EPS 259 Final project report: Dimensional analysis on the flash heating weakening mechanism

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1 Introduction and motivation

Earthquake occurs due to the unstable sliding on the frictional fault surface. The term "unstable" corresponds to some weakening processes, that is, the friction stress/fault strength decreases as the sliding happens. Because of this weakening process, the accumulated pre-stress on fault keeps exceeding the friction thus the initial sliding finally evolves into an earthquake.

Various weakening mechanisms and models have been proposed by previous studies. For example, the slip weakening model [1, 6, 7] relates the friction to the dislocation/slip u that the friction decreases from the static friction to kinematic friction within a critical slip D_C. Later based on the results from laboratory experiment, it is found that the friction is related to the slip rate and some state variables [2, 4]. Therefore, the Rate-and-State Friction (RSF) [11] has been proposed and now is widely used to explain the interactions between different stages of earthquake cycles [8]. For the weakening process in the coseismic period, the RSF mainly relates the friction decrease to the slip rate v and other dimensionless state variables, which evolve with time t. Although RSF can explain most phenomenons of earthquake science, the physical mechanism of RSF is not well addressed until recently, the more physical dynamic weakening mechanism (DW) has been proposed [9, 12] and validated in the laboratory for field samples [13]. The DW mainly includes flash heating, thermal pressurization and thermal decomposition, and all these mechanisms are controlled by frictional heat, pore fluid and even chemical composition of minerals in the fault zone.

The development of fault weakening theories actually reveals a very interesting trend that people are understanding fault friction with different physics, from dynamics to thermodynamics and finally poro-thermo-chemo-dynamics. Although there are lots of brilliant studies, either theoretically or numerically, solving for weakening mechanisms with multiple physics, it is still very interesting to apply the dimensional analysis to analyze this system. With the clear understanding on the corresponding physics, the dimensional analysis can naturally serve as a powerful tool to seek for the relation between different physics.

In this project, I will apply dimensional analysis to analyze the Flash Heating Weakening (**FHW**) processes during earthquake rupture. FHW occurs between the contacting fault asperities in the μm scale. As the dislocation of two contacting asperities occurs, frictional heating is produced and increases the temperature on the contacting interface. At a critical temperature Θ_W, above which small scale (μm) fast melting occurs on the asperity interface, the shear strength τ_c thus the fault friction f decreases greatly. In this project, we only consider the case of homogeneous dry sample. Therefore, only the **dynamic** and **thermal diffusion** processes dominates in the system with some critical state variables. There are two main reasons to focus on the flash heating weakening: Firstly, the **FHW** is the dynamic weakening mechanism that has been confirmed by both theoretical and laboratory studies. Better understanding on FHW is important for improving our understandings on the earthquake mechanisms. As far as I know, the other dynamic weakening mechanisms, however, have not been well observed by laboratory experiments. Secondly and most importantly, dimensional analysis is extremely powerful to deal with the problems with reasonable number of physical governing parameters. For the thermal pressurization and thermal decomposition mechanisms, there are just too many governing parameters in the systems, and in such situations the dimensional analysis can hardly provide important insights before some simplifications, which may worth some future efforts but not in the scope of this project. Lastly, I have to emphasize that the flash heating weakening is different from another melt lubrication weakening mechanism, in which the mineral phase change and viscosity are probably very important. Due to the lack of pseudotachyliytes found by geological observations, the melt lubrication weakening is thought not dominant during earthquake rupture and is not discussed here.

The goal is to find the dimensionless form for the fault strength drop $\Delta\tau_c$

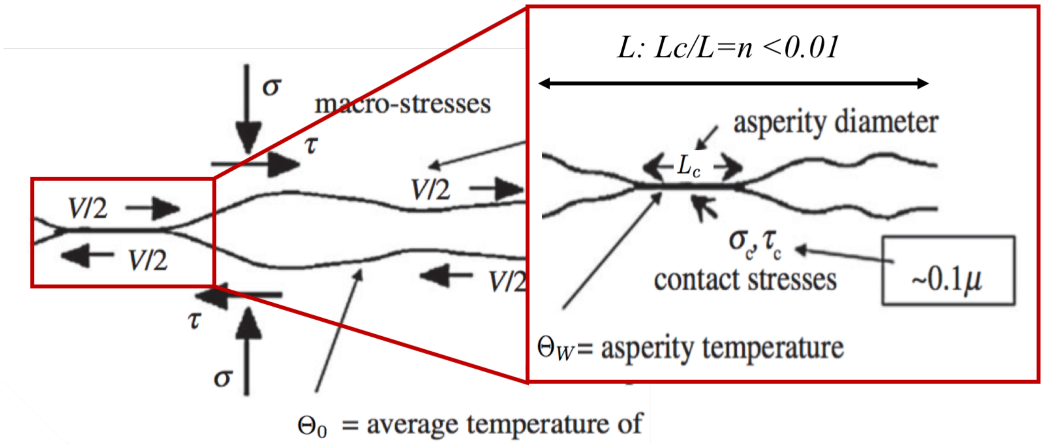


Figure 1: Mechanism of friction in various scales. It is measured from macro-stresses but determined by the asperity strengths, which are material properties. The symbol μ is the shear modulus of material. Other terms are explained in details in the text. Figure is modified based on [10].

due to FHW. I will figure out the interactions of multiple physical mechanisms (dynamics and thermal diffusion) during fault weakening and try to find some characteristic quantities for each mechanism. By comparing with the theoretical solution from previous studies [9] as well as measurements from lab experiments [5], I will validate my results from dimensional analysis and discuss the differences between them.

2 Dimensional analysis on flash heating weakening

2.1 Friction in the μm scale

To apply the dimensional analysis correctly, it is important to clarify the physics of FHW. Since the FHW is a weakening process in the μm scale, we need to consider the friction in the same scale (Fig.1). Friction in the μm scale can be quantified by the ratio f between maximum sustainable shear stress (τ_c), and normal stress (σ_c) that is at contact indentation of a stronger material. In the following sections of this report, the maximum sustainable shear and normal stress (τ_c and σ_c) are also called asperity shear and normal strength, respectively, to emphasize their differences from the exterior macroscopic loading stresses. The average length scale L_c of asperity is important because it is the size of asperity where the stresses really act. By balancing traction from the exterior macroscopic stresses (τL and σL, L is the average spatial wavelength of asperity distribution) acting on the fault with length scale L, and stresses on asperity ($\tau_c L_c$ and $\sigma_c L_c$), we have the following relation defining the friction coefficient during frictional sliding:

$$f = \frac{\tau}{\sigma} = \frac{\tau_c}{\sigma_c} \tag{1}$$

Therefore, the observed weakening (decrease of friction coefficient Δf) is just from the transmission of shear strength change $\Delta\tau_c$:

$$\Delta f = \frac{\Delta\tau_c}{\sigma_c} \tag{2}$$

In the following analysis and discussion, I will focus on the $\Delta\tau_c$.

2.2 Governing parameters and dimensional analysis

To apply dimensional analysis, I need to first clarify the assumptions and determine the governing parameters by analyzing the physics of FHW process. I have several main assumptions listed below:

1. As mentioned in the introduction, only the **dynamic** and **thermal diffusion** processes are considered here. Other mechanisms such as mineral phase change (large scale melting), thermal decomposition are ignored so I assume that the corresponding form of energy (latent heat, chemical energy etc.) are negligible.

2. Homogeneous dry medium is assumed here. All material properties except strength are constants and there is no need to consider viscosity.

3. I only consider the constant slip velocity v case, or more generally, the step-varying v cases that the slip velocity stays unchanged during most of the period.

4. Frictional heat is generated and focused at the asperity.

Table 1: **MLTK system**

Variable name	**Symbol**	**Dimension**
velocity	v	LT^{-1}
time	t	T
normal strength	σ_c	$ML^{-1}T^{-2}$
initial friction	f_0	1
asperity size	L_c	L
temperature rise	$\Delta\Theta = \Theta_W - \Theta_0$	K
heat capacity	c_p	$L^2T^{-2}K^{-1}$
thermal conductivity	c_{th}	$MLT^{-3}K^{-1}$

Furthermore, the governing parameters of this physical process have to be addressed. We need to include the slip velocity v because the heat in FHW is from the frictional sliding. As a dynamic process, we also need to include some time scale t in the analysis. Since the asperity plays the key role during FHW, the asperity size L_c, normal strength σ_c and shear strength change on asperity $\Delta\tau_c$ are also included. All these parameters are dynamic parameters. Besides, we need to include more thermal parameters to sufficiently describe the thermal diffusion process: ambient temperature Θ_0, weakening temperature Θ_W, specific heat capacity c_p, thermal conductivity c_{th}. Because only the temperature rise really matters instead of the absolute temperature for consideration of heat, I only consider the temperature rise $\Delta\Theta = \Theta_W - \Theta_0$ in the following analysis. Finally, the initial static friction coefficient f_0, as an important parameter that links the mechanical and thermal processes, should be included in the analysis. In this project, I choose the shear strength change $\Delta\tau_c$ as the targeting parameter of my dimensional analysis. Associating all other parameters and their dimensions, we can have the dimensional analysis table (Table 1) in the MLTK system. In the MLTK system, we have 8 governing parameters (v, t, σ_c, f_0, L_c, $\Delta\Theta$, c_p, c_{th}) and 4 variables with independent dimensions of mass, length, time and temperature. Therefore, there will be $8 - 4 = 4$ dimensionless variable governing the FHW process based on the Buckingham Pi theorem.

By analyzing the physics of FHW, we can actually further simplify the system. The term of temperature rise $\Delta\Theta$ here is only a quantification of the thermal energy, that is, how much heat is needed when FHW process occurs. Therefore, we can group the $\Delta\Theta$ with heat capacity (c_p) and density (ρ, which is a constant based on our homogeneous simplification of the medium)

Table 2: **MLT system**

Variable name	**Symbol**	**Dimension**
velocity	v	LT^{-1}
time	t	T
normal strength	σ_c	$ML^{-1}T^{-2}$
initial friction	f_0	1
asperity size	L_c	L
heat density	φ	$ML^{-1}T^{-2}$
thermal diffusivity	α_{th}	M^2T^{-1}

to get the heat density term $\varphi = \Delta\Theta c_p \rho$. Accordingly, we can use the thermal diffusivity $\alpha_{th} = \frac{c_{th}}{\rho c_p}$ to quantify the diffusion of thermal energy, which is equivalent to the thermal conduction but just in term of energy participation. With these simplification, we can update our dimensional analysis table (Table 2). Although the number of final dimensionless variables stays the same 4 (7 governing parameters − 3 variables with independent dimensions of MLT = 4), the system has been transformed from MLTK to MLT system and this simplification can help us build clearer understanding on the FHW physics.

Here I choose the velocity v, time scale t and normal stress σ_c as the variables with independent dimensions. The targeting quantity is the shear strength drop $\Delta\tau_c$, which has the same dimension as the normal strength σ_c ($ML^{-1}T^{-2}$) and a general functional form $\Delta\tau_c = F(v, t, \sigma_c, L_c, \varphi, \alpha_{th}, f_0)$. The dimensionless initial friction coefficient f_0 is naturally a governing dimensionless variable in the system. With simple calculation, we can easily get the dimensionless form for the shear strength drop $\Delta\tau_c$:

$$\frac{\Delta\tau_c}{\sigma_c} = \Phi(\frac{L_c}{vt}, \frac{\alpha_{th}}{v^2 t}, \frac{\varphi}{\sigma_c}, f_0) \tag{3}$$

With this choice of parameters, the shear strength drop is scaled with normal strength $\Pi = \frac{\Delta\tau_c}{\sigma_c}$, and the FHW process under my assumptions of simplification is controlled by four dimensionless parameters: $\Pi_1 = \frac{L_c}{vt}$, $\Pi_2 = \frac{\alpha_{th}}{v^2 t}$, $\Pi_3 = \frac{\varphi}{\sigma_c}$ and $\Pi_4 = f_0$. This is the dimensionless form of the flash heating weakening mechanism.

2.3 Two physical timescales

The power of dimensional analysis is not only to give us a neat but vague form of different quantities, but also to help us to find out important factors that governs the physics. In this section, I will try to start from results of dimensional analysis to find some interesting features of FHW.

To step further from the dimensional analysis results, we need to figure out the physical meanings of each dimensionless Π. The first term $\Pi_1 = \frac{L_c}{vt}$ in Eq.(3) is very clear. It describes the kinematic process during the FHW: the asperity size L_c scales with the product vt. Thus the Π_1 term naturally gives a characteristic timescale $\theta = L_c/v$ (I define θ as the kinematic timescale and call it contacting lifetime), during which the two asperities on both sides of a fault segment keep contacting and this is the necessary condition for occurrence of FHW. The second term $\Pi_2 = \frac{\alpha_{th}}{v^2 t}$ describes the heat diffusion process during frictional sliding and has another timescale $T_W \propto \frac{\alpha_{th}}{v^2}$. T_W is defined as the flash heating weakening time, which quantifies how long the asperity takes to get weakened. Either lower thermal diffusivity α_{th} (heat is hard to diffuse out) or higher slip velocity v (more frictional heat production) can shorten the weakening time T_W. $\Pi_3 = \frac{\varphi}{\sigma_c}$ is another thermal-related term and basically describes the thermal state of the system (heat density φ scaled by the normal strength σ_c). The last $\Pi_4 = f_0$ is intrinsically a dimensionless parameter that governs the physical process. Because the last 3 Πs are all related to thermal dynamics, I simply group these three terms together to "guess" the general functional form for the weakening time T_W:

$$T_W = \frac{\alpha_{th}}{v^2} g(\frac{\varphi}{\sigma_c}, f_0), \tag{4}$$

where g is an unknown function with two variables. The two timescales with different physics revealed by dimensional analysis on FHW imply that significant transition can occur in this system:

1. If $\theta < T_W$, the weakening time is longer than contacting lifetime, the asperity has no sufficient time to get weakened.

2. If $\theta > T_W$, the contacting lifetime is longer than weakening time, the asperity can sufficiently get weakened and FHW can happen.

Thus we have a critical point at which the transition begins:

$$T_W = \theta = \frac{\alpha_{th}}{v^2} g(\frac{\varphi}{\sigma_c}, f_0) = \frac{L_c}{vt}, \tag{5}$$

and we have a corresponding characteristic slip velocity V_W:

$$V_W = \frac{\alpha_{th}}{L_c} g(\frac{\varphi}{\sigma_c}, f_0) \tag{6}$$

This is a generalized form of the weakening slip velocity as a function of only material properties. Therefore, this slip velocity is determined by material itself. FHW can occur only when the actual slip velocity $v \geq V_W$ (i.e., $\theta \geq T_W$). This neat form of solution just shows the power of dimensional analysis, and is helpful for us to better understand the physics of FHW or guide the design of relevant laboratory experiments. I will discuss this in the next section.

3 Discussion

3.1 Comparison with the theoretical solution

Fortunately, we have an early theoretical study on the phenomenon of FHW [9]. In this work, the author provides a systematical study on most of heat-related weakening processes (flash heating, thermal pressurization and decomposition). Here I will first compare the dimensional analysis results with the theoretical solution. Using the equations of one-dimensional heat conduction for a planar heat source [3] with heat input into the solids on both sides of the contact interface at rate $\tau_c v/2$, we have the following theoretical solutions of T_W and V_W from [9]:

$$T_W = \frac{\pi \alpha_{th}}{v^2} (\frac{\varphi}{\tau_c})^2 = \frac{\pi \alpha_{th}}{v^2} (\frac{\varphi}{\sigma_c f_0})^2 \tag{7}$$

$$V_W = \frac{\pi \alpha_{th}}{L_c} (\frac{\varphi}{\tau_c})^2 = \frac{\pi \alpha_{th}}{L_c} (\frac{\varphi}{\sigma_c f_0})^2 \tag{8}$$

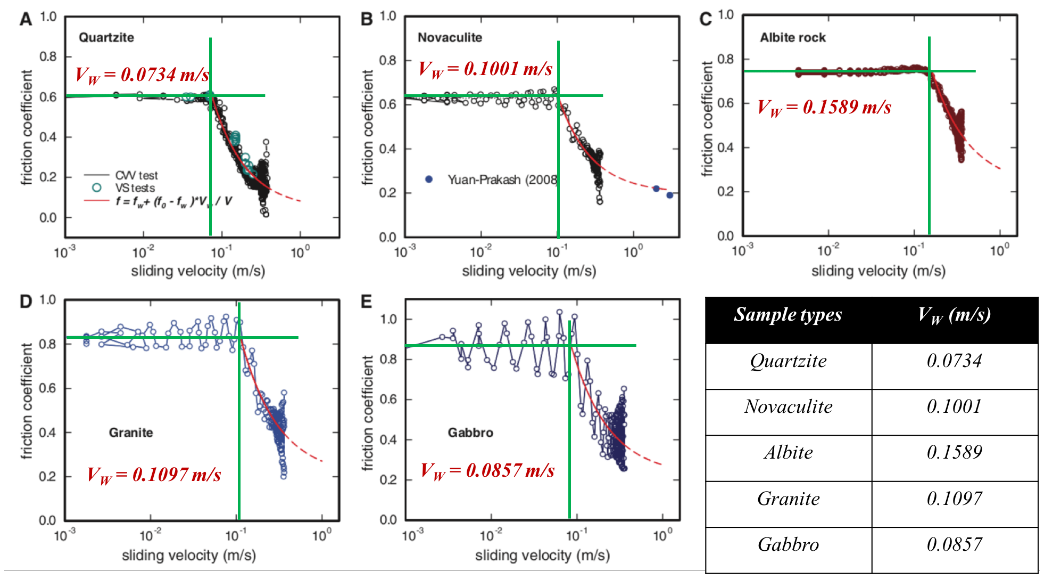
Comparing Eqs.(5)-(6) with Eqs.(7)-(8), the results from dimensional analysis have a consistent functional forms with those from theoretical solutions. With correct physical analysis, this should definitely happen, as James R. Rice said on EPS202 class, "mother nature doesn't know what units we human are using." The dimensionless quantities without units are those really governing the physical processes in nature. So it is obvious that not only this one-dimensional solution, more complex solutions (2D, 3D) should follow the dimensionless forms of Eqs.(5)-(6), if the aforementioned assumptions still hold.

On the other hand, for some problems with very clear physics but are mathematically hard to solve, dimensional analysis can provide an easier way gain physical insights. Like this FHW problem, we can directly get a generalized solution. Starting from the generalized solution, we can substitute the generalized functional forms into the equations to simplify the problem. Some examples of similarity that can help to convert partial derivative equations to ordinary derivative equations are shown during this class. As I will try in the next section, we can also use the dimensional analysis results to directly analyze the real measurements and try to infer the unknown functional form for g in Eqs.(5)-(6).

Finally, I have to mention that the dimensional analysis cannot solve every problem completely. For the FHW case, we can never get the constant π and know where to put the dimensionless f_0 without exact solutions. Besides, when I first analyze this question, I did not include the initial static friction coefficient f_0 and this incorrect choice of governing parameters gave an inconsistent results. Furthermore, dimensional analysis can not deal with physical process with lots of parameters. I also tried to apply dimensional analysis to other more complex mechanisms like thermal pressurization. For thermal pressurization, we have to consider at least two different mediums (water and rock sample) and their relevant material properties (mechanical, thermal and hydrological). There are also two different conditions of drained and undrained that can lead to different phenomenons. Including all those factors require more than 15 governing parameters and the dimensional analysis can hardly give insightful solutions.

3.2 Inferring function g from the laboratory experiments

In the last section, I will try to use the result of Eq.(6) to analyze the real measurements from rock experiments [5]. In that work, the authors measure the friction coefficients varying with slip rate for 5 different types of rock samples: quartzite, Novaculite, albite rock, granite and gabbro. In their experiment results, the rate weakening features are very clear so as the weakening velocity V_W, where a steep decrease of friction coefficient begins (Fig.2). Based on the previous study [9], the frictional contact spends a fraction T_W/θ $(= V_W/v)$ of its lifetime at the initially high strength $\sigma_c f_0$ and the remaining fraction at the weakened strength $\sigma_c f_0 - \Delta\tau_c = f_W \sigma_c$, so that the



Sample types	V_W (m/s)
Quartzite	0.0734
Novaculite	0.1001
Albite	0.1589
Granite	0.1097
Gabbro	0.0857

Figure 2: Friction coefficients varying with slip velocity in the high-speed sliding experiments for different rocks. Green lines indicate the visual picking of weakening velocity V_W and all picked V_Ws are listed in the table. Figure is modified from [5].

average contact shear strength during its lifetime is:

$$f\sigma_c = f_0\sigma_c\frac{T_W}{\theta} + f_W\sigma_c(1 - \frac{T_W}{\theta}) = (f_0 - f_W)\sigma_c\frac{V_W}{v} + f_W\sigma_c \tag{9}$$

When $v \leq V_W$, $f = f_0$ and the FHW does not begin; when $v > V_W$ and FHW occurs, we can have the functional form of friction coefficient varying with slip velocity v by substituting Eq.(6):

$$f = \begin{cases} f_0 \quad (v \leq V_W) \\ \\ (f_0 - f_W)\frac{\alpha_{th}}{L_c v}g(\frac{\varphi}{\sigma_c}, f_0) + f_W \quad (v > V_W) \end{cases} \tag{10}$$

By fitting this functional form with the experiment data, we can try to find the weakening slip velocity V_W for different kinds of rocks. Once the best-fit V_W has been found, we can further apply the relation Eq.(6):

$$\frac{V_W L_c}{\alpha_{th}} = g(\frac{\varphi}{\sigma_c}, f_0), \tag{11}$$

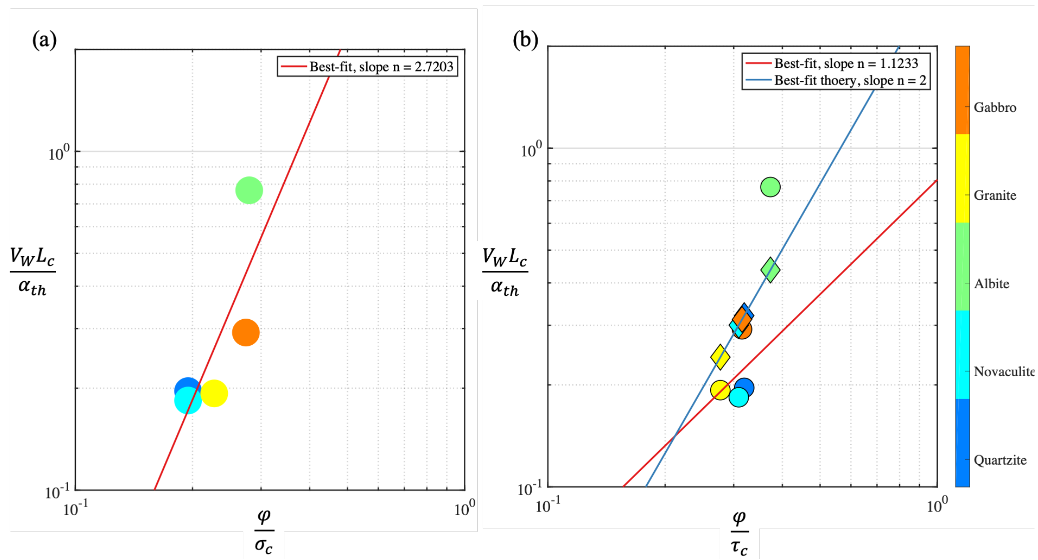


Figure 3: Dimensionless parameter fitting in the log-log space. (a) Fitting between $\frac{\varphi}{\sigma_c}$ and $\frac{V_W L_c}{\alpha_{th}}$; (b) Fitting between $\frac{\varphi}{\tau_c} = \frac{\varphi}{\sigma_c f_0}$ and $\frac{V_W L_c}{\alpha_{th}}$. Circles are from laboratory measurements and those red lines show the best linear-regression results. Diamonds and blue line are based on theoretical solution Eq.(8). All the material parameters are from [5] (also see the supplement table).

to see if we can find the trend of unknown function g.

Fig.2 shows the my data-fitting (intersecting points of two green lines) and all the visually picked weakening velocity V_Ws are listed in the table. Using the same material parameters used in that paper [5], I plot all the points $(\frac{\varphi}{\sigma_c}, \frac{V_W L_c}{\alpha_{th}})$ in the log-log space (Fig.3 (a)). The variation of $\frac{\varphi}{\sigma_c}$ and $\frac{V_W L_c}{\alpha_{th}}$ are within the same range of $0.1 - 1$. Because the initial friction coefficient f_0 is also in the same range ($0.6 - 0.8$), both $\frac{\varphi}{\sigma_c}$ and f_0 are not negligible for function g. The increasing trend indicates that the function g is an increasing function of $\frac{\varphi}{\sigma_c}$ at fixed f_0. Applying the linear regression in the log-log space gives a slope of $n = 2.7$, although the variation of data points from the best-fitted line is still obvious. Moreover, I checked the experiment data using the theoretical solution (Eq.(8)). In this theoretical solution, we know the exact form $g(\frac{\varphi}{\sigma_c}, f_0) = \pi(\frac{\varphi}{\sigma_c f_0})^2 = \pi(\frac{\varphi}{\tau_c})^2$. However, the fitting result from laboratory measurements is still not perfect (Fig.3 (b)). The linear regression in log-log space gives an even poorer fitting with slope $n = 1.1$, which is far from the theoretical solution. In my opinion, there are several reasons for this poor fitting:

1. for the $\frac{\varphi}{\sigma_c}$ and $\frac{V_W L_c}{\alpha_{th}}$ fitting (Fig.3 (a)), I have not included the f_0 terms and this implies that f_0 stays the same for all those rock types. However, the value of f_0 is very different for those rocks, varying from $0.61 - 0.87$ (supplement table).

2. even when I include f_0 in the fitting (Fig.3 (b)), the result is still very different from theoretical solution. This is probably because the theoretical solution from [9] is based on one-dimension solution and may not be applicable to the 2D or 3D cases of the real experiments.

3. besides, in this project I only focus on fitting the experiment data with FHW mechanisms and assume FHW is the only process. But in reality different weakening mechanisms can occur at the same time.

4. finally, as mentioned in the experiment paper [5], there are still lots of inevitable uncertainties in the material parameters (supplement table).

Possibly with a wide coverage of material types, more information can be obtained from this analysis. Nevertheless, this kind of dimensional analysis still have great potential to provide us valuable information of those physical process.

4 Conclusion

In this project, I apply the dimensional analysis to the flash heating weakening mechanisms. With certain assumptions, we can obtain a dimensionless form for the change of asperity shear strength $\Delta\tau_c$. The two timescales (contacting lifetime and weakening time) in this dimensionless form naturally imply a transition can occur in this system. Equating the two timescales can directly give us a critical weakening slip velocity as function of material properties. I further compare the dimensional analysis result with the theoretical solution [9] and laboratory experiment results [5]. The dimensional analysis result under my assumptions is consistent with theoretical solution. However, neither can well fit the laboratory data due to certain factors listed before, and can be improved by more laboratory experiments. In general, the dimensional analysis is a powerful tool to deal with a wide range of physical problems and can help us gain special insights from a physical system, but still has inevitable limitations.

5 Acknowledgement

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Supplementary table of the material properties used in the calculations

Sample types	V_W *(m/s)*	α_{th} ($10^{-6} m^2/s$)	L_C (μm)	φ ($10^9 J/m^3$)	σ_c *(GPa)*	f_0
Quartzite	0.0734	1.19	3.2	2.92	15	0.61
Novaculite	0.1001	1.19	2.2	2.92	15	0.63
Albite	0.1589	0.83 ~~*0.35 paper~~	4	2.66	9.5	0.75
Granite	0.1097	1.25	2.2	2.16	11	0.82
Gabbro	0.0857	1.29	4.4	2.63	9.5	0.87

- **Density of $2700\ kg/m^3$ is assumed for all minerals**
- **Weakening temperature rise of $\Delta\Theta_W = 1000\text{K}$ (?)**

Parameters are calculated based on Goldsby & Tullis (2011)