

ELECTRICITY MARKET REFORM: Market Design and Extended LMP

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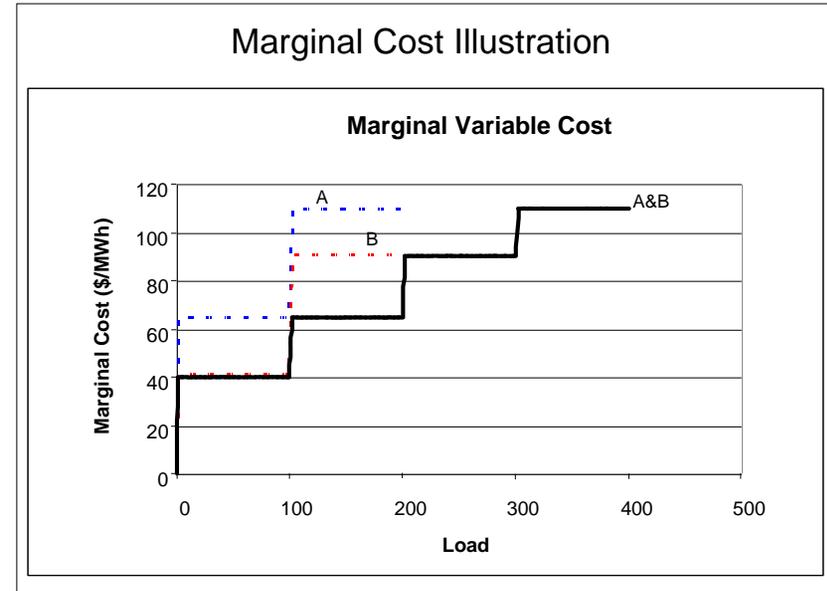
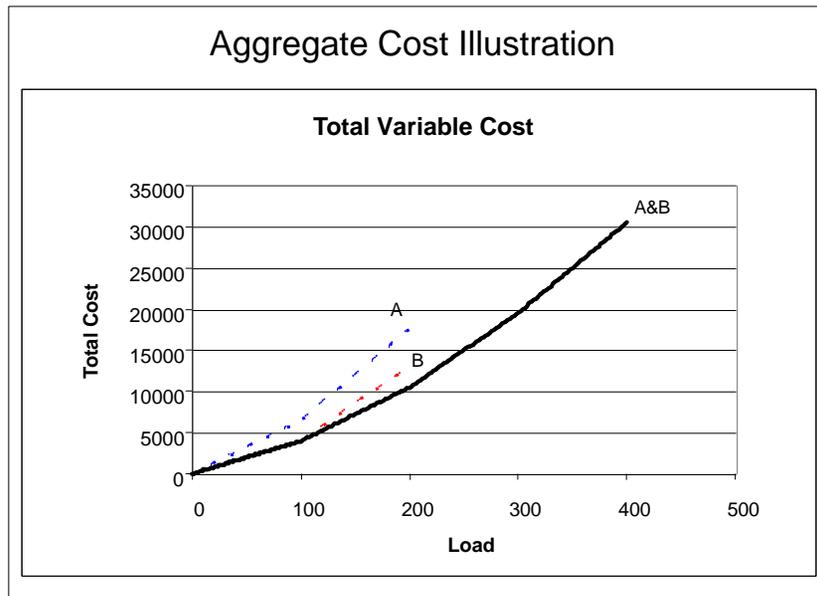
Energy dispatch is continuous but unit commitment requires discrete decisions. Bid-based, security constrained, combined unit commitment and economic dispatch presents a challenge in defining market-clearing prices.

- **Continuous convex economic dispatch**
 - Electric power systems are almost convex, and use convex approximations for dispatch.¹
 - System marginal costs provide locational, market-clearing, linear prices.
 - Linear prices support the economic dispatch.

- **Discrete, economic, unit commitment and dispatch**
 - Start up and minimum load restrictions enter the model.
 - System marginal costs not always well-defined.
 - There may be no linear prices that support the commitment and dispatch solution.

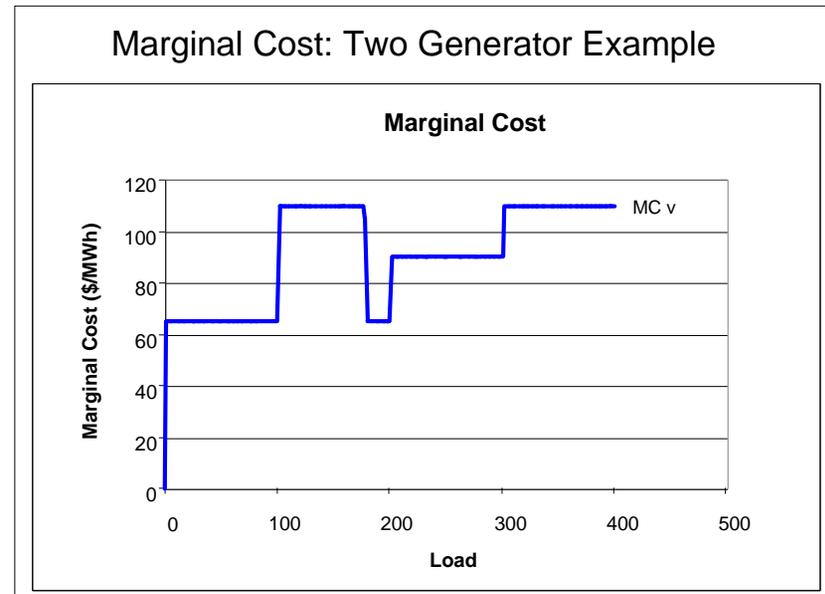
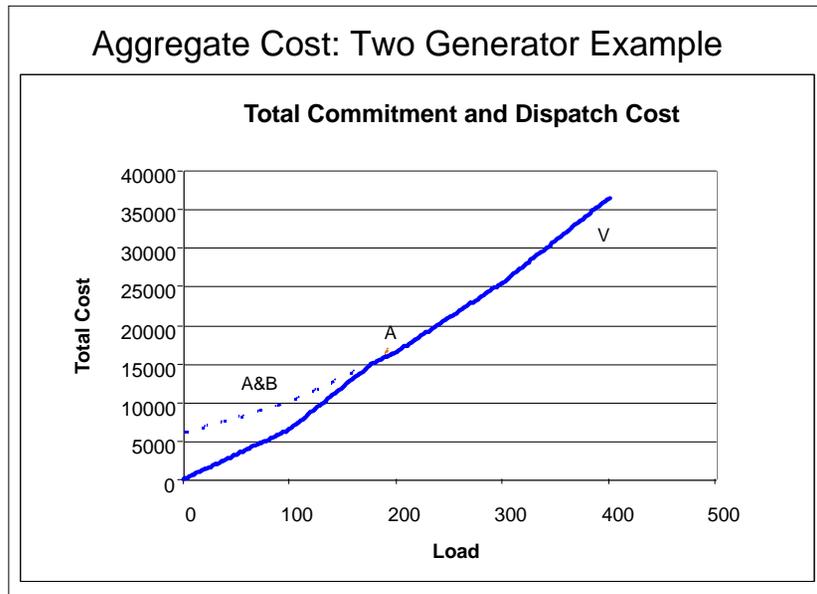
¹ J. Lavaei and S. H. Low, “Zero duality gap in optimal power flow problem,” *IEEE Transactions on Power Systems*. To appear, 2011.

Energy dispatch is continuous, convex and yields linear prices.² A simplified example with two generating units illustrates both total and marginal costs.



² Paul R. Gribik, William W. Hogan, and Susan L. Pope, "Market-Clearing Electricity Prices and Energy Uplift," Harvard University, December 31, 2007, available at www.whogan.com.

Unit commitment requires discrete decisions. Now the second unit (B) has a startup cost.



Marginal cost-based linear prices cannot support the commitment and dispatch. The solution has been to make “uplift” payments to assure reliable and economic unit commitment.

Selecting the appropriate approximation model for defining energy and uplift prices involves practical tradeoffs. All involve “uplift” payments to guarantee payments for bid-based cost to participating bidders (generators and loads), to support the economic commitment and dispatch.

Uplift with Given Energy Prices=Optimal Profit – Actual Profit

- **Restricted Model (r)**
 - Fix the unit commitment at the optimal solution.
 - Determine energy prices from the convex economic dispatch.
- **Dispatchable Model (d)**
 - Relax the discrete constraints and treat commitment decisions as continuous.
 - Determine energy prices from the relaxed, continuous, convex model.
- **Extended Locational Marginal Pricing (ELMP) Model (h)**
 - Equivalent formulations
 - Select the energy prices from the convex hull of the cost function.
 - Select the energy prices from the Lagrangean relaxation (i.e., usual dual problem for pricing the joint constraints).
 - Resulting energy prices minimize the total uplift.

A formulation that separates out the discrete variables (u) serves to distinguish the modeling approaches.³ Here assume that the problem is convex but for the integer restriction on the commitment variables. The optimal commitment is u^o .

- Unit commitment and dispatch

$$\begin{aligned} v(y) = \underset{(x,u) \in X}{\text{Min}} \quad & f(x,u) \\ \text{s.t.} \quad & g(x) = y \\ & u = 0,1. \end{aligned}$$

- Restricted Model (r)

$$\begin{aligned} v^r(y) = \underset{(x,u) \in X}{\text{Min}} \quad & f(x,u) \\ \text{s.t.} \quad & g(x) = y \\ & u = u^o. \end{aligned}$$

- Dispatchable Model (d)

$$\begin{aligned} v^d(y) = \underset{(x,u) \in X}{\text{Min}} \quad & f(x,u) \\ \text{s.t.} \quad & g(x) = y \\ & 0 \leq u \leq 1. \end{aligned}$$

³ For a further discussion of dual price functions, see M. Bjørndal and K. Jörnsten, "Equilibrium prices supported by dual price functions in markets with non-convexities," *European Journal of Operational Research*, vol. 190, no. 3, pp. 768–789, 2008.

Economic commitment and dispatch is a special case of a general optimization problem.

$$v(y) = \underset{x \in X}{\text{Min}} \quad f(x)$$
$$\text{s.t.} \quad g(x) = y.$$

From the perspective of a price-taking bidder, uplift is the difference between actual and optimal profits.

$$\text{Actual profits:} \quad \pi(p, y) = py - v(y)$$
$$\text{Optimal Profits:} \quad \pi^*(p) = \underset{z}{\text{Max}} \{pz - v(z)\}$$
$$\text{Uplift}(p, y) = \pi^*(p) - \pi(p, y)$$

Classical Lagrangean relaxation and pricing creates a familiar dual problem.

$$L(y, x, p) = f(x) + p(y - g(x))$$
$$\hat{L}(y, p) = \underset{x \in X}{\text{Inf}} \{f(x) + p(y - g(x))\}$$
$$v^h(y) = \underset{p}{\text{Sup}} \hat{L}(y, p) = \underset{p}{\text{Sup}} \left\{ \underset{x \in X}{\text{Inf}} \{f(x) + p(y - g(x))\} \right\}$$

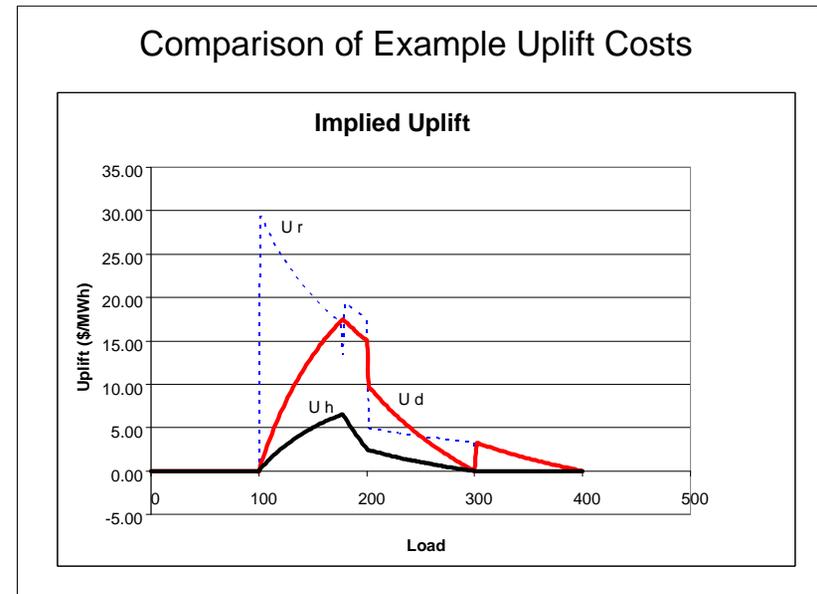
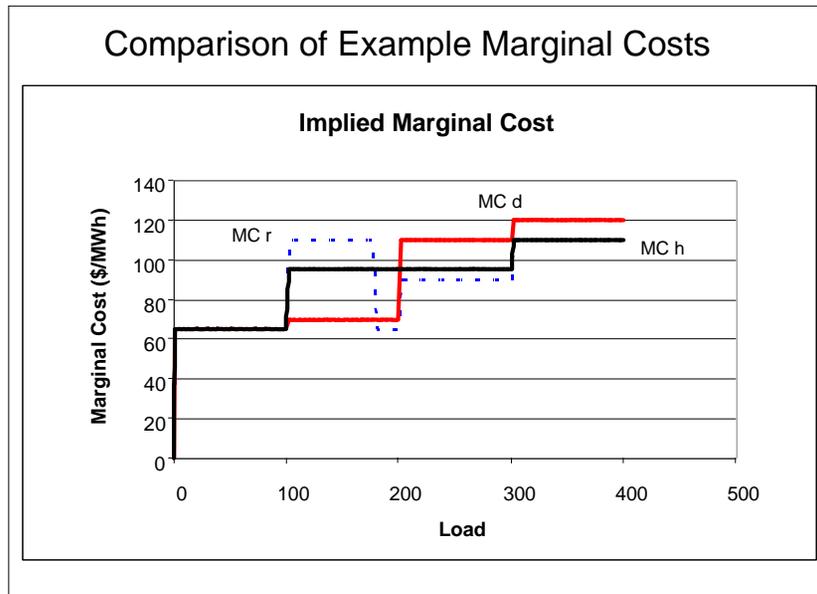
The optimal dual solution minimizes the uplift, and the “duality gap” is equal to the minimum uplift.

$$v(y) - v^h(y) = \underset{p}{\text{Inf}} \text{Uplift}(p, y) = \text{Uplift}(p^h, y).$$

In general, the solutions can be such that:

$$v^d(y) < v^h(y) < v^r(y) = v(y),$$
$$p^d \neq p^h \neq p^r.$$

Comparing illustrative energy pricing and uplift models.



Both the relaxed dispatchable and ELMP models produce “standard” implied supply curve. The ELMP model produces the minimum uplift.

Alternative pricing models have different features and raise additional questions.

- **Computational Requirements.** Dispatchable model is the easiest case, ELMP model the hardest. But not likely to be a significant issue. Approximate solutions (e.g., NYISO model) may be workable.
- **Network Application.** All models compatible with network pricing and reduce to standard LMP in the convex case.
- **Operating Reserve Demand.** All models compatible with existing and proposed operating reserve demand curves.
- **Solution Independence.** Restricted model sensitive to actual commitment. Relaxed and ELMP models (largely) independent of actual commitment and dispatch.
- **Financial Transmission Rights.** Transmission revenue collected under the market clearing solution would be sufficient to meet the obligations under the FTRs. However, this may not be true for the revenues under the economic dispatch, which is not a market clearing solution at the ELMP prices, even though the FTRs are simultaneously feasible. The FTR uplift amount included in the decomposition of the total uplift that is minimized by the ELMP prices. This uplift payment would be enough to ensure revenue adequacy of FTRs under ELMP pricing.⁴
- **Day-ahead and real-time interaction.** With uncertainty in real-time and virtual bids, expected real-time price is important, and may be similar under all pricing models.

⁴ Michael Cadwalader , Paul Gribik , William Hogan , Susan Pope, “Extended LMP and Financial Transmission Rights,” Working Paper, June 9, 2010, available at www.whogan.com.

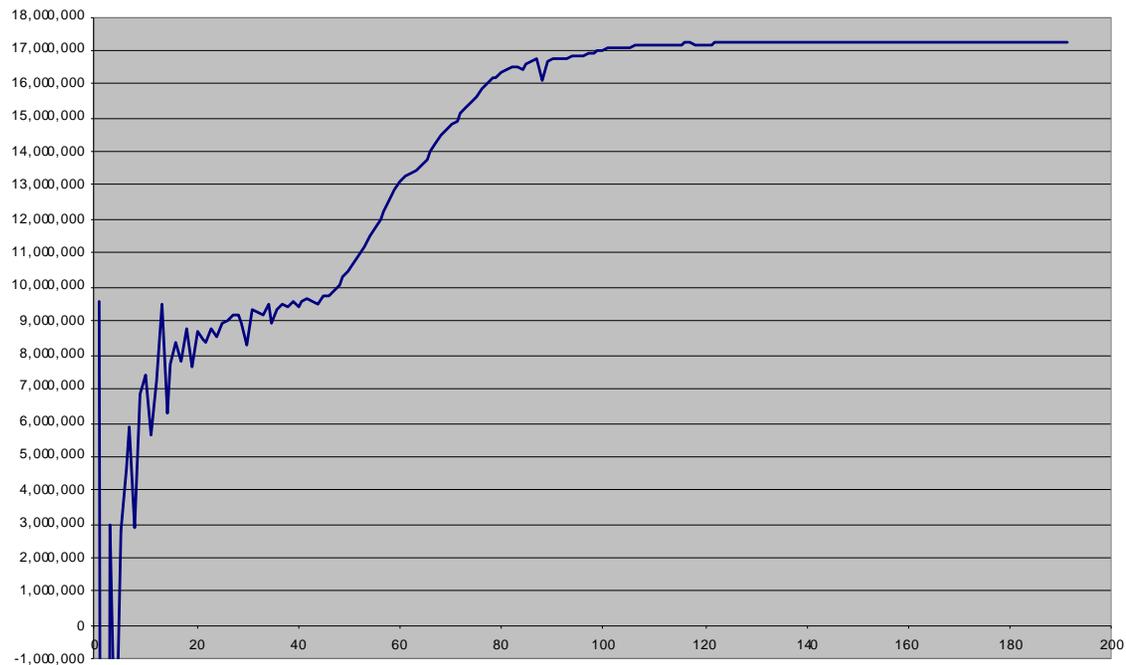
A “proof of concept” network example from the Midwest Independent System Operator illustrates ELMP in a larger scale dynamic day-ahead problem.

Example Problem

- Example problem from November 2007.
 - Day-Ahead market (energy only).
 - Number of generators: 1009
 - Number of transmission constraints modeled: 18
 - Study period: 24 hours.
- Computer used: 64 bit desk top, 2.33GHz, 4GB of RAM running Windows XP 64 bit.
- Performance summary:
 - Number of iteration: 191
 - CPU time: 223s
- The software was for proof of concept. It was not optimized for speed.
 - LPs solved using MATLAB. No performance enhancement methods such as hot-starting the LPs using solution from previous iteration were used.
 - All indications are that problem can be solved in practice.

A “proof of concept” network example from the Midwest Independent System Operator (cont.).

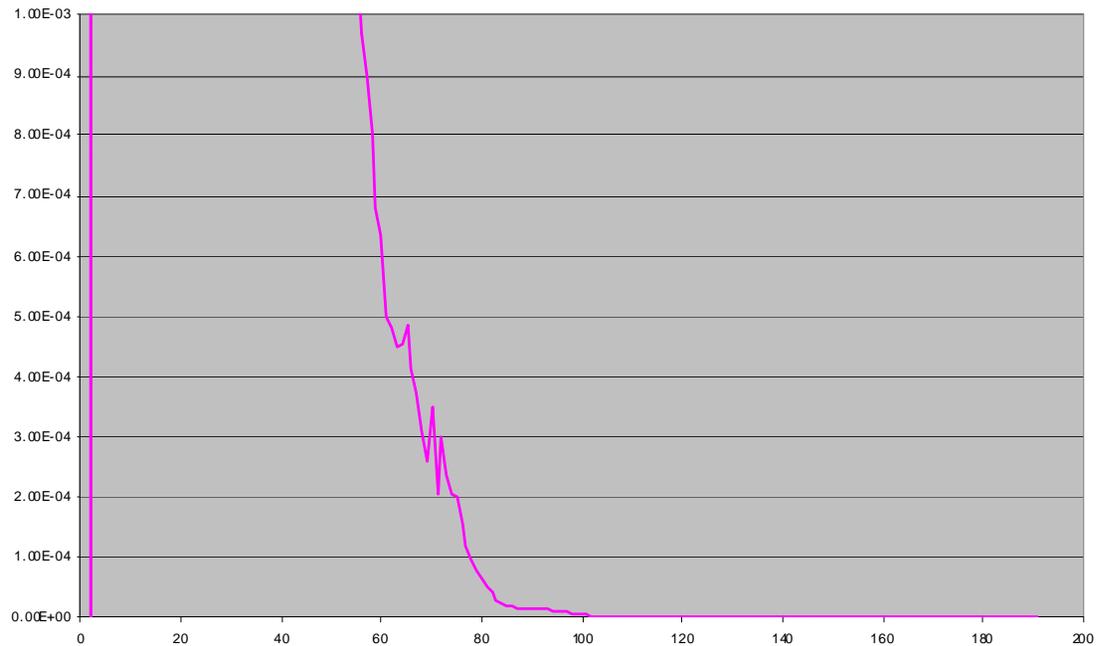
Dual Objective Function Value by Iteration



Source: Paul Gribik, “Investigation of Convex Hull Pricing at Midwest ISO,” MISO, September 2009.

A “proof of concept” network example from the Midwest Independent System Operator (cont.).

Norm of Difference between Dual Solution Prices and Optimal Prices



Source: Paul Gribik, “Investigation of Convex Hull Pricing at Midwest ISO,” MISO, September 2009.

A “proof of concept” network example from the Midwest Independent System Operator (cont.).

Comparison of Convex Hull Prices and Current Prices

- Convex hull prices at reference node compared to current prices at reference node:

Hour	1	2	3	4	5	6	7	8	9	10	11	12
Current Price	17.9	16.6	16.7	16.7	16.8	16.2	16.4	20.5	20.4	21.2	22.4	21.1
CHP	20.6	19.1	18.5	18.1	18.8	18.5	19.6	20.5	22.9	23.8	24.6	24.3
Difference	2.7	2.5	1.8	1.4	2.0	2.3	3.2	0.0	2.5	2.6	2.2	3.2

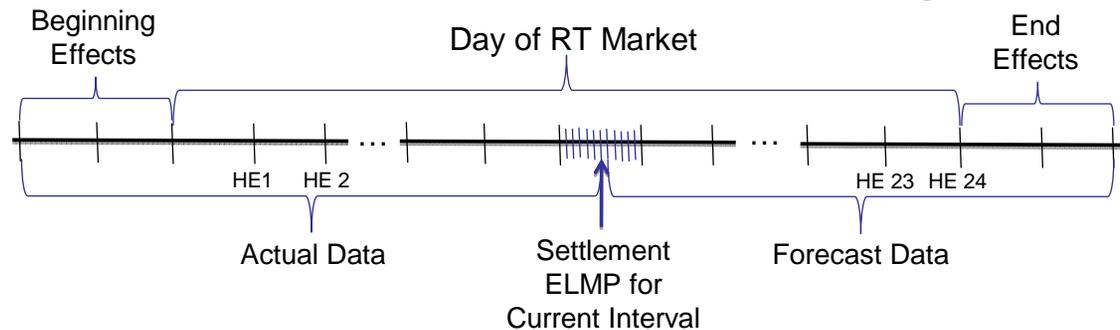
Hour	13	14	15	16	17	18	19	20	21	22	23	24	Average
Current Price	20.8	20.5	20.4	20.6	20.5	23.2	48.3	40.0	26.3	21.3	21.1	18.4	21.8
CHP	23.1	22.2	20.5	20.2	21.0	25.1	57.0	35.9	29.0	25.5	24.2	23.2	24.0
Difference	2.3	1.7	0.1	-0.4	0.5	1.9	8.7	-4.1	2.7	4.2	3.1	4.8	2.2

- The average convex hull price at the reference node is \$24.0/MWh compared to the average reference node price of \$21.8/MWh using current pricing structure.
 - The increase of \$2.2/MWh reflects the effects of including start-up and no-load costs in setting prices.



A real-time pricing model involves multiple periods and look ahead. Prices updated throughout the day on a rolling basis.

Calculation of RT ELMPs on Rolling Basis



- Calculate ELMPs for a window containing the day of the RT Market.
- As day progresses, current hour is the hour of interest in which we model 5 minute intervals.
 - Other hours may be modeled as single hour long intervals.
 - Could model 5 minute intervals in next hour to provide 5-minute interval-level informational ELMPs.
- Use actual data for past intervals and forecast data for future intervals.
 - ELMPs calculated for current interval used for settlements.
 - ELMPs for other intervals are used for calculation only.
 - Past ELMPs frozen for settlement and future ELMPs are for informational purposes only.

A real-time pricing model involves multiple periods and look ahead. Applying and ELMP framework involves choices about what is fixed and what is variable. Natural principles include:

- **Real-time quantity anchor.** Conditioning to reflect evolving economic dispatch and commitment. For example, the pricing dispatch would account for ramping limits that constrain the degree that the pricing dispatch could deviate from the actual dispatch to ensure that the price market-clearing dispatch would always be feasible conditioned on the actual dispatch.
- **Real-time price consistency.** Given that the actual conditions equal the forecast conditions, the methodology produces the same set of prices.

For actual commitment and dispatch, past decisions are sunk and real-time quantity anchors apply.

The pricing model could employ more flexibility. The Restricted model can meet both conditions by always ignoring fixed costs. But ELMP in general incorporates intertemporal constraints and reflects fixed costs of units not committed.

A stylized version of the unit commitment and dispatch problem for a fixed demand y is formulated in (Gribik, Hogan, and Pope 2007) as:

$v(\{y_t\}) = \inf_{g,d,on,start} \sum_t \sum_i (StartCost_{it} \cdot start_{it} + NoLoad_{it} \cdot on_{it} + GenCost_{it}(g_{it}))$ <p>subject to</p> $m_{it} \cdot on_{it} \leq g_{it} \leq M_{it} \cdot on_{it} \quad \forall i,t$ $-ramp_{it} \leq g_{it} - g_{i,t-1} \leq ramp_{it} \quad \forall i,t$ $start_{it} \leq on_{it} \leq start_{it} + on_{i,t-1} \quad \forall i,t$ $start_{it} = 0 \text{ or } 1 \quad \forall i,t$ $on_{it} = 0 \text{ or } 1 \quad \forall i,t$ $\mathbf{e}^T (\mathbf{g}_t - \mathbf{d}_t) - LossFn_t(\mathbf{d}_t - \mathbf{g}_t) = 0 \quad \forall t$ $Flow_{kt}(\mathbf{g}_t - \mathbf{d}_t) \leq \bar{F}_{kt}^{\max} \quad \forall k,t$ $\mathbf{d}_t = \mathbf{y}_t \quad \forall t.$ <p>\mathbf{y}_t = vector of nodal loads in period t m_{it} = minimum output from unit i in period t if unit is on M_{it} = maximum output from unit i in period t if unit is on $ramp_{it}$ = maximum ramp from unit i between period t-1 and period t $StartCost_{it}$ = Cost to start unit i in period t $NoLoad_{it}$ = No load cost for unit i in period t if unit is on \bar{F}_{kt}^{\max} = Maximum flow on transmission constraint k in period t.</p>	<p>Variables:</p> $start_{it} = \begin{cases} 0 & \text{if unit i is not started in period t} \\ 1 & \text{if unit i is started in period t} \end{cases}$ $on_{it} = \begin{cases} 0 & \text{if unit i is off in period t} \\ 1 & \text{if unit i is on in period t} \end{cases}$ <p>g_{it} = output of unit i in period t \mathbf{d}_t = vector of nodal demands in period t.</p>
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The ELMP is a solution p for the dual or convex hull problem with the loss and transmission limits included as constraints. A “market-clearing” solution is a solution to the inner problem for given prices p .

$$\begin{aligned}
 v^h(\{\mathbf{y}_t\}) \equiv & \\
 & \left. \begin{aligned}
 & + \sum_t \mathbf{p}_t^T \mathbf{y}_t \\
 & \inf_{g, d, on, start} \left(\begin{aligned}
 & \sum_t \sum_i (StartCost_{it} \cdot start_{it} + NoLoad_{it} \cdot on_{it} + GenCost_{it}(g_{it})) \\
 & - \sum_t \mathbf{p}_t^T \mathbf{d}_t
 \end{aligned} \right) \\
 & \text{subject to} \\
 & m_{it} \cdot on_{it} \leq g_{it} \leq M_{it} \cdot on_{it} \quad \forall i, t \\
 & -ramp_{it} \leq g_{it} - g_{i,t-1} \leq ramp_{it} \quad \forall i, t \\
 & start_{it} \leq on_{it} \leq start_{it} + on_{i,t-1} \quad \forall i, t \\
 & start_{it} = 0 \text{ or } 1 \quad \forall i, t \\
 & on_{it} = 0 \text{ or } 1 \quad \forall i, t \\
 & \mathbf{e}^T (\mathbf{g}_t - \mathbf{d}_t) - LossFn_t(\mathbf{d}_t - \mathbf{g}_t) = 0 \quad \forall t \\
 & Flow_{kt}(\mathbf{g}_t - \mathbf{d}_t) \leq \bar{F}_{kt}^{\max} \quad \forall k, t
 \end{aligned} \right\} \\
 \sup_p & + \left. \right\}
 \end{aligned}$$

A sufficient condition for real-time price consistency in ELMP is that all commitment and dispatch variables that are in the economic dispatch or are assigned an uplift payment from the market-clearing solution be included in the pricing model. This allows for slowly pruning those offers that were not committed in either the economic commitment or the market-clearing commitment and are subsequently excluded from retroactive starts (*Excluded_τ*). Hence, the conditional dual pricing model at time τ could take as determined prices the prior periods $p_1^*, p_2^*, \dots, p_{\tau-1}^*$:

$$v^\tau(\{\mathbf{y}_t\}) \equiv \left. \begin{array}{l} + \sum_t \mathbf{p}_t^T \mathbf{y}_t \\ \inf_{\mathbf{g}, \mathbf{d}, \text{on}, \text{start}} \left(\begin{array}{l} \sum_t \sum_i (\text{StartCost}_{it} \cdot \text{start}_{it} + \text{NoLoad}_{it} \cdot \text{on}_{it} + \text{GenCost}_{it}(\mathbf{g}_{it})) \\ - \sum_t \mathbf{p}_t^T \mathbf{d}_t \end{array} \right) \\ \text{subject to} \\ m_{it} \cdot \text{on}_{it} \leq \mathbf{g}_{it} \leq M_{it} \cdot \text{on}_{it} \quad \forall i, t \\ -\text{ramp}_{it} \leq \mathbf{g}_{it} - \mathbf{g}_{i,t-1} \leq \text{ramp}_{it} \quad \forall i, t \\ \text{start}_{it} \leq \text{on}_{it} \leq \text{start}_{it} + \text{on}_{i,t-1} \quad \forall i, t \\ \text{start}_{it} = 0 \text{ or } 1 \quad \forall i, t \\ \text{on}_{it} = 0 \text{ or } 1 \quad \forall i, t \\ \mathbf{e}^T (\mathbf{g}_t - \mathbf{d}_t) - \text{LossFn}_t(\mathbf{d}_t - \mathbf{g}_t) = 0 \quad \forall t \\ \text{Flow}_{kt}(\mathbf{g}_t - \mathbf{d}_t) \leq \bar{F}_{kt}^{\max} \quad \forall k, t \\ \text{start}_{it} = 0, i \in \text{Excluded}_\tau \\ p_t = p_t^*, t \leq \tau - 1 \end{array} \right\} \end{array}$$

This model also has an interpretation as the ELMP model for a two stage dualization of the complicating constraints. First we fix the prices for prior periods and price out the constraints to include them as part of the objective function. Then we dualize this reduced model to find the remaining prices. The corresponding statement of the conditional dispatch problem for which we find the ELMP going forward is:

$$v^\tau(\{\mathbf{y}_t\}) \equiv \inf_{g, d, on, start} \left(\sum_t \sum_i (StartCost_{it} \cdot start_{it} + NoLoad_{it} \cdot on_{it} + GenCost_{it}(g_{it})) - \sum_{t \leq \tau-1} \mathbf{p}_t^{*T} (\mathbf{d}_t - \mathbf{y}_t) \right)$$

subject to

$$m_{it} \cdot on_{it} \leq g_{it} \leq M_{it} \cdot on_{it} \quad \forall i, t$$

$$-ramp_{it} \leq g_{it} - g_{i,t-1} \leq ramp_{it} \quad \forall i, t$$

$$start_{it} \leq on_{it} \leq start_{it} + on_{i,t-1} \quad \forall i, t$$

$$start_{it} = 0 \text{ or } 1 \quad \forall i, t$$

$$on_{it} = 0 \text{ or } 1 \quad \forall i, t$$

$$\mathbf{e}^T (\mathbf{g}_t - \mathbf{d}_t) - LossFn_t(\mathbf{d}_t - \mathbf{g}_t) = 0 \quad \forall t$$

$$Flow_{kt}(\mathbf{g}_t - \mathbf{d}_t) \leq \bar{F}_{kt}^{\max} \quad \forall k, t$$

$$start_{it} = 0, i \in Excluded_\tau$$

$$\mathbf{d}_t = \mathbf{y}_t \quad \forall t \geq \tau.$$

Another approach for real-time price consistency in ELMP is to fix past decisions in the inner “market clearing” solution, as well as fixing the prices. Hence, the conditional market-clearing pricing model at time τ would take the determined prices $p_1^*, p_2^*, \dots, p_{\tau-1}^*$ and market clearing dispatch $z^\tau = \{g_t, d_t, on_t, start_t\}_1^\tau$ for the prior periods as fixed and solve as the pricing model:

$$v^\tau(\{y_t\}) \equiv \sup_p \left\{ + \sum_t p_t^T y_t + \inf_{g,d,on,start} \left(\begin{array}{l} \sum_t \sum_i (StartCost_{it} \cdot start_{it} + NoLoad_{it} \cdot on_{it} + GenCost_{it}(g_{it})) \\ - \sum_t p_t^T d_t \end{array} \right) \right\}$$

subject to

$$m_{it} \cdot on_{it} \leq g_{it} \leq M_{it} \cdot on_{it} \quad \forall i,t$$

$$-ramp_{it} \leq g_{it} - g_{i,t-1} \leq ramp_{it} \quad \forall i,t$$

$$start_{it} \leq on_{it} \leq start_{it} + on_{i,t-1} \quad \forall i,t$$

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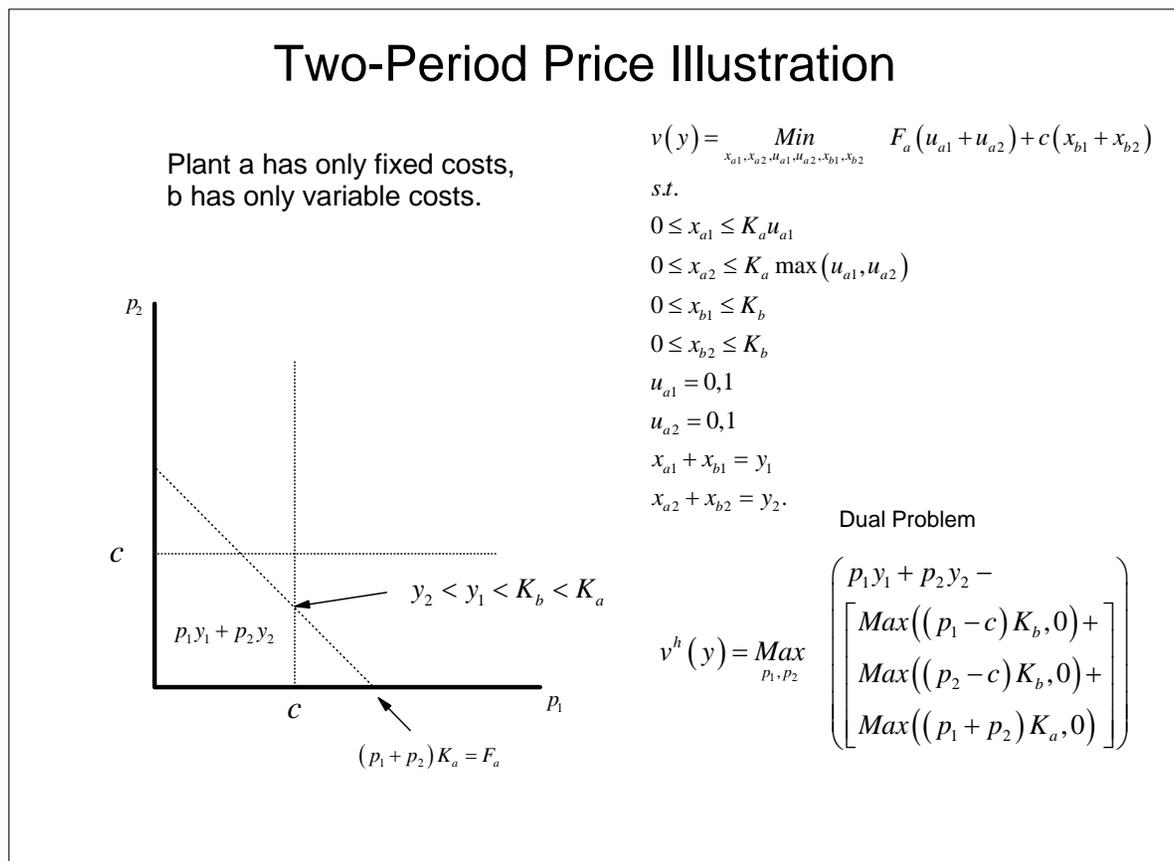
$$e^T (g_t - d_t) - LossFn_t(d_t - g_t) = 0 \quad \forall t$$

$$Flow_{kt}(g_t - d_t) \leq \bar{F}_{kt}^{\max} \quad \forall k,t$$

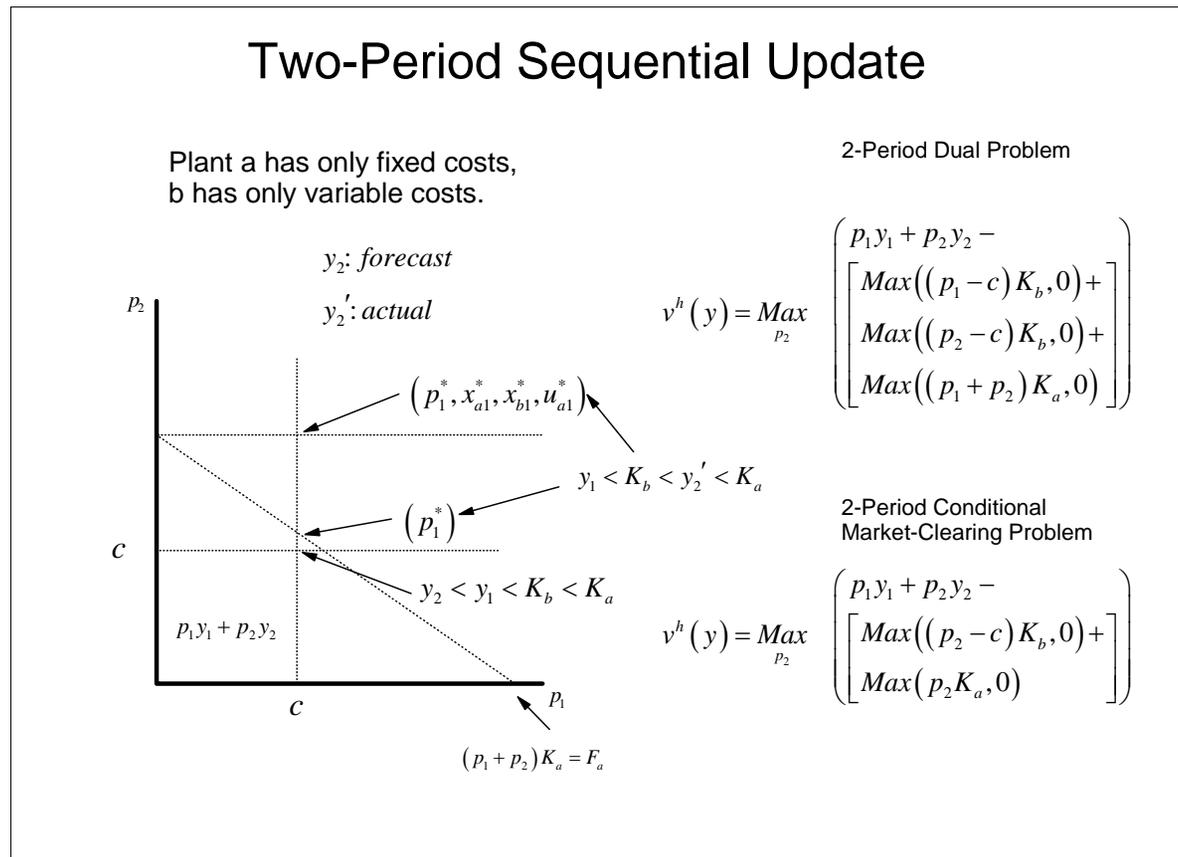
$$z^{\tau-1} = z^{*\tau-1}$$

$$p_t = p_t^*, t \leq \tau-1$$

A two period example illustrates the solution and properties of pricing model. The simplified structure with only fixed costs for one plant and variable costs for the other allows us to determine the solution from the graph of critical regions in the dual space of the prices.



A related version with different assumptions about the relation of costs illustrates the different solutions that can arise in the conditional dual and conditional market-clearing pricing problems in the second period.



Approximations of an ELMP real-time pricing model would include.

- **Block Loaded Units.** Variablize the average cost of units that are all on or off.
- **Fixed Cost Allocation.** The UK Pool solution with on and off peak periods.
- **Relaxation Variants.** Combine relaxed formulation with ad hoc fixed cost allocations.
- **Price Conditioning.** Set different windows and horizons for price consistency objective.

An objective is to obtain a workable pricing model.

- **Integrated With Day-Ahead.** Support a two-settlement system and virtual bids equating day-ahead and expected real-time prices.
- **Rolling Real-Time Pricing Updates:** Simplified separable approximations are easiest to implement. Intertemporal interactions and conditional price models present other tradeoffs.
- **Stakeholder Testing and Verification.** Simple simulations to understand market impacts.⁵

⁵ For further details on MISO implementation effort, see <https://www.midwestiso.org/Library/MeetingMaterials/Pages/ELMPTT.aspx>.

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