Relating teleseismic backprojection images to earthquake kinematics

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1-minute Summary

❖ Linear BP image ~ Smoothed slip motion distribution

❖ BP evolution ~ Kinematic representation of rupture

❖ Global BP physical resolution

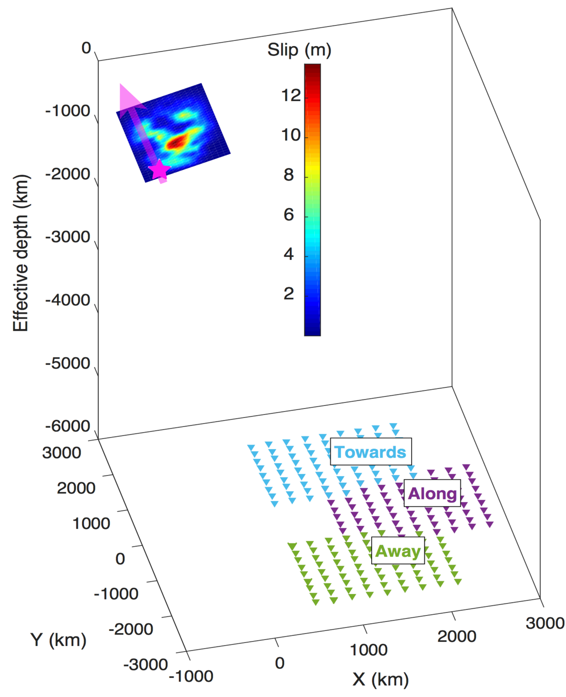
I. Theoretical formulation

Displacement seismograms:

(Radiation pattern) *(Slip rate function)*

$$d_k(t) = \sum_{n=1}^{N} \frac{R^P_{kn}}{4\pi\rho\alpha^3} \frac{\mu\Delta S}{r_{kn}} \dot{u}_n(t - t_{kn})$$

(Geometrical spreading)



Matrix form for the array spectral data:

$$\begin{bmatrix} D_1(\omega) \\ D_2(\omega) \\ \vdots \\ D_K(\omega) \end{bmatrix} = \mathbf{A}(\omega) \begin{bmatrix} \dot{U}_1(\omega) \\ \dot{U}_2(\omega) \\ \vdots \\ \dot{U}_N(\omega) \end{bmatrix},$$

(Array data) *(Source slip rate)*

$$\mathbf{A}(\omega) = \frac{\mu\Delta S}{4\pi\rho\alpha^3} \begin{bmatrix} \frac{R^P_{11}}{r_{11}} e^{-i\omega t_{11}} & \dots & \frac{R^P_{1N}}{r_{1N}} e^{-i\omega t_{1N}} \\ \frac{R^P_{21}}{r_{21}} e^{-i\omega t_{21}} & \dots & \frac{R^P_{2N}}{r_{2N}} e^{-i\omega t_{2N}} \\ \vdots & \ddots & \vdots \\ \frac{R^P_{K1}}{r_{K1}} e^{-i\omega t_{K1}} & \dots & \frac{R^P_{KN}}{r_{KN}} e^{-i\omega t_{KN}} \end{bmatrix}_{K\times N}$$

(Wave Propagation)

Linear BP operator in frequency domain:

$$\tilde{\mathbf{A}}(\omega) = \mathbf{W} \begin{bmatrix} e^{i\omega t_{11}} & e^{i\omega t_{21}} & \dots & e^{i\omega t_{K1}} \\ e^{i\omega t_{12}} & e^{i\omega t_{22}} & \dots & e^{i\omega t_{K2}} \\ \vdots & \ddots & \ddots & \vdots \\ e^{i\omega t_{1N}} & e^{i\omega t_{2N}} & \dots & e^{i\omega t_{KN}} \end{bmatrix}_{N\times K}$$

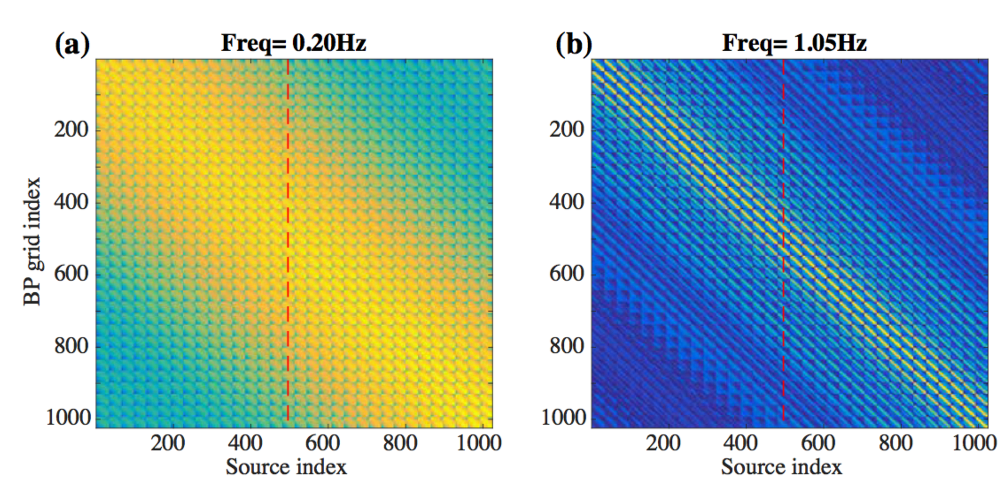
(Phase shift + stacking)

❖ **The linear BP image is the slip motion after spatial smoothing with a frequency dependent resolution matrix F(ω).**

$$\mathbf{D}^{BP}(\omega) = \tilde{\mathbf{A}}(\omega)\mathbf{D}(\omega) = \tilde{\mathbf{A}}(\omega)\mathbf{A}(\omega)\dot{\mathbf{U}}(\omega) = \mathbf{F}(\omega)\dot{\mathbf{U}}(\omega)$$

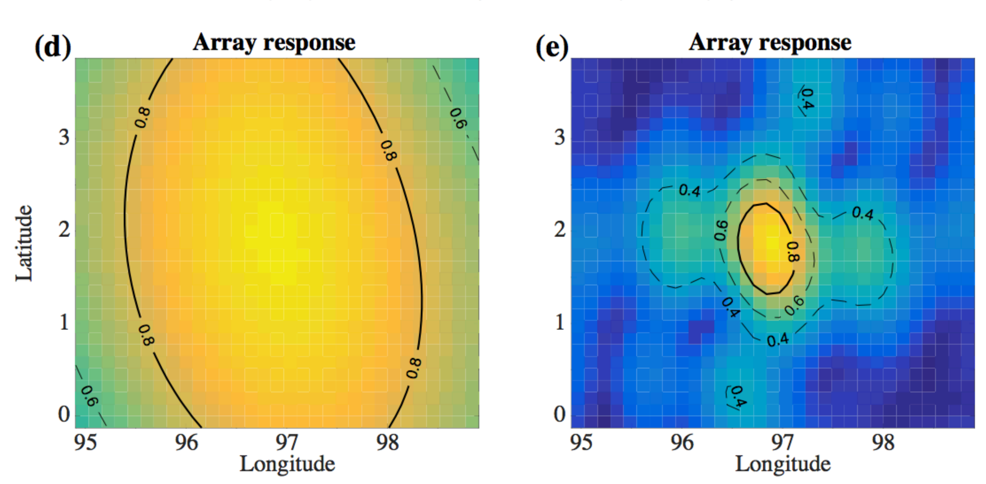
Quantification of BP resolution:

$\mathbf{F}(\omega)$



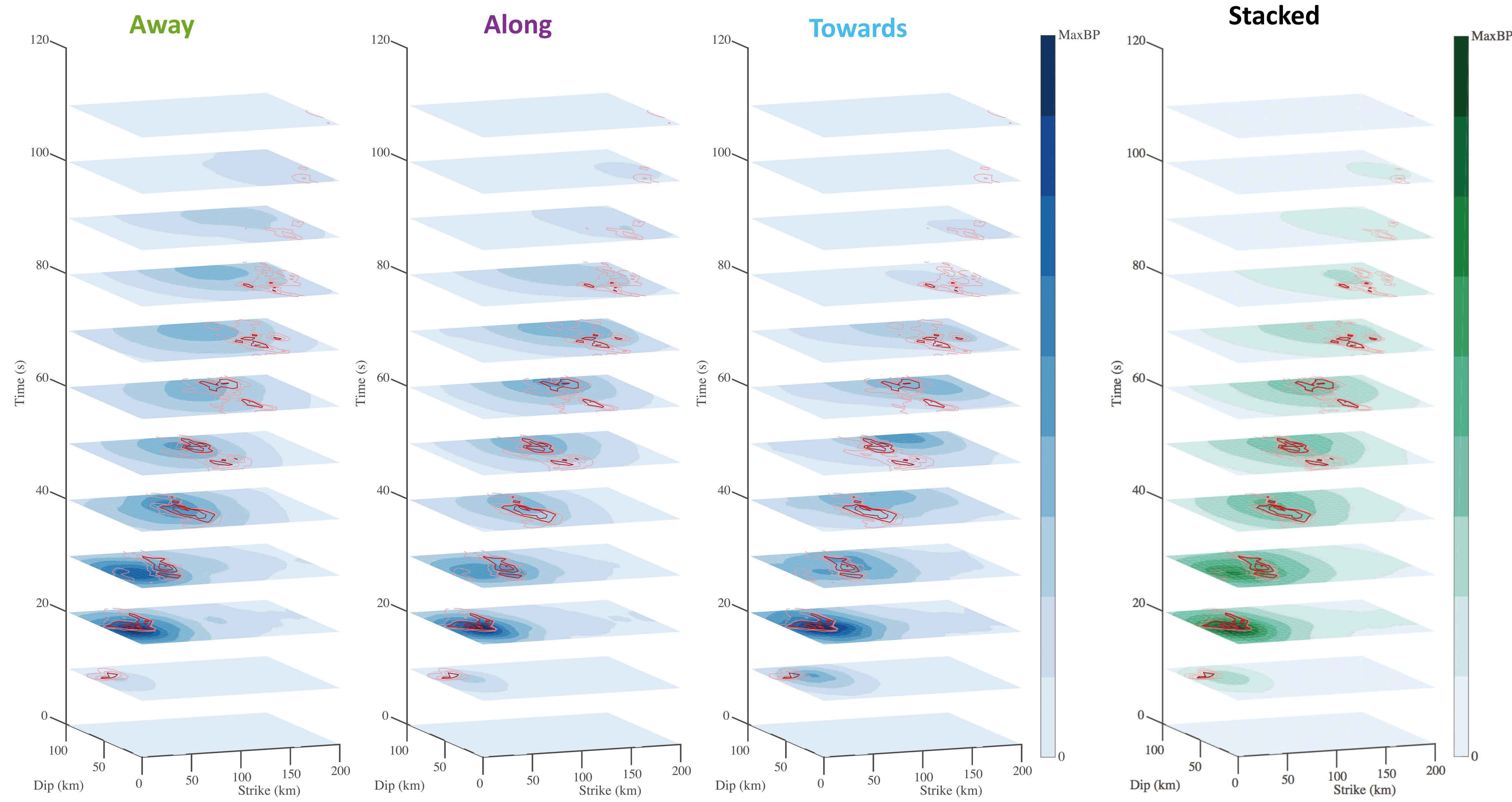
A. Resolvability: $\epsilon_I(\omega) = |corr2(\mathbf{F}, \mathbf{I})|$

Array Response

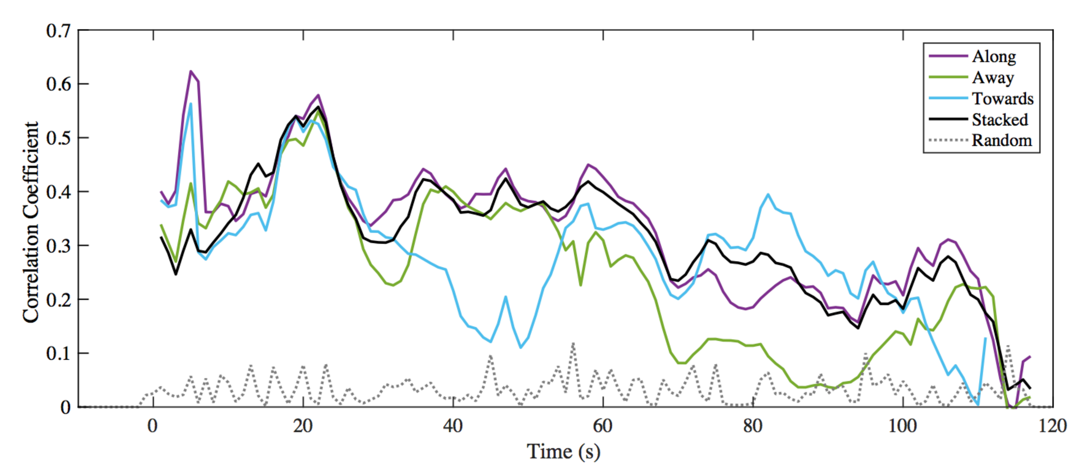


B. Resolvable area: Area within 80% of the AR maxima

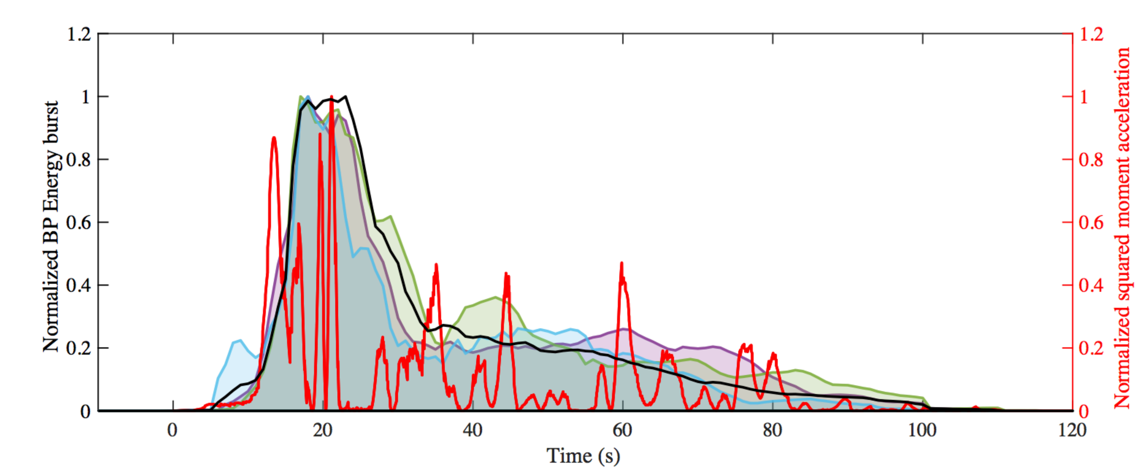
II. Validation with synthetic earthquake source



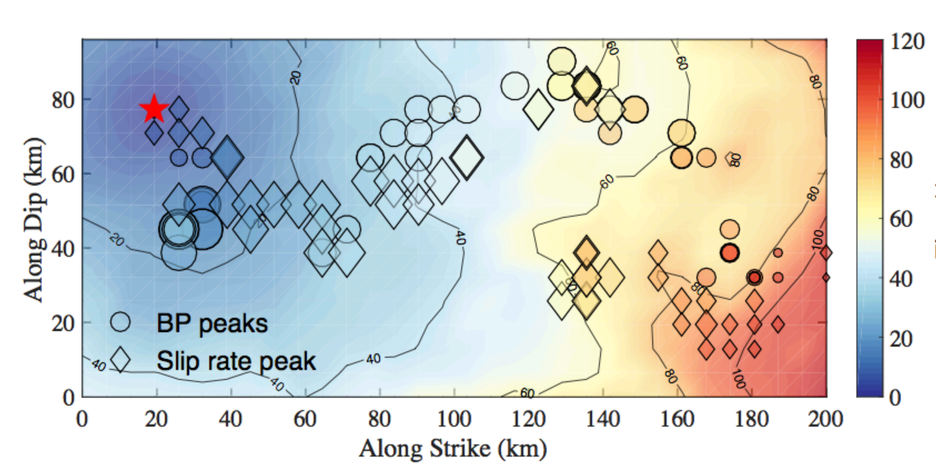
↑ *kinematic source model (red contours: slip rate) compared with BP results (blue and green images) .*

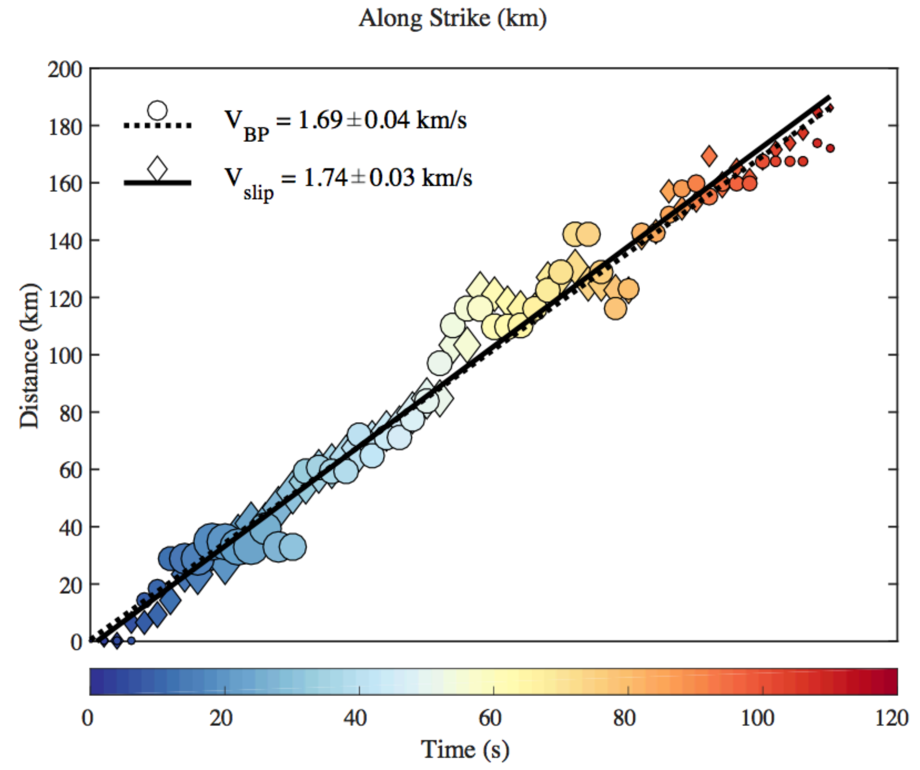


↑ *Correlation coefficients between the slip rate distribution and BP image varying with time.*



↑ *Normalized peak BP energy (color patches) vs the normalized radiated energy evolution (squared slip acceleration, red line)*





❖**General pattern (20-50%) of rupture can be well recovered from BP**

❖**Peak BP energy evolution is consistent with the normalized radiated energy evolution.**

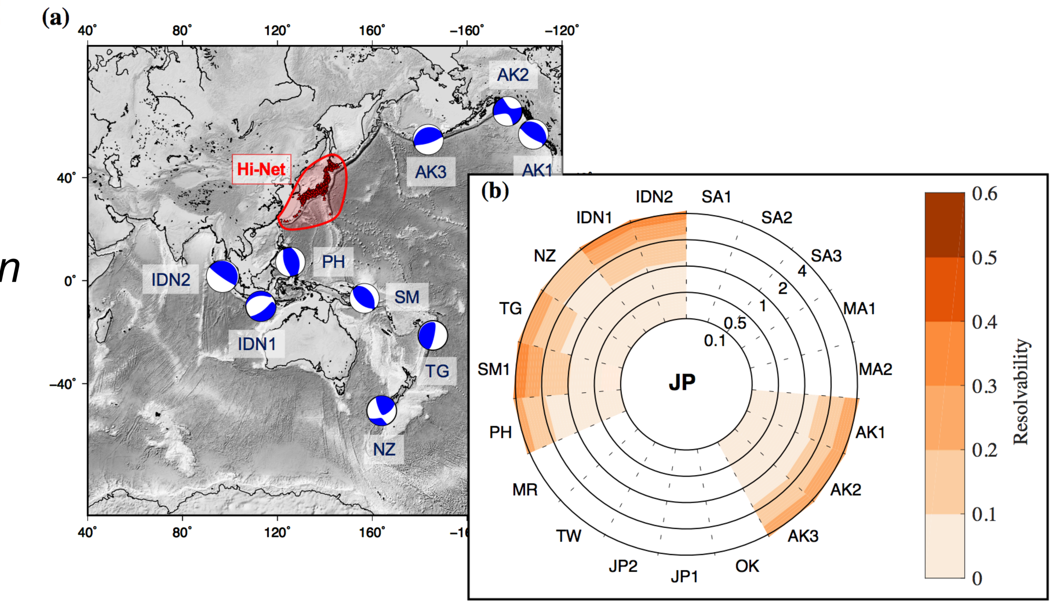
❖**The lower limit of rupture velocity can be well constrained.**

← *(Top) Spatiotemporal distribution of BP peaks (circles) compared with slip rate peaks (diamonds). Background image shows the onset time distribution from the model.*

← *(Bottom) Average rupture velocity estimated from BP and slip rate peaks, respectively.*

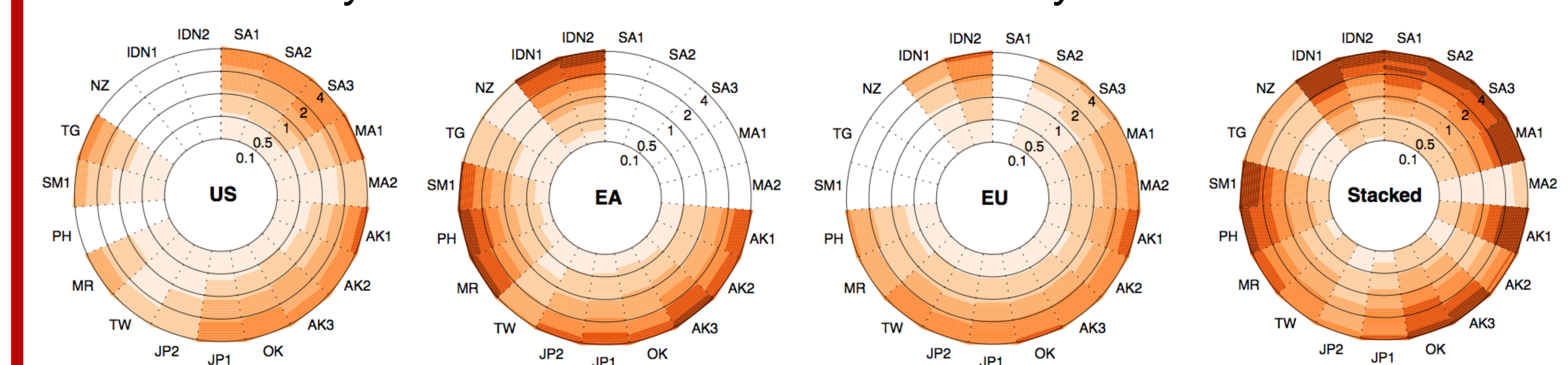
III. Global BP physical resolution

Global resolvability:



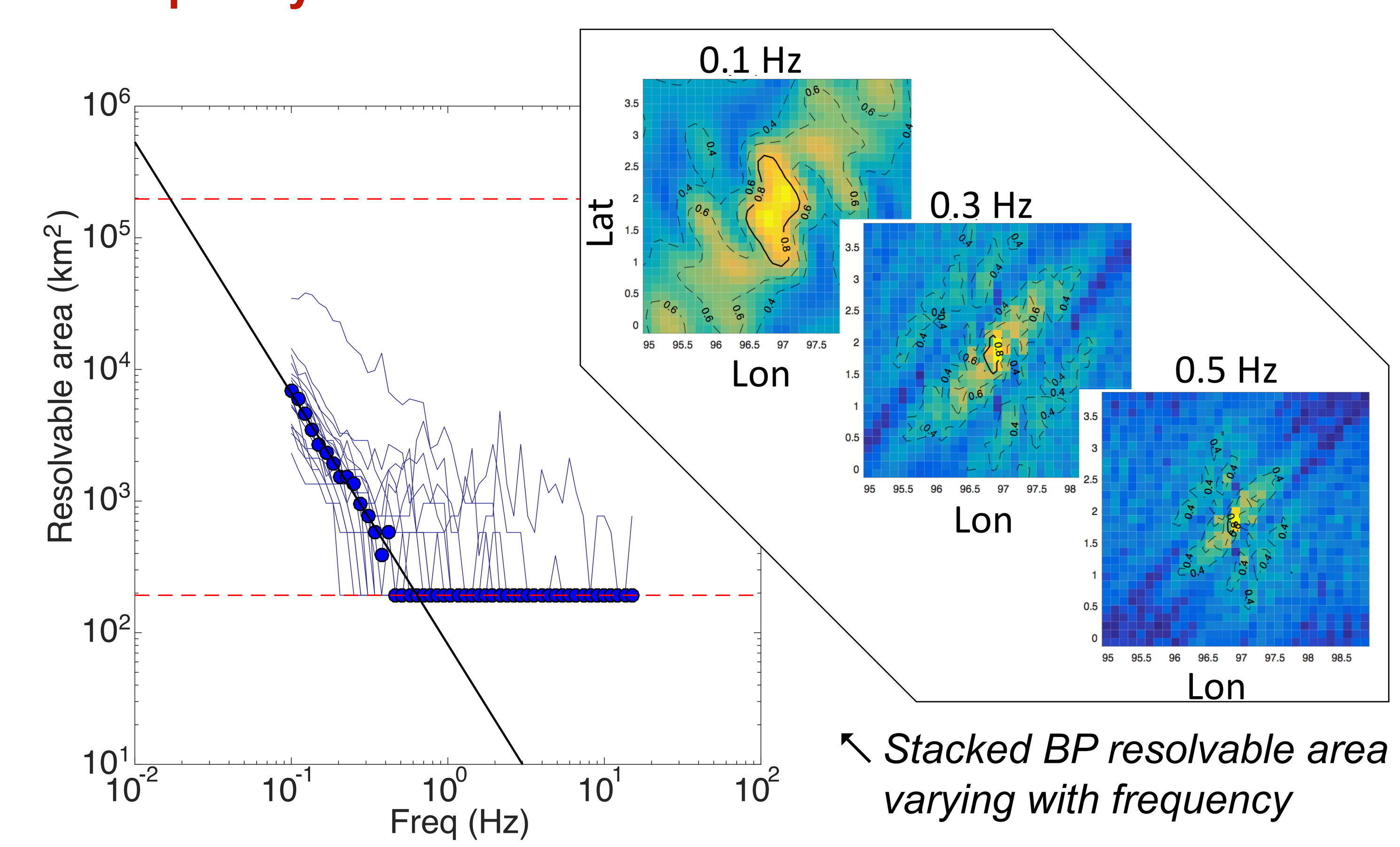
→ *Hi-net resolvability towards various source region within the teleseismic distance*

↓ *Resolvability distribution of different seismic arrays*



❖ **Modifying the radiation pattern and geometrical spreading terms, we can estimate the resolvability for any array–source configuration.**

BP frequency resolution:



↖ *Stacked BP resolvable area varying with frequency*

Empirical relation:

$$Area \approx 10^{1.91} f^{-1.91}$$

Scaling relation (Wells & Coppersmith, 1994):

$$Area \approx 10^{(-3.42+0.9M_w)}$$

❖ **Lowest BP frequency:**

$$f^{BP}_{min} \approx 10^{(2.79-0.47M_w)}$$

- **Mw 8 ~ 0.11 Hz**
- **Mw 7 ~ 0.32 Hz**
- **Mw 6 ~ 0.93 Hz**

Conclusion

1. **We propose a theoretical formulation to clarify the relation between the backprojection image and earthquake kinematics.**
2. **We apply BP to physically complex pseudo-dynamic sources to validate the theoretical formulation.**
3. **We provide a systematical evaluation on the BP resolution for the global seismic arrays**