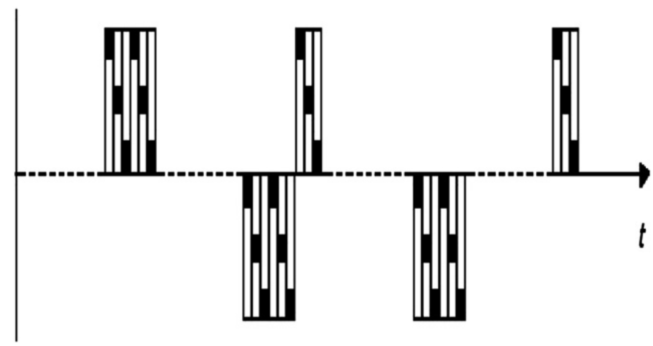

Fig. 2. A sequence of twinborn pulses operating on the four-level atom in a cycle to suppress the phase damping. The "t" axis denotes the passage of time. Among the bars above the "t" axis, the one with the top black represents the $\frac{\pi}{2}$-pulse at frequency ω_{32}, while the one with the middle black denotes the $\frac{\pi}{2}$-pulse at frequency ω_{21} and the one with the bottom black represents the $\frac{\pi}{2}$-pulse at frequency ω_{10}. Their adjoint pulses are drawn upsidedown under the "t"-axis. All these pulses are rotation operations about "x"-axis.

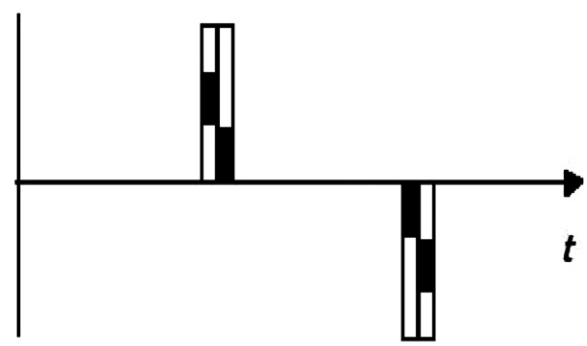

Fig. 3. A sequence of twinborn pulses operating on the four-level atom in a cycle to suppress the amplitude damping. The "t"-axis denotes the passage of time. Among the bars above the "t"-axis, the one with the middle black stands for the π-pulse at frequency ω_{21}, and the one with the bottom black denotes the π-pulse at frequency ω_{10}. Their adjoint pulses are drawn upsidedown under the "t"-axis. All these pulses are the rotations about "z"-axis.

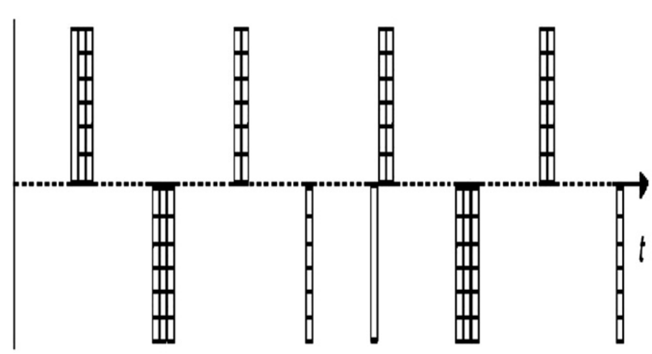

Fig. 4. A sequence of twinborn pulses operating on the four-level atom in a cycle to suppress the phase and amplitude damping simultaneously. The "t"-axis denotes the passage of time. Among the bars above the "t"-axis, hollow one represents the combined pulse set v_1 while the one with five levels embedded denotes the combined pulse set g_1. Their adjoint pulses are drawn upsidedown under the "t"-axis.

Fig. 2 shows a single cycle of the evolution of the four-level atom system under the decoupler and when $\alpha = 1$ and $\beta = 0$, i.e., the atom is exposed to an adiabatic reservoir. The evolution can be also rewritten in the following operator sequence: $g_2(t_4)^\dagger U_4(t_3+6\tau_p,t_4)g_2(t^3)\cdot g_1(t_3)^\dagger U_3(t_2+9\tau_p,t_3)g_1(t_2)\cdot g_3(t_2)^\dagger U_2(t_1+6\tau_p,t_2)g_3(t_1)\cdot U_1(t_0,t_1)$. Here we assume that the duration for the pulses is τ_p, which results in the duration for the operators $\{g_k, k = 1,2,3\}$ to be $3\tau_p$ or $6\tau_p$, and we use $U(t_i,t_j)$ to denote the free evolution of the system under the Hamiltonian H' within the time interval t_i and t_j. For we have assumed that the BB controls can be turned on/off in negligible time τ_p ($\tau_p \ll \Delta t$) so we can neglect the effect of τ_p when we calculate $U(t_i,t_j)$.

Fig. 3 shows a single cycle of the evolution of the four-level atom system under the decoupler and when $\alpha = 1$ and $\beta = 0$, i.e., the atom is exposed to a thermal reservoir. The evolution can be also rewritten in the following operator sequence: $v_1(t_2)^\dagger U_2(t_1+2\tau_p,t_2)\, v_1(t_1)\cdot U_1(t_0,t_1)$.

Fig. 4 shows a single cycle of the evolution of the four-level atom system under the decoupler and when $\alpha = 1$ and $\beta = 1$, i.e., the atom is exposed to a general reservoir. The evolution can be also rewritten in the following operator sequence: $g_1^\dagger(t_8)U(t_7,t_8)g_1(t_7)g_2^\dagger(t_7)U(t_6,\ t_7)g_2(t_6)g_3^\dagger(t_6)U(t_5,\ t_6)g_3(t_5)v_1^\dagger(t_5)U(t_4,\ t_5)v_1(t_4)v_1^\dagger(t_4)g_1^\dagger(t_4)U(t_3,\ t_4)g_1(t_3)v_1(t_3)v_1^\dagger(t_3)g_2^\dagger(t_3)U(t_2,\ t_3)g_2(t_2)v_1(t_2)v_1^\dagger(t_2)g_3^\dagger(t_2)U(t_1,t_2)g_3(t_1)v_1(t_1)U(t_0,t_1)$.

4. The numerical simulation in the non-ideal conditions

In the last section, we show the BB decoupling method to suppress the amplitude and phase decoherence and the general decoherence as well under ideal condition. Our results show that in the ideal limits the BB decoupling method can suppress all these kinds of decoherence completely. In this section, we will compare the free evolvement of $\tilde\rho(t)$ with the evolution under BB control. Then we will discuss the BB decoupling scheme in the case when the number N of the decoupling operation in a fixed duration cycle is finite. Because the typical involvement time scales of the amplitude damping is much longer than decoherence mechanisms, we can neglect the effect of this errors [13]. For the advantage of showing a clear picture of the decoherence properties, we consider the effect of the BB decoupling scheme to suppress the phase damping in the four-level atom in the Ξ-configuration.

We turn to the interaction picture. The interaction Hamiltonian H_I can be rewritten as

$$\begin{aligned}\widetilde{H}_I &= \hbar\sum_{k1}\sigma_z^{(1,0)}(j_{k1}a_{k1}^\dagger e^{i\omega_{k1}t}+j_{k1}^*a_{k1}e^{-i\omega_{k1}t})\\&+\hbar\sum_{k2}\sigma_z^{(2,1)}(j_{k2}a_{k2}^\dagger e^{i\omega_{k2}t}+j_{k2}^*a_{k2}e^{-i\omega_{k2}t})\\&+\hbar\sum_{k3}\sigma_z^{(3,2)}(j_{k3}a_{k3}^\dagger e^{i\omega_{k3}t}+j_{k3}^*a_{k3}e^{-i\omega_{k3}t}).\end{aligned}\tag{12}$$

According to Section 3, a cycle of the unitary evolution of the atom system under the decoupling operations in the interaction picture has the form

$$\begin{aligned}\widetilde{U}(t_{(0)},t_{(1)}) = &\tilde g_2(t_4)^\dagger\widetilde{U}_4(t_3+3\tau_p,t_4)\tilde g_2(t_3)\\&\cdot\tilde g_1(t_3)^\dagger\widetilde{U}_3(t_2+3\tau_p,t_3)\tilde g_1(t_2)\\&\cdot\tilde g_3(t_2)^\dagger\widetilde{U}_2(t_1+6\tau_p,t_2)\tilde g_3(t_1)\\&\cdot\widetilde{U}_1(t_0,t_1),\end{aligned}\tag{13}$$

where $t_{(0)}$ and $t_{(1)}$ are the initial and terminal points of the first cycle and $t_{(0)} = t_0$, $t_{(1)} = t_4 = t_{(0)} + 4\Delta t$. The operators in this sequence take the following form

$$\begin{aligned}\widetilde{U}(\Delta t) = &\exp(\sigma_z^{(1,0)}\sum_{k1}[a_{k1}^\dagger e^{i\omega_{k1}t_i}\xi_{k1}(\Delta t)-h.c.])\times\exp(\sigma_z^{(2,1)}\\&\times\sum_{k2}[a_{k2}^\dagger e^{i\omega_{k2}t_i}\xi_{k2}(\Delta t)-h.c.])\times\exp(\sigma_z^{(3,2)}\\&\times\sum_{k3}[a_{k3}^\dagger e^{i\omega_{k3}t_i}\xi_{k3}(\Delta t)-h.c.]),\end{aligned}\tag{14}$$

where $\Delta t = t_i - t_j$ is inter-operation period during which the system undergoes free evolution and $\xi_{ki}(\Delta t)=\frac{j_{ki}}{\omega_{ki}}(1-e^{i\omega_{ki}\Delta t})$. Look at the D.F. field, which is used to offer BB operators, during τ_p we can define

$$H_{\rm D.F.}(\omega,t) = V_i\sigma_x^{(n,n-1)}\cos[\omega_{n\ n-1}(t)].\tag{15}$$

In the interaction picture, by rotating wave approximation (RWA) method, We get

$$\widetilde{H}_{\rm D.F.}(\omega,t) = e^{\frac{i}{\hbar}H_0t}H_{\rm D.F.}(\omega,t)e^{-\frac{i}{\hbar}H_0t}\approx\frac{1}{2}V_i\sigma_x^{(n,n-1)}.\tag{16}$$

The evolution operator during τ_p is

$$\widetilde{U}_p\approx e^{-\frac{i}{\hbar}\widetilde{H}_{\rm D.F.}(\omega,t)\tau_p}\approx e^{-\frac{i}{2\hbar}V_i\sigma_x^{(n,n-1)}\tau_p}.\tag{17}$$

Choosing appropriate τ_p to satisfy $\frac{V_i\sigma_x^{(n,n-1)}\tau_p}{2\hbar}=\pm\pi$ and arrange appropriate the order of i, we get $\tilde g_i(t_j)$ and $\tilde g_i^\dagger(t_j)$ in the interaction picture. For example

$$\tilde g_1(t_j) = \exp\left(i\frac{\pi}{2}\sigma_x^{(1,0)}\right)\exp\left(i\frac{\pi}{2}\sigma_x^{(2,1)}\right)\exp\left(i\frac{\pi}{2}\sigma_x^{(3,2)}\right).\tag{18}$$

So

$$\begin{aligned}&\tilde g_i(t_j) = g_i\\&\tilde g_i^\dagger(t_i) = g_i^\dagger.\end{aligned}\tag{19}$$

After N cycles of decoupling operations performed on the atom, we obtain its evolution operator

$$\widetilde{U}^{(N)}(t_{(0)},t_{(N)}) = \widetilde{U}(t_{(N-1)},t_{(N)})\cdots\widetilde{U}(t_{(1)},t_{(2)})\widetilde{U}(t_{(0)},t_{(1)}),\tag{20}$$

where $t_{(N)} = t_{(0)} + 4N\Delta t + 24N\tau_p$, and $t_{(N)}$ is the ending time of the whole N decoupling operation cycles.

To observe the suppression effect of the BB decoupling operations on the atom of the adiabatic decoherence, we call for the reduced density matrix of the atom $\tilde\rho_{23}^s(t) = {\rm Tr}_E\{\langle 2|\tilde\rho_{SE}(t)|3\rangle\}$ with $t = t_{(N)} - t_{(0)} = 4N(\Delta t + 6\tau_p)$ and set $t_{(0)} = 0$. We assume the composed system of the atom and the environment is initially in the product state $\tilde\rho_S(0)\otimes\tilde\rho_E(0)$, where $\tilde\rho_E(0)$ is a kind of thermal equilibrium state which can be factorized into the tensor product of the density operators of each mode

$$\tilde\rho_E(0) = \prod_k\theta_k\tag{21}$$

with

$$\begin{aligned}Z_k &= \left[1-\exp\left(-\frac{\hbar\omega_k}{\kappa_B T}\right)\right]^{-1}\\\theta_k &= Z_k^{-1}\exp\left(-\frac{\hbar\omega_k a_k^\dagger a_k}{\kappa_B T}\right),\end{aligned}$$

where k_B is the Boltzman constant and T is the temperature of the bath.

Using the formula $U^\dagger\exp(iA)U = \exp(iU^\dagger AU)$ and with Eqs. (13)–(21), we obtain

$$\begin{aligned}\tilde\rho_{23}^s(t) &= {\rm Tr}_E\{\langle 2|\widetilde{U}(t_{(0)},t_{(N)})\tilde\rho_S(0)\otimes\tilde\rho_E(0)\widetilde{U}^\dagger(t_{(0)},t_{(N)})|3\rangle\}\\&= \tilde\rho_{23}^s(0){\rm Tr}_E\left[\tilde\rho_E(0)\exp\left\{\sum_{k1}a_{k1}^\dagger e^{i\omega_{k1}t_0}\eta_1(k_1,\Delta t,\tau_p)\right.\right.\\&\quad\left.\left.+\sum_{k2}a_{k2}^\dagger e^{i\omega_{k2}t_0}\eta_2(k_2,\Delta t,\tau_p)+\sum_{k3}a_{k3}^\dagger e^{i\omega_{k3}t_0}\eta_3(k_3,\Delta t,\tau_p)-h.c.\right\}\right]\\&= \tilde\rho_{23}^s(0)\exp\{-\Gamma_1(\Delta t,\tau_p)-\Gamma_2(\Delta t,\tau_p)-\Gamma_3(\Delta t,\tau_p)\},\end{aligned}\tag{22}$$

where

$$\Gamma_i(\Delta t,\tau_p)=\frac{1}{2}\sum_{ki}|\eta_i(k_i,\Delta t,\tau_p)|^2\coth\frac{\omega_{ki}}{2T},\tag{23}$$

$$\eta_i(k_i,\Delta t,\tau_p)=\sum_{n=1}^{N}e^{i\omega_{ki}4(n-1)(\Delta t+6\tau_p)}\cdot\xi_{ki}(\Delta t)\chi_{ki}(\Delta t,\tau_p),\tag{24}$$

and

$$\chi_{k1}(\Delta t,\tau_p)=-2e^{i\omega_{k1}(\Delta t+6\tau_p)}+e^{i\omega_{k1}(2\Delta t+15\tau_p)}+e^{i\omega_{k1}(3\Delta t+21\tau_p)},$$
$$\chi_{k2}(\Delta t,\tau_p)=1+e^{i\omega_{k2}(\Delta t+6\tau_p)}-2e^{i\omega_{k2}(3\Delta t+21\tau_p)},$$
$$\chi_{k3}(\Delta t,\tau_p)=-2+e^{i\omega_{k3}(2\Delta t+15\tau_p)}+e^{i\omega_{k3}(3\Delta t+21\tau_p)}.\tag{25}$$

On the other hand, the density matrix without the BB controls is

$$\tilde\rho^{sw}_{23}(t)=\tilde\rho^{s}_{23}(0)\exp\{-\Gamma'_1(\Delta t)-\Gamma'_2(\Delta t)-\Gamma'_3(\Delta t)\},\tag{26}$$

where

$$\Gamma'_1(\Delta t)=0,$$
$$\Gamma'_2(\Delta t)=\frac{1}{2}\sum_{k2}\coth\left(\frac{\omega_{k2}}{2T}\right)\left|\frac{j_{k2}}{\omega_{k2}}(1-e^{i\omega_{k2}4N\Delta t})\right|^2,$$
$$\Gamma'_3(\Delta t)=2\sum_{k3}\coth\left(\frac{\omega_{k3}}{2T}\right)\left|\frac{j_{k3}}{\omega_{k3}}(1-e^{i\omega_{k3}4N\Delta t})\right|^2.\tag{27}$$

We can define the finite cut-off frequency of each mode of the environment is ω_c such that the spectral density $I(\omega)\to 0$ for $\omega>\omega_c$ [13]. In the continuum limit of the bath mode, we take the following transformation [13,30]

$$\sum_k\mapsto\int_0^{\omega_c}d\omega I(\omega)\frac{1}{|j(\omega_k)|^2}.\tag{28}$$

Then Eqs. (23) and (27) change into

$$\Gamma_i(\Delta t,\tau_p)=\frac{1}{2}\int_0^{\omega_c}d\omega_{ki}I(\omega_{ki})\coth\left(\frac{\omega_{ki}}{2T}\right)\cdot\left|\frac{\chi_{ki}(\Delta t,\tau_p)}{\omega_{ki}}(1-e^{i\omega_{ki}\Delta t})\sum_{n=1}^{N}e^{i\omega_{ki}4(n-1)(\Delta t+6\tau_p)}\right|^2,$$
$$\Gamma'_2(\Delta t)=\frac{1}{2}\int_0^{\omega_c}d\omega_{k2}I(\omega_{k2})\coth\left(\frac{\omega_{k2}}{2T}\right)\cdot\left|\frac{1}{\omega_{k2}}(1-e^{i\omega_{k2}4N\Delta t})\right|^2,$$
$$\Gamma'_3(\Delta t)=2\int_0^{\omega_c}d\omega_{k3}I(\omega_{k3})\coth\left(\frac{\omega_{k3}}{2T}\right)\cdot\left|\frac{1}{\omega_{k3}}(1-e^{i\omega_{k3}4N\Delta t})\right|^2.\tag{29}$$

In order that the BB decoupling scheme works, we need

$$\Gamma_1+\Gamma_2+\Gamma_3<\Gamma'_1+\Gamma'_2+\Gamma'_3.\tag{30}$$

Considering the case where the bath is the generally considered Ohmic bath and we have the spectral density for each mode of the bath to be $I(\omega)=\frac{\alpha}{4}\omega^n e^{-\omega/\omega_c}$, where α measures the strength of the system-environment interaction and $n=1$ [13,31–33]. Noting that Γ_i and Γ'_i are independent on the difference in ω_{ki}, we change ω_{ki} in Eqs. (25) and (30) to ω. As a result, $\chi_{ki}(\Delta t)$ changes to $\chi_i(\omega,\Delta t,\tau_p)$ and Eq. (30) becomes

$$|\chi_1(\omega,\Delta t,\tau_p)|^2+|\chi_2(\omega,\Delta t,\tau_p)|^2+|\chi_3(\omega,\Delta t,\tau_p)|^2<5|1+e^{i\omega\Delta t}+e^{i\omega2\Delta t}+e^{i\omega3\Delta t}|^2.\tag{31}$$

We can get $cos(\omega_c\Delta t)>0.6506$ from Eqs. (23)–(31) when $\tau_p\ll\Delta t$. So when the time scale Δt of the inter-operation period of the operations satisfies $\omega_c\Delta t\in[0,\arccos(0.6506)]$, the sum of the decoherence factors $\Gamma_1+\Gamma_2+\Gamma_3$ under the effect of the BB decoupling operations is much smaller than the one without the BB decoupling operations $\Gamma'_1+\Gamma'_2+\Gamma'_3$.

Figs. 5 and 6 show that we can use BB decoupling operations to suppress the decoherence effectively. The duration of the D.F. pulse (τ_p) can be as short as 100 ns. The delay between the pulses (Δt) is adjustable. T = 190 K which is the temperature used in the experiment [22]. The value of α has been used in [13]. The value of ω_c depends on the specific physical system, which has been discuss in detail in [14]. N is an adjustable parameter. When the total evolution duration t is fixed, N should satisfy the condition $\omega_c t/N|\mathscr{G}|\in[0,\arccos(0.6506)]$ to ensure the efficiency of the decoupling operations.

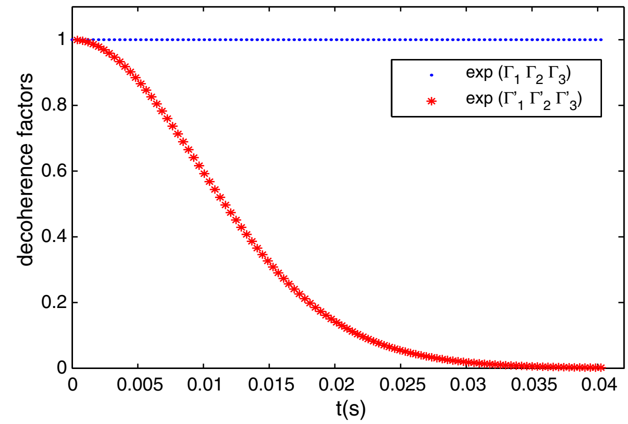

Fig. 5. The dotted pattern show the evolvement of $e^{-\Gamma_1-\Gamma_2-\Gamma_3}$ with t under the effect of the BB decoupling operations. The starred pattern show the evolvement of $e^{-\Gamma'_1-\Gamma'_2-\Gamma'_3}$ with t freely. The correlated parameters are Δt = 100 μs, τ_p = 100 ns, N = 100, T = 190 K, α = 0.35 and ω_c = 100 Hz.

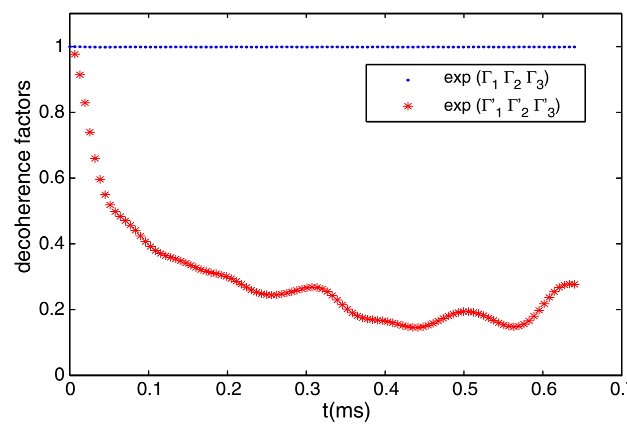

Fig. 6. The dotted pattern show the evolvement of $e^{-\Gamma_1-\Gamma_2-\Gamma_3}$ with t under the effect of the BB decoupling operations. The starred pattern show the evolvement of $e^{-\Gamma'_1-\Gamma'_2-\Gamma'_3}$ with t freely. The correlated parameters are Δt = 1 μs, τ_p = 100 ns, N = 100, T = 190 K, α = 0.35 and ω_c = 100 KHz.

We numerically simulate $\log(1-e^{-\Gamma_1-\Gamma_2-\Gamma_3})$, the function of the minus power of the sum of the decoherence factors, under different values of the parameters in the fixed duration t in Figs. 7–9.

Fig. 7 shows how the different values of ω_c affect decoupling effect. It is clear that the value of $\log(1-e^{-\Gamma_1-\Gamma_2-\Gamma_3})$ becomes bigger, which corresponds to a worse decoupling effect, and the needed cycles times in the fixed time duration t becomes larger in order that the decoupling operations take effect, as ω_c increases. This accords with our theory.

Fig. 8 shows the effect of the different values of T on decoupling effect. We see that the higher the temperature, the worse the decoupling effect is, which is in accord with the low temperature condition needed in our discussion.

Fig. 9 shows the decoupling effect varies with α. Since α measures the strength of the system-environment interaction, it is easy

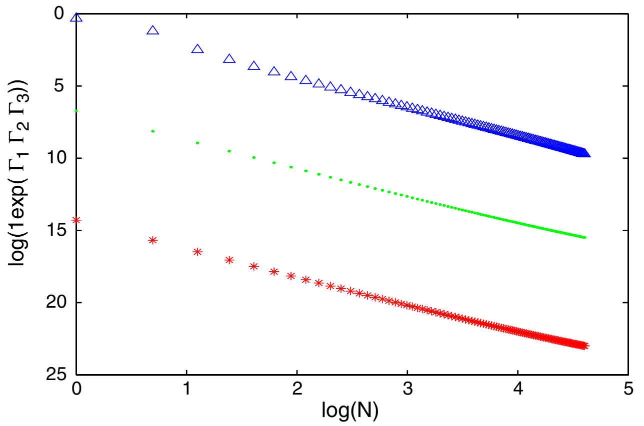

Fig. 7. The logarithm of $1-e^{-\Gamma_1-\Gamma_2-\Gamma_3}$ under the effect of the BB decoupling operations. The correlated parameters are $\tau_p = 100$ ns, $t = 1$ ms, $T = 190$ K, $\alpha = 0.35$ and $\omega_c = 100$ Hz for the starred pattern, $\omega_c = 1$ kHz for the dotted pattern, $\omega_c = 100$ kHz for the triangled pattern.

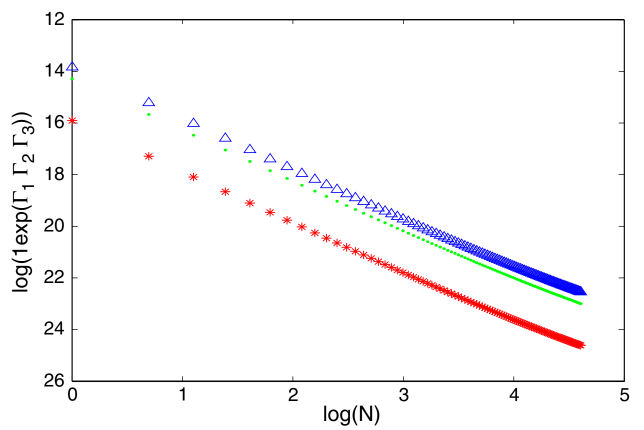

Fig. 8. The logarithm of $1-e^{-\Gamma_1-\Gamma_2-\Gamma_3}$ under the effect of the BB decoupling operations. The correlated parameters are $\tau_p = 100$ ns, $t = 1$ ms, $\omega_c = 100$ Hz, $\alpha = 0.35$ and $T = 20$ K for the starred pattern, $T = 190$ K for the dotted pattern, $T = 300$ K for the triangled pattern.

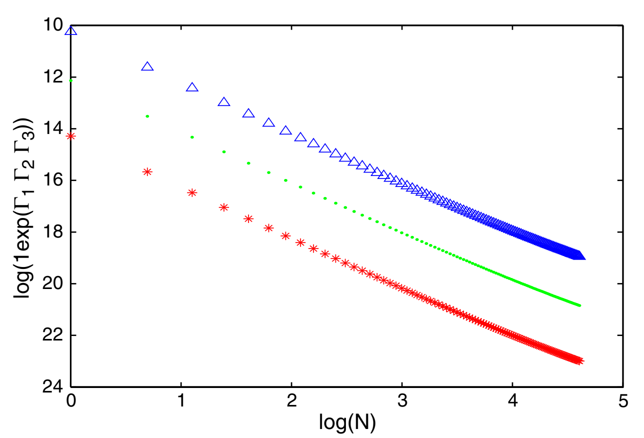

Fig. 9. The logarithm of $1-e^{-\Gamma_1-\Gamma_2-\Gamma_3}$ under the effect of the BB decoupling operations. The correlated parameters are $\tau_p = 100$ ns, $t = 1$ ms, $\omega_c = 100$ Hz, $T = 190$ K and $\alpha = 0.35$ for the starred pattern, $\alpha = 3$ for the dotted pattern, $\alpha = 20$ for the triangled pattern.

to understand $\log(1-e^{-\Gamma_1-\Gamma_2-\Gamma_3})$ becomes bigger and we need to perform more cycles of BB decoupling operations to obtain the same decoupling effect as α increases.

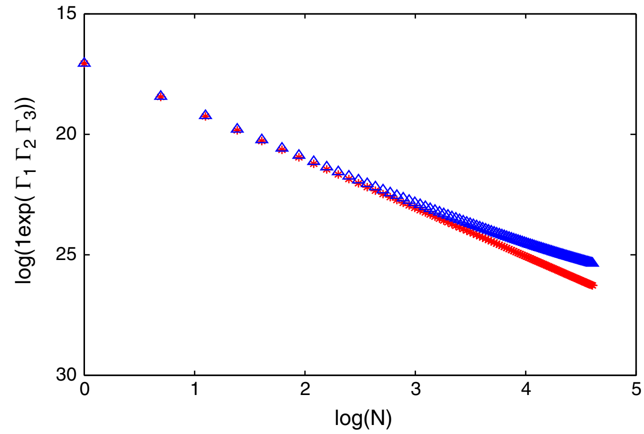

Fig. 10. The logarithm of $1-e^{-\Gamma_1-\Gamma_2-\Gamma_3}$ under the effect of the BB decoupling operations. The correlated parameters are $t = 1$ ms, $\omega_c = 100$ Hz, $T = 190$ K, $\alpha = 0.35$ and $\tau_p = 0$ s for the starred pattern, $\tau_p = 100$ ns for the triangled pattern.

Fig. 10 shows that duration of the pulse (τ_p) almost has no influence when $\tau_p \ll \Delta t$, so we can neglect it in the permissible error range. But when t is fixed, as N becomes bigger, the Δt approaches the same order as τ_p. Then we should consider the effect of τ_p.

All these figures show that in the permissible error range, when $\tau_p \ll \Delta t$ and N is large enough to let $\omega_c \Delta t \in [0, \arccos(0.6506)]$, $\log(1-\exp(-\Gamma_1-\Gamma_2-\Gamma_3))$ decreases linearly as $\log N$ increase. It is obvious that the larger the cycle times "N" in the fixed duration "t", the closer to 1 the value of " $\exp(-\Gamma_1-\Gamma_2-\Gamma_3)$", and the better the effect of suppression of the decoherence as well. Especially, in the ideal limits, $\tau_p \to 0$, $\Delta t \to 0$ ($N \to \infty$), the decoherence is completely suppressed.

5. Summary

In this paper, we have proposed a decoupling bang–bang scheme for the suppression of the phase damping, the amplitude damping and the general decoherence in a four-level Ξ-configuration atom system. We have designed sequences of periodic twin-born pulses to suppress each kind of decoherence. The detailed discussion shows that the proposed BB decoupling operations can effectively suppress all these three kinds of decoherence. Furthermore, our numerical simulation shows that we can get better suppression effect if we enlarge the operation frequency. Furthermore, in the ideal limit $\tau_p \to 0$, $\Delta t \to 0 (N \to \infty)$, we can suppress the decoherence completely. What is more, the decoupling effect will be better when ω_c, T and α decrease. We also give the effective time scale of the BB decoupling operations, which is helpful to the experimental consideration. Provided that the decoupling operation sequence is performed properly, the coherence and entanglement of the system can be preserved.

Compared to the case with three-level quantum systems, the BB control sequence becomes more complex. Under general decoherence, the number of BB control operation are 6 and 8 for the three-level and four-level atom, respectively. It will be interesting to see how the number of BB control operation increase as the number of levels increases in a quantum system.

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