ON MINIMUM-UPLIFT PRICING FOR ELECTRICITY MARKETS

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Optimality and equilibrium principles serve to characterize pricing and settlement rules in electricity market designs. Practical electricity markets include both approximations and nonconvexities that deviate from the pure case of the simple equilibrium pricing model. Recent results on equilibrium price characterizations for a class of nonconvex optimization problems provide new insight of direct relevance to electricity markets. The day-ahead electricity market application illustrates the key innovation in equilibrium pricing. A minimum-uplift pricing approach provides a related theoretical framework that is closer to actual practice.

Introduction

This paper examines alternative pricing frameworks for dealing with nonconvexities and other complications in models used in day-ahead electricity markets. The analysis follows recent work by O'Neill et al.[1] who consider a more general problem with nonconvexities, but the detail here emphasizes an application for pricing and settlement systems needed for electricity markets.

The work by O'Neill et al. provides an innovative way to recast the pricing problem for a nonconvex model into a form that is familiar and subject to relatively easy analysis. The existing pricing models applied in day-ahead electricity markets use approaches that have some features in common with the results of O'Neill et al., and this suggests that the reformulated pricing model might provide a stronger theoretical foundation for pricing methods applied in the existing electricity market models. The discussion below considers these models and compares them with an alternative approach that provides another window on the existing practice.

After describing a general model that captures the principal nonconvexities of the day-ahead electricity market model, we summarize the application of the results from O'Neill et al. for a linear integer programming model with efficient prices that would support an equilibrium solution. For simplicity, refer to these as the IP prices. This is followed by further examination of the formulation and the resulting prices. A comparison

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[1] Richard P. O'Neill, Paul M. Sotkiewicz, Benjamin F. Hobbs, Michael H. Rothkopf, William R. Stewart, Jr., "Efficient Market-Clearing Prices in Markets with Nonconvexities," December 9, 2002, available on the web page of the Harvard Electricity Policy Group (http://ksgwww.harvard.edu/hepg/).

with the pricing analysis developed by Ring, recast here as minimum-uplift pricing, provides an alternative equilibrium approach.[2] A few conclusions emerge:

- The IP formulation of the price-directed competitive equilibrium provides efficient prices that support the optimal solution in the presence of nonconvexities.
- The particular formulation of a Walrasian equilibrium in O'Neill et al. can be cast as a competitive partial equilibrium in the standard economic model, and this highlights the payments required in a settlements system.
- The IP prices require an "uplift" payment and the efficient prices may lead to large uplift requirements.
- The IP prices would modify incentives in bid-based models by converting the payments scheme towards a pay-as-bid model.
- An alternative minimum-uplift pricing and settlement approach applying a similar pricing insight would be more consistent with the incentives and principles of a market-clearing price for a settlements system.
- A minimum-uplift pricing and settlement rule is close to the existing practice.

Economic Unit Commitment and Dispatch

The problem of interest is the unit commitment and dispatch problem for a day-ahead electricity market. Real-time electricity markets typically do not have as many nonconvexities, and the focal point is on the day-ahead when units can be either turned on or not, i.e. "committed" and then dispatched optimally given the configuration of units. Suppose we have a day-ahead market with T periods (say 24 hours, or 48 half-hours, etc.) for which we have both bid-I demand and bid-in supply. For the sake of this discussion, ignore the complications of security commitments required by different views about the possible level of demand.[3] Let:

$B_{it}(d_{it})$	Bid-based, well-behaved concave benefit function of demand for customer i in period t.
$C_{jt}(g_{jt})$	Bid-based, well-behaved convex cost function for output of generator j in period t.
S_j	Bid-based startup cost for generator j.

2 Brendan J. Ring, "Dispatch Based Pricing in Decentralized Power Systems," Ph. D. thesis, Department of Management, University of Canterbury, Christchurch, New Zealand, 1995. (see the HEPG web page at http://ksgwww.harvard.edu/hepg/)

3 Reliability scheduling may commit additional units for expected differences between bid-in and forecast load. Michael D. Cadwalader, Scott M. Harvey, and William W. Hogan, "Reliability, Scheduling Markets, and Electricity Pricing," Center for Business and Government, Harvard University, May 1998.

m_j, M_j — Bid-based minimum and maximum output for generator j if committed.

z_j — Integer variable ($z_j = 0,1$) modeling commitment decision for generator j.

y_t — Vector of net load at each location, $y_{\varphi t} = \sum_{i \in \varphi} d_{it} - \sum_{j \in \varphi} g_{jt}$, for location φ.

$L_t(y_t)$ — Losses in period t, with net demand $\iota^t y_t$, where $\iota^t = (1 \quad 1 \quad \cdots \quad 1)$.

$K_t(y_t)$ — Transmission constraints for net load y_t in period t.

$R_{jt}(g_{jt}, g_{jt-1})$ — Ramping or other dynamic limits for generator j.

Then the stylized economic unit commitment and dispatch problem considered here is:

$$
\begin{aligned}
&\underset{d_{it}, g_{jt}, y_t, z_j}{Max} \sum_{t=1}^{T} \left(\sum_i B_{it}(d_{it}) - \sum_j C_{jt}(g_{jt}) \right) - \sum_j S_j z_j \\
&s.t. \\
&\qquad L_t(y_t) + \iota^t y_t = 0, \\
&\qquad y_t = d_t - g_t, \forall t, \\
&\qquad g_{jt} \geq z_j m_j, \forall jt, \\
&\qquad g_{jt} \leq z_j M_j, \forall jt, \\
&\qquad R_{jt}(g_{jt}, g_{jt-1}) \leq 0, \forall jt, \\
&\qquad K_t(y_t) \leq 0, \forall t, \\
&\qquad z_j = 0 \; or \; 1, \forall j.
\end{aligned}
\tag{1}
$$

The nonconvexities arise because of the integer variables. This statement of the general problem as an integer programming problem (IP) will be useful below in that it allows consideration of both dynamics over multiple periods and transmission constraints.

To focus first on the nonconvexities, consider a simplification to a one-period problem with no transmission constraints, no losses and a fixed level of demand at a single location. This reduces (1) to a least-cost commitment and dispatch problem:

$$
\begin{aligned}
&\underset{g_j, z_j}{Min} \sum_j C_j\left(g_{jt}\right) + \sum_j S_j z_j \\
&s.t. \\
&\qquad \sum_j g_j = d, \\
&\qquad g_j \geq z_j m_j, \forall j, \\
&\qquad g_j \leq z_j M_j, \forall j, \\
&\qquad z_j = 0 \; or \; 1, \forall j.
\end{aligned} \tag{2}
$$

This least-cost commitment and dispatch problem is essentially the model from O'Neill et al. specialized to the day-ahead electricity market. Reintroducing the dynamics and transmission constraints presents no problem in principle, but the simpler model highlights the role of the integer constraints.

A familiar problem with such integer programming models is that the usual formulation of prices for the generation commodities would not support an equilibrium solution in decentralized decisions. In other words, if we decompose the problem into the optimization problems of separate generators, given only the price of the commodity determined by the dual variable for the first constraints, the decentralized problem would not necessarily include a profit maximizing solution for the generators that is consistent with the global optimum.[4]

The insight of O'Neill et al. is to expand the commodity space in a natural way, from g to the vector (g, z). The idea is to think of the market as involving "commodities" expanded from just electric energy to include something like commitment tickets for generators. Suppose that we have the optimal solution to the least-cost commitment and dispatch problem (g^*, z^*). Then consider the following continuous variable approximation to the least-cost dispatch problem:

$$
\begin{aligned}
&\underset{g_j, z_j}{Min} \sum_j C_j\left(g_j\right) + \sum_j S_j z_j \\
&s.t. \\
&\qquad \sum_j g_j = d, \\
&\qquad g_j \geq z_j m_j, \forall j, \\
&\qquad g_j \leq z_j M_j, \forall j, \\
&\qquad z_j = z_j^*, \forall j.
\end{aligned} \tag{3}
$$

The formulation in (3) treats the commitment variables as continuous, but requires the result to be equal to the optimal integer solution. Since formulating (3) requires

[4] There may be many optimal solutions for the decentralized problem, but all that is asked here is that at least one decentralized solution is consistent with the global optimum. Hence, this is a characterization of the solution but not a method for finding the solution.

knowing the optimal solution, it is not intended as an approach for finding the optimal commitment. However, it does provide a framework for defining and interpreting the associated prices.

In particular, (3) presents a well-behaved problem and will produce shadow prices for the constraints with the usual properties. Suppose that we have prices ($p, \lambda_j, \theta_j, \pi_j$) for the four types of constraints. With $C_j'(g_j) = {}^{dC_j(g_j)}\!/_{dg_j}$, note that the first order conditions for (3) would imply that:

$$\begin{aligned} &p = C_j'(g_j) - \lambda_j + \theta_j, \\ &\pi_j = S_j + \lambda_j m_j - \theta_j M_j, \\ &\lambda_j \geq 0, \theta_j \geq 0. \end{aligned} \tag{4}$$

These yield the IP prices, (p, π_j), applied to each generator. The commodity price (p) is equal to marginal production cost plus the marginal rent on generator capacity. The price for a commitment ticket (π_j) is the startup cost less the marginal rent on capacity applied to the full capacity.

Then consider the decentralized optimization problem for each generator:

$$\begin{aligned} &\underset{g_j, z_j}{Max}\ pg_j + \pi_j z_j - C_j(g_j) - S_j z_j \\ &s.t. \\ &\qquad g_j \geq z_j m_j, \\ &\qquad g_j \leq z_j M_j \\ &\qquad z_j = 0 \ or \ 1. \end{aligned} \tag{5}$$

In the case of piecewise linear costs for the generators, O'Neill et al. establish that (g_j^*, z_j^*) is an optimal solution for this decentralized problem. Further, with linear costs the optimal profit for each generator is exactly zero.[5]

O'Neill et al. point out that this conforms to a Walrasian decentralized, price-directed equilibrium in the presence of nonconvexities. As in the more familiar case of continuous problems with constant returns to scale, the zero profit condition ensures that both those who are committed and those who are not face prices that support these commitment and dispatch decisions. Generators committed to be on would find the commitment and dispatch as profit maximizing. Similarly, generators not committed would find it profit maximizing not to enter the commitment and dispatch at these prices. Hence with price-taking generators the prices support the optimal solution.

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[5] O'Neill et al., pp. 18-24, where the problem is more generally a block diagonal linear structure with complicating constraints of the type analyzed in G. B. Dantzig and P. Wolfe, "Decomposition Principle for Linear Programs," Operations Research, Vol. 8, No. 1, January-February 1960, pp. 101-111.

Note that there may be more than one set of equilibrium solutions, in both quantities and prices. The possibility of multiple quantities for a given set of prices is not important here. The multiple prices for a given set of quantities is important and will be examined further below.

In the convex cost case here, decentralized optimality also follows immediately. Simply drop the integrality constraints on z_j in (5), to obtain the relaxed continuous problem:

$$\begin{aligned}
&\underset{g_j, z_j}{Max}\; pg_j + \pi_j z_j - C_j\left(g_j\right) - S_j z_j \\
&s.t. \\
&\qquad g_j \geq z_j m_j, \\
&\qquad g_j \leq z_j M_j.
\end{aligned} \tag{6}$$

Given the convexity assumptions, this is a well-behaved optimization problem and the first order conditions using (λ_j, θ_j) imply the optimality of (g_j^*, z_j^*) for this relaxed problem in (6). Since by construction this solution also satisfies the integrality condition, it must be a solution for the generator problem in (5).

In the linear case, we have:

$$C_j\left(g_j^*\right) = C_j'\left(g_j^*\right) g_j^* .$$

Hence, for the linear case the decentralized solution satisfies a zero profit condition for each generator:

$$\begin{aligned}
\Pi_j &= pg_j^* + \pi_j z_j^* - C_j\left(g_j^*\right) - S_j z_j^* \\
&= pg_j^* + \pi_j z_j^* - C_j'\left(g_j^*\right) g_j^* - S_j z_j^* \\
&= \left(C_j'\left(g_j^*\right) - \lambda_j + \theta_j\right) g_j^* + \left(S_j + \lambda_j m_j - \theta_j M_j\right) z_j^* - C_j'\left(g_j^*\right) g_j^* - S_j z_j^* \\
&= 0.
\end{aligned} \tag{7}$$

The last condition follows from complementary slackness so that, for example, if $\theta_j > 0$ it must be that $g_j^* = M_j$ and $z_j^* = 1$.

In the more general nonlinear case, by convexity we have:

$$C_j\left(g_j^*\right) \leq C_j'\left(g_j^*\right) g_j^* .$$

The corresponding profit condition is:

$$\begin{aligned}
\Pi_j &= pg_j^* + \pi_j z_j^* - C_j\left(g_j^*\right) - S_j z_j^* \\
&\geq pg_j^* + \pi_j z_j^* - C_j'\left(g_j^*\right) g_j^* - S_j z_j^* \\
&\geq \left(C_j'\left(g_j^*\right) - \lambda_j + \theta_j\right) g_j^* + \left(S_j + \lambda_j m_j - \theta_j M_j\right) z_j^* - C_j'\left(g_j^*\right) g_j^* - S_j z_j^* \\
&\geq 0.
\end{aligned} \tag{8}$$

Importantly for the discussion below, if the cost function is convex but not linear, the inequality can be strict. Furthermore, there can be strictly positive IP prices and strictly positive profits. The zero profit condition, therefore, is a special case following from the linear cost model formulation.

This characterization of the IP equilibrium is an elegant innovation that should have application in many areas. Here the interest is on the implications for electricity markets and the degree to which this pricing framework should guide the development of electricity pricing and settlement systems.

Equilibrium IP Model

The IP prices support the price-directed equilibrium solution defined by the Walrasian metaphor of an auctioneer who announces prices and the decentralized generators who respond. This is consistent with supporting the efficient solution in the bid-based unit commitment and dispatch problem. However, this Walrasian formulation does not explain where the auctioneer gets the money, or how the pricing framework relates to the settlement system.

An equilibrium formulation for the IP economic dispatch problem that mimics the usual economic partial equilibrium model highlights the importance of the "uplift" payments required by these IP prices (p, π_j) that must be obtained through the settlement system. The commitment prices π_j require net payments to or from the market coordinator separate from the market-clearing commodity price in p. The typical electricity settlement system employs some averaging of these payments across users, with the term "uplift" borrowed from the original UK pool terminology for charges not collected as part of the market-clearing commodity price.

To identify the role of the uplift payments, we complete the partial equilibrium formulation to include the market coordinator and customers, as well as the generators. In addition to the generator problems in (5), let the system operator (market coordinator) problem be defined as:

$$\begin{aligned} &\underset{g_j, d_i, z_j}{Max}\; p\left(\sum_i d_i - \sum_j g_j\right) - \sum_j \pi_j z_j \\ &s.t. \\ &\qquad \sum_j g_j = \sum_i d_i. \end{aligned} \tag{9}$$

The system operator acts as an intermediary that trades electricity with generators and loads, and purchase commitment tickets. The profit for the system operator as transmission provider is Π_0. Although the system operator is a monopoly, the partial equilibrium formulation here assumes that the system operator is regulated to act as though it were a price-taking profit maximizer.

Introduce the share holdings of profits for consumers ($s_{ij} \geq 0, \quad \sum_i s_{ij} = 1$) along with wealth endowments ($\tilde{w}_i$) for the numeraire good with consumption level (c_i). Then the consumer optimization problem is:

$$
\begin{aligned}
& \underset{d_i, c_i}{Max}\, B_i\left(d_i\right) + c_i \\
& s.t. \\
& \quad pd_i + c_i \leq \tilde{w}_i + \sum_{j \geq 0} s_{ij} \Pi_j .
\end{aligned}
\tag{10}
$$

The familiar definition of a competitive partial equilibrium is a set of prices and quantities ($p, \pi_j, d_i^*, g_j^*, z_j^*$) that simultaneously solves (5), (9), and (10).[6]

The analysis above establishes the IP prices as applicable for (5). The consumer problem is conventional and the same price (p) will support the solution of (10). The difficulty is with the system operator problem, where for any nonzero commitment ticket prices (π_j) the solution is unbounded. Hence, expanding the commodity space to include the commitment tickets is not quite enough. The further insight of O'Neill et al. is to impose the added requirement that $z_j = z_j^*$ to obtain the modified version of the system operator problem:

$$
\begin{aligned}
& \underset{g_j, d_i, z_j}{Max}\; p\left(\sum_i d_i - \sum_j g_j\right) - \sum_j \pi_j z_j \\
& s.t. \\
& \quad \sum_j g_j = \sum_i d_i, \\
& \quad z_j = z_j^*, \forall j.
\end{aligned}
\tag{11}
$$

In this sense, we have the IP competitive partial equilibrium as a set of prices and quantities ($p, \pi_j, d_i^*, g_j^*, z_j^*$) that simultaneously solve (5), (10), and (11). The IP prices support this version of the partial equilibrium model. Of course, we could never use this decentralized formulation as a means for actually finding a solution, since we must know the commitment solution in order to formulate the problem. But this version of the partial equilibrium problem provides a way to think about pricing rules and settlement protocols.

Using these IP prices, the equilibrium profit for the system operator is:

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[6] The standard partial equilibrium assumptions are that electricity is a small part of the overall economy with consequent small wealth effects, and prices of other goods and services are approximately unaffected by changes in the electricity market. For further details, see Mas-Colell, A., M.D. Whinston, and J.R. Green, Microeconomic Theory, Oxford University Press, 1995, pp. 311-343.

$$\Pi_0 = p\left(\sum_i d_i^* - \sum_j g_j^*\right) - \sum_j \pi_j z_j^* . \qquad (12)$$

In this case with no transmission effects, this reduces to $\Pi_0 = -\sum_j \pi_j z_j^*$, the net payments to the generators for commitment tickets. As seen below, the commitment ticket prices π_j may be positive or negative and, in turn, the net profit of the system operator may be negative or positive. In this formal equilibrium model, the net payment is recovered from consumers in the form of a fixed charge that is independent of the level of consumption through the consumers' ownership shares of the system operator net losses or profits.

In the real system, such fixed transfers are not available. This net payment from customers appears typically as an "uplift" charge allocated by some rule across final consumption. In the simple case of a single location and no transmission effects, collection for the uplift is much the same as for the commodity payments at price *p*. However, in the more general case with transmission constraints, the distributional effects across locations and customers could be quite important. Hence, the distinction between direct commodity charges and uplift payments could be important.

As illustrated below, the split between the commodity price and the uplift can be volatile as a function of the load. Small changes in the load produce large changes in incremental costs, IP commodity prices, and IP commitment ticket prices. This volatility itself could be a problem, and the IP commodity price could be technically correct as the infinitesimal marginal cost but misleading as a signal of the cost for relatively small but discrete increments of load.

For example, in the electricity system in New York there are many "block loaded" combustion turbine units for which the minimum and maximum outputs are the same. Hence, these units are either off, or on at full capacity. The running cost is relatively high. When these units are committed regularly to meet incremental load, the IP commodity price typically would reflect the (much) lower marginal cost of some other unit that must then be dispatched at less than capacity. Under IP pricing, most of the total incremental cost of the block-loaded units would then fall into the category of "uplift" and would provide a muted signal for the locational prices.

In the New York system, this was viewed as a problem. An ad hoc fix for pricing block-loaded units was to compute the day-ahead prices by calculating a hypothetical dispatch in which the "block loaded" units were committed but treated as dispatchable, meaning that the lower bound on output would be set to zero in the model. This allows these units to set the marginal commodity price and leaves much less to be covered by the residual uplift.[7] Of course, the actual dispatch would not have these units as dispatchable. This particular approach is computationally simple in that it uses the same dispatch software for computing prices and does not require any special pricing software, issues discussed further below.

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[7] For a discussion and critique of the New York approach, see Steven Stoft, Power System Economics, Wiley-Interscience, 2002, pp. 277-285.

Another feature of the equilibrium IP prices is the possibility, even likelihood, that many of the commitment ticket prices (π_j) could be negative. In other words, the generator must make a payment in order to have the unit committed. In the linear case, this payment is just the amount needed to leave the generator with zero profits. In the more general case, this negative payment would be sufficient to capture all the scarcity rents associated with the generators capacity constraint, leaving only the infra-marginal rent on the nonlinear cost function.

In effect, therefore, application of strict IP prices would fashion the system more as a pay-as-bid mechanism than paying the market-clearing price. Although there would be a single market-clearing IP commodity price p, the IP commitment ticket price π_j would take back the bulk of the infra-marginal scarcity rent from the generator capacity constraints. This seems inconsistent with the most attractive incentive properties of the market-clearing auction in the short run, and inconsistent with the incentives to introduce new lower cost generation in the long run.

O'Neill et al. identify these negative commitment ticket prices as problematic.[8] The zero-profit condition emulates the long-run perfect competitive equilibrium where there may be many potential entrants. In the short-run, where the number of generators of each type is fixed, O'Neill et al. suggest the possibility of allowing positive profits but do not address the treatment needed to maintain the equilibrium conditions for uncommitted units. The discussion below further addresses these and related settlement issues.

Incentives and IP Prices

The equilibrium analysis above assumes accuracy of the bids. An objective for electricity auctions that utilize market-clearing rather than pay-as-bid pricing is to promote a better approximation of incentive-compatible bidding. Absent the integer constraints, the idea is that an infra-marginal generator has an incentive to act as a price-taker and bid its true marginal cost. Since the payment to the generator will be the market-clearing commodity price at its location, the bid affects only whether the generator is dispatched, not what it is paid.

This system is not strictly incentive compatible because there is always some generator on the margin that is setting the market price.[9] Since the marginal generator's bid can affect the market price, there is an incentive for the marginal generator to bid other than its true costs. However, if there are many bidders and some uncertainty about who will be the marginal bidder in advance of knowing everything about supply and demand, the expectation is that most bidders, most of the time, would act as infra-marginal suppliers and bid their true costs. In addition to improving the efficiency of the dispatch,

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[8] O'Neill et al., pp. 24-25

[9] The usual electricity market auction formulation is not as a fully incentive-compatible second price or Vickrey auction but rather as a first-price, non-discriminatory auction. In the presence of binding transmission constraints, there would be more than one marginal generator.

this practice greatly simplifies both bidding for generators and the dispatch problem for the system operator when dealing with the complexities of transmission losses and constraints.[10]

The use of market-clearing prices also provides incentives for efficient long-run entry. A new supplier can expect to capture the rents associated with its lower cost generation. However, were these rents to be foregone through the pricing rules, there would be a significantly reduced incentive to enter the market. As a matter of policy, therefore, it would be desirable to maintain the practice of paying the market-clearing price rather than the bid price for each generator.

This analysis of commodity price bidding incentives does not apply strictly to the case of IP equilibrium problem because of the integer constraints, startup costs, and commitment ticket prices. By construction, the commitment ticket price is $\pi_j = S_j + \lambda_j m_j - \theta_j M_j$. This includes the bid-based startup cost S_j. Hence, there is an inherent pay-as-bid feature that is difficult to avoid. Every commitment ticket is unique, and applies to a unique plant. There is no obvious "market-clearing" commitment ticket price produced in the IP equilibrium model. The higher the bid for startup cost, the higher the payment to the generator under the IP commitment ticket price. Furthermore, for a given commodity price (p) the higher the bid-in marginal cost the lower would be the rent (θ_j) on capacity and, therefore, the higher the commitment ticket price. Hence, the generator has an incentive to increase the startup bid and marginal cost bid to levels that make the system operator just indifferent to the commitment and dispatch decisions. This seems to carry with it all the perverse effects of the pay-as-bid auction, making it harder for the generator to bid and harder for the dispatcher to decide on the most economic dispatch.

In practice, markets that include similar commitment payments and associated uplift charges have adopted a simple rule that solves most but not all these problems. In effect, in terms of the IP equilibrium, the practice would be to pay the commitment ticket price to the generator when it is positive and required to bring the generator to zero profits, but never to charge the generator for a commitment ticket that is consistent with the economic dispatch and would reduce profits to zero.

This has an immediate attractive effect. As for the case with no startup costs, this settlement policy at least mitigates, if not eliminates, the incentive for an infra-marginal generator to increase its marginal cost bid. There is a remaining incentive to increase the marginal cost bids, but the incentive is now limited by the amount of the average startup bid for that generator. If π_j is negative, increasing the marginal cost bid reduces reported θ_j but only further reduces the chance of being dispatched without increasing the

[10] The problems of bidding and dispatching with losses and transmission constraints are almost never raised in discussions of pay-as-bid auctions, but may be the most serious difficulties. In the presence of transmission constraints, prices could vary dramatically even though individual generator marginal costs remain fixed.

commitment payment. Only when π_j is positive is there an incentive to increase the marginal cost bid in order to reduce reported θ_j. Hence, the average startup cost (S_j / M_j) is an upper bound on the added deviation from true costs that could have an effect in increasing the generator's commitment ticket price. There is still an incentive to increase the startup bid, as this is still unique to the generator and there is no market-clearing price commitment ticket price. However, in the real application, the startup cost is easier to monitor than the sometimes highly variable energy cost. There could be rules, such as with PJM, requiring that the startup bid cannot change frequently or be subject to some other form of light-handed regulation.[11] This mitigates but does not eliminate the deviations from pure incentive compatible bidding.

It appears, therefore, that the pragmatic rule of not charging for commitment tickets, but only paying in the case where the payment is need to eliminate losses at the IP commodity price, would restore approximately the beneficial incentive effects. Only in the case where the generator is near the zero profit condition is there much of an incentive to deviate from true marginal cost bidding, and such bidding above marginal cost carries with it the risk of loss by not being dispatched.

This raises the question of how this pragmatic rule relates to the IP equilibrium pricing framework developed by O'Neill et al. Here we can exploit the observations above that there might be many equilibrium pricing sets that would support the equilibrium solution. It is clear that not charging for the negative commitment ticket prices would upset the IP equilibrium in the case of the possible uncommitted plants that might find it profitable to operate at the commodity IP price p. Furthermore, the condition that the IP prices support (6) without the integer constraints suggests that some available information in the multi-part bidding of unit commitment remains unexploited.

Minimum-Uplift Equilibrium Model

An alternative pricing rule in the same spirit defines a commitment payment to generators that replaces the commitment ticket price. Rather than using the shadow prices from a continuous approximation of the economic commitment and dispatch model, go directly to the available as-bid profit calculations. This is the approach proposed by Ring who analyzed many different applications of the idea dealing with deviations from perfect optimization with the simple perfectly convex equilibrium model.[12]

Ring emphasized the linear case and focused on the uplift payments, with the idea of minimizing these payments as a "best compromise" to provide workable pricing methods in the presence of deviations from the idealized model. Following Ring's idea,

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[11] "The start-up and no-load offers for price-based units can only occur during the open enrollment periods, which are once every six months … ". Section 8, Managing Unit Start-up & No Load Data , PJM Scheduling Manual, PJM Manual M11 - Version 18. (www.pjm.com)

[12] Brendan J. Ring, "Dispatch Based Pricing in Decentralized Power Systems," Ph. D. thesis, Department of Management, University of Canterbury, Christchurch, New Zealand, 1995.

define the nonnegative commitment payments in terms of uplift requirements needed to make committed and uncommitted generators at least indifferent. In particular, for the general nonlinear case suppose that we solve the day-ahead unit commitment and dispatch problem in (2). Given the solution (g^*, z^*) and an arbitrary candidate for the market-clearing commodity price (p), define a commitment related payment to each generator as:

$$\begin{aligned}
&\tilde{\pi}_j(p) = Max\left(0, \Pi_j^+ - \Pi_j^*\right), \\
&where \\
&\Pi_j^* = pg_j^* - C_j\left(g_j^*\right) - S_j z_j^*, \\
&\Pi_j^+ = \left\{ \begin{array}{l} \underset{g_j, z_j}{Max}\; pg_j - C_j\left(g_j\right) - S_j z_j \\ s.t. \\ \qquad g_j \geq z_j m_j, \\ \qquad g_j \leq z_j M_j \\ \qquad z_j = 0 \; or \; 1. \end{array} \right\}.
\end{aligned}$$

Although the rule could extend easily to include price sensitive demands, to simplify the notation here consider the case where demand is fixed. Then the pricing rule is to pay each generator an amount $\tilde{\pi}_j(p)$ in addition to the payment for the commodity produced. The payment is conditional on accepting the commitment and dispatch solution.

In the case of a committed generator, the commitment payment $\tilde{\pi}_j(p)$ is the loss on the optimal solution plus any opportunity cost given the commodity price p. If the actual dispatch is profitable and optimal at these prices, the payment is zero, allowing the committed generator to keep any profits. For the uncommitted generation, the payment $\tilde{\pi}_j(p)$ is the opportunity cost at the commodity price p, if any. The payments are never negative.

With this formulation and settlement rule for the commitment payments, we could restate the profit-maximizing, price-taking generator's problem as:

$$\begin{aligned}
&\underset{g_j, z_j}{Max}\; pg_j + \tilde{\pi}_j(p)\delta\left(g_j, z_j, g_j^*, z_j^*\right) - C_j\left(g_j\right) - S_j z_j \\
&s.t. \\
&\qquad g_j \geq z_j m_j, \\
&\qquad g_j \leq z_j M_j \\
&\qquad z_j = 0 \; or \; 1.
\end{aligned} \tag{13}$$

Here

$$\delta\left(g_j, z_j, g_j^*, z_j^*\right)=\begin{cases}1, & \text{for } z_j = z_j^*, g_j = g_j^* \\ 0, & \text{otherwise}\end{cases}.$$

Hence, the commitment payment is made only if the generator matches the desired commitment and dispatch solution. By construction, the payment is the minimum amount that would make the generator indifferent between accepting the day-ahead solution versus responding optimally to the commodity price p.

This pricing rule is similar in spirit but not identical to the pricing rules in place in New York and PJM. Note this set of commitment payments $\tilde{\pi}_j(p)$ could include payments both to units that are committed and dispatched, and those that are excluded from the day-ahead dispatch. The latter would receive the foregone profits that would accrue to the generator if it had been allowed to commit and dispatch at commodity price p. These payments are necessary to maintain the IP-like condition of decentralized equilibrium for all existing units. With minimum output restrictions forcing production from committed plants, it is not possible to guarantee that all uncommitted units are unprofitable given price p.[13]

With this set of commitment payments and settlement rules, it is apparent that ($p, \tilde{\pi}_j(p)$) supports the competitive market solution. The difference from the IP prices can be explained by referring to the analysis above. There are many prices that can support the decentralized solution. The IP prices (p, π_j), in particular, are sufficient to support the continuous version of the competitive equilibrium using (6) while ignoring the integer constraints. By contrast this may not be true of ($p, \tilde{\pi}_j(p)$), which depend on the integer constraints. However, the new prices and payments do support the decentralized equilibrium with the integer constraints, and for a given p the commitment payment $\tilde{\pi}_j(p)$ for each generator is the minimum nonnegative payment with this property. The resulting generator profit would be:

$$\Pi_j = \tilde{\pi}_j(p) + \Pi_j^* = \tilde{\pi}_j(p) + p g_j^* - C\left(g_j^*\right) - S_j z_j^* \geq 0. \tag{14}$$

As with the IP prices, the total uplift charge that must be collected from customers in the settlements process is $UP(p) = \sum_j \tilde{\pi}_j(p)$.

There is no unique commodity price and for each price there would be a different total uplift payment $UP(p)$. This suggests a further rule intended to choose a commodity price that comes close to a "market-clearing" commodity price in the sense of minimizing the total uplift payment. Hence, following Ring and given the solution to the commitment

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13 Given the history of problems with constrained off units in context of transmission constraints and zonal pricing, there should be a caution flag at this point. However, these circumstances are not exactly the same. There is a simple solution to the transmission case in nodal pricing. In addition, precluding payments in the commitment case presents its own incentive problems.

problem in (2), define MinUP prices as those ($\tilde{p}, \tilde{\pi}_j(\tilde{p})$) that minimize the total uplift requirement. This in turn maximizes the amount of total payment based on the market-clearing commodity price.

This rule is a heuristic that follows the same intuition behind the treatment of the "block-loaded" units in New York. The discrete incremental cost of additional load may be high but the exact marginal cost is low. Rather than setting the commodity price equal to the exact marginal cost, the idea is to consider the tradeoff between the discrete incremental cost, the exact marginal cost, and the total uplift to give a commodity price that is more consistent with the intuition about the average cost of incremental load. In the presence of integer constraints, there is no perfect solution to this problem, but the MinUP pricing and settlement rule is an alternative to the IP rule that has similar short-run theoretical efficiency given the bids and better incentive properties to support improved bidding. This minimum-uplift pricing moves as close as possible to full reliance on the usual "one-part" commodity prices while maintaining the principles of non-discrimination with respect to the commodity price.[14] Some discrimination remains in the commitment payments, but these are as small as possible while still preserving the equilibrium condition that all bidders accept the optimal outcome as maximizing given the prices. This is the pricing philosophy that Ring labeled as the "best compromise." However, as Ring suggested, the "best" property depends on further analysis of the long-run incentive effects. The label as minimum-uplift pricing is simply descriptive.

An Example

The paper by O'Neill et al. illustrates IP pricing using an example taken from Scarf.[15] The model consists of two types of plants, "Smokestack" and "High Tech." For purposes of this discussion, a modified example includes a "Med Tech" plant that has a minimum output level to capture a common feature in electricity markets. The technology summary for these three types of facilities includes:

Production Characteristics

	Smokestack	**High Tech**	**Med Tech**
Capacity	16	7	6
Minimum Output	0	0	2
Startup Cost	53	30	0
Marginal Cost	3	2	7
Average Cost at Capacity	6.3125	6.2857	7

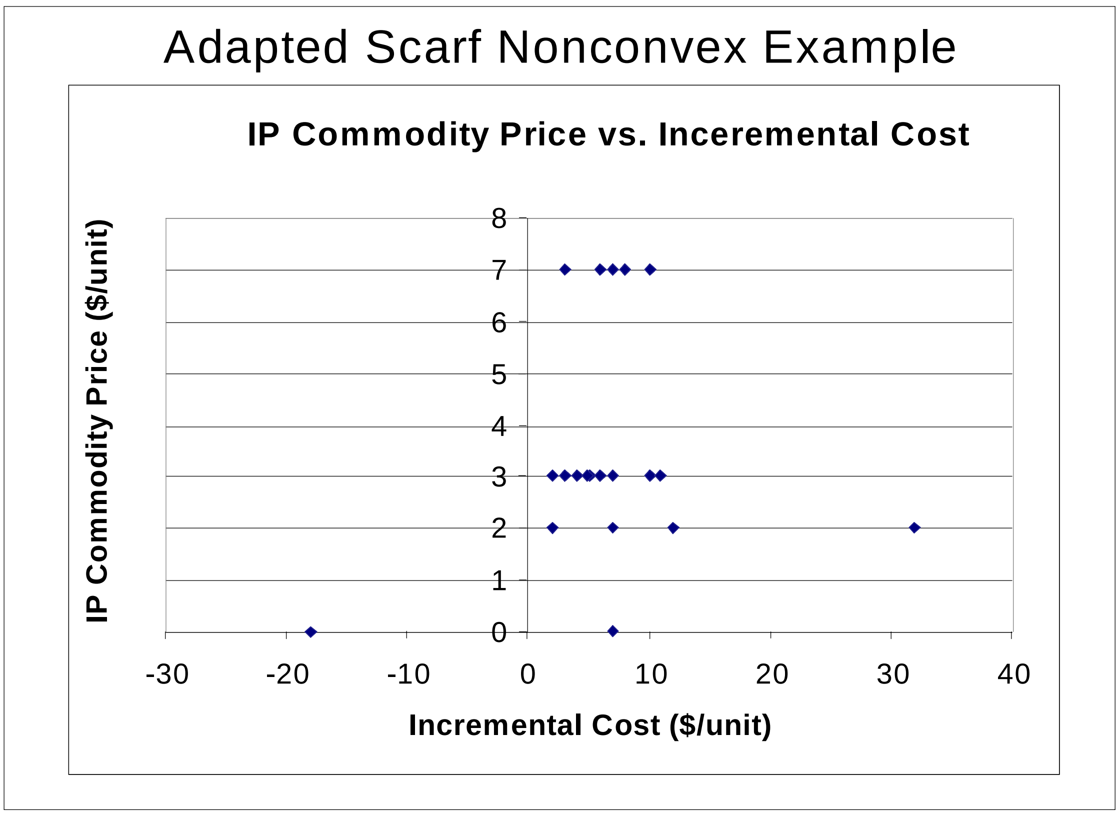
---

[14] One-part Ramsey pricing would require using the demand elasticities to discriminate among consumers.

[15] H. E. Scarf, "The Allocation of Resources in the Presence of Indivisibilities," Journal of Economic Perspectives, Vol. 8, No. 4, Fall 1994, pp. 111-128.

The model is configured not with an infinite number of each type but with 6, 5, and 5 generators of each type of generator, respectively. With these plants, the maximum feasible demand is 161 units.

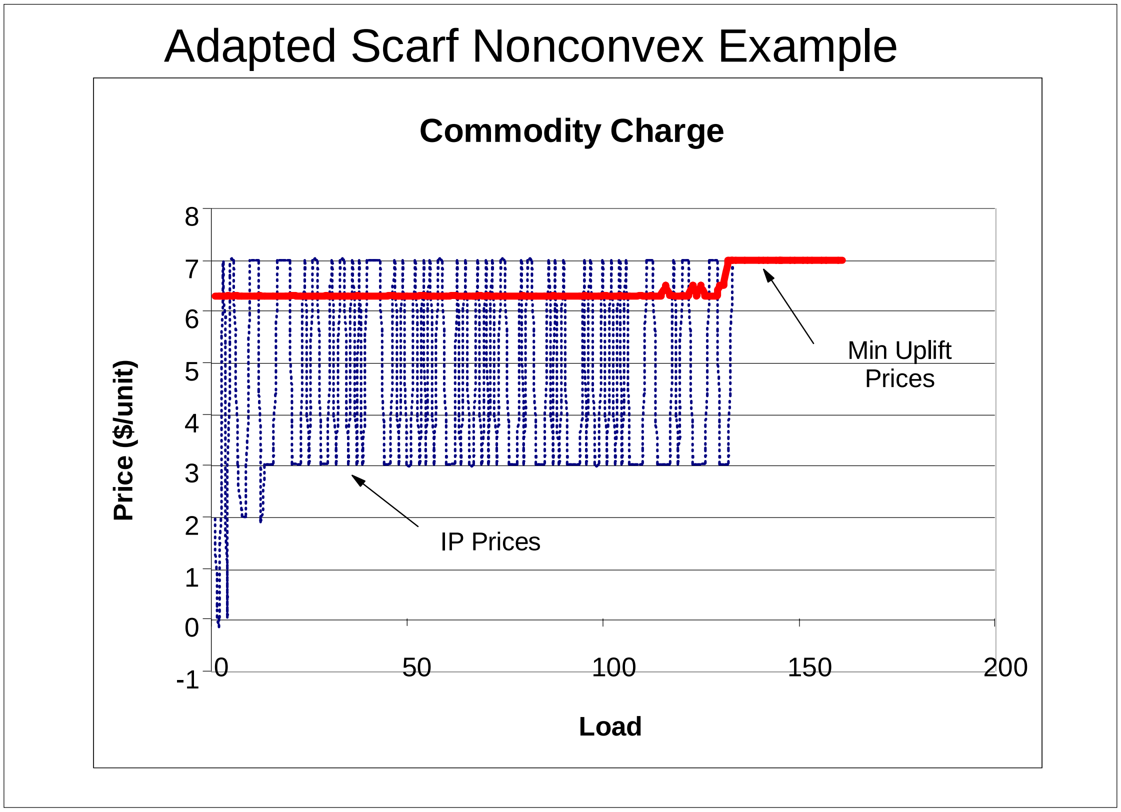
The table appended below reports the solution of (2) for each load level from 1 to 161 units. The report includes demand, number of each type committed, collective output of each type, IP commodity price, IP total uplift cost, MinUP commodity price, and the MinUP total uplift cost.[16] In the case of the IP commodity price, where more than one price is possible, the table simply reports the (extreme point) solution actually selected by the solver.



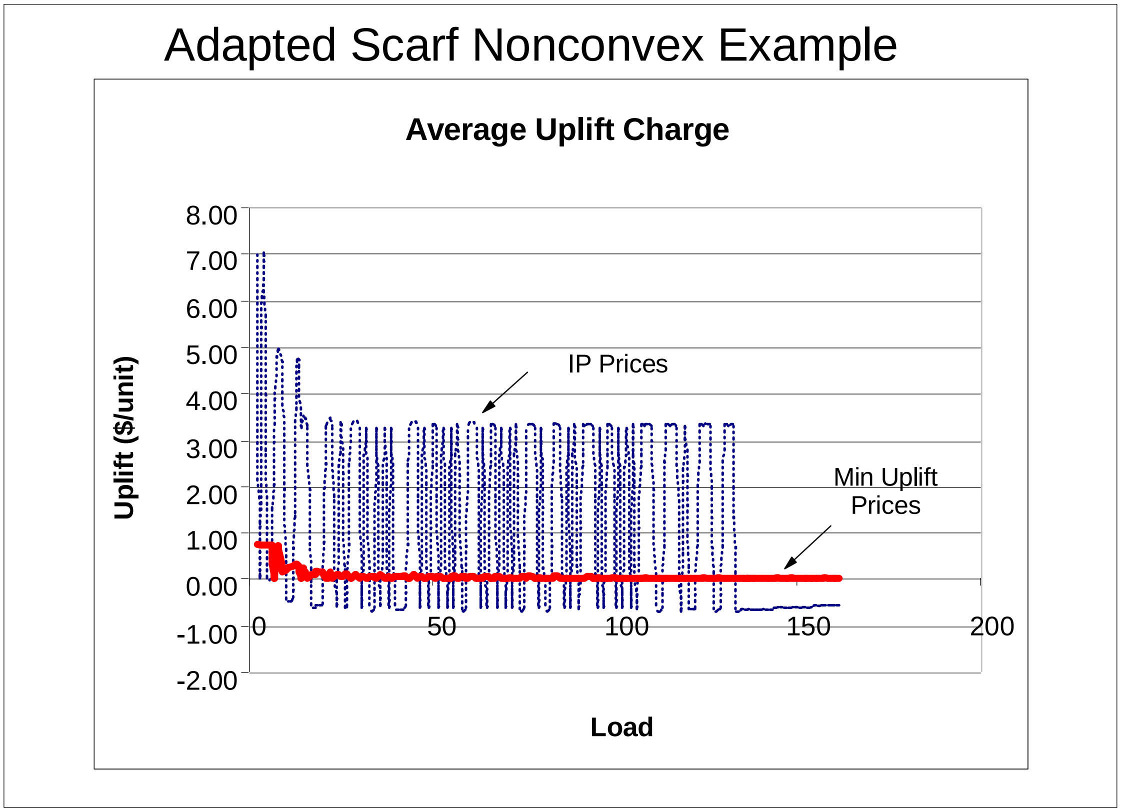
The graphic above first compares the result for the incremental cost defined as the difference in total cost for each unit of load and the IP commodity price. Ideally we would find that the IP commodity price served as a good proxy for the incremental cost. However, as shown in the graph, there is not much of a correlation. Because of the integer and linear nature of the problem, there are only a few possible extreme point values for the IP commodity price, and the incremental cost is always an integer. However, the resulting lattice of 161 points reveals little relationship, and the $R^2 = 0.29$. By design, the

16 The detailed calculations were with the Excel Solver and the spreadsheet is available on request.

MinUP commodity prices would not track incremental costs exactly, but should come closer on average.

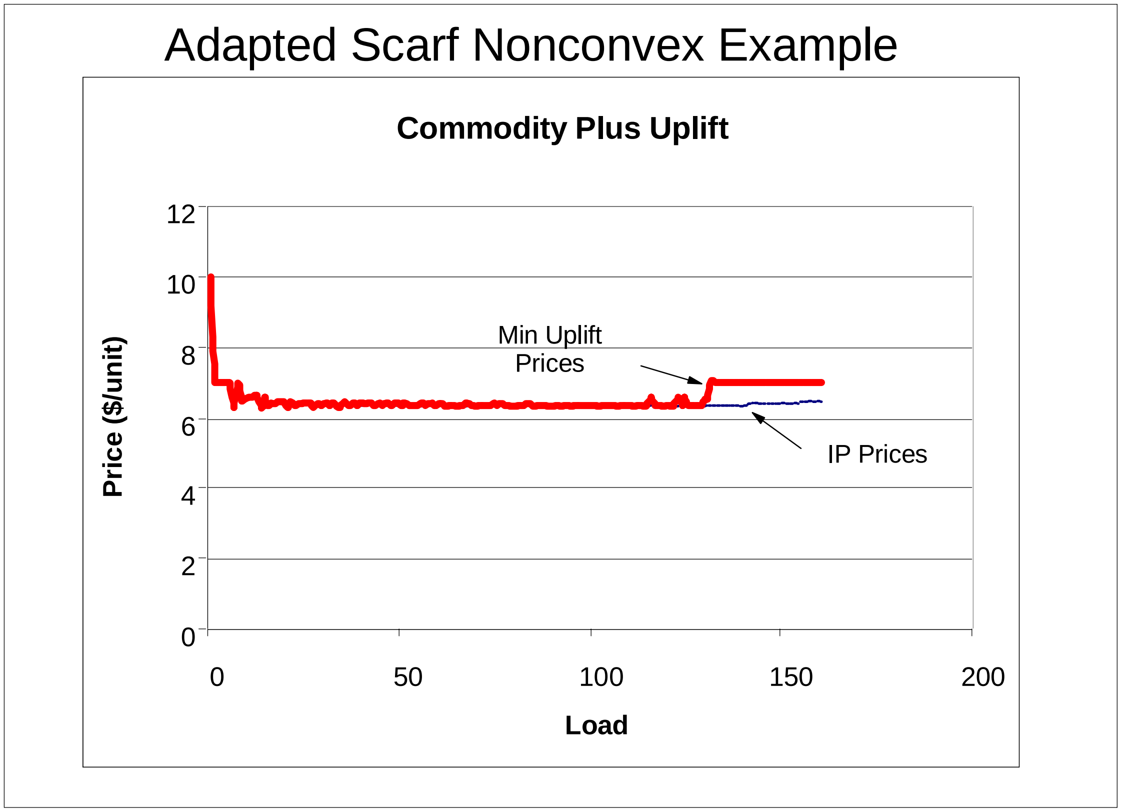


The graphic above compares the IP commodity charge with the MinUP commodity charge. As in O'Neill et al., the IP commodity charge is volatile and barely upward sloping on average. The average IP commodity charge over the whole range is 5.10. By comparison, the MinUP commodity charge is more stable and generally flat or upward sloping. The average MinUP commodity charge is 6.44 over the whole range. This compares with the average incremental cost of 6.43. Note that both the IP and MinUP commodity charges equal the marginal cost of 7 for the "Med Tech" units once load reaches 134 and all the lumpy integer effects become inframarginal.



A similar comparison of the uplift charge per unit of load mirrors the effects of the commodity charges. The IP commitment ticket prices, and the corresponding uplift charges, are very sensitive to the load level. Further the charges are often negative, implying payments by the generators to meet the zero profit condition. By construction, the MinUP generator commitment payments and corresponding uplift charges are nonnegative and small.

Putting the two charges together, as shown in the following graphic, illustrates that the net effect on consumers is almost the same in this example, although not quite identical. In the case of the IP total price, by construction this is the same as the average as-bid cost. This is the pay-as-bid property of the strict application of the IP pricing rule. By contrast, the MinUP rule reverts to the standard market-clearing price when the lumpy integer effects no longer dominate the solution.



In summary, for this example the total charges for much of the load range would be similar under the IP and MinUP rules, but the decomposition of prices into market-clearing commodity prices and uplift payments would be quite different. Both pricing rules support an appropriately defined version of a competitive partial equilibrium. By construction, the MinUP rule has the lowest uplift payment. Further, the MinUP rule provides bidding incentives more like those of a market-clearing auction and allows for short-term profits for inframarginal generators. This incentive effect would be most pronounced over the range of load where the integer effects give way to a conventional market-clearing price under the MinUP rule.

Multiple Periods

The basic idea of selecting a consistent set of market-clearing prices and associated uplift payments extends immediately to a day-ahead market with many periods. A generic version of the day-ahead model for commitment and dispatch allows for location specific loads and generation, multiple periods, transmission constraints, and ramping or other dynamic limits. Given the bids for load and generation, the economic commitment and dispatch problem from (1) is:

$$\begin{aligned}
&\underset{d_{it},g_{jt},y_t,z_j}{Max} \sum_{t=1}^{T}\left(\sum_i B_{it}\left(d_{it}\right)-\sum_j C_{jt}\left(g_{jt}\right)\right)-\sum_j S_j z_j \\
&s.t. \\
&\quad L_t\left(y_t\right)+\iota^t y_t=0, \\
&\quad y_t=d_t-g_t, \forall t, \\
&\quad g_{jt}\geq z_j m_j, \forall jt, \\
&\quad g_{jt}\leq z_j M_j, \forall jt, \\
&\quad R_{jt}\left(g_{jt},g_{jt-1}\right)\leq 0, \forall jt, \\
&\quad K_t\left(y_t\right)\leq 0, \forall t, \\
&\quad z_j=0 \; or \; 1, \forall j.
\end{aligned} \tag{15}$$

Again, let an optimal solution for this problem include the generation and commitment decisions (g^*, z^*). The corresponding version of the continuous problem in (3) as proposed by O'Neill et al. would be:

$$\begin{aligned}
&\underset{d_{it},g_{jt},y_t,z_j}{Max} \sum_{t=1}^{T}\left(\sum_i B_{it}\left(d_{it}\right)-\sum_j C_{jt}\left(g_{jt}\right)\right)-\sum_j S_j z_j \\
&s.t. \\
&\quad L_t\left(y_t\right)+\iota^t y_t=0, \\
&\quad y_t=d_t-g_t, \forall t, \\
&\quad g_{jt}\geq z_j m_j, \forall jt, \\
&\quad g_{jt}\leq z_j M_j, \forall jt, \\
&\quad R_{jt}\left(g_{jt},g_{jt-1}\right)\leq 0, \forall jt, \\
&\quad K_t\left(y_t\right)\leq 0, \forall t, \\
&\quad z_j=z^*_j, \forall j.
\end{aligned} \tag{16}$$

In general, due to losses and transmission constraints, the feasible set may not be convex. However, as is typically the case for electricity applications, assume the problem is sufficiently regular to ensure the existence of shadow prices satisfying the necessary first-order conditions for optimality.[17] With the price of marginal losses as the price at the reference or swing bus in each period, p_{st} , we have vectors of prices for each of the six types of constraint, ($p_t, \lambda_t, \theta_t, \gamma_t, \mu_t, \pi$). The price relationships from (16) and analogous to (4) become:

[17] Mokhtar S. Bazaraa, Hanif D. Sherali, and C.M. Shetty, Nonlinear Programming: Theory and Algorithms, John Wiley & Sons, 2nd. Edition, 1993, Pp. 162-163.

$$
\begin{aligned}
p_{jt} &= C'_{jt}\left(g_{jt}\right) - \lambda_{jt} + \theta_{jt} + \gamma_{jt}\frac{\partial R_{jt}\left(g^*_{jt}, g^*_{jt-1}\right)}{\partial g^*_{jt}} + \gamma_{jt+1}\frac{\partial R_{jt+1}\left(g^*_{jt+1}, g^*_{jt}\right)}{\partial g^*_{jt}} \\
&= p_{st}\left(1 + \frac{\partial L_t\left(y^*_t\right)}{\partial y_{jt}}\right) + \mu_t \nabla K_{jt}\left(y^*_t\right), \\
\pi_j &= S_j + \sum_{t=1}^{T}\left(\lambda_{jt} m_j - \theta_{jt} M_j\right), \\
&\lambda \geq 0, \theta \geq 0, \gamma \geq 0, \mu \geq 0.
\end{aligned} \tag{17}
$$

Again the IP prices are (p, π_j), where $p = \left(p_{jt}\right)$ the location and period specific prices. The locational commodity price (p) is equal to marginal production cost plus the marginal rents on capacity and ramping. In turn, the locational price is equal to the price at the reference bus plus marginal losses and transmission rents. The price for a commitment ticket (π_j) is the startup cost less the marginal rent on capacity applied to the full capacity over the full period.

Then consider a decentralized optimization problem for each generator:

$$
\begin{aligned}
&\underset{g_{jt}, z_j}{Max} \sum_{t=1}^{T}\left(p_{jt} g_{jt} - C_{jt}\left(g_{jt}\right)\right) + \pi_j z_j - S_j z_j \\
&s.t. \\
&\qquad g_{jt} \geq z_j m_j, \\
&\qquad g_{jt} \leq z_j M_j, \\
&\qquad R_{jt}\left(g_{jt}, g_{jt-1}\right) \leq 0, \\
&\qquad z_j = 0 \; or \; 1.
\end{aligned} \tag{18}
$$

As before for each generator, the IP prices support the optimal solution for the continuous version of this problem as in (6), and therefore for the integer version where (18) is analogous to (5). In the case of the system operator problem, the IP prices would at least satisfy the first-order necessary conditions for optimality.

To formulate the uplift requirement for a generator over the day-ahead schedule, redefine $\tilde{\pi}_j\left(p\right)$ as

$$
\begin{aligned}
&\tilde{\pi}_j(p) = Max\left(0, \Pi_j^+ - \Pi_j^*\right), \\
&where \\
&\Pi_j^* = \sum_{t=1}^{T}\left(p_{jt} g_{jt}^* - C_{jt}\left(g_{jt}^*\right)\right) - S_j z_j^*, \\
&\Pi_j^+ = \left\{ \begin{aligned}
&\underset{g_j^t, z_j}{Max} \sum_{t=1}^{T}\left(p_j^t g_{jt} - C_{jt}\left(g_{jt}\right)\right) - S_j z_j \\
&s.t. \\
&\qquad g_{jt} \geq z_j m_j, \\
&\qquad g_{jt} \leq z_j M_j, \\
&\qquad R_{jt}\left(g_{jt}, g_{jt-1}\right) \leq 0, \\
&\qquad z_j = 0 \; or \; 1.
\end{aligned} \right\}.
\end{aligned}
\tag{19}
$$

Following the same argument used before, it is apparent that ($p, \tilde{\pi}_j(p)$) supports the competitive market solution. A similar construct would define uplift payments for price-sensitive load. The difference from the IP prices can be explained by referring to the analysis above. There are many prices that can support the decentralized solution. The IP prices (p, π_j), in particular, are sufficient to support the continuous version of the competitive equilibrium using (6) while ignoring the integer constraints. By contrast this may not be true of ($p, \tilde{\pi}_j(p)$), which depend on the integer constraints in:

$$
\begin{aligned}
&\underset{g_{jt}, z_j}{Max} \sum_{t=1}^{T}\left(p_{jt} g_{jt} - C_{jt}\left(g_{jt}\right)\right) + \tilde{\pi}_j(p)\delta\left(g_j, z_j, g_j^*, z_j^*\right) - S_j z_j \\
&s.t. \\
&\qquad g_{jt} \geq z_j m_j, \\
&\qquad g_{jt} \leq z_j M_j, \\
&\qquad R_{jt}\left(g_{jt}, g_{jt-1}\right) \leq 0, \\
&\qquad z_j = 0 \; or \; 1.
\end{aligned}
\tag{20}
$$

However, the new prices and commitment payments do support the decentralized equilibrium with the integer constraints, and for each generator $\tilde{\pi}_j(p)$ is the minimum nonnegative payment with this property for a given set of locational prices p .

As with the static problem, in the full day-ahead problem there could be multiple sets of market-clearing prices and associated uplift payments. The same idea applied then to define the MinUP prices and payments ($\tilde{p}, \tilde{\pi}_j(\tilde{p})$) as the choice of market clearing locational prices that minimize the total uplift payments across all generators.

Approximate Optimization

The notion of uplift payments to support economic commitment and dispatch is a natural application of the similar approach in dealing with approximate optimizations. The idealized version of the economic commitment and dispatch problem in (1) presents a complicated challenge to the system operator. Many constraints are only approximate, such as interface constraints to deal with stability or voltage problems in a model with only a real power made explicit. The real dispatch may require operator judgments and occasional use of plants "out-of-merit" compared to the perfect optimization that would apply with enough time and enough information. Typically the deviations from an optimal solution would be small, but these approximations themselves may create another area where additional payments would be required in order to support the market solution, at least in principle.

For example, with a less than perfect dispatch reflecting exactly all the transmission constraints, there may be no solution to the loss and congestion pricing equations that is also consistent with the dispatch for the generators. The (slight) deviations would then raise the possibility for paying the opportunity costs when the market clearing prices don't exactly support the decentralized equilibrium.[18]

The basic model above allows for an easy reinterpretation of the problem of extending uplift payments to cover opportunity costs that follow from less than an optimal dispatch in the presence of transmission constraints. Now, even without any integer variables, the definition in (19) could give rise to uplift payments $\tilde{\pi}_j(p)$. To preserve the optimality conditions for transmission and losses, we could restate the minimum-uplift problem in terms of marginal losses and transmission constraints. The resulting minimum-uplift pricing problem would be:

$$\begin{aligned}
&\underset{p_{st}, p_t, \hat{\mu}_t}{Min} \sum_j \tilde{\pi}_j(p) \\
&s.t. \\
&\qquad \hat{\mu}_t \geq 0, \forall t, \\
&\qquad p_t = p_{st}\left(1 + \nabla L_t\left(y_t^*\right)\right) + \hat{\mu}_t \nabla \hat{K}_t\left(y_t^*\right), \forall t.
\end{aligned} \tag{21}$$

In other words, (21) selects the locational prices to be consistent with the marginal losses and binding constraints ($\nabla \hat{K}_t$) of the dispatch power flow but that also minimize the total uplift payments over the day-ahead schedule.

This broader flexibility to include other than just nonconvexities would be essential in practice. It is to be expected that the actual dispatch will be only an approximation of the optimal dispatch. Even a small deviation from optimality could have significant effects on the exact pricing solutions such as in the IP case. There would have to be some

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[18] Ring, 1995. See also William W. Hogan, E. Grand Read and Brendan J. Ring, "Using Mathematical Programming for Electricity Spot Pricing," International Transactions in Operational Research, IFORS/Elsevier, Vol. 3, No. 3/4 1996, pp. 209-221.

approximation for ex post calculation of a consistent set of prices, leading back to a search for an appropriate criterion. The minimum-uplift approach applies just as well with the actual dispatch in the role of (g^*, z^*), even if this is not quite the optimal solution.

Applications

Real applications with other than simple one-part market-clearing prices either deal with special cases or approximate the solution implied by minimum-uplift pricing. In general, the problem in (21) would require specialized software to determine minimum-uplift prices consistent with the actual dispatch, which itself may be only an approximate solution of (1). However, there should be nothing in principle that precludes creation of such software for the general case.

Observe that for given bids, commitment solution with dispatch, and commodity prices p, an evaluation of $UP(p)$ through (19) reduces to a series of relatively simple problems separable by generator. Further, each evaluation produces a linear constraint on p that could be used to build up an approximation of $UP(p)$. This suggests use of an outer approximation algorithm for iterative solution of (21) if this is needed for the general case. [19]

In some instances, existing software would suffice to solve the approximation version in (21). For example, consider the procedure that has been in use since 1998 in PJM to compute ex post locational prices given the actual dispatch. Ignoring any integer constraints but allowing for suboptimal dispatch, the locational pricing algorithm in PJM linearizes the piecewise-quadratic generator cost bids at the levels implied by the actual dispatch and then applies what is essentially the best compromise formulation of Ring, which computes prices to minimize the uplift that might be paid.[20] This utilizes essentially the same software needed for the actual dispatch but applies the software to the linearized data with appropriate bounds on the allowed deviations from the actual dispatch needed to get a consistent set of prices. In PJM the implied minimum-uplift payments are usually small and are not actually paid.

In the case of unit commitment and integer constraints, any pricing method that attempts to treat some or all of the fixed costs as variable is close to the objective of finding minimum-uplift prices. The case of New York with block loaded units treated as variable for purposes of computing locational prices could be seen as an approximation of (21) that can be implemented with the same dispatch software already in use. The prices may not be exactly the same as the minimum-uplift prices, but they will be close and quite different from the IP prices.

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[19] Arthur M. Geoffrion, "Elements of Large-Scale Mathematical Programming, Parts I and II," Management Science, Vol. 16, No. 11, July 1970, pp. 652-691.

[20] See PJM training web page for the "FERC Presentation on LMP," 1998, p. 29. Ring 1995, p. 121.

An earlier application of this approach can be found in the rules of the original UK pool. In effect, the UK rules ignored transmission constraints but then solved approximately a complicated day-ahead commitment and dispatch problem of the form of (1). Given the commitment and dispatch, the day-head periods were separated into two groups, labeled Table A and Table B, that corresponded roughly to high and low load conditions over the day. For the Table A periods, the variable cost bids were increased by the amount needed to cover the fixed costs, and these were then treated as variable in setting the market-clearing prices.[21]

As in New York, PJM computes the locational prices and then determines any uplift required to ensure that committed generators to do not lose money by following the commitment and dispatch decisions. In the case of New York, the commitment payment is based on the day-ahead solution.[22] In the case of PJM, the commitment payment is based on the net of payments in both the day-ahead and real-time markets.[23] In neither the New York case nor the PJM case are opportunity costs paid for uncommitted generators. It can occur that an uncommitted generator would be profitable at the announced commodity price, but the usual experience is that this is rare and relatively a small opportunity cost. The PJM and New York rules do not require this payment, so strictly speaking these do not meet the equilibrium conditions. Of course, if the uncommitted units were dispatched, prices would change. Apparently the slight deviation from the pure equilibrium condition is accepted as workable as part of the advantage of having the day-ahead market with multi-part bids and unit commitment.

The system operators in New York and PJM emphasize the reliability benefits of making the uplift payments to committed units in order to ensure their commitment and actual availability real time. Stoft discusses this issue and suggests that the uplift payments may not be necessary as the natural incentive under a one-part pricing system would be to commit exactly when the plant would be most needed in real time.[24] However, the argument that market incentives with just commodity prices would be sufficient to address the reliability concerns appears inconsistent with the practice of making reliability

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[21] Electricity Pool of England and Wales. "User's Guide to the Pool Rules," Issue 1.00. June 1993, pp. 28-30. (see the HEPG web page) See also Robert Thomson, "Economic Dispatch and a Competitive Electricity Market: A Comparative Review," Harvard Electricity Policy Group, 8 January 1995.

[22] New York Independent System Operator, Inc. FERC Electric Tariff, Original Volume No. 2, Attachment C, Formulas For Determining Minimum Generation And Start-Up And Curtailment Initiation Cost Payments, May 1, 2001. (www.nyiso.com)

[23] "The total resource offer amount for generation, including startup and no-load costs as applicable, is compared to its total energy market value during the day. If the total value is less than the offer amount, the difference is credited to the PJM Member." Section 5, Operating Reserves Accounting Overview, PJM Operating Agreement Accounting Manual, PJM Manual M28 -Version 23. (www.pjm.com)

[24] Steven Stoft, Power System Economics, Wiley-Interscience, 2002, pp. 300-302.

commitments intended to account for differences between forecast and bid-in demand, as is an important part of the New York and PJM day-ahead markets.[25]

As Ring[26] and Stoft[27] point out, the full long run incentive effects of these pricing rules to include the complications of multi-part bids, market power, and investment are not well understood.

Conclusion

Extending partial equilibrium analysis to consider applications that do not conform to the perfect generic convex case provides insights about the implications of alternative pricing and settlement rules. The perfect marginal cost analogy derived as IP pricing provides an innovative theoretical approach in terms of an expanded market model with commitment tickets. Literal application of this model in the electricity market to include infinitesimal marginal cost pricing would be problematic. A minimum-uplift pricing approach provides an alternative but related theoretical framework that is both closer to actual practice and that appears to have better incentive effects. Given that both frameworks attempt to balance many competing objectives, the choice of a workable pricing method involves tradeoffs that would benefit from further investigation.

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[25] For NYISO: "Bulk Power System Forecast Load Commitment. The next pass commits any additional units that may be needed to supply the forecast load. Load bids (physical and virtual) and Virtual Supply bids are not considered in Pass #2. At the beginning of this pass, generator limits and commitment statuses are modified to ensure that the units selected in pass #1 will not be decommitted or dispatched below their pass #1 value. Generating units selected in pass #1 may be dispatched higher, and additional units may be committed and dispatched. Since Pass #2 is used to assure that sufficient capacity is committed to supply forecast load it considers only incremental uplift costs and does not consider energy costs. Pass #2 also secures the bulk power system." NYISO Technical Bulletin #49: Pass #2. (www.nyiso.com) For PJM: "…the [Resource Scheduling & Commitment (RSC)] is the primary tool used to determine any change in steam unit commitment status based on minimizing the additional startup costs and costs to operate steam units at economic minimum in order to provide sufficient operating reserves to satisfy the PJM Load Forecast (if greater that cleared total demand in the Day-ahead Market) and adjusted operating reserve requirements." Section 5, Scheduling Philosophy & Tools, PJM Scheduling Manual, PJM Manual M11 - Version 18. (www.pjm.com)

[26] Ring, 1995, p. 213.

[27] Stoft, 2002, p. 302.

Adapted Scarf Example

Demand	Smoke-stack Number	Smoke-stack Output	High Tech Number	High Tech Output	Med Tech Number	Med Tech Output	Total Cost	IP Commodity Price	IP Uplift	Min Uplift Commodity Price	Min Uplift
1	0	0	1	1	0	0	32	2.000	30.000	6.286	25.714
2	0	0	0	0	1	2	14	0.000	14.000	6.286	1.429
3	0	0	0	0	1	3	21	7.000	0.000	6.286	2.143
4	0	0	0	0	2	4	28	0.000	28.000	6.286	2.857
5	0	0	0	0	2	5	35	7.000	0.000	6.286	3.571
6	0	0	0	0	1	6	42	7.000	0.000	6.286	4.286
7	0	0	1	7	0	0	44	3.000	23.000	6.286	0.000
8	0	0	1	6	1	2	56	2.000	40.000	6.286	5.714
9	0	0	1	7	1	2	58	2.000	40.000	6.286	1.429
10	0	0	1	7	1	3	65	7.000	-5.000	6.286	2.143
11	0	0	1	7	2	4	72	7.000	-5.000	6.286	2.857
12	0	0	1	7	2	5	79	7.000	-5.000	6.286	3.571
13	0	0	2	13	0	0	86	2.000	60.000	6.286	4.286
14	0	0	2	14	0	0	88	3.000	46.000	6.286	0.000
15	1	15	0	0	0	0	98	3.000	53.000	6.286	3.714
16	1	16	0	0	0	0	101	3.000	53.000	6.286	0.429
17	0	0	2	14	1	3	109	7.000	-10.000	6.286	2.143
18	1	16	0	0	1	2	115	7.000	-11.000	6.286	1.857
19	1	16	0	0	1	3	122	7.000	-11.000	6.286	2.571
20	1	16	0	0	1	4	129	7.000	-11.000	6.286	3.286
21	0	0	3	21	0	0	132	3.000	69.000	6.286	0.000
22	1	15	1	7	0	0	142	3.000	76.000	6.286	3.714
23	1	16	1	7	0	0	145	3.000	76.000	6.286	0.429
24	0	0	3	21	1	3	153	7.000	-15.000	6.286	2.143
25	1	16	1	7	1	2	159	3.000	84.000	6.286	1.857
26	1	16	1	7	1	3	166	7.000	-16.000	6.286	2.571
27	1	16	1	7	2	4	173	7.000	-16.000	6.286	3.286
28	0	0	4	28	0	0	176	3.000	92.000	6.286	0.000
29	1	15	2	14	0	0	186	3.000	99.000	6.286	3.714
30	1	16	2	14	0	0	189	3.000	99.000	6.286	0.429
31	0	0	4	28	1	3	197	7.000	-20.000	6.286	2.143
32	2	32	0	0	0	0	202	3.000	106.000	6.286	0.857
33	1	16	2	14	1	3	210	7.000	-21.000	6.286	2.571
34	2	32	0	0	1	2	216	7.000	-22.000	6.286	2.286
35	0	0	5	35	0	0	220	3.000	115.000	6.286	0.000
36	2	32	0	0	1	4	230	7.000	-22.000	6.312	3.688
37	1	16	3	21	0	0	233	3.000	122.000	6.312	0.375
38	0	0	5	35	1	3	241	7.000	-25.000	6.312	2.063
39	2	32	1	7	0	0	246	3.000	129.000	6.312	0.750
40	1	16	3	21	1	3	254	7.000	-26.000	6.312	2.438
41	2	32	1	7	1	2	260	7.000	-27.000	6.312	2.125
42	2	32	1	7	1	3	267	7.000	-27.000	6.312	2.813
43	2	32	1	7	1	4	274	7.000	-27.000	6.312	3.500
44	1	16	4	28	0	0	277	3.000	145.000	6.312	0.188
45	2	31	2	14	0	0	287	3.000	152.000	6.312	3.875
46	2	32	2	14	0	0	290	3.000	152.000	6.312	0.563
47	1	16	4	28	1	3	298	7.000	-31.000	6.312	2.250

Demand	Smoke-stack Number	Smoke-stack Output	High Tech Number	High Tech Output	Med Tech Number	Med Tech Output	Total Cost	IP Commodity Price	IP Uplift	Min Uplift Commodity Price	Min Uplift
48	3	48	0	0	0	0	303	3.000	159.000	6.312	0.938
49	2	32	2	14	1	3	311	7.000	-32.000	6.312	2.625
50	3	48	0	0	1	2	317	3.000	167.000	6.312	2.313
51	1	16	5	35	0	0	321	3.000	168.000	6.312	0.000
52	3	48	0	0	1	4	331	7.000	-33.000	6.312	3.688
53	2	32	3	21	0	0	334	3.000	175.000	6.312	0.375
54	1	16	5	35	1	3	342	7.000	-36.000	6.312	2.063
55	3	48	1	7	0	0	347	3.000	182.000	6.312	0.750
56	2	32	3	21	1	3	355	7.000	-37.000	6.312	2.438
57	3	48	1	7	1	2	361	3.000	190.000	6.312	2.125
58	3	48	1	7	1	3	368	7.000	-38.000	6.312	2.813
59	3	48	1	7	1	4	375	7.000	-38.000	6.312	3.500
60	2	32	4	28	0	0	378	3.000	198.000	6.312	0.188
61	3	47	2	14	0	0	388	3.000	205.000	6.312	3.875
62	3	48	2	14	0	0	391	3.000	205.000	6.312	0.563
63	2	32	4	28	1	3	399	7.000	-42.000	6.312	2.250
64	4	64	0	0	0	0	404	3.000	212.000	6.312	0.938
65	3	48	2	14	1	3	412	7.000	-43.000	6.312	2.625
66	4	64	0	0	1	2	418	3.000	220.000	6.312	2.313
67	2	32	5	35	0	0	422	3.000	221.000	6.312	0.000
68	4	64	0	0	1	4	432	7.000	-44.000	6.312	3.688
69	3	48	3	21	0	0	435	3.000	228.000	6.312	0.375
70	2	32	5	35	1	3	443	7.000	-47.000	6.312	2.063
71	4	64	1	7	0	0	448	3.000	235.000	6.312	0.750
72	3	48	3	21	1	3	456	7.000	-48.000	6.312	2.438
73	4	64	1	7	1	2	462	3.000	243.000	6.312	2.125
74	4	64	1	7	1	3	469	7.000	-49.000	6.312	2.813
75	4	64	1	7	1	4	476	7.000	-49.000	6.312	3.500
76	3	48	4	28	0	0	479	3.000	251.000	6.312	0.188
77	4	63	2	14	0	0	489	3.000	258.000	6.312	3.875
78	4	64	2	14	0	0	492	3.000	258.000	6.312	0.563
79	3	48	4	28	1	3	500	7.000	-53.000	6.312	2.250
80	5	80	0	0	0	0	505	3.000	265.000	6.312	0.938
81	4	64	2	14	1	3	513	7.000	-54.000	6.312	2.625
82	5	80	0	0	1	2	519	7.000	-55.000	6.312	2.313
83	3	48	5	35	0	0	523	3.000	274.000	6.312	0.000
84	4	63	3	21	0	0	533	3.000	281.000	6.312	3.688
85	4	64	3	21	0	0	536	3.000	281.000	6.312	0.375
86	3	48	5	35	1	3	544	7.000	-58.000	6.312	2.063
87	5	80	1	7	0	0	549	3.000	288.000	6.312	0.750
88	4	64	3	21	1	3	557	7.000	-59.000	6.312	2.438
89	5	80	1	7	1	2	563	3.000	296.000	6.312	2.125
90	5	80	1	7	1	3	570	7.000	-60.000	6.312	2.813
91	4	63	4	28	0	0	577	3.000	304.000	6.312	3.500
92	4	64	4	28	0	0	580	3.000	304.000	6.312	0.188
93	5	79	2	14	0	0	590	3.000	311.000	6.312	3.875
94	5	80	2	14	0	0	593	3.000	311.000	6.312	0.563
95	4	64	4	28	1	3	601	7.000	-64.000	6.312	2.250
96	6	96	0	0	0	0	606	3.000	318.000	6.312	0.938
97	5	80	2	14	1	3	614	7.000	-65.000	6.312	2.625

Demand	Smoke-stack Number	Smoke-stack Output	High Tech Number	High Tech Output	Med Tech Number	Med Tech Output	Total Cost	IP Commodity Price	IP Uplift	Min Uplift Commodity Price	Min Uplift
98	6	96	0	0	1	2	620	3.000	326.000	6.312	2.313
99	4	64	5	35	0	0	624	3.000	327.000	6.312	0.000
100	6	96	0	0	1	4	634	7.000	-66.000	6.312	3.688
101	5	80	3	21	0	0	637	3.000	334.000	6.312	0.375
102	4	64	5	35	1	3	645	7.000	-69.000	6.312	2.063
103	6	96	1	7	0	0	650	3.000	341.000	6.312	0.750
104	5	80	3	21	1	3	658	7.000	-70.000	6.312	2.438
105	6	96	1	7	1	2	664	3.000	349.000	6.312	2.125
106	6	96	1	7	1	3	671	7.000	-71.000	6.312	2.813
107	5	79	4	28	0	0	678	3.000	357.000	6.313	3.500
108	5	80	4	28	0	0	681	3.000	357.000	6.313	0.188
109	6	95	2	14	0	0	691	3.000	364.000	6.313	3.875
110	6	96	2	14	0	0	694	3.000	364.000	6.313	0.563
111	5	80	4	28	1	3	702	7.000	-75.000	6.313	2.250
112	6	96	2	14	1	2	708	7.000	-76.000	6.313	1.938
113	6	96	2	14	1	3	715	7.000	-76.000	6.313	2.625
114	5	79	5	35	0	0	722	3.000	380.000	6.313	3.313
115	5	80	5	35	0	0	725	3.000	380.000	6.313	0.000
116	6	95	3	21	0	0	735	3.000	387.000	6.520	3.480
117	6	96	3	21	0	0	738	3.000	387.000	6.313	0.375
118	5	80	5	35	1	3	746	7.000	-80.000	6.313	2.063
119	6	96	3	21	1	2	752	3.000	395.000	6.313	1.750
120	6	96	3	21	1	3	759	7.000	-81.000	6.313	2.438
121	6	96	3	21	1	4	766	7.000	-81.000	6.313	3.125
122	6	96	3	21	1	5	773	7.000	-81.000	6.313	3.813
123	6	95	4	28	0	0	779	3.000	410.000	6.520	1.844
124	6	96	4	28	0	0	782	3.000	410.000	6.313	0.188
125	6	95	4	28	1	2	793	3.000	418.000	6.520	2.805
126	6	96	4	28	1	2	796	3.000	418.000	6.312	1.563
127	6	96	4	28	1	3	803	7.000	-86.000	6.312	2.250
128	6	96	4	28	1	4	810	7.000	-86.000	6.312	2.938
129	6	96	4	28	2	5	817	7.000	-86.000	6.312	3.625
130	6	95	5	35	0	0	823	3.000	433.000	6.520	0.207
131	6	96	5	35	0	0	826	3.000	433.000	6.520	0.000
132	6	95	5	35	1	2	837	3.000	441.000	7.000	0.688
133	6	96	5	35	1	2	840	7.000	-91.000	7.000	0.000
134	6	96	5	35	1	3	847	7.000	-91.000	7.000	0.000
135	6	96	5	35	1	4	854	7.000	-91.000	7.000	0.000
136	6	96	5	35	1	5	861	7.000	-91.000	7.000	0.000
137	6	96	5	35	1	6	868	7.000	-91.000	7.000	0.000
138	6	96	5	35	2	7	875	7.000	-91.000	7.000	0.000
139	6	96	5	35	2	8	882	7.000	-91.000	7.000	0.000
140	6	96	5	35	2	9	889	7.000	-91.000	7.000	0.000
141	6	96	5	35	2	10	896	7.000	-91.000	7.000	0.000
142	6	96	5	35	2	11	903	7.000	-91.000	7.000	0.000
143	6	96	5	35	2	12	910	7.000	-91.000	7.000	0.000
144	6	96	5	35	3	13	917	7.000	-91.000	7.000	0.000
145	6	96	5	35	3	14	924	7.000	-91.000	7.000	0.000
146	6	96	5	35	3	15	931	7.000	-91.000	7.000	0.000
147	6	96	5	35	3	16	938	7.000	-91.000	7.000	0.000

Demand	Smoke-stack Number	Smoke-stack Output	High Tech Number	High Tech Output	Med Tech Number	Med Tech Output	Total Cost	IP Commodity Price	IP Uplift	Min Uplift Commodity Price	Min Uplift
148	6	96	5	35	3	17	945	7.000	-91.000	7.000	0.000
149	6	96	5	35	3	18	952	7.000	-91.000	7.000	0.000
150	6	96	5	35	4	19	959	7.000	-91.000	7.000	0.000
151	6	96	5	35	4	20	966	7.000	-91.000	7.000	0.000
152	6	96	5	35	4	21	973	7.000	-91.000	7.000	0.000
153	6	96	5	35	4	22	980	7.000	-91.000	7.000	0.000
154	6	96	5	35	4	23	987	7.000	-91.000	7.000	0.000
155	6	96	5	35	4	24	994	7.000	-91.000	7.000	0.000
156	6	96	5	35	5	25	1001	7.000	-91.000	7.000	0.000
157	6	96	5	35	5	26	1008	7.000	-91.000	7.000	0.000
158	6	96	5	35	5	27	1015	7.000	-91.000	7.000	0.000
159	6	96	5	35	5	28	1022	7.000	-91.000	7.000	0.000
160	6	96	5	35	5	29	1029	7.000	-91.000	7.000	0.000
161	6	96	5	35	5	30	1036	7.000	-91.000	7.000	0.000

Endnotes

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